Ch5

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目录

1 Euler's Method

1.1 Ex

Given the initial-value problem

$$y' = -y + t + 1, 0 \le t \le 2, y(0) = 1$$

with exact solution $y(t) = e^{-t} + t$

- Approximate $y(5)$ using Euler's Method with h=0.2,h=0.1,and h= 0.05
- Determine the optimal value of h to use in computing y(5) ,assuming $\delta = 10^{-6}$ and that Eq.(5.14) is valid

1.2 Code

- Euler

```
function w =  Eulers(f,a,b,y0,N)
h = (b-a)/N;
w =zeros(1,N+1);
w(1) = y0;
T = a:h:b;
t = T(1);
w = y0;
for i = 1:N
    w(i+1) = w(i) + h*f(t,w(i));
    t = T(i+1);
end
w = [T;w];
end
```

- main Code

```
f = @(x,y) - y + x +1;
format long
A1 = Eulers(f,0,5,1,25);
ans1 = A1(2,26);
A2 = Eulers(f,0,5,1,50);
ans2 = A2(2,51);
A3 = Eulers(f,0,5,1,100);
ans3 = A3(2,101);
g = @(x) exp(-x) + x;
ture = ones(1,3)*g(5);
Anss = [ans1 ans2 ans3];
Anly = [Anss;ture ; ture - Anss]
```

1.3 Analyse

approximate	5.003777893186296	5.005153775207321	5.005920529220334
true	5.003777893186296	5.005153775207321	5.005920529220334
error	5.003777893186296	5.005153775207321	5.005920529220334

2 Runge-Kutta Methods

2.1 Ex

1. Use the Modified Euler method to approximate the solutions to each of the following initial value problem,and compare the results to the actual values

$$y' = -(y+1)(y+3), 0 \leq t \leq 2, y(0) = -2$$

   with h = 0.2; actual solution y(t) = t tan(lnt)

2. Use the results of last question and linear interpolation to approximate values of y(t),and compare the results to the actual values.

$$y(1.3) and y(1.93)$$

3. repeat 1 using Heun's Method
4. repeat 2 using the results of E3
5. repeat 1 using the Midpoint Method
6. repeat 2 using the results of 5
7. repeat 1 using the Runge-Kutta method of order four
8. use the results of 7 and Cubic Hermite interpolation to approximate values of y(t)

2.2 code

- Midpoint Method

```
function y = Midpoint_Method(f,a,b,y0,N)
h = (b-a)/N;
T = a:h:b;
w = zeros(1,N+1);
w(1) = y0;
t = T(1);
for i = 1:N
    w(i+1) = w(i) + h*f(t+h/2,w(i) + (h/2)*f(t,w(i)));
    t = T(i+1);
end
y = [T;w];
end
```

- Modified Euler

```
function y = Modified_Euler(f,a,b,y0,N)
h = (b-a)/N;
w = zeros(1,N+1);
T = a:h:b;
w(1) = y0;
for i = 1:N
    w(i+1) = w(i) + (h/2)*(f(T(i), w(i)) + f(T(i+1) ,w(i) + h*f(T(i),w(i))));
```

```
end
y = [T;w];
end
```

- Heun

```
function y = Heun(f,a,b,y0,N)
h = (b-a)/N;
T = a:h:b;
w = zeros(1,N+1);
w(1) = y0;
for i = 1:N
    w(i+1)=w(i)+(h/4)*(f(T(i),w(i))+3*f(T(i)+2*h/3,w(i)+2*h/3*f(T(i),w(i))));
end
y = [T;w];
end
```

- Runge-Kutta(four order)

```
function y = Runge_4(f,a,b,y0,N)
h = (b-a)/N;
T = a:h:b;
w = zeros(1,N+1);
w(1) = y0;
t=T(1);
for i = 1:N
    K1 = h*f(t,w(i));
    K2 = h*f(t+h/2, w(i)+K1/2);
    K3 = h*f(t+h/2,w(i)+K2/2);
    K4 = h*f(t+h,w(i)+K3);
    w(i+1) = w(i) + (K1+2*K2+2*K3+K4)/6;
    t = T(i+1);
end
y = [T;w];
end
```

- real

```
function y = reals(f,a,b,y0,N)
h = (b-a)/N;
t = a:h:b;
w = zeros(1,N+1);
w(1) = y0;
for i = 1:N
    w(i+1) = f(t(i+1));
end
y = [t;w];
end
```

