Introduction

The **Assignment Problem** is a special type of **Linear Programming Problem (LPP)** that deals with **assigning tasks or jobs to agents or machines** in such a way that the **total cost or time is minimized** (or profit is maximized), subject to the condition that each task is assigned to exactly one agent and vice versa.

It is widely applied in:

- Job scheduling
- Task allocation
- Resource assignment
- Staff rostering
- Machine loading

Characteristics of the Assignment Problem

- Number of tasks = number of agents
- Each agent is assigned to exactly one task
- Each task is assigned to exactly one agent
- The objective is to **minimize cost or maximize profit**

Mathematical Formulation of the Assignment Model

Let:

- n: number of agents/tasks (assumed equal)
- c_{ij}: cost of assigning agent i to task j
- x_{ij}: 1 if agent i is assigned to task j, 0 otherwise

Objective Function:

Minimize:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to Constraints:

1. **Each agent is assigned to one task:**

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for all } i$$

2. **Each task is assigned to one agent:**

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for all } j$$

3. **Binary decision variables:**

$$x_{ij} = \begin{cases} 1, & \text{if agent } i \text{ is assigned to task } j \\ 0, & \text{otherwise} \end{cases}$$

Solution of Assignment Problem

The **Hungarian Method** is the most widely used method to solve the assignment problem efficiently in polynomial time.

Steps in the Hungarian Method:

1. **Construct the cost matrix.**

- If it's a maximization problem, convert it to a minimization problem (e.g., by subtracting each value from the maximum element).

2. **Row Reduction:**
    - Subtract the smallest value in each row from every element in that row.
3. **Column Reduction:**
    - Subtract the smallest value in each column from every element in that column.
4. **Cover all zeros using a minimum number of lines.**
    - Use horizontal or vertical lines.
5. **Check:**
    - If the number of lines = nnn, an optimal assignment is possible among zeros.
6. **If not optimal:**
    - Find the smallest uncovered value, subtract it from all uncovered elements, and add it to elements at intersections of lines.
    - Repeat the covering process until the optimal number of assignments is possible.
7. **Make assignments:**
    - Choose zero elements in such a way that no row or column has more than one assignment.