Filters

- What is a filter
- Passive filters
- Some common filters

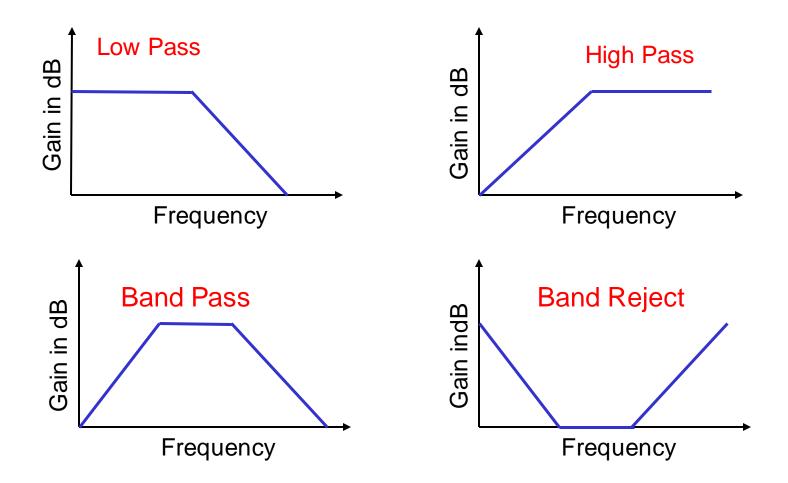
What are filters?

 Filters are electronic circuits which perform signal processing functions, specifically intended to remove unwanted signal components and/or enhance wanted ones.

Common types of filters:

- Low-pass: deliver low frequencies and eliminate high frequencies
- High-pass: send on high frequencies and reject low frequencies
- Band-pass: pass some particular range of frequencies, discard other frequencies outside that band
- Band-rejection: stop a range of frequencies and pass all other frequencies (e.g., a special case is a notch filter)

Bode Plots of Common Filters



Passive vs. Active filters

- Passive filters: RLC components only, but gain < 1
- Active filters: op-amps with RC elements, and gain > 1

Gain (in Decibel) =
$$20 \log_{10}(Vout/Vin)$$

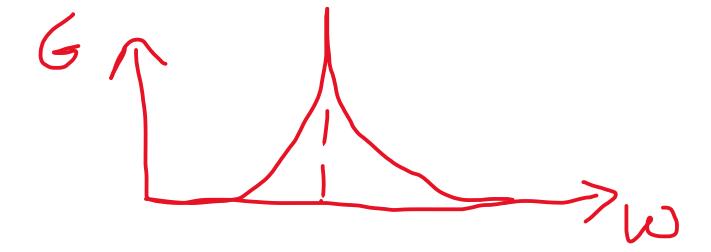
*
$$-3 dB = 20\log_{10}(0.707)$$

Passive Filters

• Passive filters use R, L, C elements to achieve the desired filter

Some Technical Terms:

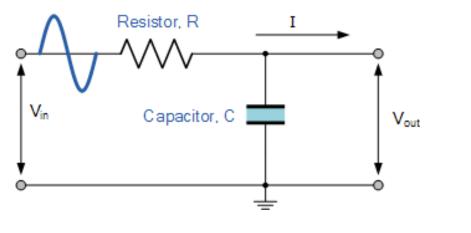
- The half-power frequency is the same as the break frequency (or corner frequency) and is located at the frequency where the magnitude is 1/√2 of its maximum value
- The resonance frequency, ω_0 , is also referred to as the center frequency



LOW-PASS FILTER

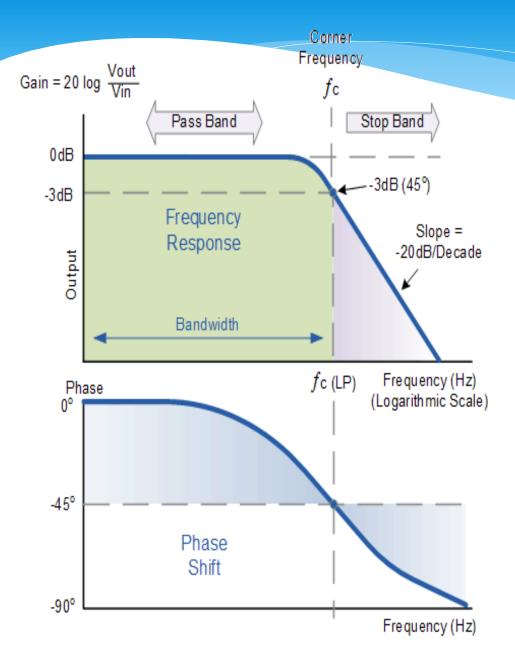
A low-pass filter allows for easy passage of low-frequency signals from source to load, and difficult passage of high-frequency signals.

