CHAPTER 5

ADVANCED ENCRYPTION STANDARD

"It seems very simple."

"It is very simple. But if you don't know what the key is it's virtually indecipherable."

—*Talking to Strange Men*, Ruth Rendell

KEY POINTS

- AES is a block cipher intended to replace DES for commercial applications. It uses a 128-bit block size and a key size of 128, 192, or 256 bits.
- AES does not use a Feistel structure. Instead, each full round consists of four separate functions: byte substitution, permutation, arithmetic operations over a finite field, and XOR with a key.

The Advanced Encryption Standard (AES) was published by the National Institute of Standards and Technology (NIST) in 2001. AES is a symmetric block cipher that is intended to replace DES as the approved standard for a wide range of applications. Compared to public-key ciphers such as RSA, the structure of AES and most symmetric ciphers is quite complex and cannot be explained as easily as many other cryptographic algorithms. Accordingly, the reader may wish to begin with a simplified version of AES, which is described in Appendix 5B. This version allows the reader to perform encryption and decryption by hand and gain a good understanding of the working of the algorithm details. Classroom experience indicates that a study of this simplified version enhances understanding of AES.[1] One possible approach is to read the chapter first, then carefully read Appendix 5B, and then re-read the main body of the chapter.

Appendix H looks at the evaluation criteria used by NIST to select from among the candidates for AES, plus the rationale for picking Rijndael, which was the winning candidate. This material is useful in understanding not just the AES design but the criteria by which to judge any symmetric encryption algorithm.

5.1 FINITE FIELD ARITHMETIC

In AES, all operations are performed on 8-bit bytes. In particular, the arithmetic operations of addition, multiplication, and division are performed over the finite field $GF(2^8)$. Section 4.7 discusses such operations in some detail. For the reader who has not studied Chapter 4, and as a quick review for those who have, this section summarizes the important concepts.

In essence, a field is a set in which we can do addition, subtraction, multiplication, and division without leaving the set. Division is defined with the following

[1]However, you may safely skip Appendix 5B, at least on a first reading. If you get lost or bogged down in the details of AES, then you can go back and start with simplified AES.

rule: $a/b = a(b^{-1})$. An example of a finite field (one with a finite number of elements) is the set Z_p consisting of all the integers $\{0, 1, \ldots, p - 1\}$, where p is a prime number and in which arithmetic is carried out modulo p.

Virtually all encryption algorithms, both conventional and public-key, involve arithmetic operations on integers. If one of the operations used in the algorithm is division, then we need to work in arithmetic defined over a field; this is because division requires that each nonzero element have a multiplicative inverse. For convenience and for implementation efficiency, we would also like to work with integers that fit exactly into a given number of bits, with no wasted bit patterns. That is, we wish to work with integers in the range 0 through $2^n - 1$, which fit into an n-bit word. Unfortunately, the set of such integers, Z_{2^n}, using modular arithmetic, is not a field. For example, the integer 2 has no multiplicative inverse in Z_{2^n}, that is, there is no integer b, such that $2b \bmod 2^n = 1$.

There is a way of defining a finite field containing 2^n elements; such a field is referred to as $\mathrm{GF}(2^n)$. Consider the set, S, of all polynomials of degree $n - 1$ or less with binary coefficients. Thus, each polynomial has the form

$$f(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x + a_0 = \sum_{i=0}^{n-1} a_i x^i$$

where each a_i takes on the value 0 or 1. There are a total of 2^n different polynomials in S. For $n = 3$, the $2^3 = 8$ polynomials in the set are

0	x	x^2	$x^2 + x$
1	$x + 1$	$x^2 + 1$	$x^2 + x + 1$

With the appropriate definition of arithmetic operations, each such set S is a finite field. The definition consists of the following elements.

1. Arithmetic follows the ordinary rules of polynomial arithmetic using the basic rules of algebra with the following two refinements.
2. Arithmetic on the coefficients is performed modulo 2. This is the same as the XOR operation.
3. If multiplication results in a polynomial of degree greater than $n - 1$, then the polynomial is reduced modulo some irreducible polynomial $m(x)$ of degree n. That is, we divide by $m(x)$ and keep the remainder. For a polynomial $f(x)$, the remainder is expressed as $r(x) = f(x) \bmod m(x)$. A polynomial $m(x)$ is called **irreducible** if and only if $m(x)$ cannot be expressed as a product of two polynomials, both of degree lower than that of $m(x)$.

For example, to construct the finite field $\mathrm{GF}(2^3)$, we need to choose an irreducible polynomial of degree 3. There are only two such polynomials: $(x^3 + x^2 + 1)$ and $(x^3 + x + 1)$. Addition is equivalent to taking the XOR of like terms. Thus, $(x + 1) + x = 1$.

A polynomial in $\mathrm{GF}(2^n)$ can be uniquely represented by its n binary coefficients $(a_{n-1}a_{n-2} \ldots a_0)$. Therefore, every polynomial in $\mathrm{GF}(2^n)$ can be represented by an n-bit number. Addition is performed by taking the bitwise XOR of the two n-bit elements. There is no simple XOR operation that will accomplish multiplication in

GF(2^n). However, a reasonably straightforward, easily implemented, technique is available. In essence, it can be shown that multiplication of a number in GF(2^n) by 2 consists of a left shift followed by a conditional XOR with a constant. Multiplication by larger numbers can be achieved by repeated application of this rule.

For example, AES uses arithmetic in the finite field GF(2^8) with the irreducible polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$. Consider two elements $A = (a_7 a_6 \ldots a_1 a_0)$ and $B = (b_7 b_6 \ldots b_1 b_0)$. The sum $A + B = (c_7 c_6 \ldots c_1 c_0)$, where $c_i = a_i \oplus b_i$. The multiplication $\{02\} \cdot A$ equals $(a_6 \ldots a_1 a_0 0)$ if $a_7 = 0$ and equals $(a_6 \ldots a_1 a_0 0) \oplus (00011011)$ if $a_7 = 1$.

To summarize, AES operates on 8-bit bytes. Addition of two bytes is defined as the bitwise XOR operation. Multiplication of two bytes is defined as multiplication in the finite field GF(2^8), with the irreducible polynomial[2] $m(x) = x^8 + x^4 + x^3 + x + 1$. The developers of Rijndael give as their motivation for selecting this one of the 30 possible irreducible polynomials of degree 8 that it is the first one on the list given in [LIDL94].

