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2017
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Summary Sheet

Optimizing the Passenger Throughput at an Airport Security Checkpoint

Summary

With the ever-increasing speed of the demands air traffic, the congestion at airport checkpoint is a big problem nowadays. How to optimize the airport checkpoint has become a global concern ever since.

At the beginning, we establish the Simplified Double-layer Multi-servers System(SDMS) which takes the special structure of the security checkpoint into consideration. Our model is based on the traditional M/M/s model. We regard the average waiting time and its variance are closely related to throughput of the checkpoint.By using the computer simulation, we analyze the bottlenecks and optimize the throughput of the airport checkpoint.

Then, we develop the Sorted Queue Model to analyze the influence of the priority of passengers in detail.We find out that if passengers are ideally sorted by service time from low to high in a queue, then the average waiting time and its variance will be reduced. However, it is hard to do that in the real-time situation. So we develop the Pre-Queue Model to let fast passengers go first as more as possible. In this way, we reduce the variance in waiting time.

Next, Single Queue Multi Sever Model(SQMS) is proposed to analyze the impact on the form of the queue to the throughput. We compare the difference between traditional multi queue multi sever and SQMS, the average of waiting time decrease in SQMS but the variance in waiting time doesn't decrease significantly. But SQMS is more convenient to manage.

Finally, we make a conclusion about the development and improvement of our models, including sensitivity analysis based on different culture and the evaluation of strengths and weaknesses. Meanwhile, we make a recommendation to the security managers.

Optimizing the Passenger Throughput at an Airport Security Checkpoint

Contents

1 Introduction

1.1 Restatement

Nowadays, with the development of economy, more and more people choose to take the airplane as their means of transportation for the sake of efficiency and convenience. Unfortunately, many terrorists attempted to hijack an aeroplane[1], which seriously threat social harmony and stability. Therefore, it is vital to keep passengers safe by security check in the airport.

However, the strict checkpoint policies bring a lot of inconvenience for passengers.The main reason is that it makes passengers wait in queue for an unnecessary long time.It is hard for passengers to make a decision about arriving unnecessarily early or take a potentially risk of missing scheduled flight. To avoid it, TSA introduced a Pre-Check policy, but it seems not work well.So,how to improve the efficacy of airport security check is becoming more urgent problem.

Figure 1 is the simplified illustration of the TSA security screening process.

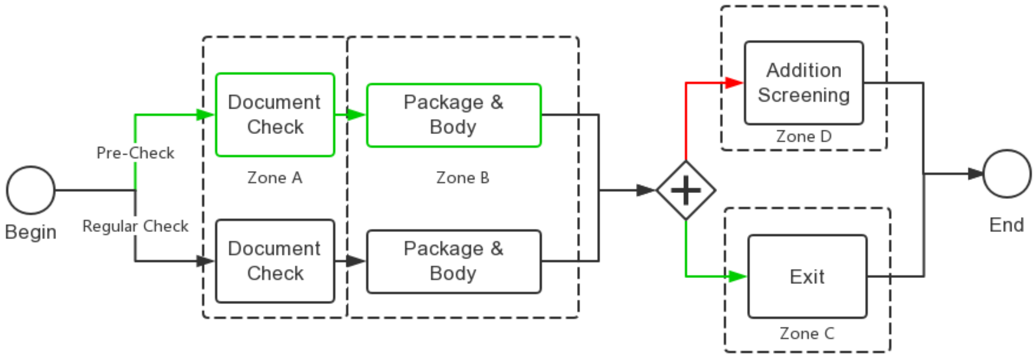

Figure 1: Illustration of the TSA Security Screening Process

We are asked to optimize the airport security system as following tasks:

- Explore the flow of passengers through a security check point and identify bottlenecks.Clearly identify where problem areas exist in the current process.
- Develop two or more potential modifications to the current process to improve passenger throughput and reduce variance in wait time. Model these changes to demonstrate how our modifications impact the process.
- Take cultural norms that shape the local rules of social interaction into account while checkout of the model or optimise the model.
- Propose policy and procedural recommendations for the security managers based on our model.

1.2 Previous Research

How to describe the a queueing system has been focused for a long time. In 1909, Agner Krarup Erlang firstly come up with Queuing Theory to describe the telephone service time and service queue[2], then it was widely used in traffic and the design of shops .etc.[3].After the 1940s queueing theory became an area of research interest to mathematicians[4]. In 1953 David George Kendall solved the GI/M/k queue[5] and introduced the modern notation for queues, now known as Kendall's notation. Meanwhile, the study of optimizing the queueing system are still work on. In 2005, Dickson et al. discussed the application of virtual queue in the real word[6]. They illuminated the virtual queue can optimize the waiting time and it can improve customer satisfaction.

In addition, petri net is also an effective method to solve this problem. The petri net was firstly put forward in 1939 by Carl Adam Petri[7]. Xun Lu used the Petri net to construct a simulation system of packages flow to illustrate the effectiveness of his model[8].

