(1) What is degeneracy in transportation problems? Explain how to resolve degeneracy in transportation problem.

Degeneracy in a transportation problem occurs when the number of basic variables in a feasible solution is less than $(m+n-1)$. where m is the number of supply points (origin) and n is the number of demand points (destination).

Degeneracy can occur in two cases.

1. Initial Degeneracy: when the basic feasible solution has fewer than $(m+n-1)$ allocated cells.

2. Degeneracy in Iterations: when during the optimality checking process, a loop formation is not possible due to a shortage of basic variables.

Methods to Resolve Degeneracy:

we use the concept of artificial allocations (dummy allocations) by a very small quantity Δ close to zero to one or more unoccupied cell so to get m+n-1 number of occupied cell.

Steps to Resolve Degeneracy.

1. Identify the Degenerate solution:
- 2. Select an empty cell: choose an unoccupied cell that does not distrub the feasibily of the solution.

3 Assign a small value (Δ)

4 Continue with the optimality check.

Q1) obtain the initial basic feasible solution of transportation problem by Vogel's approximation method.

	D1	D2	D3	D4	supply
O1	1	2	1	4	30
O2	3	3	2	1	50
O3	4	2	5	9	20
demand	20	40	30	10	

Solution since, $\sum a_i = 21 = \sum b_j$ there exists a feasible solution exists a feasible solution.

	D1	D2	D3	D4	supply
O1	1	2	1	4	30
O2	3	3	2	1	50
O3	4	2	5	9	20
demand	20	40	30	10	100

Step 1 : Take the difference between the lowest and second lowest cost entries for each row and column.

	D1	D2	D3	D4	supply	P1
O1	1	2	1	4	30	0
O2	3	3	2	1 (10)	~~50~~ 40	1
O3	4	2	5	9	20	2
demand	20	40	30	~~10~~ 0	100	
P1	2	0	1	3		

Step 2 : select the row or column for which the penalty is the largest. allocate the maximum possible amt to the cell with the lowest cost.

Here $(2,4) \rightarrow 10$ (allocated)

since demand is 0 cross the column

since demand is 0 cross the column and repeat these step untill all the demand is fulfilled.

	D1	D2	D3	D4	supply	P_1	P_2	P_3
O_1	1 (20)	2 (10)	1	4	~~30~~ ~~10~~ 0	0	1	1
O_2	3	3 (10)	2 30	1 (10)	~~50~~ ~~40~~ ~~0~~ ~~10~~	0	1	1
O_3	4	2 (20)	5	9	~~20~~ 0	2	3	x
demand	~~20~~ 0	~~40~~ ~~20~~ 0	~~30~~ 0	~~10~~ 0				
P_1	2 ↑	0	1	x				
P_2	x	0	1					
P_3	x	1	1 ↑					

Now, here : Total number of allocation: 6

and $m+n-1 \Rightarrow 4+3-1 = 6$

Hence, the solution is non-degenerate basic feasible.

Initial cost $\Rightarrow 20\times1 + 2\times10 + 3\times10 + 2\times20 + 2\times30 + 10 = 180$

Now, If required to find optimal solution.

Now, using UV method,

we determine a set of numbers u_i and v_j for each row and column.

v_j for each row and c...
such that $c_{ij} = u_i + v_j$ for each occupied cell.

Set the row/column having maximum number of allocation $\Rightarrow 0$. (O_2 column)

$\rightarrow c_{ij} = u_i + v_j$

	D1	D2	D3	D4	
O1	1 20	2 10	1 $\boxed{0}$	4 $\boxed{-1}$	$u_1 = -2$
O2	3 $\boxed{3}$	3 10	2 30	1 10	$u_2 = 0$
O3	4 $\boxed{4}$	2 20	5 $\boxed{3}$	9 $\boxed{2}$	$u_3 = 1$
	$v_1 = 3$	$v_2 = 3$	$v_3 = 2$	$v_4 = 1$	

$u_2 = 0$, For occupied cells

$C_{22} = u_2 + v_2 \Rightarrow v_2 = 3$

$C_{11} = u_1 + v_1$

$v_1 \Rightarrow 3$

$C_{23} = u_2 + v_3 \Rightarrow v_3 = 2$

$C_{12} = u_1 + v_2 \Rightarrow u_1 = -2$

$C_{24} = u_2 + v_4 \Rightarrow v_4 = 1$

$C_{32} = u_3 + v_2 \Rightarrow u_3 = 1$

For unoccupied cells:

we find the sum of u_i and v_j.

draw the above table too.