5.2 AES STRUCTURE

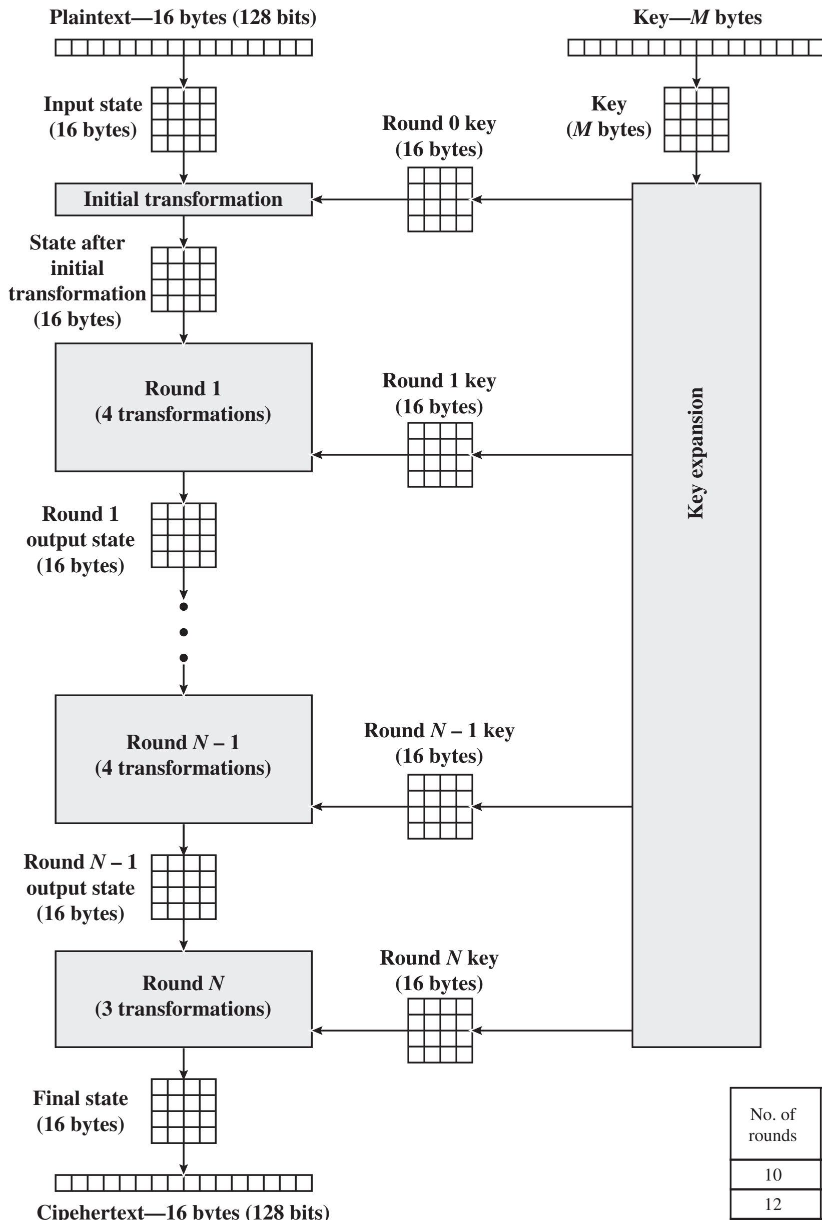
General Structure

Figure 5.1 shows the overall structure of the AES encryption process. The cipher takes a plaintext block size of 128 bits, or 16 bytes. The key length can be 16, 24, or 32 bytes (128, 192, or 256 bits). The algorithm is referred to as AES-128, AES-192, or AES-256, depending on the key length.

The input to the encryption and decryption algorithms is a single 128-bit block. In FIPS PUB 197, this block is depicted as a 4×4 square matrix of bytes. This block is copied into the **State** array, which is modified at each stage of encryption or decryption. After the final stage, **State** is copied to an output matrix. These operations are depicted in Figure 5.2a. Similarly, the key is depicted as a square matrix of bytes. This key is then expanded into an array of key schedule words. Figure 5.2b shows the expansion for the 128-bit key. Each word is four bytes, and the total key schedule is 44 words for the 128-bit key. Note that the ordering of bytes within a matrix is by column. So, for example, the first four bytes of a 128-bit plaintext input to the encryption cipher occupy the first column of the **in** matrix, the second four bytes occupy the second column, and so on. Similarly, the first four bytes of the expanded key, which form a word, occupy the first column of the **w** matrix.

The cipher consists of N rounds, where the number of rounds depends on the key length: 10 rounds for a 16-byte key, 12 rounds for a 24-byte key, and 14 rounds for a 32-byte key (Table 5.1). The first $N - 1$ rounds consist of four distinct transformation functions: SubBytes, ShiftRows, MixColumns, and AddRoundKey, which are described subsequently. The final round contains only three transformations, and there is a initial single transformation (AddRoundKey) before the first round,

[2] In the remainder of this discussion, references to GF(2^8) refer to the finite field defined with this polynomial.



No. of rounds	Key Length (bytes)
10	16
12	24
14	32

Figure 5.1 AES Encryption Process

which can be considered Round 0. Each transformation takes one or more 4×4 matrices as input and produces a 4×4 matrix as output. Figure 5.1 shows that the output of each round is a 4×4 matrix, with the output of the final round being the ciphertext. Also, the key expansion function generates $N + 1$ round keys, each of which is a distinct 4×4 matrix. Each round key serve as one of the inputs to the AddRoundKey transformation in each round.

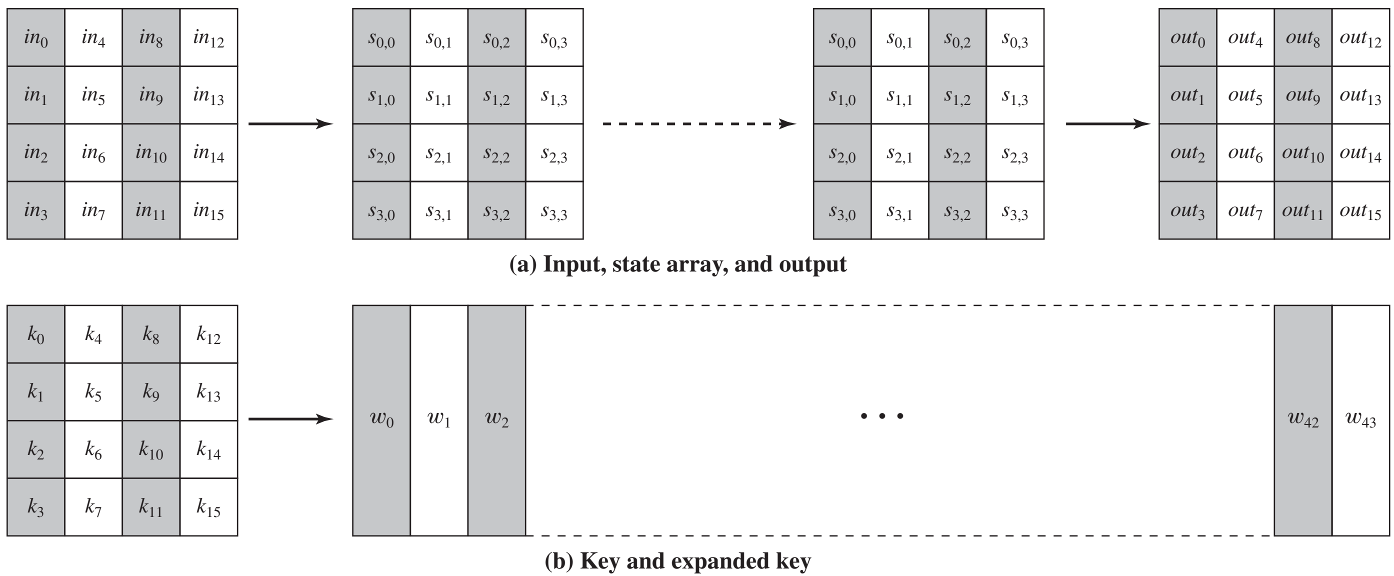


Figure 5.2 AES Data Structures

Table 5.1 AES Parameters

Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240

Detailed Structure

Figure 5.3 shows the AES cipher in more detail, indicating the sequence of transformations in each round and showing the corresponding decryption function. As was done in Chapter 3, we show encryption proceeding down the page and decryption proceeding up the page.

