
Queuing Theory

Introduction

Queuing Theory is the mathematical study of waiting lines or queues. It analyzes the behavior of queues formed by entities (like customers, jobs, or data packets) waiting for service from one or more servers. The goal is to understand and optimize system performance, such as minimizing wait times and maximizing service efficiency.

It is widely used in operations research, telecommunications, manufacturing, health services, and computing.

Definition of Terms in Queuing Models

- **Arrival Rate (λ):** The average number of arrivals per time unit.
- **Service Rate (μ):** The average number of services completed per time unit.
- **Queue Discipline:** The rule by which waiting customers are served (e.g., First In First Out - FIFO).
- **System Capacity:** The maximum number of customers allowed in the system (including those waiting and being served).
- **Population Source:**
 - Finite: A limited number of potential arrivals.
 - Infinite: An unlimited or very large number of potential arrivals.
- **Utilization Factor (ρ):** Given by $\rho = \lambda / \mu$, it indicates how busy the server is.
- **Queue Length:** The average number of customers waiting in line.
- **System Length:** The total number of customers in the queue and in service.
- **Waiting Time:** The time a customer spends in the queue or in the system.

Single-Channel Infinite Population Model (M/M/1)

Assumptions:

- One server (single channel).
- Infinite population of customers.
- Poisson arrivals and exponential service times.
- Unlimited queue capacity.
- First-Come-First-Served service discipline.

Performance Metrics:

- Average number in the system (L):
$$L = \lambda / (\mu - \lambda)$$
- Average number in the queue (Lq):
$$Lq = (\lambda^2) / [\mu(\mu - \lambda)]$$
- Average time in the system (W):
$$W = 1 / (\mu - \lambda)$$
- Average waiting time in queue (Wq):
$$Wq = \lambda / [\mu(\mu - \lambda)]$$

Multi-Channel Service Infinite Queue Model (M/M/c)

Assumptions:

- Multiple servers (c channels).
- Infinite queue capacity and population.
- Poisson arrivals and exponential service times.

- FIFO service discipline.

This model applies to environments like banks, help desks, and hospitals with multiple service counters.

Note: Calculations require Erlang-C formulas, which consider the number of servers, arrival rate, and service rate.

Finite Population Model

Features:

- Only a limited number of customers (N) in the population.
- The arrival rate depends on the number of customers not currently in the system.
- Suitable for repair stations, machine shops, or clinics with a small number of users.

Implications:

- Arrival rate is not constant—it decreases as more customers are in the system.
 - Special models and formulas (like birth-death processes) are used for analysis.
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Applications of Queuing Theory

- Service centers (e.g., banks, customer support)
- Assembly lines in manufacturing
- Hospital emergency rooms
- Cloud computing and data networks
- Traffic systems and toll booths
- Call centers and help desks

Discuss Different Attitudes of Customers in Waiting Line

The attitude of customers in a waiting line plays a significant role in their overall satisfaction, even if the actual wait time is short or reasonable. Understanding these attitudes helps in designing better queuing systems.

Below are the **different attitudes customers show while waiting in line**:

1. Unoccupied Time Feels Longer than Occupied Time

- When customers are not doing anything while waiting, time seems to pass slowly.
 - **Example:** People waiting in silence at a bank counter feel more impatient than those watching a TV screen.
 - **Solution:** Engage customers with displays, music, or information boards.
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2. Uncertain Wait is More Frustrating than a Known Wait

- Customers get more anxious if they don't know how long the wait will be.
 - **Example:** Waiting for a bus without any schedule feels longer than waiting with a display showing time.
 - **Solution:** Inform customers about estimated wait times.
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3. Unexplained Wait is More Annoying than an Explained Wait

- People feel more frustrated if they don't know why the wait is happening.
 - **Example:** No update in a hospital waiting room may cause anger.
 - **Solution:** Provide reasons for delay (e.g., "Doctor is in emergency surgery").
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4. Unfair Wait is More Agonizing than a Fair Wait

- Customers are more irritated when they feel the order of service is unfair.
- **Example:** Someone who came later being served earlier creates frustration.
- **Solution:** Maintain **first-come, first-served (FIFO)** discipline or transparent priority rules.

5. The More Valuable the Service, the More Willing the Customer is to Wait

- Customers tolerate longer waits if the service is important or high quality.
- **Example:** People may wait longer for a specialist doctor than a general one.
- **Solution:** High-value services can afford longer waits if quality is assured.

6. Solo Waiting Feels Longer than Group Waiting

- Waiting alone feels more tiring and frustrating.
- **Example:** Waiting in a queue with friends feels quicker than waiting alone.
- **Solution:** Create a comfortable and social environment where possible.

7. Anxiety Makes Wait Time Feel Longer

- If a customer is anxious (e.g., waiting for test results), time seems to pass more slowly.
- **Solution:** Reduce anxiety through reassurance or updates.

Conclusion:

Customer attitudes in waiting lines depend not just on the actual wait time but on psychological and emotional factors. Understanding these behaviors helps improve **customer experience** and reduce perceived wait times.