1.3 A Brief Description of Our Approaches

Firstly, we analyze the official Excel sheet in the following statement.

We consider each time in columns A-B as passengers' arrival time point. By subtracting arrival time point one by one, we can get a message that Pre-Checker comes every 9 seconds and Regular passenger comes every 13 seconds. And columns C-D(ID Check Process Time) are regarded as passengers' service time at zone A. We let column H represent columns E-G in the calculation to be the passengers' service time at zone B, because H(Time to get scanned property) is longer than the sum of columns E-G. And passengers can get Milimeter Wave Scan while post-xray scans their belongings.

Based on previous research, we use the χ^2 Test[9] on these data. However,the data in official Excel sheet cannot meet the χ^2 Test, it may be caused by the deficiency of data. From the precious analyze, Chunling Cheng's research[10] and Keller's research[11] had proved that the passengers occurrence governed by a Poisson process.

So, we apply Queueing Theory in the whole system, and give three sub models.

2 Specification of Our Models

2.1 Assumptions and Justifications

- **Take no account of some objective factors that cause the delay of time, such as the breakdown of the machine, or the overfatigue of the officers.** We regard these unpredictable factors as extraneous variables.

- **Ignore the influence of flight schedule.** Even though the flight schedule can influence the throughput at the checkpoint, our goal is to optimize the structure of the checkpoint. Therefore, we can take no consideration of the flight schedule.

- **We take no account of the wasting time between process stations.** Assume that after people check their documents, they go ahead checking their body and packages without wasting time.
- **Passengers occurrence governed by Poisson process.** Even though the occurrence of passengers do not perfectly match the Poisson process, however, it is a good assumption to simulate the real-world situation.
- **The passengers' service time is governed by the exponential distribution.** All the service time have an exponential distribution with parameter μ, which does not change over time.
- **Passengers never go back.** Passengers must move from zone A to B, and then, move from zone B to C.
- **Cannot serve two passengers at the same time in single queue single server.** If one is in service, the others in the same queue should wait.
- **The passengers have to catch the plane.** No passenger abandons his flight and go home.

2.2 Variable Declaration

Table 1 shows the key variables declaration:

Table 1: Variable Declaration

Symbol	Definition
n	The number of the people in system
λ	The average arrival rate
μ	the average service rate of a single service
p_n	The probability of n passengers in system
L_q	The average queue length
W_q	The average waiting time
D_{wait}	The variance of the passenger's waiting time
$T_{wait}(i)$	The waiting time of the i^{th} passenger

2.3 Model Overview

Figure 2 shows the model overview.

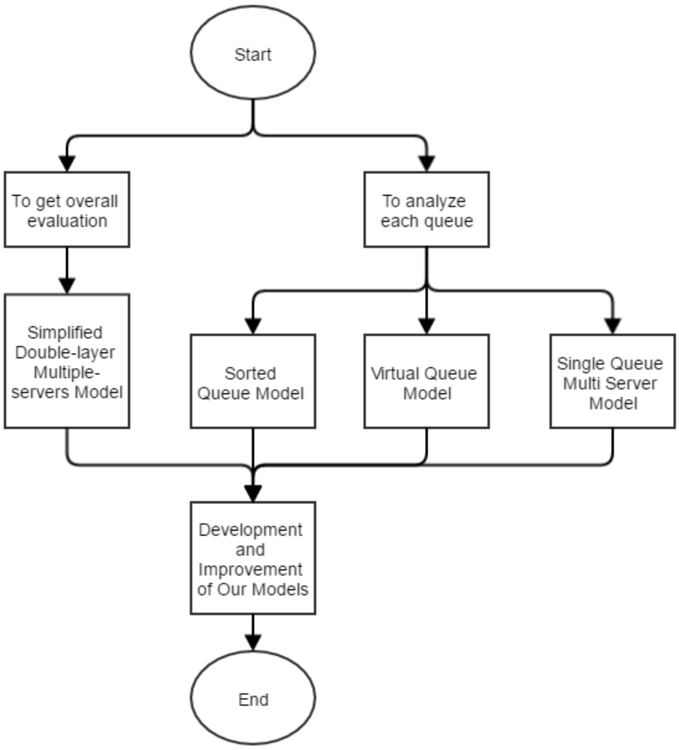

Figure 2: Model Overview

3 Simplified Double-layer Multiple-servers Model(SDMS)

In order to optimize the passenger throughput at the airport security checkpoint, we have to model the highly complex system. A large amount of factors ought to take into consideration to quantify the airport security system, such as the density of the passengers, the rate of the service, the occurrence of the sudden events, the personal behavior, and etc. No need to say we cannot take all the factors into account. Based on the M/M/s model and the structure of the airport checkpoint, we establish the Simplified Double-layer Multiple-servers Model(SDMS)to simulate the real-world airport security checkpoint.

3.1 Structure of the SDMS

The Figure 3 is the structure of the SDMS.

