IntersectNET: Traffic Flow? Managed

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***Abstract*—Urban traffic congestion poses significant challenges to city planners and commuters worldwide. Efficient traffic management systems are essential to alleviate congestion, reduce travel time, and enhance the overall quality of urban life. This paper presents a comprehensive mathematical model for simulating and optimizing urban traffic flow using the Cell Transmission Model (CTM) coupled with Genetic Algorithms (GA) for optimizing traffic signal transitions at intersections. The model discretizes both space and time to simulate traffic dynamics accurately and employs evolutionary optimization techniques to minimize total travel time across the network.**

I. Introduction

Urbanization has led to a significant increase in the number of vehicles on city roads, resulting in chronic traffic congestion. Efficient traffic management is crucial for reducing travel time, minimizing fuel consumption, and lowering greenhouse gas emissions. Mathematical models play a pivotal role in understanding traffic dynamics and devising strategies for congestion mitigation.

This paper integrates the Cell Transmission Model (CTM) with Genetic Algorithms (GA) to simulate and optimize traffic flow. CTM discretizes roads into cells and time into intervals, providing a computationally efficient method for simulating traffic dynamics. GA, inspired by natural selection, optimizes traffic signal transition matrices at intersections to minimize total travel time (TTT) across the network.

II. Cell Transmission Model (CTM)

A. Model Formulation

The CTM is a discrete approximation of the Lighthill-Whitham-Richards (LWR) macroscopic traffic flow model. It represents roads as sequences of cells and time as discrete steps.

B. Discretization

Time and space are discretized as follows:

- Space: $x_i = i\Delta x$, for $i = 0, 1, 2, \ldots, N$.
- Time: $t^n = n\Delta t$, for $n = 0, 1, 2, \ldots, M$.

III. Traffic Network Representation

The urban traffic network is modeled as a directed graph $G = (V, E)$, where:

- V: Set of nodes representing intersections.
- E: Set of directed edges representing road segments.

Each edge $e \in E$ has attributes:

$$\begin{aligned} L_e &: \text{Length of road segment } e \text{ (m)},\\ C_e &: \text{Capacity of road segment } e \text{ (vehicles/s)},\\ v_f^e &: \text{Free-flow speed on } e \text{ (m/s)},\\ w^e &: \text{Congestion wave speed on } e \text{ (m/s)},\\ \rho_{\max}^e &: \text{Maximum density on } e \text{ (vehicles/m)}. \end{aligned}$$

IV. Flow Dynamics

A. Sending and Receiving Functions

For each cell i on edge e at time step n:

$$S_i^n = \min\left(v_f^e \rho_i^n \Delta t, C_e \Delta t\right), \tag{1}$$
$$R_i^n = \min\left(w^e(\rho_{\max}^e - \rho_i^n)\Delta t, C_e \Delta t\right). \tag{2}$$

B. Flow Between Cells

The flow from cell i to cell $i+1$ at time step n is:

$$q_i^n = \min\left(S_i^n, R_{i+1}^n\right). \tag{3}$$

C. Density Update

The density in each cell is updated based on inflow and outflow:

$$\rho_i^{n+1} = \rho_i^n + \frac{q_{i-1}^n - q_i^n}{\Delta x}. \tag{4}$$

V. Intersection Modeling

A. Transition Matrices

At each intersection, the distribution of flows from incoming edges to outgoing edges is governed by a transition matrix T. For an intersection with m incoming edges and n outgoing edges, T is an $n \times m$ matrix where:

$$\begin{aligned} T_{jk} = {} & \text{Probability that a vehicle from incoming edge } k \\ & \text{exits through outgoing edge } j. \end{aligned} \tag{5}$$

Subject to:

$$\sum_{j=1}^{n} T_{jk} = 1 \quad \forall k = 1, 2, \ldots, m. \tag{6}$$

VI. Optimization Framework

A. Genetic Algorithms (GA) Overview

Genetic Algorithms are evolutionary optimization techniques that mimic natural selection. They operate on a population of candidate solutions, evolving them over generations to optimize a fitness function.

B. Fitness Function

The fitness function evaluates how well a set of transition matrices minimizes the total travel time (TTT) across the network:

$$\text{Fitness} = -\sum_{e \in E} \sum_{i=1}^{N_e} \rho_i^e \Delta x, \tag{7}$$

where N_e is the number of cells in edge e.

C. Genetic Operators

- **Selection:** Select top-performing individuals based on fitness scores.
- **Crossover:** Combine transition matrices from two parents using a weighted average:

  $$T_{\text{child}} = \alpha T_{\text{parent1}} + (1 - \alpha) T_{\text{parent2}}, \tag{8}$$

  where $\alpha \in [0, 1]$.
- **Mutation:** Introduce Gaussian noise to transition probabilities:

  $$T_{\text{child}} = T_{\text{child}} + \mathcal{N}(0, \sigma^2). \tag{9}$$

  Normalize after mutation:

  $$\sum_{j=1}^{n} T_{jk} = 1 \quad \forall k. \tag{10}$$

VII. Simulation Procedure

1) **Network Initialization:** Define edges and intersections with their respective parameters.
2) **Vehicle Initialization:** Distribute vehicles across network cells based on initial densities.
3) **Flow Calculation:** For each time step, compute sending and receiving flows for each cell.
4) **Intersection Flow Distribution:** Use transition matrices to distribute flows from incoming to outgoing edges.
5) **Density Update:** Update the density in each cell based on inflow and outflow.
6) **Optimization:** Use GA to iteratively adjust transition matrices to minimize TTT.
7) **Visualization:** Generate plots to analyze traffic dynamics and optimization effects.

VIII. Conclusion

The integration of the Cell Transmission Model with Genetic Algorithms provides a robust framework for simulating and optimizing urban traffic flow. CTM effectively models traffic dynamics through discretization of space and time, while GA optimizes traffic signal transitions to minimize total travel time. This combined approach facilitates the development of efficient traffic management strategies, contributing to reduced congestion and improved urban mobility.

References

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