Before delving into details, we can make several comments about the overall AES structure.

1. One noteworthy feature of this structure is that it is not a Feistel structure. Recall that, in the classic Feistel structure, half of the data block is used to modify the other half of the data block and then the halves are swapped. AES instead processes the entire data block as a single matrix during each round using substitutions and permutation.
2. The key that is provided as input is expanded into an array of forty-four 32-bit words, $\mathbf{w}[i]$. Four distinct words (128 bits) serve as a round key for each round; these are indicated in Figure 5.3.
3. Four different stages are used, one of permutation and three of substitution:
   - **Substitute bytes:** Uses an S-box to perform a byte-by-byte substitution of the block
   - **ShiftRows:** A simple permutation
   - **MixColumns:** A substitution that makes use of arithmetic over $GF(2^8)$
   - **AddRoundKey:** A simple bitwise XOR of the current block with a portion of the expanded key
4. The structure is quite simple. For both encryption and decryption, the cipher begins with an AddRoundKey stage, followed by nine rounds that each includes all four stages, followed by a tenth round of three stages. Figure 5.4 depicts the structure of a full encryption round.
5. Only the AddRoundKey stage makes use of the key. For this reason, the cipher begins and ends with an AddRoundKey stage. Any other stage, applied at the beginning or end, is reversible without knowledge of the key and so would add no security.
6. The AddRoundKey stage is, in effect, a form of Vernam cipher and by itself would not be formidable. The other three stages together provide confusion, diffusion, and nonlinearity, but by themselves would provide no security because they do not use the key. We can view the cipher as alternating operations of XOR

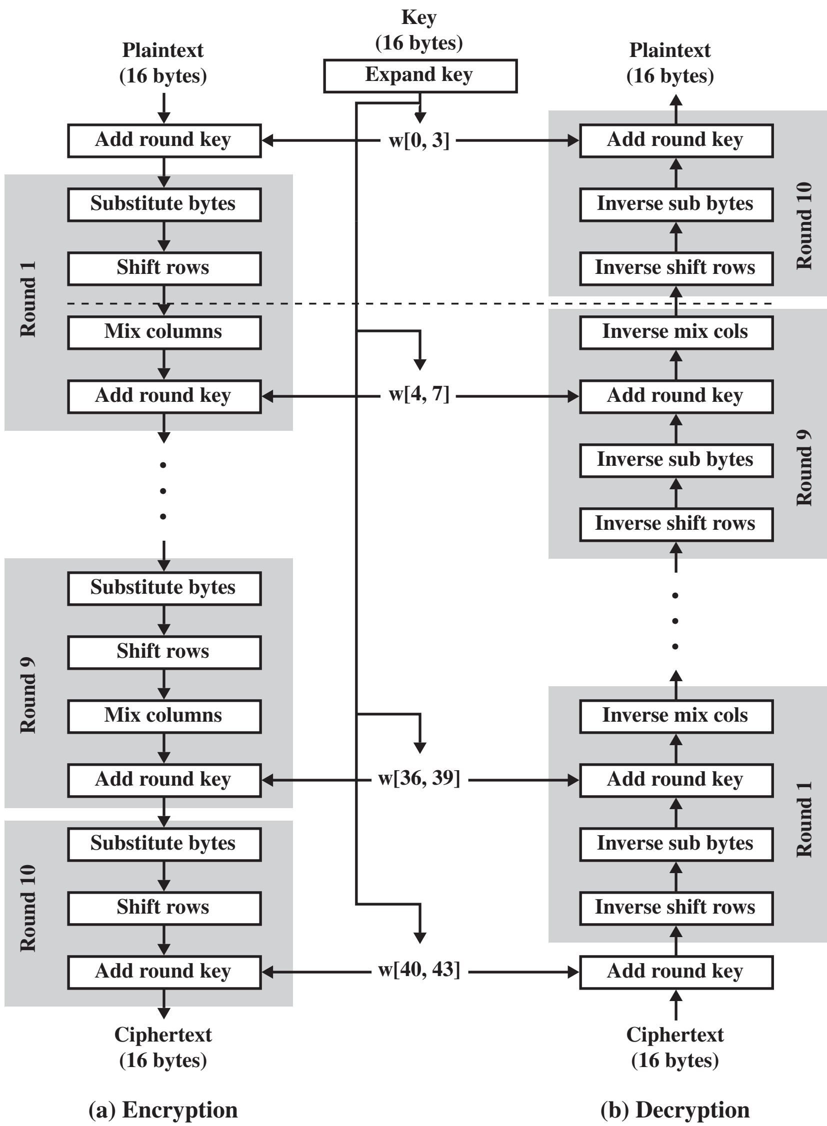


Figure 5.3 AES Encryption and Decryption

encryption (AddRoundKey) of a block, followed by scrambling of the block (the other three stages), followed by XOR encryption, and so on. This scheme is both efficient and highly secure.

7. Each stage is easily reversible. For the Substitute Byte, ShiftRows, and MixColumns stages, an inverse function is used in the decryption algorithm. For the AddRoundKey stage, the inverse is achieved by XORing the same round key to the block, using the result that $A \oplus B \oplus B = A$.
8. As with most block ciphers, the decryption algorithm makes use of the expanded key in reverse order. However, the decryption algorithm is not