* The cutoff frequency for a low-pass filter is that frequency at which the output (load) voltage equals 70.7% of the input (source) voltage. Above the cutoff frequency, the output voltage is lower than 70.7% of the input, and vice versa.



$$V_0 = V_C$$

$$V_0 = \frac{X_c}{e+X_c}$$
. Vin



Cutoff freq. is also known as 3-dB frequency. Because gain at cutoff freq. is -3dB.

Cut-off Frequency and Phase Shift

$$fc = \frac{1}{2\pi RC}$$

Phase Shift $\varphi = -\arctan(2\pi fRC)$

$$\phi = -\tan^{-1}(\omega RC)$$

$$\omega = 0 \implies -\tan^{-1}(0) = 0^{\circ}$$

$$\omega = \omega_{c} \implies -\tan^{-1}(1) = -45^{\circ}$$

$$\omega = \infty \implies -\tan^{-1}(\infty) = -90^{\circ}$$

$$-3 dB = 20\log_{10}(0.707)$$

NOTE: we used the modulus of quantities here as we only need the ratio of values to be $(1/\sqrt{2})$, i.e., 0.707. We don't need phases.

$$|\frac{V_0 ut}{vin}|^{2} \frac{|x_c|}{\sqrt{R^2 + x_c^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{\frac{1}{\omega c}}{\sqrt{R^2 + (\frac{1}{\omega c})^2}} \qquad \Rightarrow \omega = \frac{1}{R^c}$$

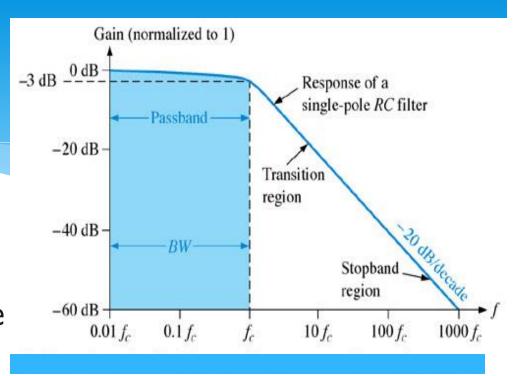
$$\frac{1}{2} = \frac{(\frac{1}{\omega c})^2}{R^2 + (\frac{1}{\omega c})^2} \Rightarrow \omega = \frac{1}{R^c}$$

$$\frac{V_0}{V_{in}} = \frac{X_c}{X_c + R} = \frac{1 + \rho X_c}{1 + \rho X_c} = \frac{1 +$$

consider the phase to find "phi".

Passband of a filter is the range of frequencies that are allowed to pass through the filter with minimum attenuation (usually defined as less than -3 dB of attenuation).

Transition region shows the area where the fall-off occurs.



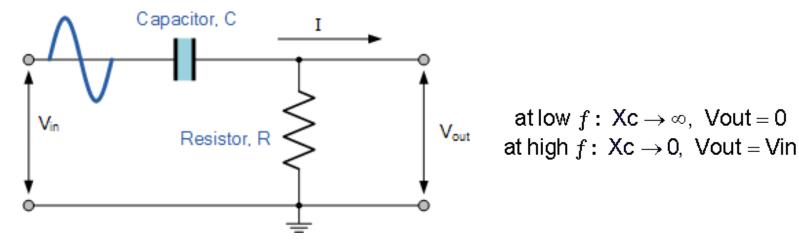
Stopband is the range of frequencies that have the most attenuation.

Critical frequency, f_c , (also called the cutoff frequency) defines the end of the passband and normally specified at the point where the response drops – 3 dB (70.7%) from the passband response.

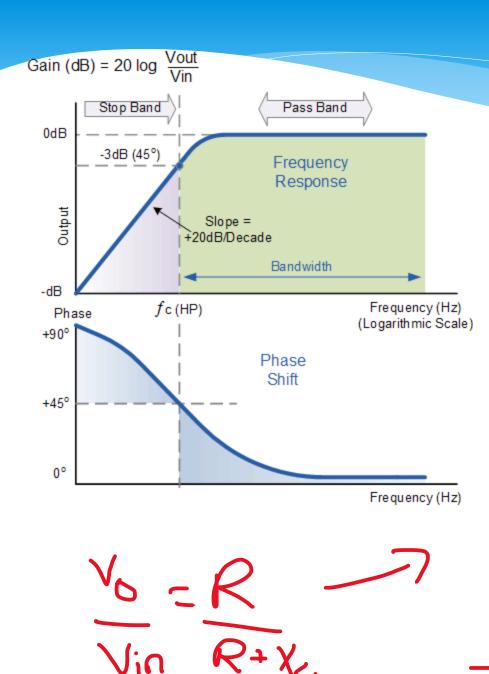
HIGH-PASS FILTER

A high-pass filter allows for easy passage of high-frequency signals from source to load, and difficult passage of low-frequency signals.