- Lagrange interpolation

```
function g = Lagrange(X,F,a)
n = size(X,2);
L = zeros(1,n);
T = a*ones(1,n);
for i = 1:n
    temp = X(1,i)*ones(1,n);
    A = X;
    B = X;
    A(1,i) = a-1;
    B(1,i) = X(1,i) -1;
    L(1,i) = prod(T - A)/prod(temp - B);
end
g = dot(F,L);
end
```

- Cubic Hermite interpolation

```
function q = Hermite(X,F,F1)
n = size(X,2);
Q = zeros(2*n);
Z = zeros(1,2*n);
q = zeros(1,2*n);
for i = 1:n
    Z(1,2*i-1) = X(i);
    Z(1,2*i) = X(i);
    Q(2*i-1,1) = F(i);
    Q(2*i,1) = F(i);
    Q(2*i,2) = F1(i);
    if i > 1
        Q(2*i-1,2)=(Q(2*i-1,1)-Q(2*i-2,1))/(Z(1,2*i-1)-Z(1,2*i-2));

    end
for i = 3:2*n
    for j = 3:i
    Q(i,j) = (Q(i,j-1) - Q(i-1,j-1))/(Z(i)-Z(i-j+1));
    end
end
end
for i = 1:2*n
    q(i) = Q(i,i);
end
end
```

- main Code

```
f = @(x,y) -(y+1)*(y+3);
Temper = [];
```

```
g = @(t) -3 + 2*(1+exp(-2*t))^(-1);
Real = reals(g,0,2,-2,10);
M_E_m = Modified_Euler(f,0,2,-2,10);
Temper = [Temper; Real;M_E_m(2,:)];
y1 = Lagrange(M_E_m(1,7:8),M_E_m(2,7:8),1.3);
y2 = Lagrange(M_E_m(1,10:11),M_E_m(2,10:11),1.93);
Interp = [g(1.3) g(1.93) ; y1 y2 ];
H_m = Heun(f,0,2,-2,10);
Temper = [Temper;H_m(2,:)];
y1 = Lagrange(H_m(1,7:8),H_m(2,7:8),1.3);
y2 = Lagrange(H_m(1,10:11),H_m(2,10:11),1.93);
Interp = [Interp ; y1 y2 ];
Mid_m = Midpoint_Method(f,0,2,-2,10);
Temper = [Temper;Mid_m(2,:)];
y1 = Lagrange(Mid_m(1,7:8),Mid_m(2,7:8),1.3);
y2 = Lagrange(Mid_m(1,10:11),Mid_m(2,10:11),1.93);
Interp = [Interp ; y1 y2 ];
Runge_4_m = Runge_4(f,0,2,-2,10);
Temper = [Temper;Runge_4_m(2,:)]
F1 = [f(0,Runge_4_m(2,7)) , f(0,Runge_4_m(2,8)) ];
C = Hermite(Runge_4_m(1,7:8),Runge_4_m(2,7:8),F1);
y1= C(1) + C(2)*(1.3 - 1.2) + C(3)*(1.3 -1.2)^2 +C(4)* (1.3 - 1.2)^2*(1.3-1.4);
F2 = [f(0,Runge_4_m(2,10)) , f(0,Runge_4_m(2,11)) ];
C = Hermite(Runge_4_m(1,10:11),Runge_4_m(2,10:11),F2);
y2=C(1)+C(2)*(1.93-1.8)+C(3)*(1.93-1.8)^2+C(4)*(1.93-1.8)^2*(1.93-2.0);
Interp = [Interp ; y1 y2 ]
```

2.3 OUTPUT

X	real value	Modified Euler	hune's	Midpoint	Runge
0	-2.000000000000000	-2.000000000000000	-2.000000000000000	-2.000000000000000	-2.000000000000000
0.200000000000000	-1.802624679775096	-1.804000000000000	-1.802666666666667	-1.802000000000000	-1.802627392079468
0.400000000000000	-1.620051037744775	-1.622920593716224	-1.620503746898940	-1.619296562227232	-1.620057638804308
0.600000000000000	-1.462950433001964	-1.467240245458374	-1.464254798809155	-1.462766901437550	-1.462962838790012
0.800000000000000	-1.335963229732151	-1.341320216751159	-1.338293908391554	-1.336789994698356	-1.335982382711483
1.000000000000000	-1.238405844044235	-1.244290314141912	-1.241586576320713	-1.240246987950836	-1.238430738149290
1.200000000000000	-1.166345392987845	-1.172210612839059	-1.169993230887207	-1.168897587973531	-1.166373542889978
1.400000000000000	-1.114648351797737	-1.120076330215134	-1.118361779329033	-1.117516487371196	-1.114676938045796
1.600000000000000	-1.078331445593529	-1.083078446495682	-1.081805369225573	-1.081178817189693	-1.078358205408060
1.800000000000000	-1.053193987153732	-1.057169890718042	-1.056250581139051	-1.055798721566814	-1.053217547197435