3.2 M/M/s model

The main idea of the SDMS is to separate the whole airport security process as two connected M/M/s queueing system.In queueing theory,the M/M/s model can be

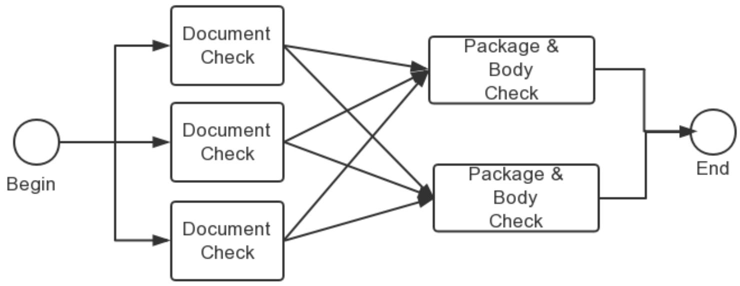

Figure 3: Structure of the SDMS

described as a continuous time Markov chain with transition rate matrix

$$Q = \begin{pmatrix} -\lambda & \lambda & & & & \\ \mu & -(\mu+\lambda) & \lambda & & & \\ & 2\mu & -(2\mu+\lambda) & \lambda & & \\ & & 3\mu & \ddots & & \\ & & & & \lambda & \\ & & & & -(c\mu+\lambda) & \lambda \\ & & & & \ddots & \end{pmatrix} \tag{1}$$

on the state space$(0, 1, 2, 3, \ldots)$ .The model is a type of birth-death process. In general,it is difficult to get the distribution of $p_n(t) = P\{N(t) = n\}$. However, we can get the solutions after the system reach the statistical equilibrium. The stationary equations at any time are as follows

$$\begin{cases} \mu_1 p_1 = \lambda_0 p_0 \\ \lambda_0 p_0 + \mu_2 p_2 = (\lambda_1 + \mu_1) p_1 \\ \lambda_{n-1} p_{n-1} + \mu_{n+1} p_{n+1} = (\lambda_n + \mu_n) p_n \end{cases} \tag{2}$$

From the stationary equation above, we can solve that

$$\begin{cases} p_1 = \dfrac{\lambda_0}{\mu_1} p_0 \\ p_2 = \dfrac{\lambda_1}{\mu_2} p_1 + \dfrac{1}{\mu_2}(\mu_1 p_1 - \lambda_0 p_0) = \dfrac{\lambda_1}{mu_2} = \dfrac{\lambda_1 \lambda_0}{\mu_2 \mu_1} p_0 \\ \quad \vdots \\ p_{n+1} = \dfrac{\lambda_n}{\mu_{n+1}} p_1 + \dfrac{1}{\mu_{n+1}}(\mu_n p_n - \lambda_{n-1} p_{n-1}) = \dfrac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1} \mu_n \cdots \mu_1} p_0 \end{cases} \tag{3}$$

We assume that

$$C_n = \frac{\lambda_{n-1} \lambda_{n-2} \cdots \lambda_0}{\mu_n \mu_{n-1} \cdots \mu_1} \tag{4}$$

SO the distribution of the stationary states is

$$p_n = C_n p_0, n = 1, 2, \cdots \tag{5}$$

We can also solve the p_0 by

$$\sum_{n=0}^{\infty} p_n = 1 \tag{6}$$

So we get

$$p_0 = \frac{1}{1 + \sum_{n=1}^{\infty} C_n} \tag{7}$$

Now we find some vital parameters of the system, such as the average waiting time and the average line length.

First, we assume

$$p_n = P\{N = n\}(n = 0, 1, 2 \ldots) \tag{8}$$

is the probability distribution of the line length N after the system reaches steady state. We should mention that as for the multi-server system, we have

$$\lambda_n = \lambda, n = 0, 1, 2 \ldots \tag{9}$$

and

$$\mu_n = \begin{cases} n\mu, & n = 1, 2, \cdots, s \\ s\mu, & n = s, s+1, \cdots \end{cases} \tag{10}$$

We let the server utilization $\rho_s = \rho/s = \lambda/s\mu$, so, when ρ_s<1 , from wikipedia, we found the following equations

$$C_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!}, & n = 1, 2, \cdots, s \\ \dfrac{(\lambda/\mu)^n}{s!s^{n-s}}, & n \geqslant s \end{cases} \tag{11}$$

$$p_n = \begin{cases} \dfrac{\rho^n}{n!} p_0, & n = 1, 2, \cdots, s \\ \dfrac{\rho^n}{s!s^{n-s}} & n \geqslant s \end{cases} \tag{12}$$

$$p_0 = [\sum_{n=0}^{s-1} \frac{\rho_n}{n!} + \frac{\rho_s}{s!(1-\rho_s)}]^{-1} \tag{13}$$

$$c(s, \rho) = \sum_{n=s}^{\infty} = \frac{\rho_s}{s!(1-\rho_s)} p_0 \tag{14}$$

Where p_n is the probability that the system have n arrivals, naturally, p_0 is the probability that the system is idel, $c(s, \rho)$ is the probability that an arrival is forced to join the queue,which is referred to as Erlang's C formula. We can also get the average line length L_q is

$$L_q = \sum_{n=s+1}^{\infty} (n-s)p_n = \frac{p_0 \rho^s}{s!} \sum_{n=s}^{\infty} \rho_s^{n-s} = \frac{p_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \tag{15}$$

or

$$L_q = \frac{c(s,\rho)\rho_s}{1-\rho_s} \tag{16}$$

We can also get average waiting time W_q from Little equation

$$W_q = \frac{L_q}{\lambda} \tag{17}$$

3.3 Simulation of the SDMS Model

In order to get the reliable statistics and demonstrate the SDMS model, we use Matlab to simulate our SDMS model. The flow chart below gives us the processes of queueing system, which help us to simulate.

