1.绪论

迭代与递归

分而治之

凡治众如治寡，分数是也

The control of a large force is
the same principle as
the control of a few men:
it is merely a question of
dividing up their numbers.

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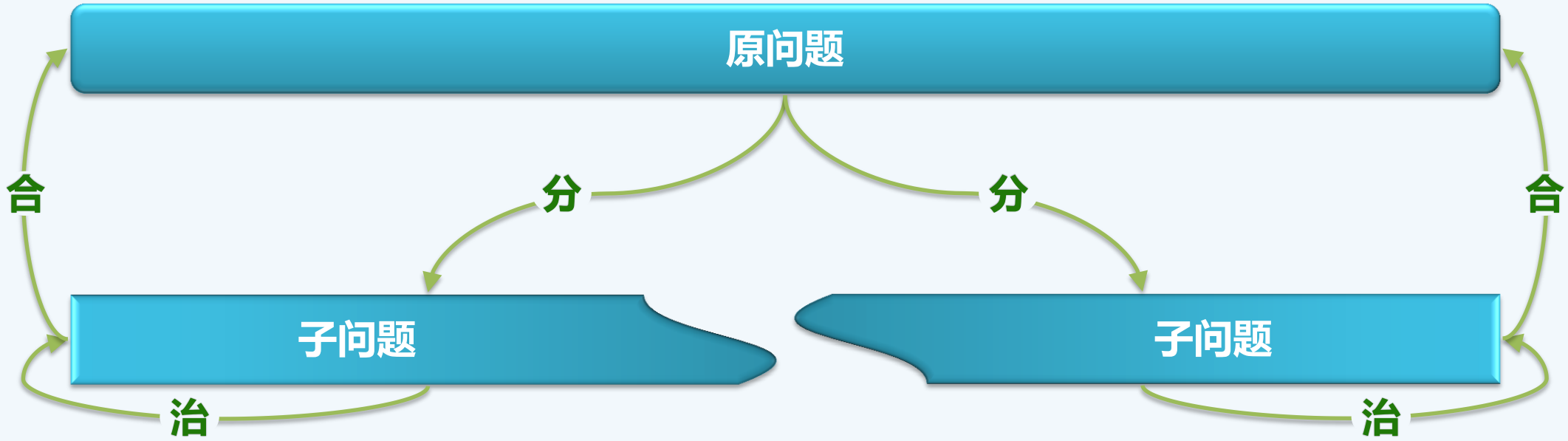
Divide-and-conquer