The cutoff frequency for a high-pass filter is that frequency at which the output (load) voltage equals 70.7% of the input (source) voltage. Above the cutoff frequency, the output voltage is greater than 70.7% of the input, and vice versa.



First order high pass filter



$$fc = \frac{1}{2\pi RC}$$

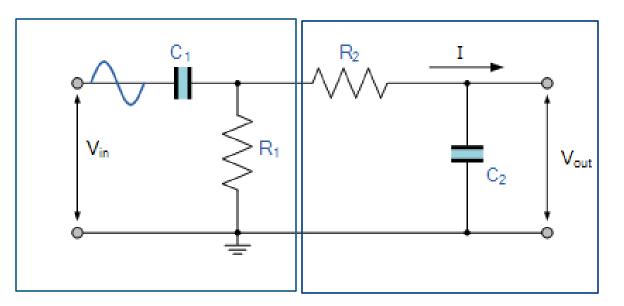
Phase Shift $\phi = \arctan \frac{1}{2\pi fRC}$

$$\frac{\sqrt{s}}{\sqrt{1+1}} = \frac{1}{\sqrt{1+1}}$$

$$\frac{\sqrt{s}}{\sqrt{1+1}} = \frac{1}{\sqrt{1+1}}$$

BAND-PASS FILTER

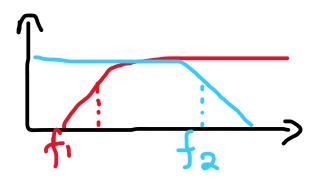
- By connecting or "cascading" together a single Low Pass Filter circuit with a High Pass Filter circuit, we can produce another type of passive RC filter that passes a selected range or "band" of frequencies that can be either narrow or wide while attenuating all those outside of this range.
- known commonly as a Band Pass Filter



To set the first band pass frequency f1. Freq of high pass part

To set the second band pass frequency f2. Freq of low pass part

Coutoff Frequency of high pass filter must be less than that of low pass filter for the cascading to be a band pass filter



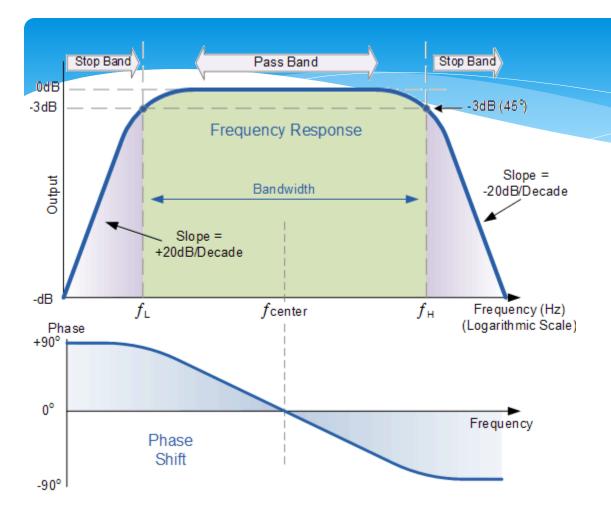
Some definition

* The bandwidth (BW) is defined as the difference between the upper critical frequency and the lower critical frequency.

Bandwidth = Fh - Fl

* The frequency about which the pass band is centered is called the center frequency, fo and defined as the geometric mean of the critical frequencies.

Centre frequency = V(Fl * Fh)



Fl and Fh are lower cutoff and higher cutoff frequencies of the filter respectively.

And f1 and f2 (they are the freq of high pass and low pass filter respectively)

So we have:

Lower cutoff = Fl = freq. of high pass filter = f1And Fh = freq. Of low pass filter = f2

Bandwidth = Fh − Fl Centre frequency = V(Fl * Fh)

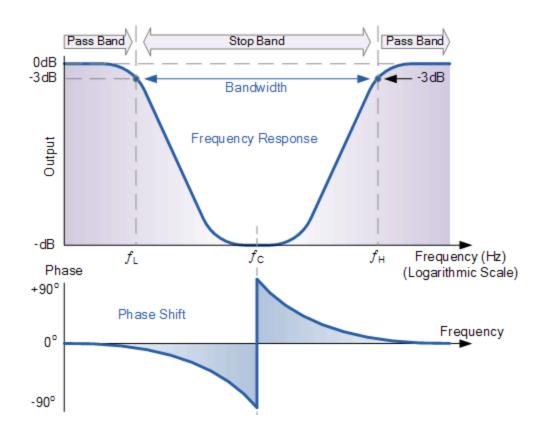
Find separately Fh and Fl by using the respective formulae of cutoff freq.

