Chap Eight

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目录

1 Discrete Least Squares Approximation

1.1 Ex

Given the data:

x	4	4.2	4.5	4.7	5.1	5.5	5.9	6.3	6.8	7.1
y	102.56	113.18	130.11	142.05	167.53	195.14	224.87	256.73	299.5	326.72

- Construst the least squares polynomial of degree 1 and compute the error
- Construst the least square polynomial of degree 2 and compute the error
- Construst the least square polynomial of degree 3 and compute the error
- Construst the least square Approximation of form be^{ax} and compute the error
- Construst the least square Approximation of the form bx^a and compute the error

1.2 Code

1.2.1 main code

注: 直接使用了一个函数, 在这个函数中的参数为 x——自变量,y——因变量,n——逼近多项式最高次数,i——种类 0 为多项式逼近,1 为 be^{ax} 形式,2 为 be^a 形式

```
x = [4 4.2 4.5 4.7 5.1 5.5 5.9 6.3 6.8 7.1];
y = [102.56 113.18 130.11 142.05 167.53 195.14 224.87 256.73 299.5 326.72];
a1 = Leastsquares(x,y,1,0);
a2 = Leastsquares(x,y,2,0);
a3 = Leastsquares(x,y,3,0);
a4 = Leastsquares(x,y,1,1);
a5 = Leastsquares(x,y,1,2);
y1 = [];y2 = [];y3 = [];y4 = [];y5 = [];
```

```
n = size(x,2);
f1 = @(x) a1(1) + a1(2)*x;
f2 = @(x) a2(1) + a2(2)*x + a2(3)*x^2;
f3 = @(x) a3(1) + a3(2)*x + a3(3)*x^2 + a3(4)*x^3;
f4 = @(x) a4(1)*exp(a4(2)*x);
f5 = @(x) a5(1)*x^(a5(2));
for i = 1:n
  y1(i) = f1(x(i));
  y2(i) = f2(x(i));
  y3(i) = f3(x(i));
  y4(i) = f4(x(i));
  y5(i) = f4(x(i));
end
plot(x,y,'o',x,y1,x,y2,x,y3,x,y4,x,y5)
y1 = y1 - y;
y2 = y2 - y;
y3 = y3 - y;
y4 = y4 - y;
y5 = y5 - y;
thetay = [y1;y2;y3;y4;y5]
```

1.2.2 function

```
function [a0 ,a1] = LinearLeastSquares(x,y)
m = size(x,2);
t11 = sum(x.^2)*sum(y) - sum(x.*y)*sum(x);
t12 = m*sum(x.^2) - (sum(x))^2;
t21 = m*sum(x.*y) - sum(x)*sum(y);
t22 = m*sum(x.^2) - (sum(x))^2;
a0 = t11/t12;
a1 = t21/t22;
end

function a = Leastsquares(x,y,n,i)
```

```
if i == 0
n = n+1;
X = [];
for i = 1:n
    for j = 1:n
    X(j,i) = sum(x.^(i+j-2));
    end
end
Y = [];
for i = 1:n
    Y(i) = sum(y.*x.^(i-1));
end
Y  = Y';
a = pinv(X)*Y;
elseif i == 1
    y = log(y);
    [b ,temp] = LinearLeastSquares(x,y);
    b = exp(b);
    a = [b temp]';
elseif i == 2
    y = log(y);
    x = log(x);
    [b ,temp] = LinearLeastSquares(x,y);
    b = exp(b);
    a = [b temp]';
end
end
```

1.3 Ans

- $f_a = -194.1382 + 72.0845x$
- $f_b = 1.2356 - 1.1435x + 6.6182x^2$
- $f_c = 3.4291 - 2.3792x + 6.8456x^2 - 0.0137x^4$