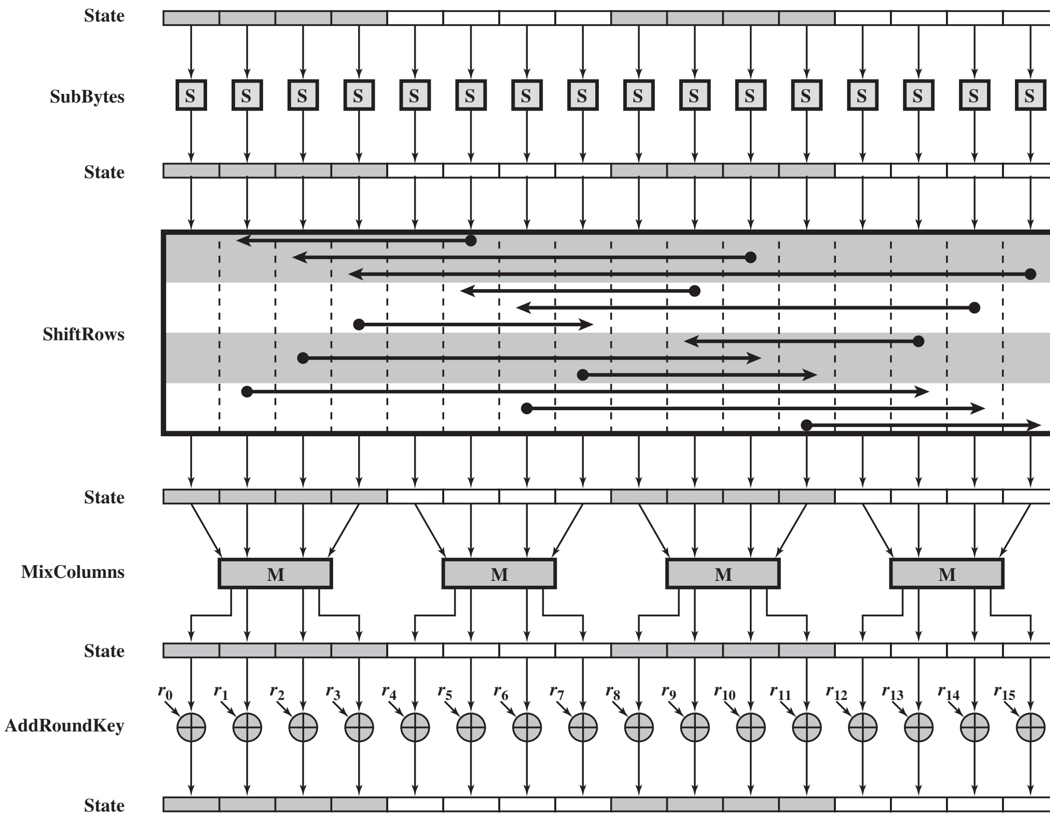


Figure 5.4 AES Encryption Round

identical to the encryption algorithm. This is a consequence of the particular structure of AES.

9. Once it is established that all four stages are reversible, it is easy to verify that decryption does recover the plaintext. Figure 5.3 lays out encryption and decryption going in opposite vertical directions. At each horizontal point (e.g., the dashed line in the figure), **State** is the same for both encryption and decryption.
10. The final round of both encryption and decryption consists of only three stages. Again, this is a consequence of the particular structure of AES and is required to make the cipher reversible.

5.3 AES TRANSFORMATION FUNCTIONS

We now turn to a discussion of each of the four transformations used in AES. For each stage, we describe the forward (encryption) algorithm, the inverse (decryption) algorithm, and the rationale for the stage.

Substitute Bytes Transformation

Forward and Inverse Transformations The **forward substitute byte transformation**, called SubBytes, is a simple table lookup (Figure 5.5a). AES defines a 16×16 matrix of byte values, called an S-box (Table 5.2a), that contains a permutation of all possible 256 8-bit values. Each individual byte of **State** is mapped into a new byte in the following way: The leftmost 4 bits of the byte are used as a row value and the rightmost 4 bits are used as a column value. These row and column values serve as indexes into the S-box to select a unique 8-bit output value. For example, the hexadecimal value[3] {95} references row 9, column 5

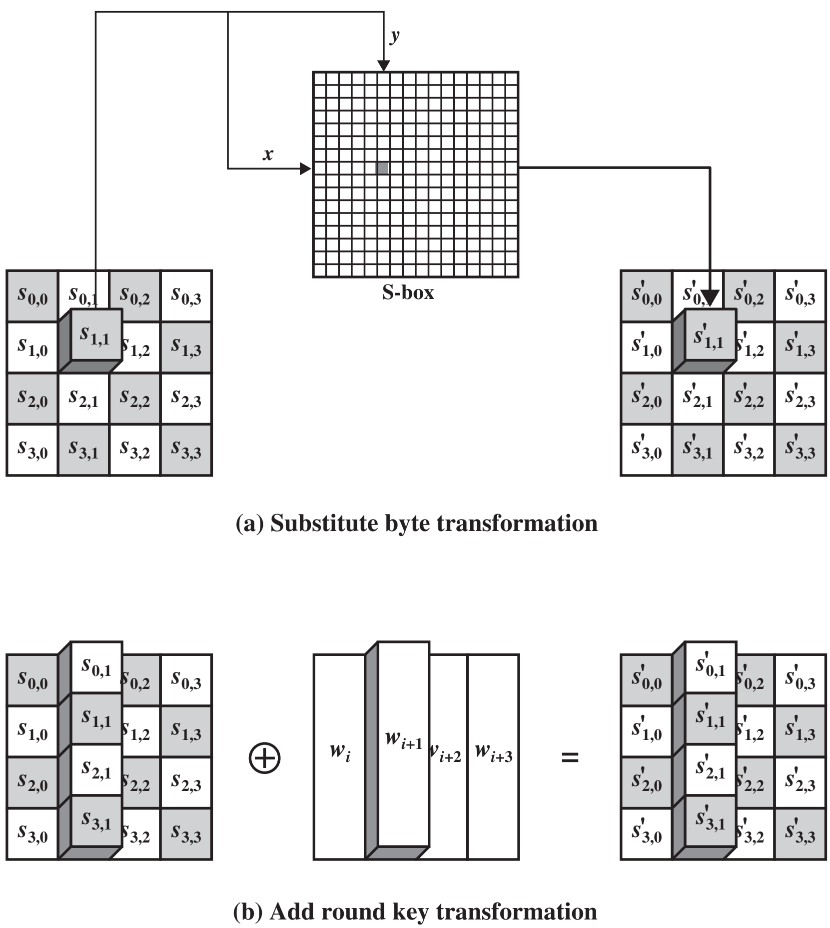


Figure 5.5 AES Byte-Level Operations

[3]In FIPS PUB 197, a hexadecimal number is indicated by enclosing it in curly brackets. We use that convention in this chapter.

Table 5.2 AES S-Boxes

		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
x	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

(a) S-box

		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
	0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
	1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
	2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
	3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
	4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
	5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
	6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
x	7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
	8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
	9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
	A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
	B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
	C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
	D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
	E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
	F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