$$fl = fi = \frac{1}{2\pi R_1 C_1}$$

$$fl = fi = \frac{1}{2\pi R_2 C_2}$$

BAND-STOP FILTER

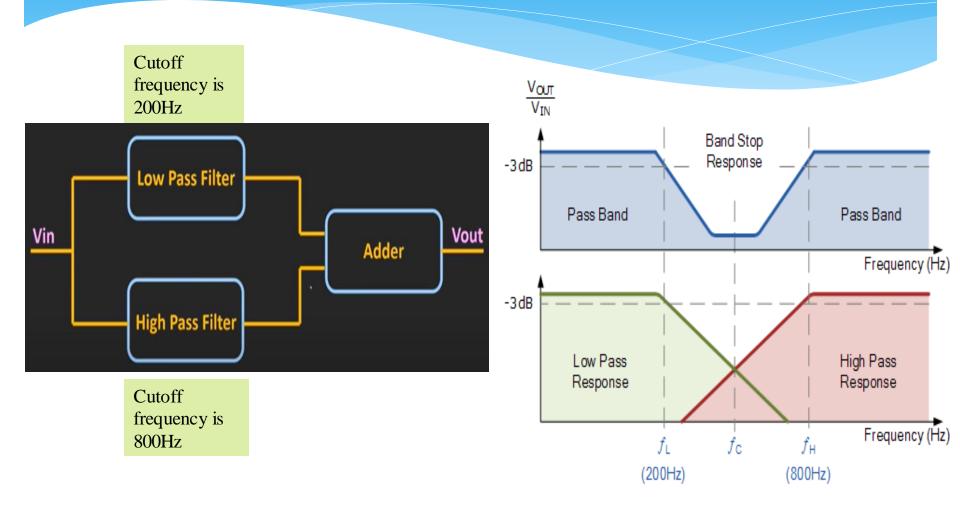
- combine the low and high pass filter to produce another kind of RC filter network
- that can block or at least severely attenuate a band of frequencies within these two cut-off frequency points.



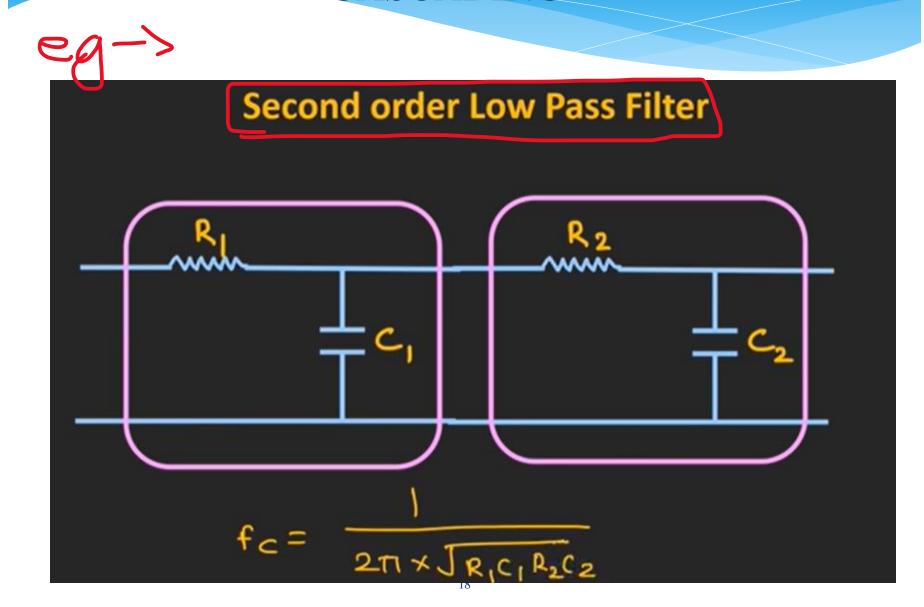
Opposite to Band-pass filter:

Coutoff Frequency of high pass filter must be greater than that of low pass filter for the cascadng to be a band-stop filter

- Band-pass filters are constructed by combining a low pass filter in series with a high pass filter
 - Band stop filters are created by combining together the low pass and high pass filter sections in a "parallel" type configuration as shown.