X	1.3	1.93
real	-1.138276840686693	-1.041266594453812
linear + Modified Euler	-1.146143471527097	-1.045485444078921
linear + hune's	-1.144177505108120	-1.044740343780932
linear+Midpoint	-1.143207037672364	-1.044374305696879
Cubic Hermite interpolation + Runge	-1.138303644117032	-1.041286207998959

3 Multistep Method

3.1 Ex

1. Use each of the Adams-Bashforth methods to approximate the solution to this initial-value problems.Use starting values obtained from the Runge-Kutta method of order fout.

$$y' = -(y+1)(y+3), 0 \leq t \leq 2, y(0) = -2,$$

   with h = 0.1;actual solution $y(t) = -3 + 2/(1 + e^{2t})$

2. Use Algorithm5.4 to approximate the solution to the initial value problems in 1

3. Change Algorithm 5.4 ,st corrector can be iterated for a given number p iterations.Repeat 1 with p = 2,3and 4.

3.2 code

- Adams-Bashforth two-Step Explicit Method

```
function y = Adams_two_E(f,a,b,W,N)
h = (b-a)/N;
w = [W zeros(1,N-1)];
t = a:h:b;
for i = 3:N+1
    w(i) = w(i-1) + (h/2)*(3*f(t(i-1),w(i-1)) - f(t(i-2),w(i-2)));
end
y = [t;w];
end
```

- Adams-Bashforth Three-Step Explicit Method

```
function y = Adams_three_E(f,a,b,W,N)
h = (b-a)/N;
w = [W zeros(1,N-2)];
t = a:h:b;
for i = 4:N+1
    w(i) = w(i-1) + (h/12)*(23*f(t(i-1),w(i-1)) ...
    - 16*f(t(i-2),w(i-2)) + 5*f(t(i-3),w(i-3)));
end
y = [t;w];
end
```

- Adams-Bashforth Four-Step Explicit Method

```
function y = Adams_four_E(f,a,b,W,N)
h = (b-a)/N;
w = [W zeros(1,N-3)];
t = a:h:b;
for i = 5:N+1
    w(i) = w(i-1) + (h/24)*(55*f(t(i-1),w(i-1)) - 59*f(t(i-2),w(i-2))...
     + 37*f(t(i-3),w(i-3)) - 9*f(t(i-4),w(i-4)));
```

```
end
y = [t;w];
end
```

- Adams-Bashforth Five-Step Explicit Method

```
function y = Adams_five_E(f,a,b,W,N)
h = (b-a)/N;
w = [W zeros(1,N-4)];
t = a:h:b;
for i = 6:N+1
    w(i) = w(i-1) + (h/720)*(1901*f(t(i-1),w(i-1))...
    - 2774*f(t(i-2),w(i-2)) + 2616*f(t(i-3),w(i-3)) - 1274*f(t(i-4),w(i-4)) + 251*f(t(
end
y = [t;w];
end
```

- Adams Fourth-Order Predictor-Corrector

```
function y =Adams_four_corrector(f,a,b,y0,N)
h = (b-a)/N;
t = a:h:b;
w = [y0 zeros(1,N)];
for i = 1:3
    K1 = h*f(t(i),w(i));
    K2 = h*f(t(i)+h/2, w(i)+K1/2);
    K3 = h*f(t(i)+h/2,w(i)+K2/2);
    K4 = h*f(t(i)+h,w(i)+K3);
    w(i+1) = w(i) + (K1+2*K2+2*K3+K4)/6;
end
for i = 5:N+1
     w(i) = w(i-1) + (h/24)*(55*f(t(i-1),w(i-1)) - 59*f(t(i-2),w(i-2))...
     + 37*f(t(i-3),w(i-3)) - 9*f(t(i-4),w(i-4)));
     w(i) = w(i-1) + (h/24)*(9*f(t(i),w(i)) + 19*f(t(i-1),w(i-1))...
      - 5*f(t(i-2),w(i-2)) + f(t(i-3),w(i-3)));
end
y = [t;w];
end
```