$C_{21} = 20$ $\quad$ $C_{31} \Rightarrow 30$

$C_{13} = 30$ $\quad$ $C_{33} \Rightarrow 40$

$c_{18} = 30$ c_{33} 4

$3,4 = 10$ $c_{34} \Rightarrow 20$

Now, we find $d_{ij} = c_{ij} - (u_i + v_j)$ for each unoccupied cell.

	D1	D2	D3	D4	
O_1	1 (20)	2 (10)	1 0 1	4 -1 5	$u_1 = -2$
O_2	3 3 0	3 (10)	2 (30)	1 (10)	$u_2 = 0$
O_3	4 4 0	2 (20)	5 3 2	9 2 7	$u_3 = -1$
	$v_1 = 3$	$v_2 = 3$	$v_3 = 2$	$v_4 = 1$	

Since, all $d_{ij} \geq 0$, the solution is optimal having alternative solution.

Therefore the total cost $\Rightarrow 20 + 20 + 30 + 40 + 60 + 10 \Rightarrow 180$

{ If, any $d_{ij} < 0$,

(i) select the most negative d_{rs} and construct a loop that starts and ends at the cell(r,s).

Refer. to notes page 16. $\boxed{Q_2}$

2) what is an integer linear programming?

ILP is a special case of LP where variables are constrained to be integers. It is commonly used in decision-making problems where discrete values are required, such as scheduling resource allocation and logistics.

Types of ILP :

1) pure integer linear programming
↳ All variable must be integer

2) mixed integer linear programming.
↳ some variables are integers, where others can be continuous.

Applications of ILP :

- supply chain optimization.
- scheduling (eg, employee shifts, project scheduling.

project scheduling.

- network design.
- capital Budgeting.

② Q $\text{Max } Z = 6x_1 + 5x_2$

subject to the constraints:

$2x_1 + 4x_2 \leq 8$

$4 + x_2/2 \leq 3$ $x_1, x_2 \geq 0$ and are integers.

using Gomory's cutting-plane Algorithm.

{ step 1: If IPP is in minization from convert it to maximization form }

Now, converting the canonical form by adding slack, surplus and artificial variable as appropriate.

for the constraint (1,2): ($\leq$)

we add slack variable s_1, s_2

Now,

$+ 5x_2 + 0s_1 + 0s_2$

Now,

$$\max Z = 6x_1 + 5x_2 + 0s_1 + 0s_2$$

ST :

$$\left.\begin{aligned} 2x_1 + 4x_2 + s_1 &= 8 \\ 4x_1 + 0.5x_2 + s_2 &= 3 \end{aligned}\right] \quad x_1, x_2, s_1, s_2 \geq 0$$

	c_j		6	5	0	0	XB/x_1
BV	CB	XB	x_1	x_2	s_1	s_2	Min Ratio
s_1	0	8	2	4	1	0	4
← s_2	0	3	$\boxed{4}$	0.5	0	1	3/4 →
	z_j		0	0	0	0	
	$c_j - z_j$		6 ↑	5	0	0	

Maximum $c_j - z_j \Rightarrow$ C. so, x_1 is P.C and minimum ratio is 3/4 so, s_2 is P.R and Pivot element : 4.

so, s_2 is replaced by x_1 as B.V.

$$R_2 = R_2 / 4 \quad , \quad R_1 = R_1 - 2R_2$$

	c_j		6	5	0	0	XB/x_2
BV	CB	XB	x_1	x_2	s_1	s_2	M.R
← s_1	0	6.5	0	$\boxed{3.75}$	1	-0.5	1.733 →
x_1	6	3/4	1	0.125	0	0.25	6
	z_j		6	0.75	0	1.5	
	$c_j - z_j$		0	4.25 ↑	0	-1.5	

$C_j - Z_j$ 4.25 ↑

Here, S_1 is replaced by x_2 as Base variable with 3.75 as piviot element.