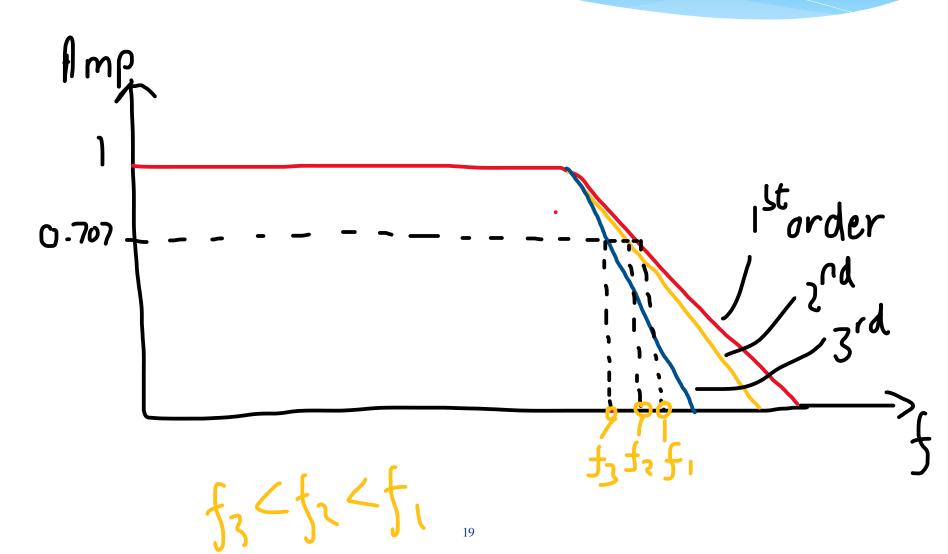


CASCADING



* Roll-off rate increases * Cutoff frequency decreases.

Example- consider General low pass response



Active Filter

Introduction

- Filters are circuits that are capable of *passing signals within a* band of frequencies while *rejecting or blocking* signals of frequencies *outside this band*. This property of filters is also called "frequency selectivity".
- > Filter can be passive or active filter.

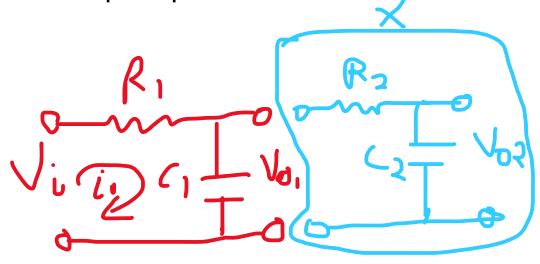
Passive filters: The circuits built using RC, RL, or RLC circuits.

Active filters: The circuits that employ one or more op-amps in the design an addition to

resistors and capacitors

Advantages of Active Filters over Passive Filters

- Active filters can be designed to provide required gain, and hence no attenuation as in the case of passive filters
- No loading problem, because of high input resistance and low output resistance of op-amp it isolates the load from input.
- Active Filters are cost effective as a wide variety of economical op-amps are available.



- -> LOADING PROBLEM: While calculating Vo1 we assume open circuit at output, for this assumption to be right any external load applied must be >> R1 (like here R2 >= 10*R1 must hold). If this is not true filter response is not as expected (cutoff freq changes).
- -> Occurs while cascading or when an external load comparable to R1 is connected at o/p.

Applications

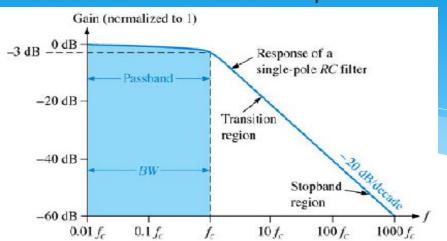
- Active filters are mainly used in communication and signal processing circuits.
- They are also employed in a wide range of applications such as entertainment, medical electronics, etc.

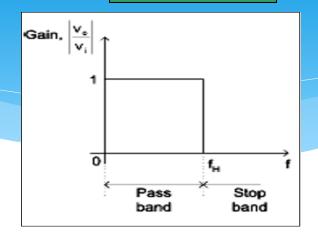
Active Filters

- ➤ There are 4 basic categories of active filters:
 - 1. Low-pass filters
 - 2. High-pass filters
 - 3. Band-pass filters
 - 4. Band-reject filters
- ➤ Each of these filters can be built by using op-amp as the active element combined with RC, RL or RLC circuit as the passive elements.

> The bandwidth of an ideal low-pass filter is equal to fc:

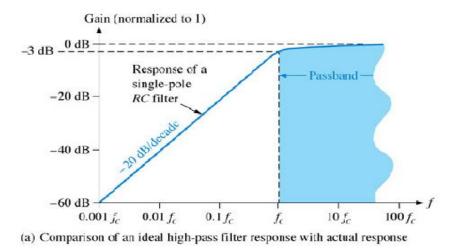


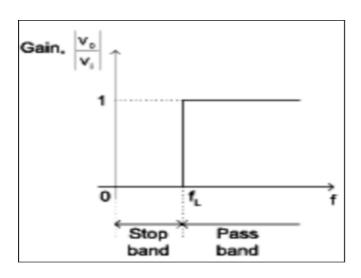




(a) Comparison of an ideal low-pass filter response with actual response

The passband of a high-pass filter is all frequencies above the critical frequency.