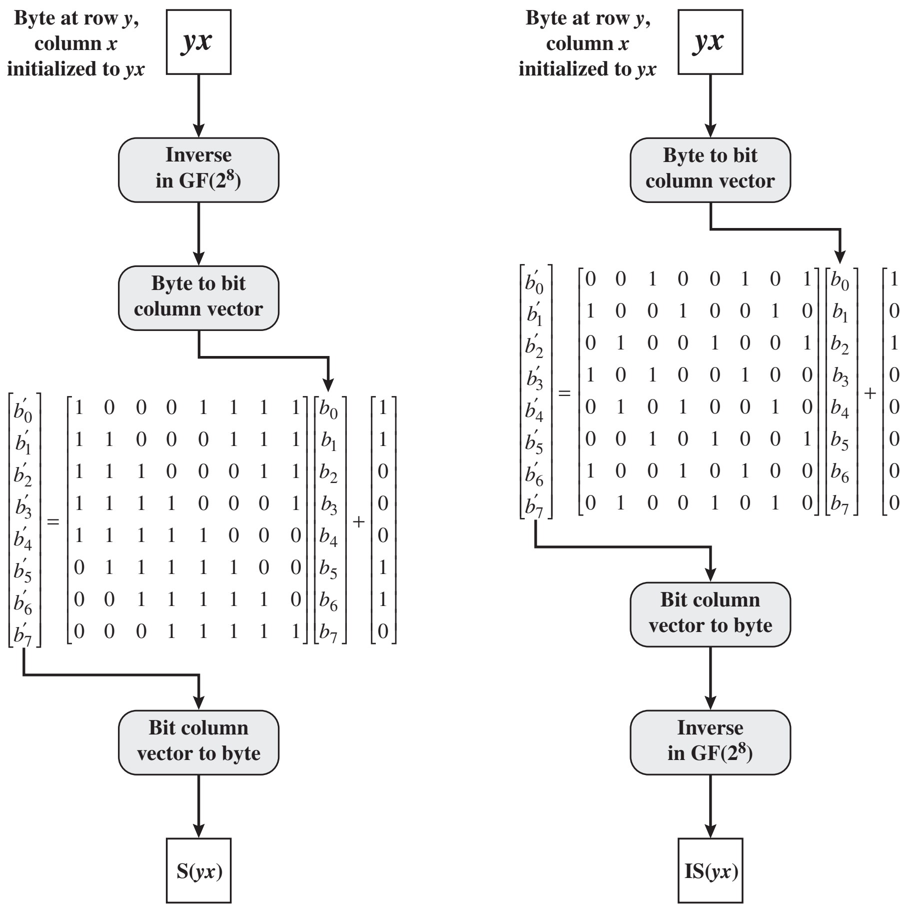
(b) Inverse S-box

of the S-box, which contains the value {2A}. Accordingly, the value {95} is mapped into the value {2A}.

Here is an example of the SubBytes transformation:

EA	04	65	85		87	F2	4D	97
83	45	5D	96		EC	6E	4C	90
5C	33	98	B0	→	4A	C3	46	E7
F0	2D	AD	C5		8C	D8	95	A6

The S-box is constructed in the following fashion (Figure 5.6a).



$$\begin{bmatrix} b_0' \\ b_1' \\ b_2' \\ b_3' \\ b_4' \\ b_5' \\ b_6' \\ b_7' \end{bmatrix} = \begin{bmatrix} 1&0&0&0&1&1&1&1 \\ 1&1&0&0&0&1&1&1 \\ 1&1&1&0&0&0&1&1 \\ 1&1&1&1&0&0&0&1 \\ 1&1&1&1&1&0&0&0 \\ 0&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&0 \\ 0&0&0&1&1&1&1&1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

(a) Calculation of byte at row *y*, column *x* of S-box

$$\begin{bmatrix} b_0' \\ b_1' \\ b_2' \\ b_3' \\ b_4' \\ b_5' \\ b_6' \\ b_7' \end{bmatrix} = \begin{bmatrix} 0&0&1&0&0&1&0&1 \\ 1&0&0&1&0&0&1&0 \\ 0&1&0&0&1&0&0&1 \\ 1&0&1&0&0&1&0&0 \\ 0&1&0&1&0&0&1&0 \\ 0&0&1&0&1&0&0&1 \\ 1&0&0&1&0&1&0&0 \\ 0&1&0&0&1&0&1&0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(a) Calculation of byte at row *y*, column *x* of IS-box

Figure 5.6 Constuction of S-Box and IS-Box

1. Initialize the S-box with the byte values in ascending sequence row by row. The first row contains {00}, {01}, {02}, . . . , {0F}; the second row contains {10}, {11}, etc.; and so on. Thus, the value of the byte at row y, column x is $\{yx\}$.
2. Map each byte in the S-box to its multiplicative inverse in the finite field GF(2^8); the value {00} is mapped to itself.
3. Consider that each byte in the S-box consists of 8 bits labeled (b_7, b_6, b_5, b_4, b_3, b_2, b_1, b_0). Apply the following transformation to each bit of each byte in the S-box:

$$b'_i = b_i \oplus b_{(i+4) \bmod 8} \oplus b_{(i+5) \bmod 8} \oplus b_{(i+6) \bmod 8} \oplus b_{(i+7) \bmod 8} \oplus c_i \tag{5.1}$$

where c_i is the ith bit of byte c with the value {63}; that is, $(c_7c_6c_5c_4c_3c_2c_1c_0) = (01100011)$. The prime (′) indicates that the variable is to be updated by the value on the right. The AES standard depicts this transformation in matrix form as follows.

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \tag{5.2}$$

Equation (5.2) has to be interpreted carefully. In ordinary matrix multiplication,[4] each element in the product matrix is the sum of products of the elements of one row and one column. In this case, each element in the product matrix is the bitwise XOR of products of elements of one row and one column. Furthermore, the final addition shown in Equation (5.2) is a bitwise XOR. Recall from Section 4.7 that the bitwise XOR is addition in GF(2^8).

As an example, consider the input value {95}. The multiplicative inverse in GF(2^8) is $\{95\}^{-1} = \{8A\}$, which is 10001010 in binary. Using Equation (5.2),

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

[4] For a brief review of the rules of matrix and vector multiplication, refer to Appendix E.

The result is {2A}, which should appear in row {09} column {05} of the S-box. This is verified by checking Table 5.2a.

The **inverse substitute byte transformation**, called InvSubBytes, makes use of the inverse S-box shown in Table 5.2b. Note, for example, that the input {2A} produces the output {95}, and the input {95} to the S-box produces {2A}. The inverse S-box is constructed (Figure 5.6b) by applying the inverse of the transformation in Equation (5.1) followed by taking the multiplicative inverse in GF(2^8). The inverse transformation is

$$b'_i = b_{(i+2) \bmod 8} \oplus b_{(i+5) \bmod 8} \oplus b_{(i+7) \bmod 8} \oplus d_i$$

where byte d = {05}, or 00000101. We can depict this transformation as follows.

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

To see that InvSubBytes is the inverse of SubBytes, label the matrices in SubBytes and InvSubBytes as **X** and **B**, respectively, and the vector versions of constants c and d as **C** and **D**, respectively. For some 8-bit vector **B**, Equation (5.2) becomes $\mathbf{B}' = \mathbf{XB} \oplus \mathbf{C}$. We need to show that $\mathbf{Y}(\mathbf{XB} \oplus \mathbf{C}) \oplus \mathbf{D} = \mathbf{B}$. To multiply out, we must show $\mathbf{YXB} \oplus \mathbf{YC} \oplus \mathbf{D} = \mathbf{B}$. This becomes

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} \oplus$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix}$$

We have demonstrated that **YX** equals the identity matrix, and the **YC** = **D**, so that **YC** $\oplus$ **D** equals the null vector.

Rationale The S-box is designed to be resistant to known cryptanalytic attacks. Specifically, the Rijndael developers sought a design that has a low correlation between input bits and output bits and the property that the output is not a linear mathematical function of the input [DAEM01]. The nonlinearity is due to the use of the multiplicative inverse. In addition, the constant in Equation (5.1) was chosen so that the S-box has no fixed points [S-box(a) = a] and no "opposite fixed points" [S-box(a) = $\overline{a}$], where $\overline{a}$ is the bitwise complement of a.

Of course, the S-box must be invertible, that is, IS-box[S-box(a)] = a. However, the S-box does not self-inverse in the sense that it is not true that S-box(a) =IS-box(a). For example, S-box({95}) = {2A}, but IS-box({95}) = {AD}.

