Advanced Automata Theory

Tutorial 01

01.02 $\to$ 08.02

In all these exercises, Σ is an alphabet.

Exercise 1 (2 points)**.** Two finite words u and v over Σ are *conjugate* if there exists two words x, y such that $u = x \cdot y$ and $v = y \cdot x$.

Prove that u and v are conjugate if and only if there exists a word z such that $u \cdot z = z \cdot v$, and conclude that conjugacy is an equivalence relation.

Exercise 2 (3 points)**.** Find a regular expression defining the words over $\{a, b, c\}$ having both an even number of a's and a number of b's divisible by 3.

Exercise 3 (2 points)**.** An *automaton with ϵ-transitions* $\mathcal{A}$ is an automaton in which $\delta_{\mathcal{A}}$ also contains some transitions $p \xrightarrow{\epsilon} q$: one can go from p to q without reading any letter.

Show that epsilon transitions do not increase expressiveness of automata: in other words, for every automaton $\mathcal{A}$ with ϵ-transition, there exists an automaton $\mathcal{B}$ without ϵ-transitions such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$.

Exercise 4 (3 points)**.** Let n be a non-zero natural number. Find the equivalence classes of the MN relation for the language $L_n := \{u \in \{a, b\} \mid |u|_a = |u|_b \mod n\}$, and construct the minimal automaton for it.

Exercise 5 (3 points)**.** Find the equivalence classes of the MN relation for the language of words over the singleton alphabet $\{a\}$ that are of prime length, and deduce that this language is not regular.

Exercise 6 (4 points)**.** Let u and v be two finite words over Σ. A *shuffle* of u and v is obtained by putting all the letters of v somewhere between the letters of u, by respecting the order.

For instance, if $u = abcabc$ and $v = beadcb$, then the word $\underline{b}ab\underline{e}\underline{a}ca\underline{d}\underline{c}bc\underline{b}$ is a shuffle of u and b: you can see that the underline letters are the ones from v.

We denote by $u \otimes v$ the language of shuffles of u and v, and, if K and L are two languages, then $K \otimes L$ stands for the language $\bigcup_{u \in K, v \in L} u \otimes v$.

Show that regular languages are stable by shuffles: if K and L are regular, then $K \otimes L$ is regular as well.

Exercise 7 (3 points)**.** Let $u = a_0 a_1 \cdots a_{n-1}$. A *permutation* of u is a word of the shape $a_{\pi(0)} a_{\pi(1)} \cdots a_{\pi(n-1)}$, where π is a permutation (meaning a bijection) of the set $\{0, 1, \ldots, n-1\}$. If L is a language, then its *permutation*, denoted by $L!$, is the languages of words that are permutations of some word u in L.

Show that regular languages are *not* stable by permutations.