Chapter 7

Project Management

A project defines a combination of interrelated activities which must be executed in a certain order before the entire task can be completed. The activities are interrelated in a logical sequence in such a way that some activities cannot start until some others are completed. An activity in a project is usually viewed as job requiring time and resources for its completion. Example of projects include, construction of a bridge, highway, power plant, repair and maintenance of an oil refinery or an air plane, design, development and marketing of a new product, research etc. The main objective before starting any project is to schedule the required activities in an efficient manner so as to complete it on or before a specified time limit at a minimum cost of its completion.

The two basic techniques for planning, scheduling, and controlling large and complex project are

1. PERT (Programme Evaluations and Review Technique)
2. CPM (Critical Path Method)

PERT: This technique is used to plan, schedule and control the activities of complex projects.

CPM: This technique is used for planning, sequencing and control where the emphasis is on optimizing resource allocation and minimizing overall cost for a given execution time.

The main differences between PERT and CPM are

1. Cost considerations
2. Computation of time estimates for activities

   CPM estimates one most likely time for completion of each activity

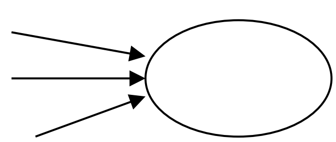
   PERT makes three time estimates for completion of each activity based on probability concepts.

PERT/CPM network consists of two major components:

1. **Events**: Events in the network diagram represent project milestones, such as the start or the completion of an activity or activities are occur at a particular instant of time at which some specific part of the project has been or is to be achieved. Events are commonly represented by circles (nodes) in the network diagram.

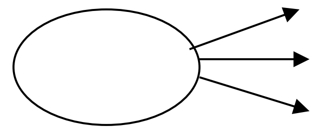
   The events can be classified into following categories

a) **Merge Event**: An event which represents the joint completion of more than one activity is known as merge event.



Merge Event

b) **Burst Event**: An event which represents the initiation of more than one activity is known as burst event



Burst Event

2. **Activities**: Activities in the network diagram represent project operations to be conducted each activity excepted dummy requires resource and takes a certain amount of time for completion. An arrow is commonly used to represent an activity with its head indicating the direction of progress in the project.
   Activities are identified by the numbers of their starting (tail) event and ending (head) event. An arrow (i, j) between two events, the tail event i represents the start of the activity and head event j represents the completion of the activity. The activities can be classified into three categories

a) **Predecessor Activity**: An activity which must be completed before one or more other activities start is known as predecessor activity.

b) **Successor Activity**: An activity which started immediately after one or more of other activities are completed is known as successor activity.

c) **Dummy Activity**: An activity which does not consume either any resource and/or time is known as dummy activity.

Critical Path Analysis

The objective of critical path analysis is to estimate the total project duration and to assign starting and finishing times to all activities involved in the project.

Notations for the purpose of calculating various times of events and activities

$(i, j) =$ Activity (i, j) with trial event i and head event j

$E_i =$ Earliest occurrence time of an event i

$L_j =$ Latest allowable time of an event j

$ES_{ij} =$ Earliest starting time for an activity (i, j)

$EF_{ij} =$ Earliest finish time for an activity (i, j)

$LS_{ij} =$ Latest starting time for an activity (i, j)

$LF_{ij} =$ Latest finishing time for an activity(i, j)

$t_{ij} =$ Duration of an activity (i, j)

For calculating earliest occurrence and latest allowable times of events two method namely forward pass method and backward pass method are used.

Float of an activity and event

The float (slack) or free time is the length of time to which a non-critical activity and/or an event can be delayed or extended without delaying the total project completion time.

Float on the events: - The float of an event is the difference between its latest occurrence time (L_i) and its earliest occurrence time (E_i)

i.e. Event float = $L_i - E_i$

a) If $L = E$ for certain events, then such events are called critical events.
b) If $L \neq E$ for certain events, then float on these events can be negative ($L < E$) or positive ($L > E$)

Total Float $TF_{ij} = \left(L_j - E_i\right) - t_{ij}$

Free Float $FF_{ij} = \left(E_j - E_i\right) - t_{ij}$

Independent Float $IF_{ij} = \left(E_j - L_i\right) - t_{ij}$

Critical Path: The critical path is the sequence of critical activities that form a continuous path between the start of a project and its completion. The length of the critical path is the sum of the individual times of all the critical activities lying on it and defines the longest time to complete the project.

Forward Pass Computations (for earliest event time)

Step1. The computations begin from the 'start' node and move towards the 'end' node. For easiness, the forward pass computations start by assuming the earliest occurrence time of zero for the initial project event.