- Algorithm 5.4 Changed

```
function y =Adams_four_corrector_any(f,a,b,W,N)
h = (b-a)/N;
n = max(size(W));
t = a:h:b;
if n < 4
    a1 = a+(n-1)*h;
    a2 = a + 3*h;
    y0 = W(n);
```

```
        N1 = 4-n;
        temp = Runge_4(f,a1,a2,y0,N1);
        w = [W temp(2,2:N1+1) zeros(1,N-4)];
    else
        w = [W zeros(1,N-3)];
    end
    for i = 5:N+1
        w(i) = w(i-1) + (h/24)*(55*f(t(i-1),w(i-1)) - 59*f(t(i-2),w(i-2))...
         + 37*f(t(i-3),w(i-3)) - 9*f(t(i-4),w(i-4)));
        w(i) = w(i-1) + (h/24)*(9*f(t(i),w(i)) + 19*f(t(i-1),w(i-1))...
         - 5*f(t(i-2),w(i-2)) + f(t(i-3),w(i-3)));
    end
    y = [t;w];
    end
```

- main Code

```
format long
f = @(x,y) -(y+1)*(y+3);
g = @(t) -3 + 2*(1+exp(-2*t))^(-1);
W = Runge_4(f,0,0.1,-2,1);W = W(2,:);
Real = reals(g,0,2,-2,20);
two = Adams_two_E(f,0,2,W,20);
two = [Real,;two(2,:);abs(Real(2,:) - two(2,:))./abs(Real(2,:))] %X值 真实值 近似值 相对误差
W = Runge_4(f,0,0.2,-2,2);W = W(2,:);
three = Adams_three_E(f,0,2,W,20);
three = [Real;three(2,:);abs(Real(2,:) - three(2,:))./abs(Real(2,:))]
W = Runge_4(f,0,0.3,-2,3);W = W(2,:);
four = Adams_four_E(f,0,2,W,20);
four = [Real;four(2,:);abs(Real(2,:) - four(2,:))./abs(Real(2,:))]
W = Runge_4(f,0,0.4,-2,4);W = W(2,:);
five = Adams_five_E(f,0,2,W,20);
five = [Real;five(2,:);abs(Real(2,:) - five(2,:))./abs(Real(2,:))]
Adams_C = Adams_four_corrector(f,0,2,-2,20);
Adams_C = [Real;Adams_C(2,:);abs(Real(2,:) - Adams_C(2,:))./abs(Real(2,:))]
W = Real(2,1:2);
Adams_2 = Adams_four_corrector_any(f,0,2,W,20);
Adams_2 = [Real;Adams_2(2,:);abs(Real(2,:) - Adams_2(2,:))./abs(Real(2,:))]
W = Real(2,1:3);
Adams_3 = Adams_four_corrector_any(f,0,2,W,20);
Adams_3 = [Real;Adams_3(2,:);abs(Real(2,:) - Adams_3(2,:))./abs(Real(2,:))]
W = Real(2,1:4);
Adams_4 = Adams_four_corrector_any(f,0,2,W,20);
Adams_4 = [Real;Adams_4(2,:);abs(Real(2,:) - Adams_4(2,:))./abs(Real(2,:))]
```

3.3 OUTCOM

- Adams with 2

X	real value	approximate	relative value
0	-2.000000000000000	-2.000000000000000	0
0.100000000000000	-1.900332005375044	-1.900332089046914	0.000000044030132
0.200000000000000	-1.802624679775096	-1.801822142917977	0.000445204631959
0.300000000000000	-1.708687387548409	-1.707216627749933	0.000860754172586
0.400000000000000	-1.620051037744775	-1.618111220057970	0.001197380601975
0.500000000000000	-1.537882842739990	-1.535700970941505	0.001418750335102
0.600000000000000	-1.462950433001964	-1.460745057186924	0.001507484987386
0.700000000000000	-1.395632222882836	-1.393585761769941	0.001466332662246
0.800000000000000	-1.335963229732151	-1.334206701351746	0.001314802938669
0.900000000000000	-1.283702129800975	-1.282311897414077	0.001082986741725
1.000000000000000	-1.238405844044235	-1.237409293476838	0.000804704348086
1.100000000000000	-1.199500978239370	-1.198887170698494	0.000511719083196
1.200000000000000	-1.166345392987845	-1.166077206205439	0.000229937704576
1.300000000000000	-1.138276840686694	-1.138302201843384	0.000022280306323
1.400000000000000	-1.114648351797737	-1.114909304845072	0.000234112441752
1.500000000000000	-1.094851746355133	-1.095290980875149	0.000401181732118
1.600000000000000	-1.078331445593529	-1.078896465335535	0.000523975948504
1.700000000000000	-1.064590929396901	-1.065236303106948	0.000606217554768
1.800000000000000	-1.053193987153732	-1.053882192382754	0.000653445839434
1.900000000000000	-1.043762541872261	-1.044463869814866	0.000671922889038
2.000000000000000	-1.035972419924183	-1.036664318933720	0.000667873966748