- $f_d = 24.2588e^{0.3724x}$
- $f_e = 6.2390x^{2.0195}$

```
    error =
-8.3602   -4.5633    0.1321    2.6090    5.9628    7.1866    6.2904    3.2642   -3.4635   -9.0582
-0.0072   -0.0020   -0.0015    0.0073    0.0133    0.0071   -0.0013   -0.0218   -0.0143    0.0206
 0.0063   -0.0008   -0.0105   -0.0042    0.0046    0.0070    0.0078   -0.0083   -0.0098    0.0079
 5.0275    2.7262   -0.5044   -2.4233   -5.4768   -7.0581   -6.5787   -3.3772    5.7020   14.5549
 5.0275    2.7262   -0.5044   -2.4233   -5.4768   -7.0581   -6.5787   -3.3772    5.7020   14.5549
```

2 Orthogonal Polynomials and Least Squares Approximation

2.1 Ex

- $f(x) = \frac{1}{x+2}$
- $f(x) = e^x$
- $f(x) = \frac{1}{2}cosx + \frac{1}{3}sin2x$

分别利用 1，x 和 Legendre 基做线性逼近和二次逼近。

2.2 code

2.2.1 main code

```
syms x
a = -1;
b = 1;
f1 = 1/(x+2);
f2 = exp(x);
f3 = 1/2*cos(x) + 1/3*sin(2*x);
f13 = Ch8LS(f1,1,a,b)
f23 = Ch8LS(f2,1,a,b)
f33 = Ch8LS(f3,1,a,b)
```

```
f14 = Ch8LS(f1,2,a,b)
f24 = Ch8LS(f2,2,a,b)
f34 = Ch8LS(f3,2,a,b)
A = [1 x];B = [1 x x^2-1/3];
f11 = CH8OL(f1,1,A)
f21 = CH8OL(f2,1,A)
f31 = CH8OL(f3,1,A)
x = -1:0.01:1;
y1 = subs(f1,x);
y2 = subs(f2,x);
y3 = subs(f3,x);
y11 = subs(f11,x);
y21 = subs(f21,x);
y31 = subs(f31,x);
f12 = CH8OL(f1,1,B)
f22 = CH8OL(f2,1,B)
f32 = CH8OL(f3,1,B)
y12 = subs(f12,x);
y22 = subs(f22,x);
y32 = subs(f32,x);
y13 = subs(f13,x);
y23 = subs(f23,x);
y33 = subs(f33,x);
y14 = subs(f14,x);
y24 = subs(f24,x);
y34 = subs(f34,x);
plot(x,y1,'r',x,y11,'b',x,y12,'b',x,y13,'b',x,y14,'b')
plot(x,y2,'r',x,y21,'b',x,y22,'b',x,y23,'b',x,y24,'b')
plot(x,y3,'r',x,y31,'b',x,y32,'b',x,y33,'b',x,y34,'b')
```

2.2.2 function

```
function p = Ch8LS(f,N,a,b)
syms x
```

```
A = [1 x x^2 x^3 x^4];
X = [];
N = N+1;
B = [];
for i = 1:N
    for j = 1:N
        X(i,j) = int(x^(i+j-2),x,a,b);
    end
end
for i = 1:N
    B(i) = int(x^(i-1)*f,x,a,b);
end
B = B';
p = pinv(X)*B;
A = A(1:N);
digits 5;
p = A*vpa(p);
end

function p = CH8OL(f,w,A)
syms x
N = size(A,2);
thate = [];
for i = 1:N
thate(i) = [2/(2*(i-1)+1)];
end
for i =1:N
    p(i) = int(w*f*A(i),x,-1,1)/thate(i);
end
p = p';
p = A*p;
end
```

2.3 Ans

其中角标 ij，i 代表第 i 小题，j 代表所用的方法，其中 j=1 意为一次正交基下，2 为二次正交基下，3 为一次平凡基下，4 为二次平凡基下的逼近。

$f_{13} = 0.54931 - 0.29584x$

$f_{23} = 1.1036x + 1.1752$

$f_{33} = 0.4354x + 0.42074$

$f_{14} = 0.15888x^2 - 0.29584x + 0.49635$

$f_{24} = 0.53672\,x^2 + 1.1036\,x + 0.99629$

$f_{34} = -0.23263\,x^2 + 0.4354\,x + 0.49828$

$f_{11} = \frac{\log(3)}{2} - x\left(\frac{3\,\log(9)}{2} - 3\right)$

$f_{21} = \frac{\mathrm{e}}{2} - \frac{\mathrm{e}^{-1}}{2} + 3\,x\,\mathrm{e}^{-1}$

