# Algolab 2016
# Winter Games

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# Objective

**Learn how to solve a problem given by a textual description.**
This includes:

- appropriate problem modeling
- choice of suitable (combinatorial) algorithms, and
- implementation.

**Your Task:**

- read the 3 problem descriptions
- sketch your approach (modeling, algorithms)

# Use them all!

## Observations

- $2 \leq n \leq 90'000$ — probably not looking for $\Omega(n^2)$
- no overlap — only "close neighbors" matter
- maximize radius — optimization problem?

## Key ideas

- for a fixed cannon, the nearest neighbor determines an upper bound on the operation range
- only the closest cannon pair matters

# Use them all! – Solution

Try them all!

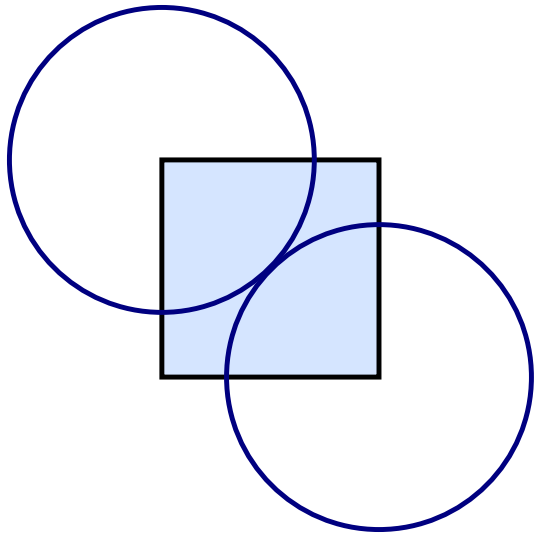
- Compute $\binom{n}{2}$ pairwise distances $\rightarrow \Theta(n^2)$

Check only the distance to the nearest neighbor for each cannon

- Compute the Delaunay triangulation $\rightarrow \Theta(n \log n)$
- Iterate over the edges of the triangulation $\rightarrow \Theta(n)$

## Implementation details

- the distance fits into a `double`
  radius $\leq \sqrt{2^{50} + 2^{50}} < 2^{50}$



# Downhill course

## Observations

- $1 \leq n \leq 10'000$ — probably $\Omega(n^2)$ is fine
- no overlap — 2 options for every cannon (on/off)
- each cannon has at most 2 neighbors — graph problem?

## Key ideas

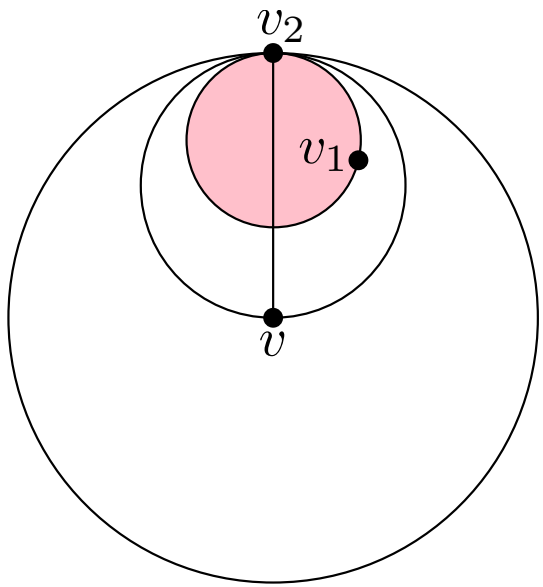
- graph problem: cannons are vertices, put an edge whenever two ranges overlap — still need to find/construct the graph
- find maximum independent set
  - in general — NP-complete
  - bipartite graphs — König's Theorem, Matching
  - special cases — trivial

# Downhill course – Solution

Construct a graph $G$ to model the dependencies: vertices are cannons and there is an edge between two vertices if the repective operation ranges overlap

- Compute $\binom{n}{2}$ pairwise distances $\rightarrow \Theta(n^2)$

- It suffices to consider
  the nearest neighbor $\rightarrow \Theta(n \log n)$
  the second nearest neighbor $\rightarrow \Theta(n \log n)$




**Lemma**

Let $v_1$ be a nearest neighbor and $v_2$ be a second nearest neighbor of $v$. Then at least one of $vv_2$ or $v_1v_2$ is an edge of the Delaunay triangulation.

- maximal degree 2

# Software update

## Observations

- $2 \leq n \leq 50$ — tiny...
- minimal radius — need solution of Exercise 2
- no overlap — radii are bounded from above
- maximize sum of the radii — optimization problem?

## Key ideas

- optimization problem — find a larger radius for each cannon
  - the improvement is linear in the radius
  - lower bounds for the radii
  - implicit upper bounds: no overlap

# Software update – Solution

Linear program with $n$ variables and $n + \binom{n}{2}$ constraints:

- *Variables*: Operation range (radius) of every snow cannon

$$r_i \geq \frac{\texttt{closest\_pair\_dist}}{2}, \quad i = 1, \ldots, n$$

- *Constraints*: Operation ranges are not overlapping

$$r_i + r_j \leq \texttt{dist}(i, j), \quad 1 \leq i, j \leq n$$

- *Objective*: Maximize the sum of the radii

$$\max \sum_i r_i$$

## Implementation details

- **IT**: Exact type with sqrt
- **ET**: Exact type with sqrt