- Adams with 3

X	real value	approximate	relative value
0	-2.000000000000000	-2.000000000000000	0
0.100000000000000	-1.900332005375044	-1.900332089046914	0.000000044030132
0.200000000000000	-1.802624679775096	-1.802624856125257	0.000000097829661
0.300000000000000	-1.708687387548409	-1.708767112050842	0.000046658331427
0.400000000000000	-1.620051037744775	-1.620243270294086	0.000118658329171
0.500000000000000	-1.537882842739990	-1.538198838736886	0.000205474687742
0.600000000000000	-1.462950433001964	-1.463379066932855	0.000292992791294
0.700000000000000	-1.395632222882836	-1.396146045779654	0.000368164970966
0.800000000000000	-1.335963229732151	-1.336526211650152	0.000421405249390
0.900000000000000	-1.283702129800975	-1.284277197655288	0.000447976084921
1.000000000000000	-1.238405844044235	-1.238960518407062	0.000447893851191
1.100000000000000	-1.199500978239370	-1.200010571664499	0.000424837857054
1.200000000000000	-1.166345392987845	-1.166793981111214	0.000384610018667
1.300000000000000	-1.138276840686694	-1.138656633640604	0.000333656049508
1.400000000000000	-1.114648351797737	-1.114958168543800	0.000277950212336
1.500000000000000	-1.094851746355133	-1.095095177144217	0.000222341326023
1.600000000000000	-1.078331445593529	-1.078515096001344	0.000170309795347
1.700000000000000	-1.064590929396901	-1.064722956444133	0.000124016693724
1.800000000000000	-1.053193987153732	-1.053283000177796	0.000084517216344
1.900000000000000	-1.043762541872261	-1.043816854832376	0.000052035743702
2.000000000000000	-1.035972419924183	-1.035999598668512	0.000026235007618

- Adams with 4

X	real value	approximate	relative value
0	-2.000000000000000	-2.000000000000000	0
0.100000000000000	-1.900332005375044	-1.900332089046914	0.000000044030132
0.200000000000000	-1.802624679775096	-1.802624856125257	0.000000097829661
0.300000000000000	-1.708687387548409	-1.708687676018630	0.000000168825627
0.400000000000000	-1.620051037744775	-1.620089945099723	0.000024016129147
0.500000000000000	-1.537882842739990	-1.537937175207833	0.000035329393328
0.600000000000000	-1.462950433001964	-1.463005338962106	0.000037530977744
0.700000000000000	-1.395632222882836	-1.395671437512254	0.000028098111218
0.800000000000000	-1.335963229732151	-1.335979246211470	0.000011988712685
0.900000000000000	-1.283702129800975	-1.283692373546538	0.000007600092117
1.000000000000000	-1.238405844044235	-1.238373355911001	0.000026233833917
1.100000000000000	-1.199500978239370	-1.199451219736751	0.000041482669479
1.200000000000000	-1.166345392987845	-1.166285129741235	0.000051668439702
1.300000000000000	-1.138276840686694	-1.138212275019395	0.000056722288455
1.400000000000000	-1.114648351797737	-1.114584644111368	0.000057154963955
1.500000000000000	-1.094851746355133	-1.094792499471376	0.000054114069740
1.600000000000000	-1.078331445593529	-1.078278907620805	0.000048721543768
1.700000000000000	-1.064590929396901	-1.064546121119952	0.000042089666286
1.800000000000000	-1.053193987153732	-1.053157083405705	0.000035039839267
1.900000000000000	-1.043762541872261	-1.043733105761996	0.000028201922453
2.000000000000000	-1.035972419924183	-1.035949689324408	0.000021941317489

