Tutorial 4

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1 Exercise 1

Show that if r and s are two series recognisable by weighted automata, then their sum $r+s$, defined by $(r+s)(w) = r(w)+s(w)$ for every $w \in \Sigma^*$, also is.

r, s are recognised are regular, so they have corensponding automata A_r, A_s. We can just make single automaton which containts both automata. and beetwen these two subautomatas there is no edge with weight diffrent than 0.

2 Exercise 2

We need to take smart product of both automatas

$$Q = Q_a \times Q_b$$

$$\lambda((x,y)) = \lambda_a(x) * \lambda_b(y)$$

$$\mu((x,y)) = \mu_a(x) * \lambda_b(y)$$

$$\delta((p,q)_l, (r,s))_l = \delta_a(p,r)_l * \delta_b(q,s)$$

$$weight(w) = \sum_{roverrunsonw} \lambda_a(p_0) * lambda_b(p_0) * (\prod_{i \in [0,n-1)} \delta_a(p_i, p_{i+1})_{w_i} * \delta_b(p_i, p_{i+1})_{w_i}) * \mu_a(p_{n-1}) * \mu_b(p_{n-1})$$

We can rearrenge it, to product of two automata A,B, because we are in ring.

3 Exercise 3

Consider the following two famous De Morgan's laws:

3.1 $\neg(\alpha \vee \psi) \equiv \neg\alpha \wedge \neg\psi$

The law dont work if there are two non zero elements which sum to 0. Example of such case is any ring. Suppose we have such elements x, y(Note that is possible that $x = y$).

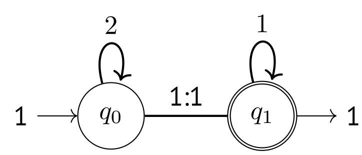
Then take $\alpha = x$ and $\psi = y$. Left side evaluates to 1. And right side evaluates to 0☕.

3.2 $\neg(\alpha \wedge \psi) \equiv \neg\alpha \vee \neg\psi$

Consider $\mathbb{Z}$ as our semiring. Suppose that $\alpha = \psi = 0$. Then left side evaluates to 1. And right side evaluates to 2 ☕.

4 Exercise 4

Construct a weighted automaton over $\Sigma = \{0, 1\}$ and $\langle \mathbb{N}, +, *, 0, 1 \rangle$ recognising the series s_bin sending a word w over Σ to the number it represents in binary.



There is only one decision in this automata, and is when to cross to accepting state. We can cross between q_0 and q_1 only when our letter is one. So the weight of run in which we crossed is 2^k, where k is number of times we looped before crossing.

5 Exercise 6

What is the series defined by the $wMSO$ formula $\forall x.\exists y.1$? Can you construct a weighted automaton recognising this series?

There are no free formulas in every subformula, so we don't need to take care of substitution. The evaluation of this formula is $\prod_{w \in \Sigma^*} \sum_{v \in \Sigma^*} 1$. Which is equal to $|\Sigma^*|^{|\Sigma^*|}$. And there is no such weighted automaton(Tutorial 5, Exercise 6).