$R_1 \Rightarrow R_1 \div 3.75$ $\quad R_2 \Rightarrow R_2 - 0.125\, R_1$

	C_j		6	5	0	0	
BV	C_B	X_B	x_1	x_2	s_1	s_2	min Ratio
x_2	5	1.733	0	1	0.2677	-0.133	
x_1	6	0.533	1	0	-0.083	0.2677	
	Z_j		6	5	1.133	0.983	
	$C_j - Z_j$		0	0	-1.133	-0.983	

Since all $C_j - Z_j \leq 0$

Hence, non-integer optimal solution is obtained with value of variable as:

$x_1 = 0.533 \quad x_2 = 1.733$

Max $z = 11.8667$

To obtain the integer value solution, we proceed to construct Gomory's fractional cut, with the help of x_2-row

$$1.733 = x_2 + 0.2667\, s_1 - 0.133\, s_2$$

$$1 + 0.733 = (1+0)x_2 + (0 + 0.2667)s_1 +$$

$1 + 0.733 = (1+0)x_2 + (0+0.2667)s_1 +$
$(-1+0.8667)s_2$

Now,
$-0.733 = g_1 - 0.2667 s_1 - 0.8667 s_2$

Now, Adding this additional constraint at the botton of optimal simplex table,

		c_j	6	5	0	0	0
BV	CB	x_B	x_1	x_2	s_1	s_2	g_1
x_2	5	1.733	0	1	0.267	−0.133	0
x_1	6	0.533	1	0	−0.033	0.266	0
g_1	0	$\boxed{-0.733}$	0	0	−0.2667	−0.8667	1
		z_j	6	5	1.133	0.933	0
		$c_j - z_j$	0	0	−1.133	$\boxed{-0.933}$	0

min negative : -0.733

$$\text{Ratio} = \min\left\{ \frac{c_j - z_j}{a_{ik}}, \; a_{ik} < 0 \right\}$$

$$= \min\left\{ \frac{-1.133}{-0.2667}, \frac{-0.9333}{-0.8667} \right\}$$

$$= \min\{ 4.25, 1.07 \}$$

Min Ratio = 1.07.

$R_3 = R_3 \div (-0.8667)$
$R_1 = R_1 - 0.1333 R_3$
$R_2 = R_2 - 0.2667 R_3$

$R_2 = R_2 - 0.2667R_3$

		c_j	6	5	0	0	0
BV	CB	XB	x_1	x_2	S_1	S_2	G_1
x_2	5	1.8462	0	1	0.3077	0	-0.1538
x_1	6	0.3077	1	0	-0.1154	0	0.3077
s_2	0	0.8462	0	0	0.3077	1	-1.1538
	z_j		6	5	0.8462	0	1.0769
	$c_j - z_j$		0	0	-0.8462	0	-1.0767

Since all $c_j - z_j \leq 0$.
and, NON-integer optimal Sol^n is obtained.

$x_1 = 0.3077$, $\boxed{x_2 = 1.8462}$

$\text{Max } z = 11.0769$

Now, to obtain integer valued Sol^n

$1.8462 = x_2 + 0.3077 S_1 - 0.1538 g_1$

$1 + 0.8462 = (1+0)x_2 + (0 + 0.3077)S_1 + (-1 + 0.8462) g_1$

$-0.8462 = g_2 - 0.3077 S_1 - 0.8462 g_2$

BV	CB	XB	x_1	x_2	S_1	S_2	g_1	g_2
		1.8462		1	0.3077	0	-0.1538	0

x_2	5	1.8462	0	1	0.3077	0	-0.1538	0
x_1	6	0.3077	1	0	-0.1154	0	0.3077	0
s_2	0	0.8462	0	0	0.3077	1	-1.1538	0
← ξ_2	0	$\boxed{-0.8462}$	0	0	-0.3077	0	$\boxed{-0.8462}$	1
	z_j		6	5	0.8462	0	1.0769	0
	$c_j - z_j$		0	0	-0.8462	0	-1.0769	0
min ration:			—	—	2.75	—	$\boxed{1.2727}$	—

min negative X_B is -0.8462

min positive ratio: 1.2727

entering variable is ξ_1

$R_4 = R_4 \div (-0.8462)$ $\quad R_1 = R_1 + 0.1538 R_3$

$R_2 = R_2 - 0.3077 R_4$ $\quad R_3 = R_3 + 1.1538 R_4$

		c_j	6	5	0	0	0	0
BV	C_B	X_B	x_1	x_2	s_1	s_2	ξ_1	ξ_2
x_2	5	2	0	1	0.3636	0	0	-0.1818
x_1	6	0	1	0	-0.2273	0	0	0.3636
s_2	0	2	0	0	0.7273	1	0	-1.3636
ξ_1	0	1	0	0	0.3636	0	1	-1.1818
	z_j		6	5	0.4545	0	0	1.2727
	$c_j - z_j$		0	0	-0.4545	0	0	-1.2727

since, all $c_j - z_j \leq 0$ ... solution since, all $c_j - z_j = 0$

Hence, integer optimal solution is obtained : $x_1 = 0,\ x_2 = 2$

$\max z = 10$ //

Q5) Dual simplex method :

$\min z = 3x_1 + x_2$

ST : $x_1 + x_2 \geq 5$ where, $x_1, x_2 \geq 0$

$2x_1 + 3x_2 \geq 2$

In order to apply the dual simplex method, convert all $\geq$ constraint to $\leq$ constraint by multiplying (-1)

$\leq \rightarrow$ slack
$\geq \rightarrow$ surplus
$\geq$ or $=$

Convert to max .