- Adams with 5

X	real value	approximate	relative value
0	-2.000000000000000	-2.000000000000000	0
0.100000000000000	-1.900332005375044	-1.900332089046914	0.000000044030132
0.200000000000000	-1.802624679775096	-1.802624856125257	0.000000097829661
0.300000000000000	-1.708687387548409	-1.708687676018630	0.000000168825627
0.400000000000000	-1.620051037744775	-1.620051464009317	0.000000263117971
0.500000000000000	-1.537882842739990	-1.537867616851528	0.000009900551615
0.600000000000000	-1.462950433001964	-1.462922613604022	0.000019015953866
0.700000000000000	-1.395632222882836	-1.395592813296701	0.000028237801829
0.800000000000000	-1.335963229732151	-1.335920420345938	0.000032043835683
0.900000000000000	-1.283702129800975	-1.283659516989099	0.000033195249028
1.000000000000000	-1.238405844044235	-1.238369304598727	0.000029505226969
1.100000000000000	-1.199500978239370	-1.199471171030549	0.000024849674458
1.200000000000000	-1.166345392987845	-1.166324435299833	0.000017968680751
1.300000000000000	-1.138276840686694	-1.138262570458784	0.000012536693535
1.400000000000000	-1.114648351797737	-1.114640966295674	0.000006625858327
1.500000000000000	-1.094851746355133	-1.094848145492648	0.000003288904180
1.600000000000000	-1.078331445593529	-1.078331786665552	0.000000316296093
1.700000000000000	-1.064590929396901	-1.064592294564497	0.000001282340060
1.800000000000000	-1.053193987153732	-1.053197257236194	0.000003104919419
1.900000000000000	-1.043762541872261	-1.043765134846931	0.000002484257258
2.000000000000000	-1.035972419924183	-1.035976006728739	0.000003462258731

- Adams Corrector

x	real value	approximate	relative value
0	-2.000000000000000	-2.000000000000000	0
0.100000000000000	-1.900332005375044	-1.900332089046914	0.000000044030132
0.200000000000000	-1.802624679775096	-1.802624856125257	0.000000097829661
0.300000000000000	-1.708687387548409	-1.708687676018630	0.000000168825627
0.400000000000000	-1.620051037744775	-1.620048210750588	0.000001745003164
0.500000000000000	-1.537882842739990	-1.537878842588273	0.000002601077017
0.600000000000000	-1.462950433001964	-1.462947169978559	0.000002230440165
0.700000000000000	-1.395632222882836	-1.395631160671330	0.000000761097006
0.800000000000000	-1.335963229732151	-1.335965152386325	0.000001439152015
0.900000000000000	-1.283702129800975	-1.283707122024276	0.000003888926555
1.000000000000000	-1.238405844044235	-1.238413444347288	0.000006137166656
1.100000000000000	-1.199500978239370	-1.199510408550067	0.000007861861614
1.200000000000000	-1.166345392987845	-1.166355781050363	0.000008906506238
1.300000000000000	-1.138276840686694	-1.138287385634645	0.000009263957215
1.400000000000000	-1.114648351797737	-1.114658418442201	0.000009031228950
1.500000000000000	-1.094851746355133	-1.094860897430323	0.000008358277931
1.600000000000000	-1.078331445593529	-1.078339431346485	0.000007405657128
1.700000000000000	-1.064590929396901	-1.064597654254872	0.000006316846955
1.800000000000000	-1.053193987153732	-1.053199468693826	0.000005204682291
1.900000000000000	-1.043762541872261	-1.043766871670757	0.000004148260090
2.000000000000000	-1.035972419924183	-1.035975731058299	0.000003196160489

- Algorithm Changed with 2

X	real value	approximate	relative value
0	-2.000000000000000	-2.000000000000000	0
0.100000000000000	-1.900332005375044	-1.900332005375044	0
0.200000000000000	-1.802624679775096	-1.802624774906202	0.000000052773662
0.300000000000000	-1.708687387548409	-1.708687598679180	0.000000123563135
0.400000000000000	-1.620051037744775	-1.620048138440666	0.000001789637512
0.500000000000000	-1.537882842739990	-1.537878776127216	0.000002644292960
0.600000000000000	-1.462950433001964	-1.462947109846071	0.000002271543737
0.700000000000000	-1.395632222882836	-1.395631107033229	0.000000799529839
0.800000000000000	-1.335963229732151	-1.335965105144497	0.000001403790392
0.900000000000000	-1.283702129800975	-1.283707080878461	0.000003856874092
1.000000000000000	-1.238405844044235	-1.238413408858117	0.000006108509515
1.100000000000000	-1.199500978239370	-1.199510378195962	0.000007836556003
1.200000000000000	-1.166345392987845	-1.166355755274272	0.000008884406360
1.300000000000000	-1.138276840686694	-1.138287363879390	0.000009244844769
1.400000000000000	-1.114648351797737	-1.114658400175034	0.000009014840672
1.500000000000000	-1.094851746355133	-1.094860882158313	0.000008344329002
1.600000000000000	-1.078331445593529	-1.078339418624741	0.000007393859509
1.700000000000000	-1.064590929396901	-1.064597643689502	0.000006306922608
1.800000000000000	-1.053193987153732	-1.053199459941324	0.000005196371855
1.900000000000000	-1.043762541872261	-1.043766864435137	0.000004141327843
2.000000000000000	-1.035972419924183	-1.035975725086961	0.000003190396495