Step2. (i) Earliest starting time of activity (i, j) is the earliest event time of the tail end event i.e., $(Es)_{ij} = E_i$

(ii) Earliest finish time of activity (i, j) is the earliest starting time + the activity time i.e.,

$(Ef)_{ij} = (Es)_{ij} + t_{ij}$ or $(Ef)_{ij} = E_i + t_{ij}$

(iii) Earliest event time for event j is the maximum of the earliest finish times of all activities ending into that event. That is

$E_j = \max_i\left[(Ef)_{ij}\ for\ all\ immediate\ predecessor\ of\ (i,j)\right]\ or\ E_j = \max_i\left[E_i + t_{ij}\right]$

The computed 'E' values are put over the respective circles representing each event.

Backward Pass Computations (for latest allowable time)

Step1. For ending event assume *E=L*.

Step2. Latest finish time for activity (i, j) is equal to the latest event time of event j, i.e., $(Lf)_{ij} = L$

Step3. Latest starting time of activity (i,j) = the latest completion time of (i,j) − the activity time

Step4. Latest event time for event i is the minimum of the latest start time of all activities originating from that event, i.e.

$$L_i = \min_j\left[(Ls)_{ij}\ for\ all\ immediate\ successors\ of\ (i,j)\right] = \min_j\left[L_j - t_{ij}\right]$$

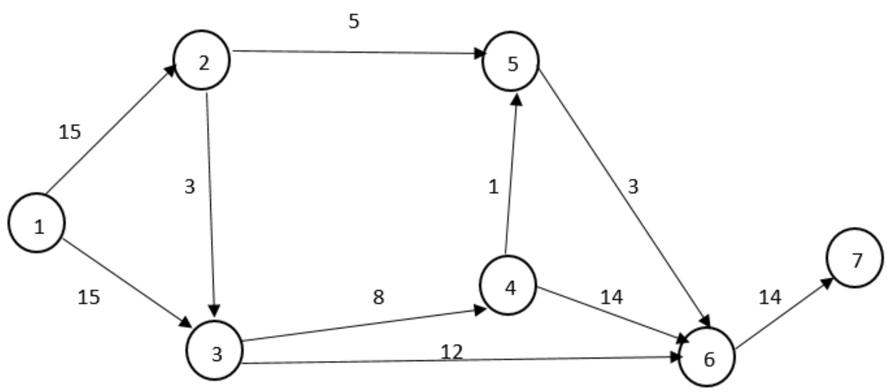
The computed 'L' values are put over the respective circles representing each event.

Q1. A small maintenance project consists of the following jobs whose precedence relationships is given below:

Job	1-2	1-3	2-3	2-5	3-4	3-6	4-5	4-6	5-6	6-7
Duration (days)	15	15	3	5	8	12	1	14	3	14

a) Draw an arrow diagram representing the project?
b) Find the total float for each activity.
c) Find the critical path and the total projection duration.

Solution: (a)



To determine critical path we compute earliest start time E_i and latest finish time L_j for each event

Forward Pass Method: $E_1 = 0$

$$E_2 = E_1 + t_{12} = 0 + 15 = 15$$

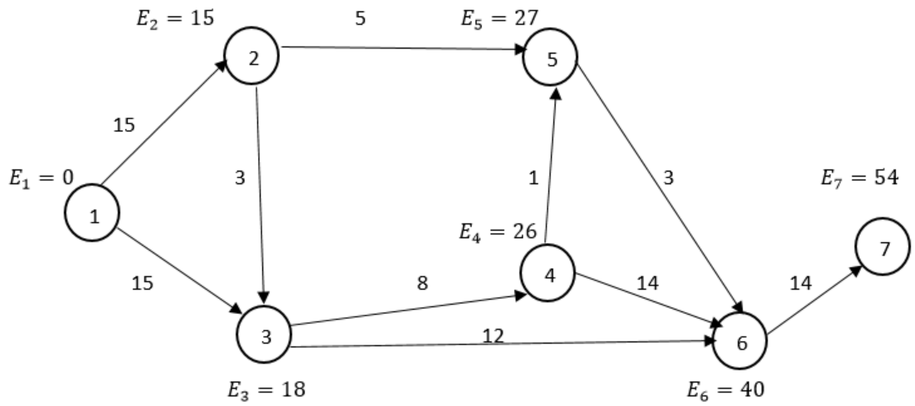
$$E_3 = max.\{E_2 + t_{23},\ E_1 + t_{13}\} = max.\{15 + 3,\ 0 + 15\} = 18$$

$$E_4 = E_3 + t_{34} = 18 + 8 = 26$$

$$E_5 = max.\{E_2 + t_{25},\ E_4 + t_{45}\} = max.\{15 + 5,\ 26 + 1\} = 27$$

$$E_6 = max.\{E_5 + t_{56},\ E_4 + t_{46},\ E_3 + t_{36}\} = max.\{27 + 3,\ 26 + 14,\ 18 + 12\} = 40$$