$f_{31} = \frac{\sin(1)}{2} - x\left(\frac{\cos(2)}{2} - \frac{\sin(2)}{4}\right)$

$f_{12} = \frac{\log(3)}{2} - x\left(\frac{3\,\log(9)}{2} - 3\right) + \left(\frac{55\,\log(3)}{6} - 10\right)\left(x^2 - \frac{1}{3}\right)$

$f_{22} = \frac{\mathrm{e}}{2} - \frac{\mathrm{e}^{-1}}{2} + 3\,x\,\mathrm{e}^{-1} + \frac{5\,\mathrm{e}^{-1}\left(x^2 - \frac{1}{3}\right)(\mathrm{e}^2 - 7)}{3}$

$f_{32} = \frac{\sin(1)}{2} + \left(x^2 - \frac{1}{3}\right)\left(5\,\cos(1) - \frac{10\,\sin(1)}{3}\right) - x\left(\frac{\cos(2)}{2} - \frac{\sin(2)}{4}\right)$

- Ex1

- Ex2

- Ex3

3 Chebyshev Polynomlals and Economization of Power Series

3.1 Ex

f(x) = sinx.Use the zeros of T_4 to construct an interpolating polynomial of degree 3

3.2 code

```
f = @(x)sin(x);
x1 = -1:2/3:1;
y1 = f(x1);
xq = -0.9:0.2:0.9;
vq1 = interp1(x1,y1,xq);
g = @(x)cos(((2*x+1)/8)*pi);
x2 = 0:3;
x2 = g(x2);
x2 = sort(x2);
y2 = f(x2);
vq2 = interp1(x2,y2,xq);
plot(xq,sin(xq),'r',xq,vq1,'b',xq,vq2,'g')
df1 = abs(vq1 - sin(xq))';
df2 = abs(vq2 - sin(xq))';
[xq'  sin(xq)'  vq1' df1 vq2' df2]
```

3.3 Ans

x	f(x) = $sinx$	P_3x	$\|sinx - P_3(x)\|$	$P_3'(x)$	$\|sinx - P_3'(x)\|$
-0.9000	-0.7833	-0.7643	0.0190	-0.7792	0.0041
-0.7000	-0.6442	-0.6100	0.0342	-0.6223	0.0219
-0.5000	-0.4794	-0.4558	0.0237	-0.4654	0.0140
-0.3000	-0.2955	-0.2945	0.0010	-0.2927	0.0028
-0.1000	-0.0998	-0.0982	0.0017	-0.0976	0.0023
0.1000	0.0998	0.0982	0.0017	0.0976	0.0023
0.3000	0.2955	0.2945	0.0010	0.2927	0.0028
0.5000	0.4794	0.4558	0.0237	0.4654	0.0140
0.7000	0.6442	0.6100	0.0342	0.6223	0.0219
0.9000	0.7833	0.7643	0.0190	0.7792	0.0041

3.4 Ex2

Find the sixth Maclaurin polynomial for sinx and use Chebyshev economization to obtain a lesser degree polynomial Approximation while keeping the error less than 0.001

3.5 code

```
clc
clear
syms x
f = x - x^3/factorial(3) + x^5/factorial(5)
T = [1 x x^2-1/2 x^3-3/4 x^4-x^2+1/8 x^5-5/4*x^3-x]
f3 = f - T(6)*(1/factorial(5))
t1 = (1/factorial(5))*(1/16)
f1 = f3 + T(4)*(5/32)
t2 = (5/32)*(1/4)
a = -1:0.001:1;
vpa(max(abs(subs(f3,x,a)-subs(f,x,a))))
vpa(max(abs(subs(f1,x,a)-subs(f,x,a))))
```

3.6 Ans

$$\frac{x^5}{120} - \frac{x^3}{6} + x$$

最终可降到三阶

$$\frac{121\,x}{120} - \frac{5\,x^3}{32}$$

当再次降阶误差为 0.2839 > 0.0100