$\max z = -3x_1 - x_2$ and $x_1, x_2 \geq 0$

ST : $-x_1 - x_2 \leq -5$

$-2x_1 - 3x_2 \leq -2$

converting into canonical form by

converting into canonical form by using surplus variable

$$Max\ Z = -3x_1 - x_2 + 0S_1 + 0S_2$$

ST:

$$-x_1 - x_2 + S_1 = -5$$
$$-2x_1 - 3x_2 + S_2 = -2$$

where, $x_1, x_2, S_1, S_2 \geq 0$

Now, using simplex method

		C_j	-3	-1	0	0
BV	CB	X_B	x_1	x_2	S_1	S_2
S_1	0	$\boxed{-5}$	-1	-1	1	0
S_2	0	-2	-2	-3	0	1
		Z_j	0	0	0	0
		$C_j - Z_j$	-3	-1	0	0
		Min Ratio	3	$\boxed{1}$ ↑		

min negative, $X_B \Rightarrow -5$ so the Row 1 is piviot Row.

$$\text{Min Ration} = \min\left\{ \frac{C_j - Z_j}{S_{1j}},\ S_{1j} < 0 \right\}$$

$$= \left\{ \frac{-3-0}{-1},\ \frac{-1-0}{-1} \right\},\ \{3, 1\}$$

$$= 1$$

∴ piviot colum ⇒ x_2 and piviot element -1 so, replacing S_1 with x_2

$$R_1(new) = R_1(old) \div -1$$
$$R_2(new)\ R_2 - 3R_1(new)$$

		c_j	-3	-1	0	0
BV	CB	XB	x_1	x_2	s_1	s_2
x_2	-1	5	1	1	-1	0
s_2	0	13	1	0	-3	1
		z_j	-1	-1	1	0
		$c_j - z_j$	-2	0	-1	0

since, $c_j - z_j <= 0$ and $x_{Bi} \geq 0$

thus the current solution is the optimal solution.

$x_1 = 0 \quad x_2 = 5$

$\text{Max } Z = -5$

$\therefore \text{min } z = 5$

(6) Explain duality theory of LP.

Duality in linear programming (LP) states that every linear programming problem (called the primal problem) has another related LP problem (called the Dual problem). The optimal solution of both problems provide the same objective function value.

Formulation of dual LPP:

suppose the primal LPP is given by :

$$\text{maximum } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

subject TC.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2$$
$$| \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad |$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m$$

let $y_1, y_2, \cdots y_m$ are dual variables. then the dual LPP is :

$$\min Z_y = b_1y_1 + b_2y_2 + \cdots + b_my_m$$

ST :

$$a_{11}y_1 + a_{21}y_2 + \cdots + a_{m1}y_m \ge c_1$$
$$a_{12}y_1 + a_{22}y_2 + \cdots + a_{m2}y_m \ge c_2$$
$$| \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad |$$
$$a_{1n}y_1 + a_{21}y_2 + \cdots + a_{mn}y_m \ge c_n$$

where $y_1, y_2 \cdots y_m \ge 0$

Q) $\min Z = 2x_1 + 3x_2 + 4x_3$

ST : $2x_1 + 3x_2 + 5x_3 \ge 2$

$3x_1 + x_2 + 7x_3 = 3$ 4 $\}$ $x_1, x_2, x_3 \}$

$3x_1 + x_2 + 7x_3 = 5$

$x_1 + 4x_2 + 6x_3 \le 5 \quad (\ge 0)$

Now,
since, objective function is min, all $\le$ constraints can be converted to $\ge$ type by multiplying by (-1)

$$\min Z_x = 2x_1 + 3x_2 + 4x_3$$

ST:

$$2x_1 + 3x_2 + 5x_3 \ge 2$$
$$3x_1 + x_2 + 7x_3 \ge 3$$
$$-3x_1 - x_2 - 7x_3 \ge -3$$
$$-x_1 - 4x_2 - 6x_3 \ge -5$$