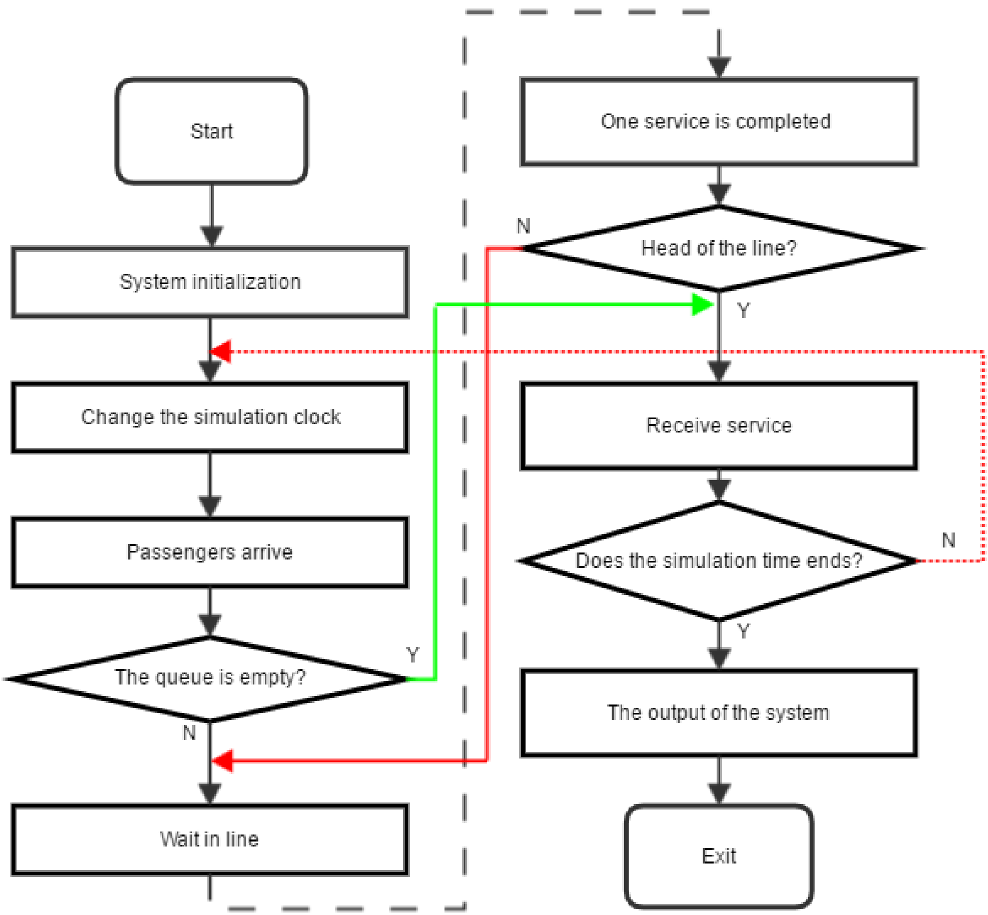

Figure 4: The Flow Chart of System

By setting some parameters, we can get the simulation outcome of the system.

The Figure 5 shows that the differences between the stationary process and non-stationary process.

Parameters	First Test	Second Test
N	500	500
λ	5	5
μ_1	4	2
μ_2	3	2
s	2	2

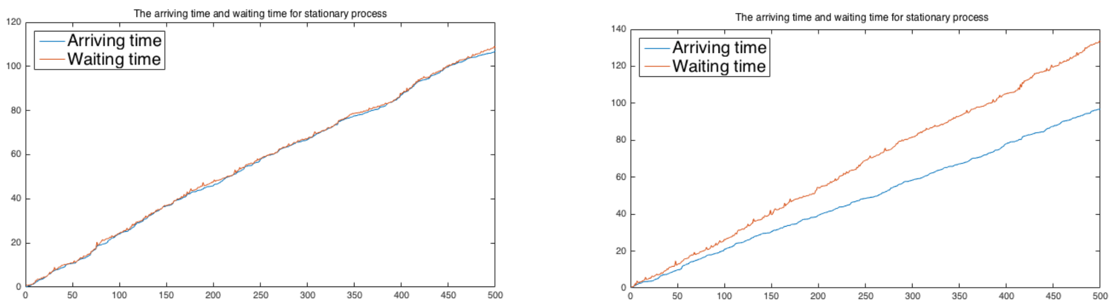

Figure 5: Differences between the stationary model and non-stationary model

4 Problems and Improvements of Queuing

4.1 Sorted Queue Model

In this model, we analyze the circumstance of single queue and single sever, which is the special case for M/M/s when $s = 1$. With the comparison of the random queue and sorted queue, we simulate 18 passengers' queuing situation from the official Excel sheet.