$$E_7 = E_6 + t_{67} = 40 + 14 = 54$$



Backward Method: $L_7 = 54$

$$L_6 = L_7 - t_{67} = 54 - 14 = 40$$

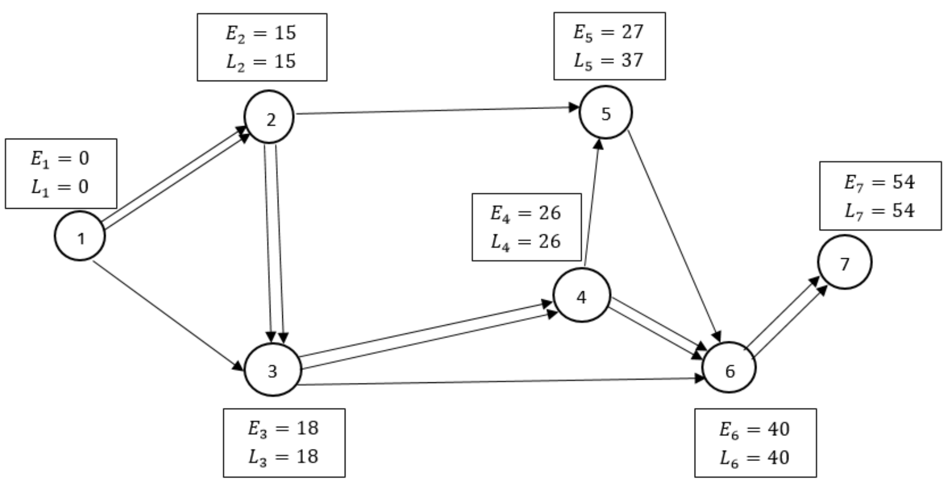
$$L_5 = L_6 - t_{56} = 40 - 3 = 37$$

$$L_4 = min.\{L_5 - t_{45}, \ L_6 - t_{46}\} = min.\{37 - 1, 40 - 14\} = 26$$

$$L_3 = min.\{L_4 - t_{34}, \ L_6 - t_{36}\} = min.\{26 - 8, 40 - 12\} = 18$$

$$L_2 = min.\{L_3 - t_{23}, \ L_5 - t_{25}\} = min.\{18 - 3, 37 - 5\} = 15$$

$$L_1 = min.\{L_2 - t_{12}, \ L_3 - t_{13}\} = min.\{15 - 15, 18 - 15\} = 0$$



The following table gives the calculation for critical path and total float

Activity (i, j)	Duration t_{ij}	Earliest Time Start E_i	Earliest Time Finish $E_i + t_{ij}$	Latest Time Start $L_j - t_{ij}$	Latest Time Finish L_j	Total Float $(L_j - t_{ij}) - E_i$
$(1,2)$	15	0	15	0	15	0
$(1,3)$	15	0	15	3	18	3
$(2,3)$	3	15	18	15	18	0
$(2,5)$	5	15	20	32	37	17
$(3,4)$	8	18	26	18	26	0
$(3,6)$	12	18	30	28	40	10
$(4,5)$	1	26	27	36	37	10
$(4,6)$	14	26	40	26	40	0
$(5,6)$	3	27	30	37	40	10
$(6,7)$	14	40	54	40	54	0

The activities$(1-2), (2-3), (3-4), (4-6)$, $(6-7)$ are critical activities and 1-2-3-4-6-7 is critical path.

The total project completion time is 54 days.

Q2. A project schedule has the following characteristics

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time(days)	4	1	1	1	6	5	4	8	1	2	5	7

a) Draw an arrow diagram representing the project?
b) Find the total float for each activity.
c) Find the critical path and the total projection duration.

Answer: 1-3-5-7-8-10 and time duration is 22 days

Q3. A research and development department is developing a new power supply for a television set. It has broken the job down into the following for:

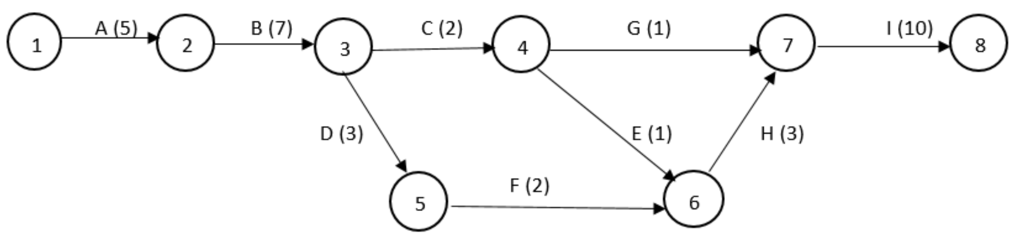
Job	Immediate Predecessors	Expected Time (days)
A	-	5
B	A	7
C	B	2
D	B	3
E	C	1
F	D	2
G	C	1
H	E, F	3
I	G, H	10

a) Draw the network diagram of activities involved in the project and indicate the critical path.

b) What is the minimum completion time for the project?

c) Find the total float for each activity.