- Algorithm Changed with 3

X	real value	approximate	relative value
0	-2.000000000000000	-2.000000000000000	0
0.100000000000000	-1.900332005375044	-1.900332005375044	0
0.200000000000000	-1.802624679775096	-1.802624679775096	0
0.300000000000000	-1.708687387548409	-1.708687508092220	0.000000070547609
0.400000000000000	-1.620051037744775	-1.620048053749162	0.000001841914571
0.500000000000000	-1.537882842739990	-1.537878698260651	0.000002694925272
0.600000000000000	-1.462950433001964	-1.462947039394377	0.000002319701004
0.700000000000000	-1.395632222882836	-1.395631044190443	0.000000844558025
0.800000000000000	-1.335963229732151	-1.335965049795591	0.000001362360430
0.900000000000000	-1.283702129800975	-1.283707032671695	0.000003819321169
1.000000000000000	-1.238405844044235	-1.238413367278721	0.000006074934580
1.100000000000000	-1.199500978239370	-1.199510342632848	0.000007806907745
1.200000000000000	-1.166345392987845	-1.166355725074797	0.000008858513965
1.300000000000000	-1.138276840686694	-1.138287338390760	0.000009222452474
1.400000000000000	-1.114648351797737	-1.114658378773074	0.000008995640033
1.500000000000000	-1.094851746355133	-1.094860864265506	0.000008327986326
1.600000000000000	-1.078331445593529	-1.078339403719845	0.000007380037324
1.700000000000000	-1.064590929396901	-1.064597631311030	0.000006295295164
1.800000000000000	-1.053193987153732	-1.053199449686824	0.000005186635281
1.900000000000000	-1.043762541872261	-1.043766855957827	0.000004133205967
2.000000000000000	-1.035972419924183	-1.035975718090894	0.000003183643355

- Algorithm Changed with 4

X	real value	approximate	relative value
0	-2.000000000000000	-2.000000000000000	0
0.100000000000000	-1.900332005375044	-1.900332005375044	0
0.200000000000000	-1.802624679775096	-1.802624679775096	0
0.300000000000000	-1.708687387548409	-1.708687387548409	0
0.400000000000000	-1.620051037744775	-1.620047941741467	0.000001911052946
0.500000000000000	-1.537882842739990	-1.537878595332317	0.000002761853865
0.600000000000000	-1.462950433001964	-1.462946946191019	0.000002383410174
0.700000000000000	-1.395632222882836	-1.395630961054740	0.000000904126514
0.800000000000000	-1.335963229732151	-1.335964976573867	0.000001307552242
0.900000000000000	-1.283702129800975	-1.283706968898228	0.000003769641835
1.000000000000000	-1.238405844044235	-1.238413312272710	0.000006030517791
1.100000000000000	-1.199500978239370	-1.199510295585869	0.000007767685619
1.200000000000000	-1.166345392987845	-1.166355685123456	0.000008824260526
1.300000000000000	-1.138276840686694	-1.138287304671467	0.000009192829371
1.400000000000000	-1.114648351797737	-1.114658350460099	0.000008970239220
1.500000000000000	-1.094851746355133	-1.094860840594841	0.000008306366353
1.600000000000000	-1.078331445593529	-1.078339384001934	0.000007361751748
1.700000000000000	-1.064590929396901	-1.064597614935364	0.000006279913043
1.800000000000000	-1.053193987153732	-1.053199436120990	0.000005173754621
1.900000000000000	-1.043762541872261	-1.043766844743066	0.000004122461415
2.000000000000000	-1.035972419924183	-1.035975708835690	0.000003174709523

4 Higher-Order Equations and Systems of Differential Equations

4.1 Ex

Use the Runge-Kutta for systems Algorithm to approximate the solutions of the followng higer-order differential equations , and compare the results to the actual solutions.