4.1.1 Model Establishment

When it comes to only one queue, we can calculate each passenger's waiting time using the flowing equation.

$$T_{wait}(i) = \begin{cases} 0 & i = 1 \\ T_{wait}(i-1) + T_{serve}(i-1) & i \geqslant 2 \end{cases} \tag{18}$$

Sum each passenger's waiting time using equation (18), we can get the average waiting time and the variance in waiting time.

$$W_q = \frac{\sum_{i=1}^{n}[(n-i)T_{serve}(i)]}{n} \tag{19}$$

$$D_{wait} = \frac{\sum_{i=1}^{n}[T_{wait}(i) - E_{wait}]^2}{n-1} \tag{20}$$

4.1.2 The Bottleneck of Order

We use the official Excel data, and consider columns C-D as passenger's service time. Compare the random order queue with the sorted order queue which let slower traveler go later. We get a contrast figure 6 and table 2.

Table 2: Contrast of waiting time

	Definition	Average	Variance
Random order	Traditional way of queuing	88.978s	3982.831
Sorted order	Sorted from low to high by service time	78.283s	3304.178

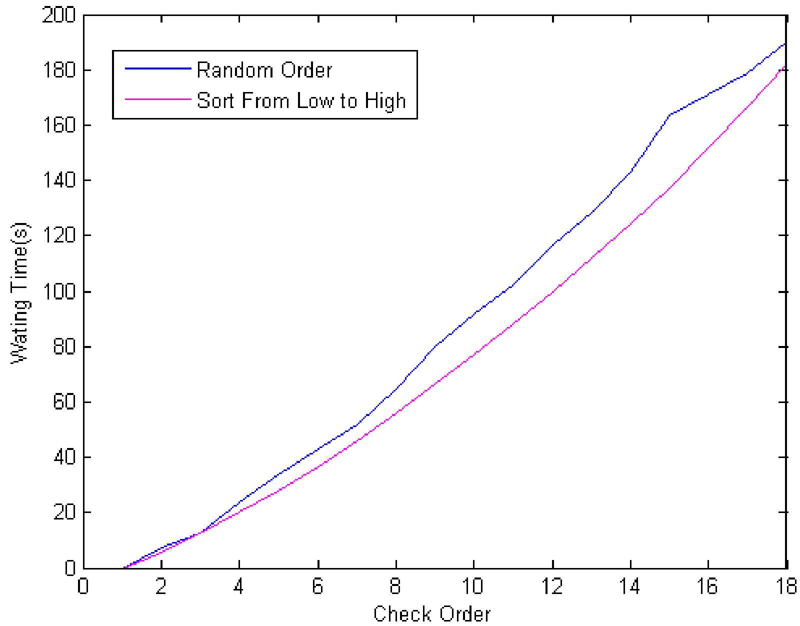

Figure 6: Random order and sorted order contrast graph

So we can get an inference that let faster travelers go first and slower traveler go last, we will get a less average waiting time and reduce variance.

However, it is a waste of time to sort the random order queue manually. So we propose an optimization in Pre-Queue Model.

4.2 Pre-Queue Model

To solve the bottleneck that we mentioned in Sorted Queue Model, we establish The Pre-Queue Model which attempt to sort the queue in a simple way. The Jianjun Ma's research[12] inspired us to build this model, and we improve his method to make it more suitable to our problem.

4.2.1 Model Establishment

While Pre-Check is also an optimization method,the passengers who are already the Pre-checker don't have the qualification to be the Pre-queuer. The prerequisite conditions of the Pre-Queue are

- **Passengers must be the native speaker**
- **Passengers must be the alone**

Here are other conditions to decide a passenger qualification of being a Pre-queuer.

- **Passengers who take an airline more than four times per year.** They are familiar with the process of security check.
- **Passengers in uniform.** Soldiers are trained and efficient, they can pass faster.
- **Passengers who will be late for flights.** Instead of opening another individual "Fast Pass Station", Pre-Queue ticket is another way to solve their problem.
- **Passengers without packages.** They can pass faster without packages scanning.

4.2.2 Results

People who get a Pre-Queue ticket don't have their own queue, and we make figure 7 to show this model. Passengers in Pre-Queue can "cut in line", and go first. (Figure 7 comes from TSA's official Checkpoint_Design_Guide[13], and we edit it using Photoshop.)