ST: $x_1, x_2, x_3 \ge 0$

Now,
let, y_1, y_2', y_2'', y_3 be dual variables.

i.e. $\max Z_y = 2y_1 + 3y_2' - 3y_2'' - 5y_3$

ST:

$$2y_1 + 3y_2' - 3y_2'' - y_3 \le 2$$
$$3y_1 + y_2' - y_2'' - 4y_3 \le 3$$
$$5y_1 + 7y_2' - 7y_2'' - 6y_3 \le 4$$

ST: $y_1, y_2', y_2'', y_3 \ge 0$

$y_1, y_2, y_2, y_3 \leq 0$

$$Max\ z_y = 2y_1 + 3(y_2' - y_2'') - 5y_3$$

st:

$$2y_1 + 3(y_2' - y_2'') - y_3 \leq 2$$
$$3y_1 + (y_2' - y_2'') - 4y_3 \leq 3$$
$$5y + 7(y_2' - y_2'') - 6y_3 \leq 4$$

$$y_1, y_2', y_2'', y_3 \geq 0$$

let $y_2 = y_2' - y_2''$

$$Max\ z_y = 2y_1 + 3y_2 - 5y_3$$

st:

$$2y_1 + 3y_2 - y_3 \leq 2$$
$$3y_1 + y_2 - 4y_3 \leq 3$$
$$5y_7 + 7y_2 - 6y_3 \leq 4$$

8) consider a problem of assigning four clerks to four tasks. The time (hours) required to complete the tasks is given below.

A	B	C	D

	A	B	C	D
1	4	7	5	6
2	–	8	7	4
3	3	–	5	3
4	6	6	4	2

Find the optimal assignment schedule

Step 1 : substract minimum of each row of the matrix, from all the elements of the respestive row.

	A	B	C	D	
1	0	3	1	2	−4
2	M_1	4	3	0	−4
3	0	M_2	2	0	−3
4	4	4	2	0	−2

Step 2 : Find out the each column minimum element and subtract it from that column.

	A	B	C	D
1	0	0	0	2
2	M_1	1	2	0
3	0	M_2	1	0
4	4	1	1	0

-0 -3 -1 -0

Step 3 : Make assignment in the cost table.

	A	B	C	D
1	0X	$\boxed{0}$	0X	2
2	M 1	1	2	$\boxed{0}$
3	$\boxed{0}$	M2	1	0X
4	4	1	1	0X

Step 4 : Number of assignment = 4, number of Rows = 4 which is not equal, so solution is not optimal.

Step 5 : Draw a set of horizontal and vertical lines to cover all the 0.

	A	B	C	D
1	0X	$\boxed{0}$	0X	2
2	M	1	2	$\boxed{0}$
3	$\boxed{0}$	M	1	0X
4	4	1	1	0X

Step 6 : Develop the new revised

Step 6 : Develop the new revised table by selecting the smallest element among the cells not covered by any line,
subtract (1) from uncovered
Add 1 on intersection

1	$\boxed{0}$	0 X	3
M	X 0	1	$\boxed{0}$
$\boxed{0}$	M	0 X	X 0
4	X 0	$\boxed{0}$	0 X

Repeat the all step (3 to 6)

Number of assignment = number of rows = 4 is optimal and so, solution has alternative solution

optimum sol^n :

work	Job	cost
1	B	7
2	D	4
3	A	3

3 A 3

4 C 4

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Alternative

1	~~0~~	$\boxed{0}$	3
M	$\boxed{0}$	1	0
$\boxed{0}$	M	~~0~~	~~0~~
4	0	~~0~~	$\boxed{0}$

Work	Job	Cost
1	C	
2	B	
	A	
	D	

4. A firm makes two products X and Y and has a total production capacity of

9 tones per day, X and Y requiring the same production capacity.
The firm has a permanent contract to supply at least 2 tones of X and at least 3 tones of Y per day to another company. Each tone of X requires 20 machine hours of production time and each tonne of Y requires 50 machine hours of production time. The daily maximum possible number of machine hours is 360. All the firm output can be sold and the profit made is Rs 80 per tonne of X and Rs 120 per tonne of Y.
It is required to determine the production schedule for maximum profit and to calculate this profit (use graphical method)

Here,
step 1: Define Decision variables
let, x = Number of tonnes of product X produced per day.
y = Number of tonnes of product Y produced per day.
Step 2: objective function:

$$Z = 80X + 120Y$$

where Z is the total profit

where z is the total profit.

step 3 : constraints.