$$y''' + 2y'' - y' - 2y = e^t, 0 \leq t \leq 3, y(0) = 1, y'(0) = 2y''(0) = 0$$

with h = 0.2, actual solution y(t) = $\frac{43}{36}e^t + \frac{1}{4}e^{-t} - \frac{4}{9}e^{-2t} + \frac{1}{6}te^t$

4.2 code

```
format long
f1 = @(t,x,y,z)y;
f2 =  @(t,x,y,z)z;
f3 = @ (t,x,y,z)exp(t) - 2*z + y+2*x;
w = [];
k = [];
t = 0;
w(1,1) = 1;
w(2,1) = 2;
w(3,1) = 0;
```

```
h = 0.2;
N = 3/0.2;
for i = 2:N+1
    k(1,1) = h*f1(t,w(1,i-1),w(2,i-1),w(3,i-1));
    k(1,2) = h*f2(t,w(1,i-1),w(2,i-1),w(3,i-1));
    k(1,3) = h*f3(t,w(1,i-1),w(2,i-1),w(3,i-1));
    k(2,1) = h*f1(t+h/2,w(1,i-1) + k(1,1)/2,w(2,i-1) + k(1,2)/2,w(3,i-1)+k(1,3)/2);
    k(2,2) = h*f2(t+h/2,w(1,i-1) + k(1,1)/2,w(2,i-1) + k(1,2)/2,w(3,i-1)+k(1,3)/2);
    k(2,3) = h*f3(t+h/2,w(1,i-1) + k(1,1)/2,w(2,i-1) + k(1,2)/2,w(3,i-1)+k(1,3)/2);
    k(3,1) = h*f1(t+h/2,w(1,i-1) + k(2,1)/2,w(2,i-1) + k(2,2)/2,w(3,i-1)+k(2,3)/2);
    k(3,2) = h*f2(t+h/2,w(1,i-1) + k(2,1)/2,w(2,i-1) + k(2,2)/2,w(3,i-1)+k(2,3)/2);
    k(3,3) = h*f3(t+h/2,w(1,i-1) + k(2,1)/2,w(2,i-1) + k(2,2)/2,w(3,i-1)+k(2,3)/2);
    k(4,1) = h*f1(t+h,w(1,i-1) + k(3,1),w(2,i-1) + k(3,2),w(3,i-1)+k(3,3));
    k(4,2) = h*f2(t+h,w(1,i-1) + k(3,1),w(2,i-1) + k(3,2),w(3,i-1)+k(3,3));
    k(4,3) = h*f3(t+h,w(1,i-1) + k(3,1),w(2,i-1) + k(3,2),w(3,i-1)+k(3,3));
    for j = 1:3
        w(j,i) = w(j,i-1) + (k(1,j) + 2*k(2,j) + 2*k(3,j) + k(4,j))/6;
    end
    t =(i-1)*h;
end
t = 0:0.2:3;
[t;w]
g = @(t) 43/36*exp(t) + 1/4*exp(-t) -4/9 * exp(-2*t) + 1/6*t*exp(t);
Real = reals(g,0,3,1,15)
abs(Real(2,:) - w(1,:))./abs(Real(2,:))
```

4.3 OUTPUT

t	real value	approximate	relative error
0	1.000000000000000	1.000000000000000	0
0.200000000000000	1.406373831994753	1.406336780612050	0.000026345329997
0.400000000000000	1.849234951704414	1.849181455163592	0.000028929012386
0.600000000000000	2.361970373123576	2.361909032700863	0.000025970022068
0.800000000000000	2.977624243641124	2.977556432805527	0.000022773469736
1.000000000000000	3.731704445368067	3.731626953085328	0.000020765921812
1.200000000000000	4.664698060443165	4.664604403001034	0.000020077921640
1.400000000000000	5.824546943591313	5.824427826233221	0.000020450922492
1.600000000000000	7.269288303374473	7.269131509917554	0.000021569299548
1.800000000000000	9.070042889451500	9.069832748305053	0.000023168704824
2.000000000000000	11.314529243558209	11.314245733682135	0.000025057151736
2.200000000000000	14.111293045972488	14.110910548085377	0.000027105800005
2.400000000000000	17.594864163212449	17.594349818173889	0.000029232680275
2.600000000000000	21.932090167341553	21.931401767283631	0.000031387799916
2.800000000000000	27.329944490437967	27.329027793513237	0.000033541850956
3.000000000000000	34.045171552636994	34.043956875177194	0.000035678406200