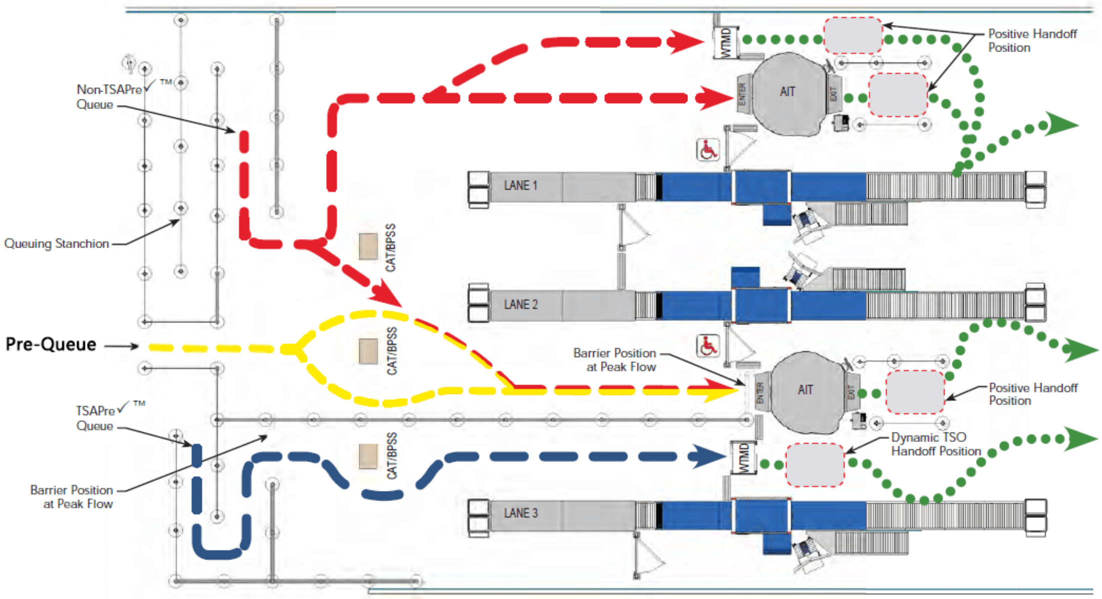

Figure 7: Pre-Queue Sketch

We cannot guarantee that the queue is completely ordered by service time from low to high.So we just let faster travelers go first as far as possible by Pre-Queue Model.

4.3 Single Queue Multi Server Model(SQMS)

4.3.1 Model Establishment

Figure 8 is the contrast of (a) traditional multi queue multi server and (b) single queue multi server. Firstly, based on the actual situation, we assume that in (a) passengers will go to the other queue if he find the other queue has fewer passengers than his own queue.

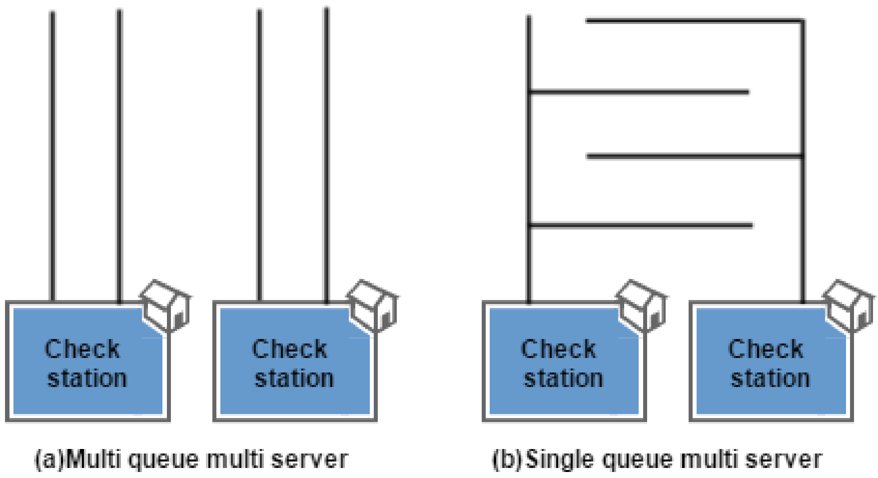

Figure 8: Sketch of traditional way and SQMS

4.3.2 Results

Based on the actual situation, we assume that three passengers arrive per minute and each check station can serve two people per minute.We simulate the two process for 700 times and gets lots of possible waiting time. Because of the different order of the queue, waiting time may be different. The results are as follows. Each possible waiting time is sorted from low to high in Figure 9, and Table 3 is the average and variance.