ShiftRows Transformation

Forward and Inverse Transformations The **forward shift row transformation**, called ShiftRows, is depicted in Figure 5.7a. The first row of **State** is not altered. For the second row, a 1-byte circular left shift is performed. For the third row, a 2-byte circular left shift is performed. For the fourth row, a 3-byte circular left shift is performed. The following is an example of ShiftRows.

87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

→

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

The **inverse shift row transformation**, called InvShiftRows, performs the circular shifts in the opposite direction for each of the last three rows, with a 1-byte circular right shift for the second row, and so on.

Rationale The shift row transformation is more substantial than it may first appear. This is because the **State**, as well as the cipher input and output, is treated as an array of four 4-byte columns. Thus, on encryption, the first 4 bytes of the plaintext are copied to the first column of **State**, and so on. Furthermore, as will be seen, the round key is applied to **State** column by column. Thus, a row shift moves an individual byte from one column to another, which is a linear

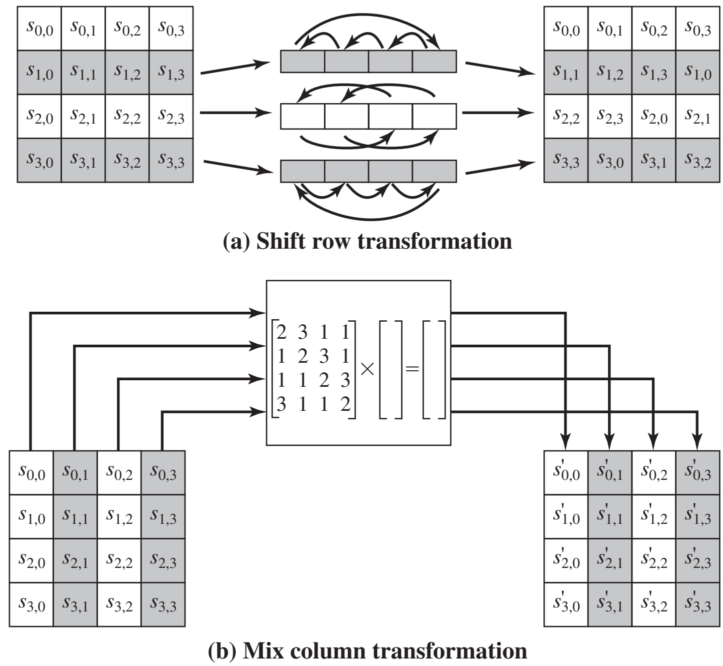


Figure 5.7 AES Row and Column Operations

distance of a multiple of 4 bytes. Also note that the transformation ensures that the 4 bytes of one column are spread out to four different columns. Figure 5.4 illustrates the effect.

MixColumns Transformation

Forward and Inverse Transformations The **forward mix column transformation**, called MixColumns, operates on each column individually. Each byte of a column is mapped into a new value that is a function of all four bytes in that column. The transformation can be defined by the following matrix multiplication on **State** (Figure 5.7b):

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix} \quad \textbf{(5.3)}$$

Each element in the product matrix is the sum of products of elements of one row and one column. In this case, the individual additions and multiplications[5] are performed

[5]We follow the convention of FIPS PUB 197 and use the symbol $\bullet$ to indicate multiplication over the finite field $GF(2^8)$ and $\oplus$ to indicate bitwise XOR, which corresponds to addition in $GF(2^8)$.

in GF(2^8). The MixColumns transformation on a single column of **State** can be expressed as

$$\begin{aligned} s'_{0,j} &= (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j} \\ s'_{1,j} &= s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j} \\ s'_{2,j} &= s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j}) \\ s'_{3,j} &= (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j}) \end{aligned} \qquad \textbf{(5.4)}$$

The following is an example of MixColumns:

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

→

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

Let us verify the first column of this example. Recall from Section 4.7 that, in GF(2^8), addition is the bitwise XOR operation and that multiplication can be performed according to the rule established in Equation (4.14). In particular, multiplication of a value by x (i.e., by {02}) can be implemented as a 1-bit left shift followed by a conditional bitwise XOR with (0001 1011) if the leftmost bit of the original value (prior to the shift) is 1. Thus, to verify the MixColumns transformation on the first column, we need to show that

$$\begin{array}{llllll} (\{02\} \cdot \{87\}) & \oplus\ (\{03\} \cdot \{6E\}) & \oplus\ \{46\} & \oplus\ \{A6\} & = & \{47\} \\ \{87\} & \oplus\ (\{02\} \cdot \{6E\}) & \oplus\ (\{03\} \cdot \{46\}) & \oplus\ \{A6\} & = & \{37\} \\ \{87\} & \oplus\ \{6E\} & \oplus\ (\{02\} \cdot \{46\}) & \oplus\ (\{03\} \cdot \{A6\}) & = & \{94\} \\ (\{03\} \cdot \{87\}) & \oplus\ \{6E\} & \oplus\ \{46\} & \oplus\ (\{02\} \cdot \{A6\}) & = & \{ED\} \end{array}$$

For the first equation, we have $\{02\} \cdot \{87\} = (0000\ 1110) \oplus (0001\ 1011) = (0001\ 0101)$ and $\{03\} \cdot \{6E\} = \{6E\} \oplus (\{02\} \cdot \{6E\}) = (0110\ 1110) \oplus (1101\ 1100) = (1011\ 0010)$. Then,

$$\begin{array}{lcl} \{02\} \cdot \{87\} & = & 0001\ \ 0101 \\ \{03\} \cdot \{6E\} & = & 1011\ \ 0010 \\ \{46\} & = & 0100\ \ 0110 \\ \{A6\} & = & \underline{1010\ \ 0110} \\ & & 0100\ \ 0111 \ = \{47\} \end{array}$$

The other equations can be similarly verified.

The **inverse mix column transformation**, called InvMixColumns, is defined by the following matrix multiplication:

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix} \qquad \textbf{(5.5)}$$

It is not immediately clear that Equation (5.5) is the **inverse** of Equation (5.3). We need to show

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix}$$

which is equivalent to showing

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.6}$$

That is, the inverse transformation matrix times the forward transformation matrix equals the identity matrix. To verify the first column of Equation (5.6), we need to show

$$\begin{aligned} (\{0E\} \cdot \{02\}) \oplus \{0B\} \oplus \{0D\} \oplus (\{09\} \cdot \{03\}) &= \{01\} \\ (\{09\} \cdot \{02\}) \oplus \{0E\} \oplus \{0B\} \oplus (\{0D\} \cdot \{03\}) &= \{00\} \\ (\{0D\} \cdot \{02\}) \oplus \{09\} \oplus \{0E\} \oplus (\{0B\} \cdot \{03\}) &= \{00\} \\ (\{0B\} \cdot \{02\}) \oplus \{0D\} \oplus \{09\} \oplus (\{0E\} \cdot \{03\}) &= \{00\} \end{aligned}$$

For the first equation, we have $\{0E\} \cdot \{02\} = 00011100$ and $\{09\} \cdot \{03\} = \{09\} \oplus (\{09\} \cdot \{02\}) = 00001001 \oplus 00010010 = 00011011$. Then

$$\begin{aligned} \{0E\} \cdot \{02\} &= 00011100 \\ \{0B\} &= 00001011 \\ \{0D\} &= 00001101 \\ \{09\} \cdot \{03\} &= \underline{00011011} \\ & 00000001 \end{aligned}$$

The other equations can be similarly verified.