❖ 为求解一个大规模的问题，可以

将其划分为若干（通常两个）子问题，规模大体相当

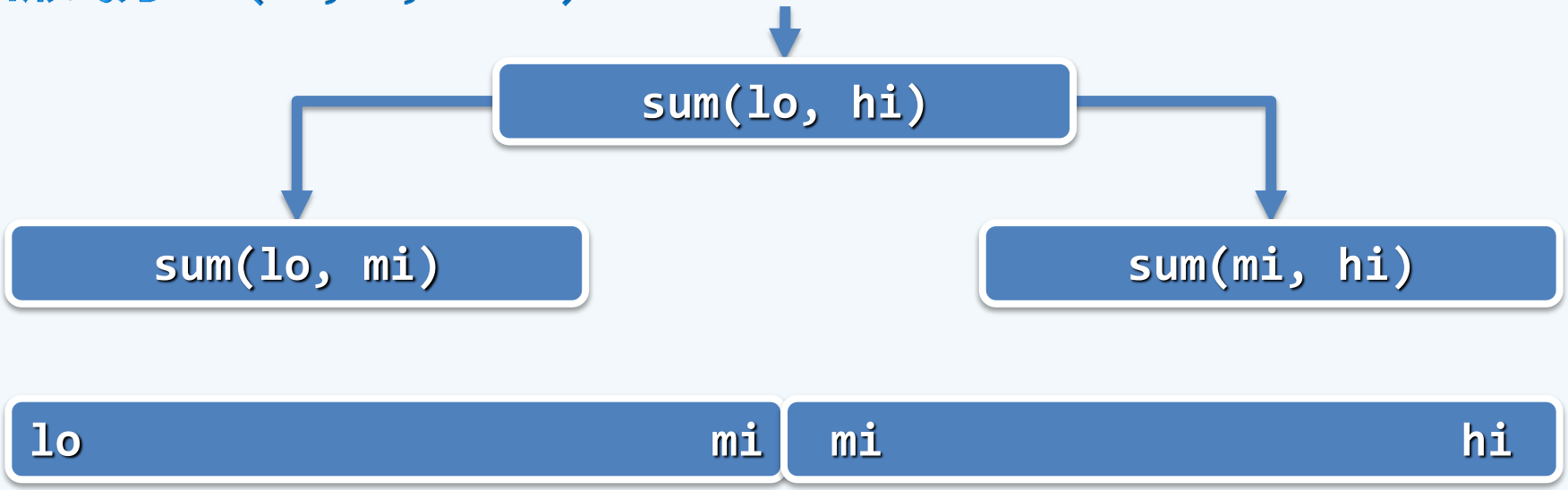
分别求解子问题

由子问题的解，得到原问题的解



Binary Recursion

```
❖sum( int A[], int lo, int hi ) { //区间范围A[lo, hi)
    if ( lo + 1 >= hi ) return A[lo];
    int mi = (lo + hi) >> 1;
    return sum( A, lo, mi ) + sum( A, mi, hi );
} //入口形式为sum( A, 0, n-1 )
```

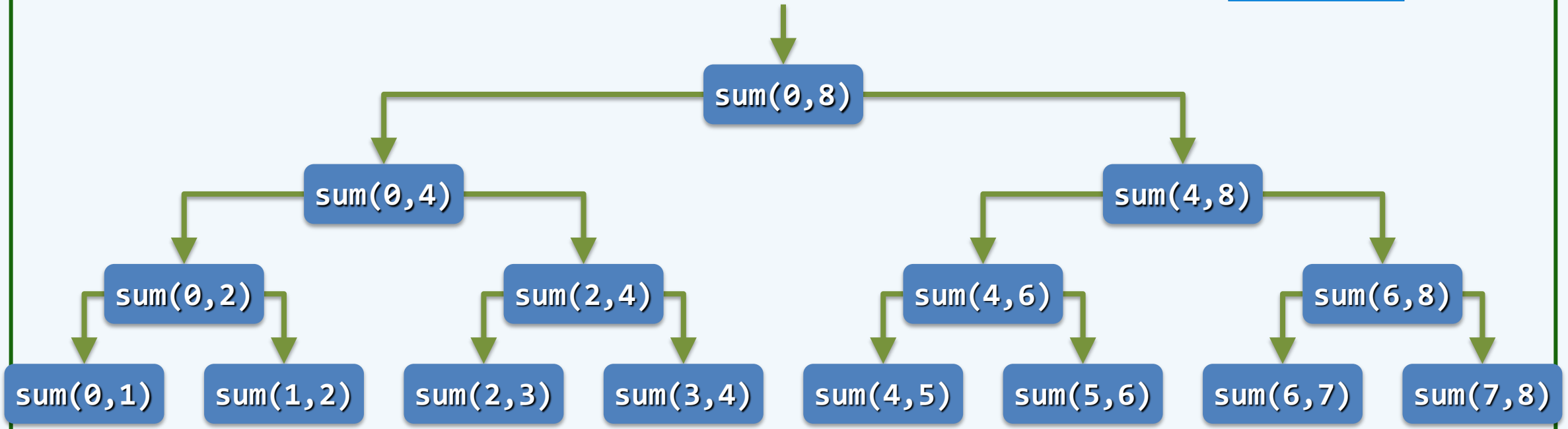


Binary Recursion: Trace

❖ T(n) = 各层递归实例所需时间之和 //递归跟踪

$= O(1) \times (2^0 + 2^1 + 2^2 + ... + 2^{\log n})$

$= O(1) \times (2^{1 + \log n} - 1) = O(n)$ //更快捷地，几何级数与最大/末项等阶



Binary Recursion: Recurrence

❖ 从递推的角度看，为求解sum(A, lo, hi)，需

- 递归求解sum(A, lo, mi)和sum(A, mi+1, hi)，进而 //2*T(n/2)
- 将子问题的解累加 //O(1)

❖ 递推关系 $T(n) = 2*T(n/2) + O(1)$

$T(1) = O(1)$ //base: sum(A, k, k)

❖ 求解 $T(n) = 2*T(n/2) + c_1$

$T(n) + c_1 = 2*(T(n/2) + c_1) = 2^2 * (T(n/4) + c_1) = ...$

$= 2^{\log n}(T(1) + c_1) = n*(c_2 + c_1)$

$T(n) = (c_1+c_2)n - c_1 = O(n)$