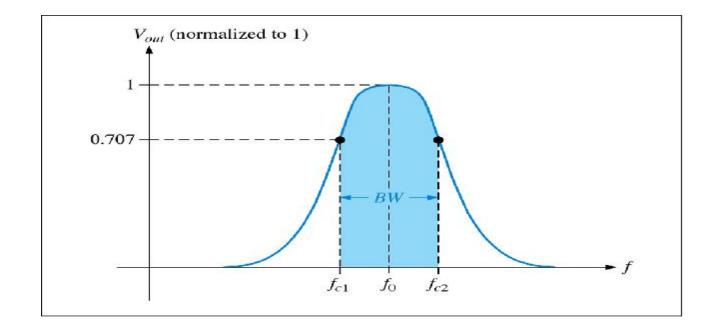




The bandwidth (BW) for a band-pass filter is defined as the difference between the upper critical frequency (f_{c2}) and the lower critical frequency (f_{c1}).

$$BW = f_{c2} - f_{c1}$$

The frequency about which the pass band is centered is called the *center frequency*, **f**_o and defined as the geometric mean of the critical frequencies.



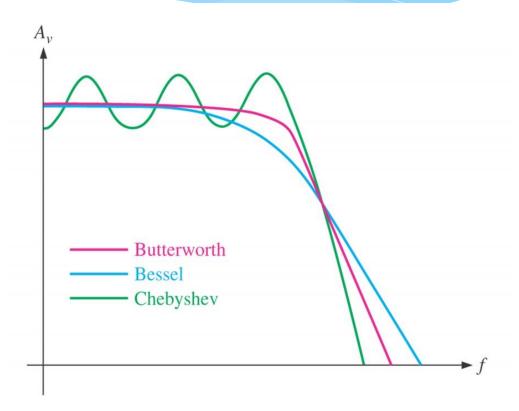
The *quality factor* (Q) of a bandpass filter is the ratio of the center frequency to the bandwidth.

$$Q = \frac{f_o}{BW}$$

- \succ The higher value of Q, the narrower the bandwidth and the better the selectivity for a given value of f_o .
- > (Q>10) as a narrow-band or (Q<10) as a wide-band
- ➤ The quality factor (Q) can also be expressed in terms of the damping factor (DF) of the filter as :

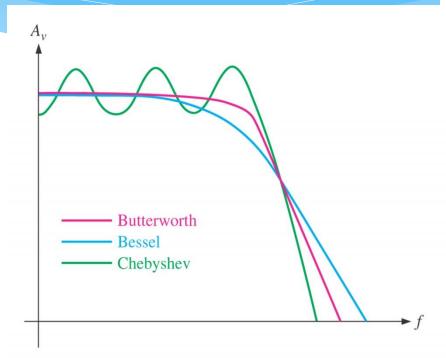
$$Q = \frac{1}{DF}$$

- There are **3** most common characteristics of filter response :
- i) **Butterworth** characteristic
- ii) **Chebyshev** characteristic
- iii) Bessel characteristic.
- ➤ Each of the characteristics is identified by the shape of the response curve.
- > Also known as
- "Filter aprroximations"
- ➤ Others are: Inverse
- Chebyshev and Elliptical

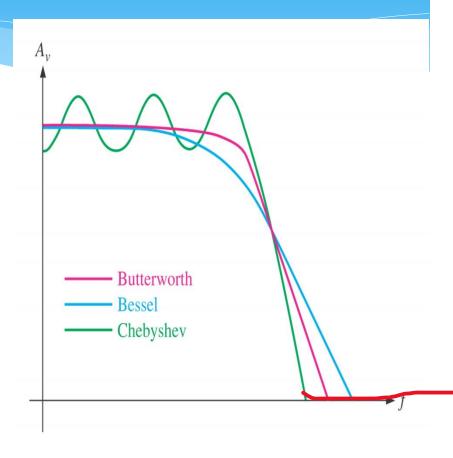


Buildinginging

- Filter response is characterized by flat amplitude response in the passband.
- ➤ Provides a roll-off rate of -20 dB/decade/pole.
- Filters with the Butterworth response are normally used when all frequencies in the passband must have the *same gain*.