Table 3: Data we get from contrast

Way	Average	Variance
Multi queue multi server	1.160min	0.892
Single queue multi server	1.032min	0.838

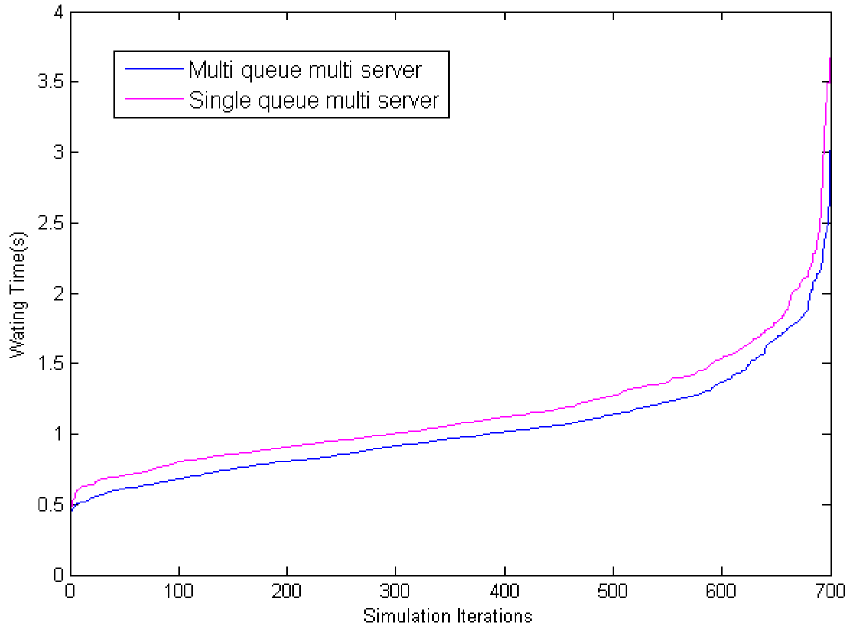

Figure 9: Contrast graph of traditional way and SQMS

It shows that (b) single queue multi server can get fewer average waiting time and reduce the variance in waiting time. Meanwhile, Ivo Adan and Jacques Resing's book[14] reach the same result.

5 Development and Improvement of Our Models

5.1 Sensitivity Analysis Based on Different Culture

There is no doubt that in different cultures, there are different social value orientations. For example,Americans are known for deeply respecting personal privacy, the Swiss are known for their awareness of the collective consciousness. By taking these cultural differences into account, we can test our model in the real-time situation.

5.1.1 The Swiss Model

In this section, we assume that the traveler style is to minimize the variance of average waiting time W_q by letting the "Tough passenger" get the service at last. By adding the random noises in the server time, we can simulate the "Tough passengers". In order to interpret the trend, we simulate the M/M/s model with the parameters sets at Table4. where α is the proportion of the "Tough passengers", μ_T is the average service rate of them.

In conclude,form the Table5 the Sorted queue model can reduce the average waiting

Table 4: Set Parameters

Parameters	Values
N	500
λ	5
μ	3
μ_T	1
s	2
α	0.05

Table 5: The Comparison

Way	Average	Variance
Normal line	0.924	0.43
After sorted	0.683	0.37

time and the variance of average waiting time.

5.1.2 The Best Service model

In this section,we suppose there is one traveler style which aim to maximize the experience for the passengers.We can quantify this need as to minimize the average waiting time W_q ,and average line length L_q. First,we set up two parameters,the max tolerance of the waiting timeW_q^{tol}, and the max tolerance of line length L_q^{tol}.These two parameters reflect to tolerance of the passengers.Our goal is to find the minimal server number s^*.Besides,we only consider the optimization procedures at M/M/s model.

$$\min s^*$$

$$s.t \begin{cases} \rho_s & = \dfrac{\lambda}{s\mu} < 1 \\ W_q & = \dfrac{L_q}{\lambda} \leq W_q^{tol} \\ L_q & = \dfrac{p_0 \rho^s \rho_s}{s!(1-\rho_s)^2} \leq L_q^{tol} \end{cases} \tag{21}$$

However, in the real time situation,it is hard to obtain the reliable statics of the W_q^{tol} and L_q^{tol}.Besides,these values are so fuzzy,which cannot to put a scale to measure.

5.2 Evaluation of Strengths and Weaknesses

5.2.1 Strengths

- We analysis the problem based on the Queue Theory and the structure of real-time security checkpoint,so our model is of great validity.

- Our model is quite flexible and extendable.Anybody can make a different multi-layer system as they like.
- We decompose the complex system into small units which have the similar form.We can focus on the essence of the problem.
- Besides, we use Matlab to help us simulate the whole system,which reduce the complexity. The out come of the simulation is vivid for us to understand the features of the system
- The Sorted Queue Model analyze the order of queue's impact to the whole system in detail, and find out the bottleneck of order.
- The Pre-Queue Model can solve the bottleneck of order as far as possible, and reduce the variance in waiting time. And it can also optimize the problem of cultural differences.

5.2.2 Weaknesses

- This model requires huge amount of data, some of which are hard to get. We have no data to check our model. Some of the data are put forward by our guessing and experiences, which are not very reliable.
- We considered the whole process is governed by the statistical law and made a lot of simplifications to establish the model. However, in the real-world situation, the uncertainties are huge.
- In our model, we take no consideration of the distribution functions of the passengers which they move from one station to another. In reality, this kind of situation should be concerned.