Solution: (a)



Forward Pass Method: $E_1 = 0$

$$E_2 = E_1 + t_{12} = 5$$

$$E_3 = E_2 + t_{23} = 5 + 7 = 12$$

$$E_4 = E_3 + t_{34} = 12 + 2 = 14$$

$$E_5 = E_3 + t_{35} = 12 + 3 = 15$$

$$E_6 = max.\{E_4 + t_{46},\ E_5 + t_{56}\} = max.\{15,\ 17\} = 17$$

$$E_7 = max.\{E_4 + t_{47},\ E_6 + t_{67}\} = max.\{14 + 1,\ 17 + 3\} = 20$$

$$E_8 = E_7 + t_{78} = 20 + 10 = 30$$

Backward Method: $L_8 = 30$

$$L_7 = L_8 - t_{78} = 20$$

$$L_6 = L_7 - t_{67} = 20 - 3 = 17$$

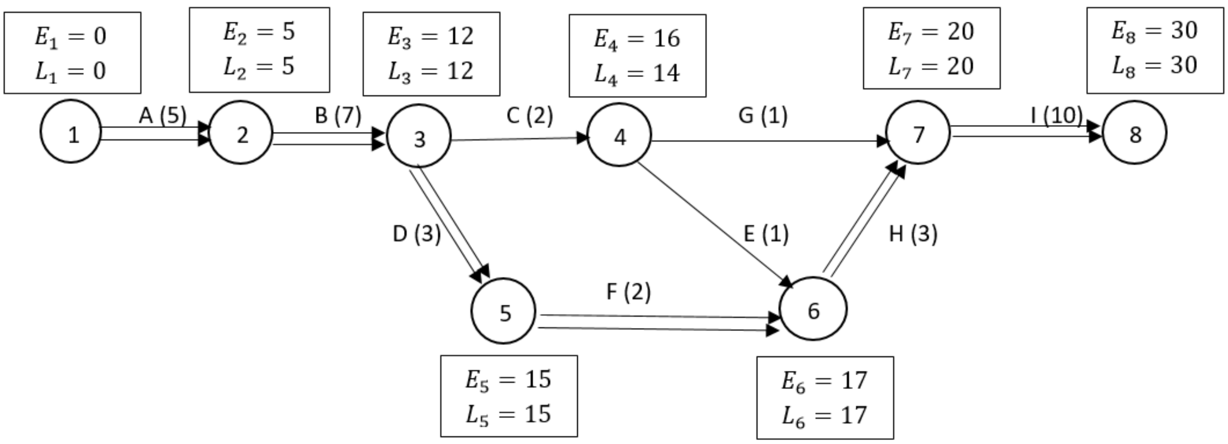
$$L_5 = L_6 - t_{56} = 17 - 2 = 15$$

$$L_4 = min.\{L_7 - t_{47}, L_6 - t_{46}\} = min.\{20 - 1, 17 - 1\} = 16$$

$$L_3 = min.\{L_4 - t_{34}, L_5 - t_{35}\} = min.\{16 - 2, 15 - 3\} = 12$$

$$L_2 = 12 - 7 = 5$$

$$L_1 = 5 - 5 = 0$$



The following table gives the calculation for critical path and total float

Activity (i,j)	Duration t_{ij}	Earliest Time		Latest Time		Total Float
		Start E_i	Finish $E_i + t_{ij}$	Start $L_j - t_{ij}$	Finish L_j	$(L_j - t_{ij}) - E_i$
(1,2)	5	0	5	0	5	0
(2,3)	7	5	12	5	12	0
(3,4)	2	12	14	14	16	2
(3,5)	3	12	15	12	15	0
(4,7)	1	14	17	19	20	5
(4,6)	1	14	17	16	17	2
(5,6)	2	15	17	15	17	0
(6,7)	3	17	20	17	20	0
(7,8)	10	20	30	20	30	0

The activities$(1-2)$, $(2-3)$, $(3-5)$, $(5-6)$, $(6-7)$, $(7-8)$ are critical activities and 1-2-3-5-6-7-8 is critical path. The total project completion time is 30 days.

Q4. Draw the network diagram from the following activities and find critical path and total float of activities

Job	Immediate Predecessors	Expected Time (days)
A	-	13
B	A	8
C	B	10
D	C	9
E	B	11
F	E	10
G	D,F	8
H	E	6
I	H	7
J	G, I	14
K	J	18

Answer: 1-2-3-5-6-8-9-10