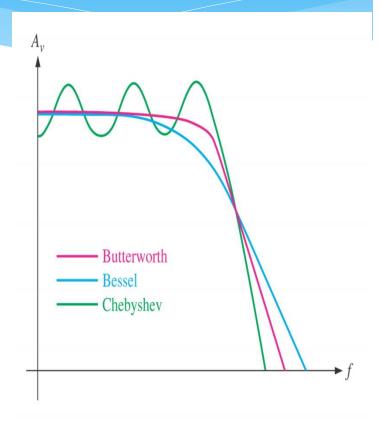


- Filter response is characterized by **overshoot** or **ripples** in the passband.
- Provides a roll-off rate greater than -20 dB/decade/pole.
- Filters with the Chebyshev response can be implemented with fewer poles and less complex circuitry for a given roll-off rate



Besiliation

- Filter response is characterized by a linear characteristic, meaning that the phase shift increases linearly with frequency.
- Filters with the Bessel response are used for filtering pulse waveforms without distorting the shape of waveform.



THE TRANSFER FUNCTION IS THE FILTER

$$V_{i} \longrightarrow V_{0}$$

$$Lot \quad M() = V_{0}() = X_{c}$$

$$V_{i}() = R + x_{c}$$

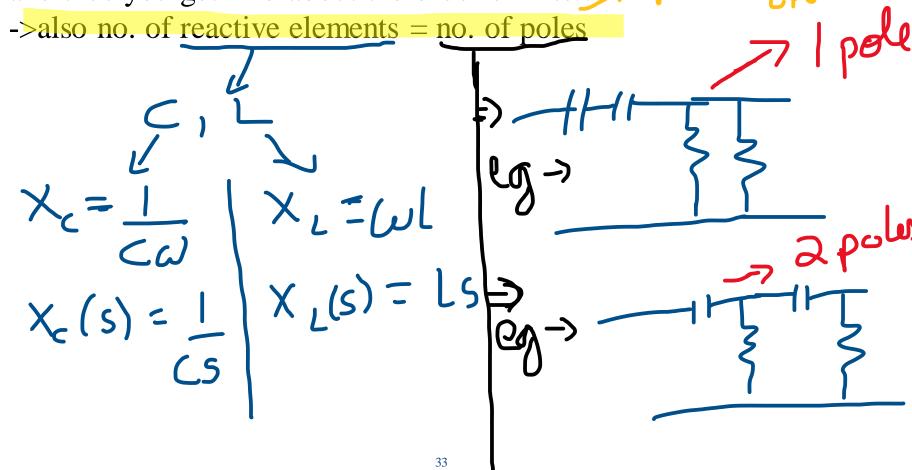
* System order is always equal to number of poles of the transfer function.

So the order of filter is equal to the no. of poles the filter has

$$H(s) = \frac{1}{5} = \frac{1}{5}$$

So only by knowing the transfer function you can get a good idea about the filter.

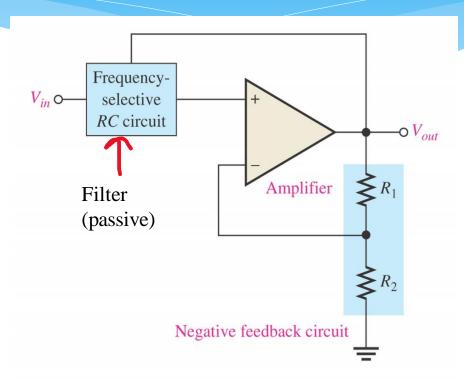
-> if you have the transfer function you can factorise the denominator and get the no. of poles and thus you get info about the order of filter.



DAMPING FACTOR

- > The damping factor (DF) of an active filter determines which response characteristic the filter exhibits.
- This active filter consists of an amplifier, a negative feedback circuit and RC circuit.
- ➤ The amplifier and feedback are connected in a non-inverting configuration.
- ➤DF is determined by the negative feedback and defined as :

$$DF = 2 - \frac{R_1}{R_2}$$



General diagram of active filter

SOME POINTS TO REMEMBER:

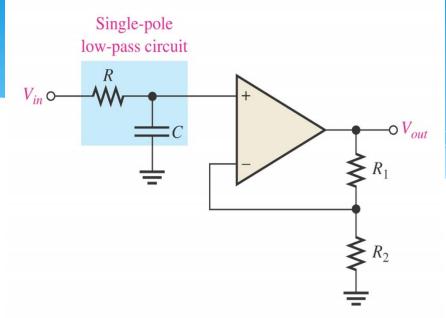
- > The value of DF required to produce a desired response characteristics depends on order (number of poles) of the filter.
- > A pole (single pole) is simply one resistor and one capacitor.
- > The more poles filter has, the faster its roll-off rate

Revision negative feedback:

Revision negative feedback:

$$V_0 = A V_1 - V_2 = V_0 R_2$$
 $V_0 = A V_1 - V_2 = V_0 R_2$
 $V_1 = V_2 = V_0 R_2$
 $V_1 = V_2 = V_0 R_2$
 $V_2 = V_1 = V_2 = V_0 R_2$
 $V_1 = V_2 = V_0 R_2$
 $V_2 = V_1 = V_2 = V_1 = V_2 = V$

CRITICAL FREQUENCY AND ROLL-OFF RATE



One-pole (first-order) low-pass filter.

