Introduction to Transportation Problem

The Transportation Problem is a special type of Linear Programming Problem (LPP) that deals with optimizing the distribution of goods or services from multiple sources (origins) to multiple destinations (e.g., warehouses to retailers) at the minimum cost while satisfying supply and demand constraints.

It is widely used in logistics, supply chain management, and operations research to minimize transportation costs or time.

Basic Structure of a Transportation Problem

1. **Sources (Supply points):**
   Each source has a certain supply capacity.

2. **Destinations (Demand points):**
   Each destination has a certain demand that must be met.

3. **Cost matrix:**
   The cost of transporting one unit from each source to each destination is known.

Objective:

To minimize the total transportation cost while:

- **Fulfilling all demands**
- **Not exceeding supplies**
- **Satisfying the constraints of supply and demand**

Types of Transportation Problems

1. **Balanced Transportation Problem:**
   Total supply equals total demand:
   $\sum a_i = \sum b_j$
2. **Unbalanced Transportation Problem:**
   Total supply ≠ total demand. Add a dummy row or column to balance it.

Applications of Transportation Problem

- **Distribution of goods from factories to warehouses**
- **Assignment of workers to tasks**
- **Shipping schedules and routing**
- **Inventory transfers among locations**
- **Scheduling buses, trains, or aircraft efficiently**

What is Degeneracy in Transportation Problems?

In a **transportation problem**, **degeneracy** occurs when the number of **basic allocations** (non-zero cells in the initial feasible solution) is **less than** the required number, which is:

$\text{Number of basic allocations} = m + n - 1$

Where:

- **m** = number of supply points (rows)

- **n** = number of demand points (columns)

If the number of occupied cells (basic variables) is **less than m + n - 1**, the solution is **degenerate**.

Why is Degeneracy a Problem?

Degeneracy creates difficulties in applying optimization methods like the **Modified Distribution Method (MODI)** or **Stepping Stone Method**, because these methods require exactly m+n−1m + n - 1 basic variables to calculate unique values of dual variables (i.e., uiu_i and vjv_j). Without this, we cannot proceed correctly with optimization steps.

How to Resolve Degeneracy in a Transportation Problem

To resolve degeneracy, **artificial allocations** (called **ε**, a very small positive number close to zero) are introduced in such a way that:

- The total number of basic variables becomes **m + n – 1**.
- These artificial allocations do **not affect** the supply and demand constraints.
- They are treated as **basic cells** during calculations but considered to carry **no actual quantity**.

Steps to Resolve Degeneracy:

1. **Check for degeneracy:**
    - Count the number of basic allocations in your initial feasible solution.
    - If the count is less than m+n−1m + n - 1, degeneracy exists.
2. **Introduce ε (epsilon):**
    - Choose one or more of the **unused cells (empty cells)** where an artificial allocation (ε) can be safely placed.

    - Ensure that placing ε **does not form a closed loop** (which could affect further optimization steps).

3. **Proceed with MODI or Stepping Stone method:**
    - Treat ε as a very small positive value during computations.
    - At the final step, if no further improvement is possible, **remove ε** or **set it to zero** to find the actual optimal solution.

Example:

Suppose a transportation problem has:

- 3 sources (m = 3)
- 4 destinations (n = 4)

Then:

m+n−1=3+4−1=6 basic allocations requiredm + n - 1 = 3 + 4 - 1 = 6 \text{ basic allocations required}

If your initial feasible solution (e.g., via Vogel's or North-West Corner method) only gives **5 allocations**, then the solution is **degenerate**.
To resolve:

- Choose a suitable empty cell (with no allocation but not in the closed loop), and insert ε.
- You now have 6 basic cells: 5 real allocations + 1 ε (artificial).