The AES document describes another way of characterizing the MixColumns transformation, which is in terms of polynomial arithmetic. In the standard, MixColumns is defined by considering each column of **State** to be a four-term polynomial with coefficients in $GF(2^8)$. Each column is multiplied modulo $(x^4 + 1)$ by the fixed polynomial $a(x)$, given by

$$a(x) = \{03\}x^3 + \{01\}x^2 + \{01\}x + \{02\} \tag{5.7}$$

Appendix 5A demonstrates that multiplication of each column of **State** by $a(x)$ can be written as the matrix multiplication of Equation (5.3). Similarly, it can be seen that the transformation in Equation (5.5) corresponds to treating each column as a four-term polynomial and multiplying each column by $b(x)$, given by

$$b(x) = \{0B\}x^3 + \{0D\}x^2 + \{09\}x + \{0E\} \tag{5.8}$$

It readily can be shown that $b(x) = a^{-1}(x) \bmod (x^4 + 1)$.

Rationale The coefficients of the matrix in Equation (5.3) are based on a linear code with maximal distance between code words, which ensures a good mixing among the bytes of each column. The mix column transformation combined with the shift row transformation ensures that after a few rounds all output bits depend on all input bits. See [DAEM99] for a discussion.

In addition, the choice of coefficients in MixColumns, which are all {01}, { 02}, or { 03}, was influenced by implementation considerations. As was discussed, multiplication by these coefficients involves at most a shift and an XOR. The coefficients in InvMixColumns are more formidable to implement. However, encryption was deemed more important than decryption for two reasons:

1. For the CFB and OFB cipher modes (Figures 6.5 and 6.6; described in Chapter 6), only encryption is used.
2. As with any block cipher, AES can be used to construct a message authentication code (Chapter 12), and for this, only encryption is used.

AddRoundKey Transformation

Forward and Inverse Transformations In the **forward add round key transformation**, called AddRoundKey, the 128 bits of **State** are bitwise XORed with the 128 bits of the round key. As shown in Figure 5.5b, the operation is viewed as a columnwise operation between the 4 bytes of a **State** column and one word of the round key; it can also be viewed as a byte-level operation. The following is an example of AddRoundKey:

47	40	A3	4C		AC	19	28	57		EB	59	8B	1B
37	D4	70	9F		77	FA	D1	5C		40	2E	A1	C3
94	E4	3A	42	⊕	66	DC	29	00	=	F2	38	13	42
ED	A5	A6	BC		F3	21	41	6A		1E	84	E7	D6

The first matrix is **State**, and the second matrix is the round key.

The **inverse add round key transformation** is identical to the forward add round key transformation, because the XOR operation is its own inverse.

Rationale The add round key transformation is as simple as possible and affects every bit of **State**. The complexity of the round key expansion, plus the complexity of the other stages of AES, ensure security.

Figure 5.8 is another view of a single round of AES, emphasizing the mechanisms and inputs of each transformation.

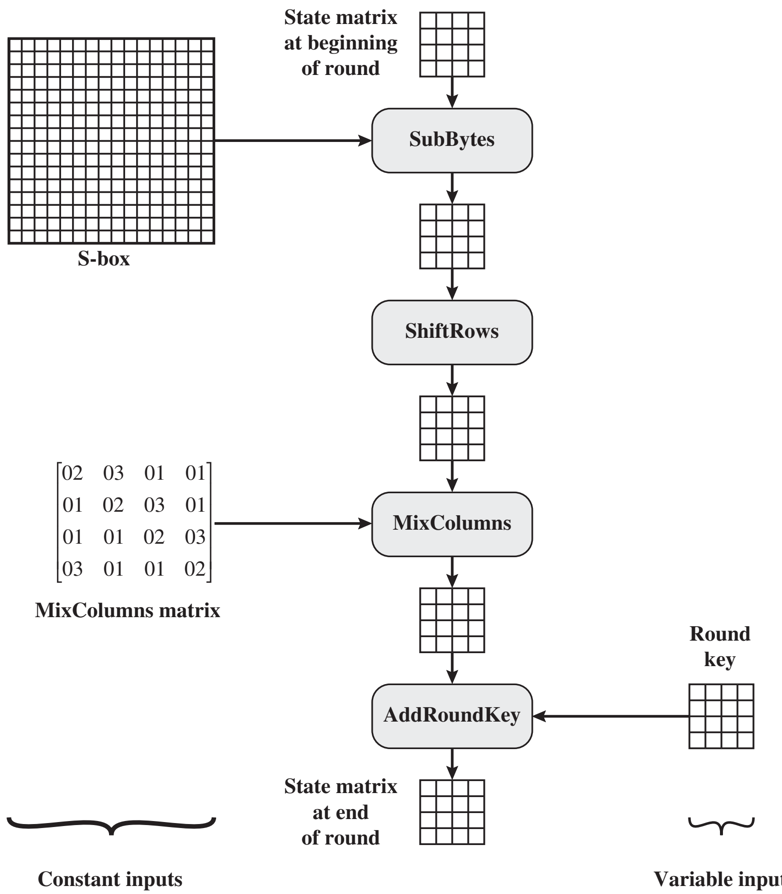


Figure 5.8 Inputs for Single AES Round

5.4 AES KEY EXPANSION

Key Expansion Algorithm

The AES key expansion algorithm takes as input a four-word (16-byte) key and produces a linear array of 44 words (176 bytes). This is sufficient to provide a four-word round key for the initial AddRoundKey stage and each of the 10 rounds of the cipher. The pseudocode on the next page describes the expansion.

The key is copied into the first four words of the expanded key. The remainder of the expanded key is filled in four words at a time. Each added word **w**[i] depends on the immediately preceding word, **w**[i − 1], and the word four positions back, w[i − 4]. In three out of four cases, a simple XOR is used. For a word whose position in the **w** array is a multiple of 4, a more complex function is used. Figure 5.9 illustrates the generation of the expanded key, using the symbol g to represent that complex function. The function g consists of the following subfunctions.

```
KeyExpansion (byte key[16], word w[44])
{
    word temp
    for (i = 0; i < 4; i++)   w[i] = (key[4*i], key[4*i+1],
                                      key[4*i+2],
                                      key[4*i+3]);
    for (i = 4; i < 44; i++)
    {
     temp = w[i − 1];
     if (i mod 4 = 0)   temp = SubWord (RotWord (temp))
                                 ⊕ Rcon[i/4];
     w[i] = w[i−4] ⊕ temp
    }
}
```