5.3 Future Work

- If there are more passengers' information such as age, country, educational background, frequency of taking a plane per year and etc.Then,we can predict their service time by using neural network, which will give the Pre-Queue model a better performance.
- If we could get data about the influence of flight schedule to passenger throughput, then we can get the relations between the flight, security check efficiency and passengers. It is a more general model to reflect the reality.
- Optimize as an aspect of dynamic system.As the security checkpoint is a highly dynamic system,the number of the severs should be depend on the dynamic situations such as the random occurrence of the passenger and the structure of the service. However,the dynamic system can be highly uncertainty and full of chaos. We don't have enough time to establish the dynamic system. If we had more time, we would take the dynamic process into consideration.

6 A Recommendation to the Security Managers

1. **Use Pre-Queue Model (let fast traveler go first) to minimize the discontents of single passengers.** Compared with single passengers, passengers in a group can take their attention away from waiting by talking with others. So single passengers are more likely to feel anxious in waiting than passengers in a group.

2. **Use Single Queue Multi Server Model on queuing.** As the results show, Single Queue Multi Server can get fewer average waiting time and reduce variance in waiting time.Meanwhile, it is more convenient for management. And single queue need fewer officers.

3. **Milimeter Wave Scanner can be improved.** Let passengers get body scanning while walking like Figure 10, because the pause at Milimeter Wave Scanner is a waste of time. But it will cost a lot of money and the technology maybe is still immature.

4. **Increase length of the post-Xray belt at both ends.** It can make the passengers have better prepared and give them more time to pick up their belongings.

5. **The bins of the X-ray machine should be colorful.** Colorful bins are more convenient for different passengers to distinguish. It can prevent customers from taking wrong bin and save time.

6. **Design the double-deck X-ray machine.** We notice that the X-ray machine is a potential bottleneck for the security process and reduce the robustness of the system. So, it is important to purchase or design a double-deck machine to increase the performance.

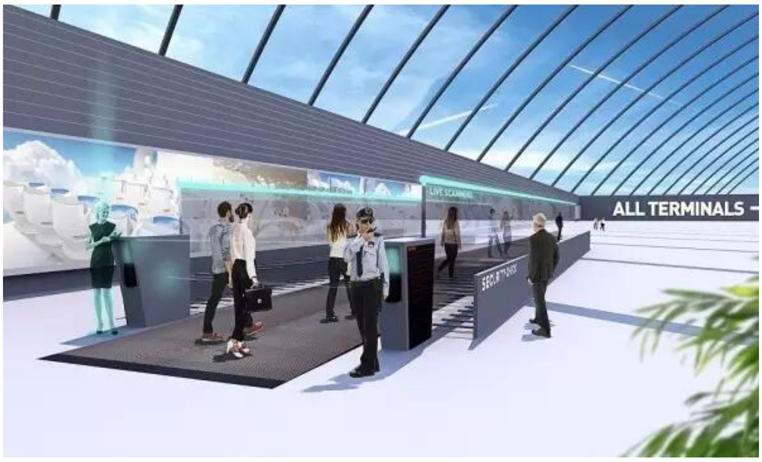

Figure 10: The Design of Milimeter Wave Scanner.

References

[1] http://blog.tsa.gov/2017/01/tsa-year-in-review-record-amount-of.html

[2] Erlang A K. Solution of some problems in the theory of probabilities of significance in automatic telephone exchanges[J]. Elektrotkeknikeren, 1917, 13: 5-13.

[3] https://en.wikipedia.org/wiki/Queueing_theory

[4] Whittle, P .(2002). "Applied Probability in Great Britain". Operations Research. 50: 227ÍC177. doi:10.1287/opre.50.1.227.17792. JSTOR 3088474.

[5] Kendall, D.G.:Stochastic processes occurring in the theory of queues and their analysis by the method of the imbedded Markov chain, Ann. Math. Stat. 1953

[6] Dickson D, Ford R C, Laval B. Managing real and virtual waits in hospitality and service organizations[J]. Cornell Hotel and Restaurant Administration Quarterly, 2005, 46(1): 52-68.

[7] http://www.scholarpedia.org/article/Petri_net

[8] Xun Lu. the planning and simulation research of the airport passengers and packages flow [D]. NUAA, 2008.

[9] LIU R, ZHANG Z. Hypothesis test in the judgment on a binomial distribution and a Poisson distribution[J]. Journal of Qinghai University (Nature Science), 2008, 1: 015.

[10] Chunling Cheng,Jun Li,Ying Wang. An Energy-Saving Task Scheduling Strategy Based on Vacation Queuing Theory in Cloud Computing.College of Computer, Nanjing University of Posts and Telecommunications

[11] T. F. Keller, Duke University,D. J. Laughhunn, Duke University. AN APPLICATION OF QUEUING THEORY TO A CONGESTION PROBLEM IN AN OUTPATIENT CLINIC*

[12] Jianjun Ma.The apply of virtual queue in the civil aviation security system[J].science and innovation, 2016 (19): 34-35.

[13] http://www.acina.org/sites/default/files/Checkpoint_Design_Guide_(CDG)_Rev_4_0.pdf

[14] Ivo Adan and Jacques Resing's Book, Queueing Systems.