(i) Total production capacity

$$x + y \leq 9$$

(ii) minimum supply

$$x \geq 2 \quad , \quad y \geq 3$$

(iii) machine hour constraint

$$20x + 50y \leq 360$$

where, x, y are ≥ 0

converting to canonical form

$x + y = 9$ — (1)

if $x = 0$, $y = 9$ so, $(0,9)$ & $(9,0)$

if $y = 0$, $x = 9$

$x = 2$ — (ii) point $(2,0)$

$y = 3$ — (iii) point $(0,3)$

$20x + 50y = 360$

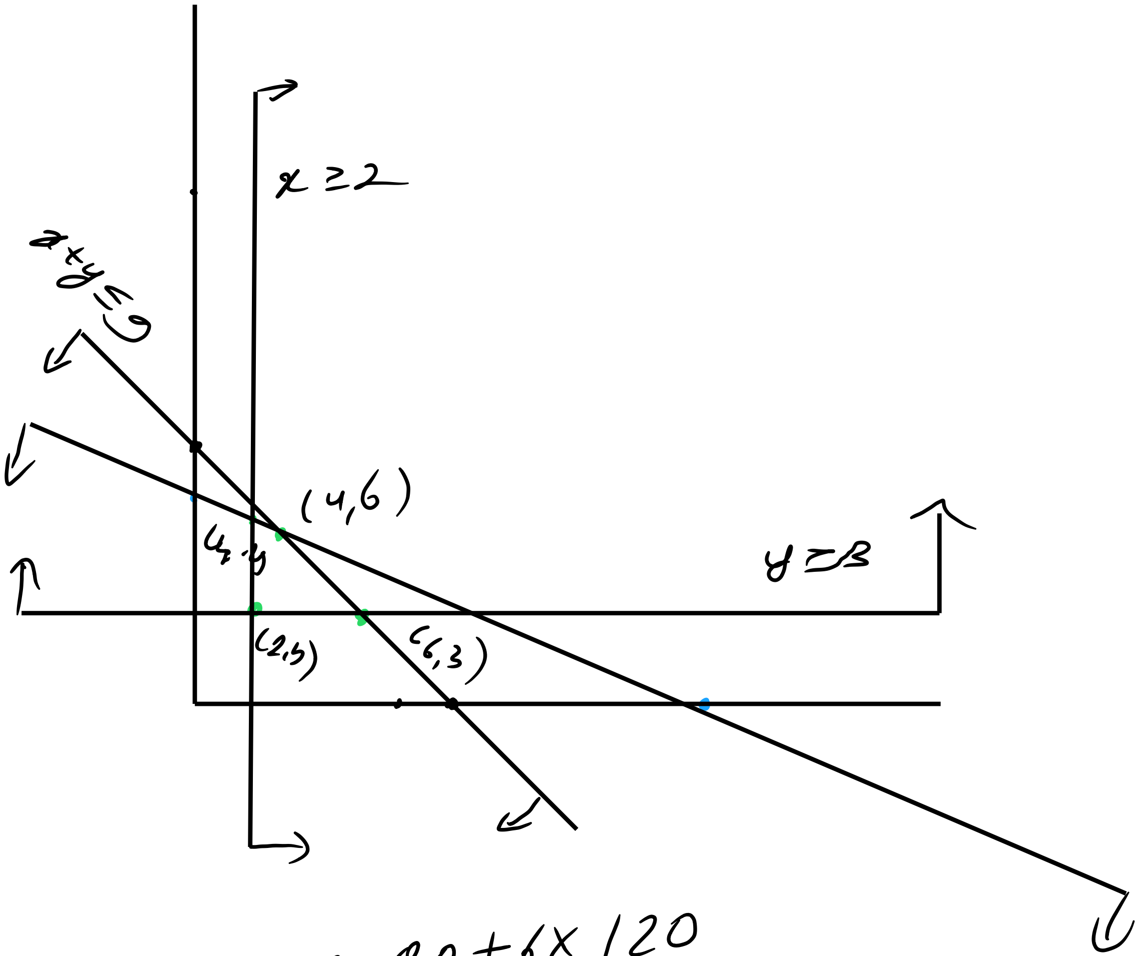
if $x = 0$,

$y = 7.2$ point$(0, 7.2)$

if $y = 0$

$x = 18$ point$(18,0)$

I



$max\, z = 3 \times 80 + 6 \times 120$

$= 960$ //