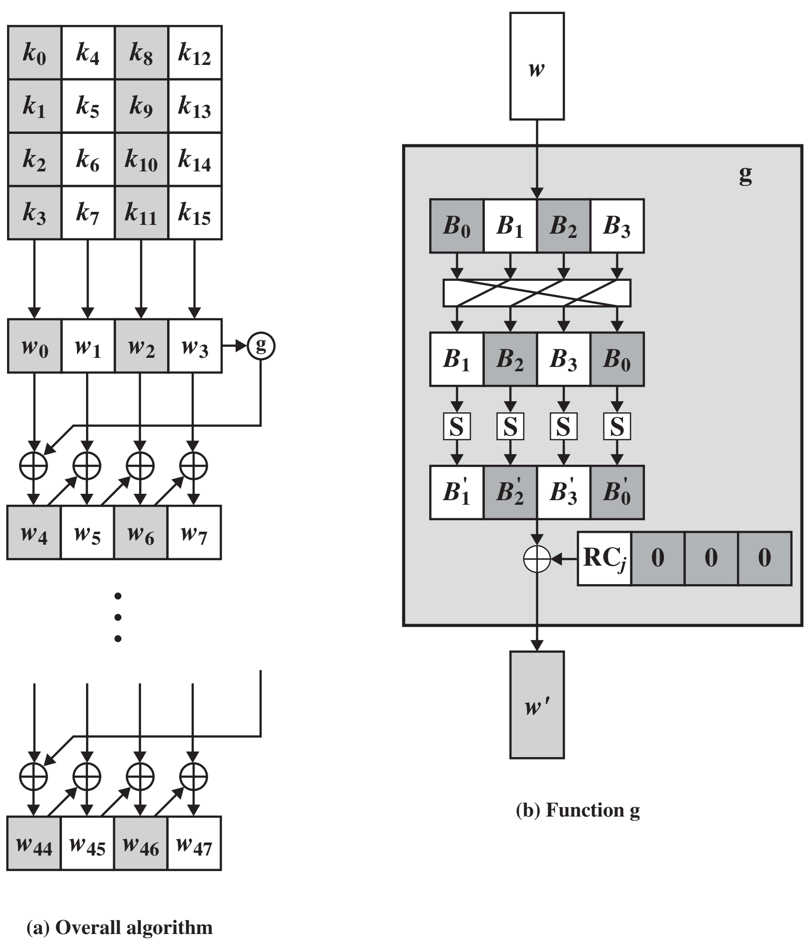


Figure 5.9 AES Key Expansion

1. RotWord performs a one-byte circular left shift on a word. This means that an input word $[B_0, B_1, B_2, B_3]$ is transformed into $[B_1, B_2, B_3, B_0]$.
2. SubWord performs a byte substitution on each byte of its input word, using the S-box (Table 5.2a).
3. The result of steps 1 and 2 is XORed with a round constant, Rcon[j].

The round constant is a word in which the three rightmost bytes are always 0. Thus, the effect of an XOR of a word with Rcon is to only perform an XOR on the left-most byte of the word. The round constant is different for each round and is defined as $\text{Rcon}[j] = (\text{RC}[j], 0, 0, 0)$, with $\text{RC}[1] = 1$, $\text{RC}[j] = 2 \cdot \text{RC}[j-1]$ and with multiplication defined over the field $GF(2^8)$. The values of RC[j] in hexadecimal are

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

For example, suppose that the round key for round 8 is

EA D2 73 21 B5 8D BA D2 31 2B F5 60 7F 8D 29 2F

Then the first 4 bytes (first column) of the round key for round 9 are calculated as follows:

i (decimal)	temp	After RotWord	After SubWord	Rcon (9)	After XOR with Rcon	w[i−4]	w[i] = temp ⊕ w[i−4]
36	7F8D292F	8D292F7F	5DA515D2	1B000000	46A515D2	EAD27321	AC7766F3

Rationale

The Rijndael developers designed the expansion key algorithm to be resistant to known cryptanalytic attacks. The inclusion of a round-dependent round constant eliminates the symmetry, or similarity, between the ways in which round keys are generated in different rounds. The specific criteria that were used are [DAEM99]

- Knowledge of a part of the cipher key or round key does not enable calculation of many other round-key bits.
- An invertible transformation [i.e., knowledge of any *Nk* consecutive words of the expanded key enables regeneration the entire expanded key (*Nk* = key size in words)].
- Speed on a wide range of processors.
- Usage of round constants to eliminate symmetries.
- Diffusion of cipher key differences into the round keys; that is, each key bit affects many round key bits.
- Enough nonlinearity to prohibit the full determination of round key differences from cipher key differences only.
- Simplicity of description.

The authors do not quantify the first point on the preceding list, but the idea is that if you know less than *Nk* consecutive words of either the cipher key or one of the round keys, then it is difficult to reconstruct the remaining unknown bits. The fewer bits one knows, the more difficult it is to do the reconstruction or to determine other bits in the key expansion.

5.5 AN AES EXAMPLE

We now work through an example and consider some of its implications. Although you are not expected to duplicate the example by hand, you will find it informative to study the hex patterns that occur from one step to the next.

For this example, the plaintext is a hexadecimal palindrome. The plaintext, key, and resulting ciphertext are

Plaintext:	`0123456789abcdeffedcba9876543210`
Key:	`0f1571c947d9e8590cb7add6af7f6798`
Ciphertext:	`ff0b844a0853bf7c6934ab4364148fb9`

Results

Table 5.3 shows the expansion of the 16-byte key into 10 round keys. As previously explained, this process is performed word by word, with each four-byte word occupying one column of the word round-key matrix. The left-hand column shows the four round-key words generated for each round. The right-hand column shows the steps

Table 5.3 Key Expansion for AES Example

Key Words	Auxiliary Function
w0 = 0f 15 71 c9 w1 = 47 d9 e8 59 w2 = 0c b7 ad w3 = af 7f 67 98	RotWord(w3) = 7f 67 98 af = x1 SubWord(x1) = d2 85 46 79 = y1 Rcon(1) = 01 00 00 00 y1 ⊕ Rcon(1) = d3 85 46 79 = z1
w4 = w0 ⊕ z1 = dc 90 37 b0 w5 = w4 ⊕ w1 = 9b 49 df e9 w6 = w5 ⊕ w2 = 97 fe 72 3f w7 = w6 ⊕ w3 = 38 81 15 a7	RotWord(w7) = 81 15 a7 38 = x2 SubWord(x4) = 0c 59 5c 07 = y2 Rcon(2) = 02 00 00 00 y2 ⊕ Rcon(2) = 0e 59 5c 07 = z2
w8 = w4 ⊕ z2 = d2 c9 6b b7 w9 = w8 ⊕ w5 = 49 80 b4 5e w10 = w9 ⊕ w6 = de 7e c6 61 w11 = w10 ⊕ w7 = e6 ff d3 c6	RotWord(w11) = ff d3 c6 e6 = x3 SubWord(x2) = 16 66 b4 83 = y3 Rcon(3) = 04 00 00 00 y3 ⊕ Rcon(3) = 12 66 b4 8e = z3
w12 = w8 ⊕ z3 = c0 af df 39 w13 = w12 ⊕ w9 = 89 2f 6b 67 w14 = w13 ⊕ w10 = 57 51 ad 06 w15 = w14 ⊕ w11 = b1 ae 7e c0	RotWord(w15) = ae 7e c0 b1 = x4 SubWord(x3) = e4 f3 ba c8 = y4 Rcon(4) = 08 00 00 00 y4 ⊕ Rcon(4) = ec f3 ba c8 = 4

(*Continued*)