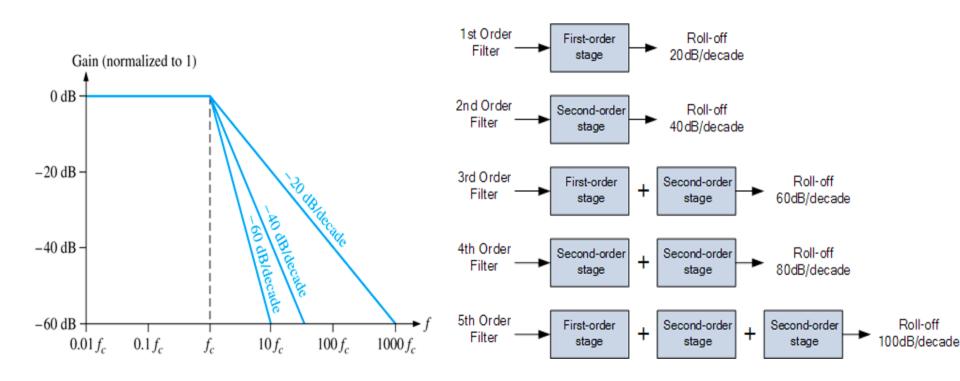
- ➤ The critical frequency, f_c is determined by the values of R and C in the frequency-selective RC circuit.
- > Each RC set of filter components represents a pole.
- Greater roll-off rates can be achieved with more poles.
- > Each pole represents a -20dB/decade increase in roll-off.

> For a single-pole (first-order) filter, the critical frequency is:

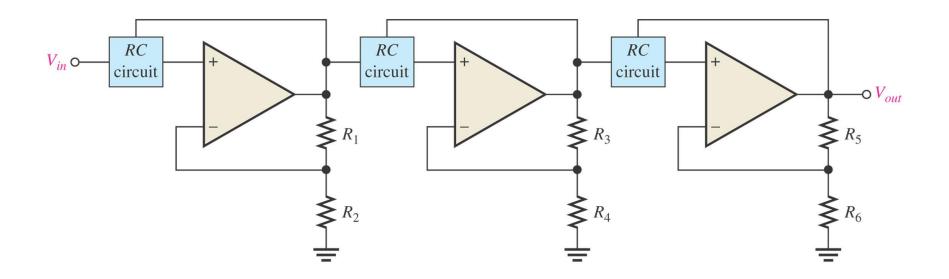
$$f_c = \frac{1}{2\pi RC}$$

> The above formula can be used for both low-pass and high-pass filters.

- ➤ The number of poles determines the roll-off rate of the filter. For example, a Butterworth response produces -20dB/decade/pole. This means that:
- One-pole (first-order) filter has a roll-off of -20 dB/decade
- Two-pole (second-order) filter has a roll-off of -40 dB/decade
- Three-pole (third-order) filter has a roll-off of -60 dB/decade



The number of filter poles can be increased by *cascading*. To obtain a filter with three poles, you can cascade 3 single-pole filters (as shown below), but more preferred way is to cascade a two-pole with one-pole filters (we will see this in later slides)



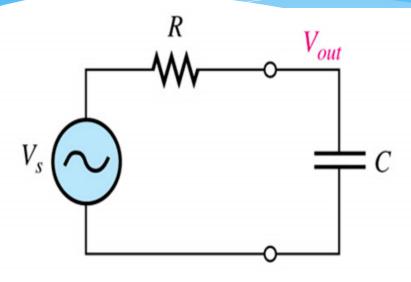
Three-pole (third-order) low-pass filter.

ACTIVE LOW-PASS FILTERS

Advantages of active filters over passive filters (R, L, and C elements only):

- 1. By containing the op-amp, active filters can be designed to provide required gain, and hence **no signal attenuation** as the signal passes through the filter.
- 2. **No loading problem**, due to the high input impedance of the op-amp prevents excessive loading of the driving source, and the low output impedance of the op-amp prevents the filter from being affected by the load that it is driving.
- 3. Easy to adjust over a wide frequency range without altering the desired response.

> Figure below shows the basic Low-Pass filter circuit



(b) Basic low-pass circuit

At critical frequency,

Resistance = Capacitance

$$R = X_c$$

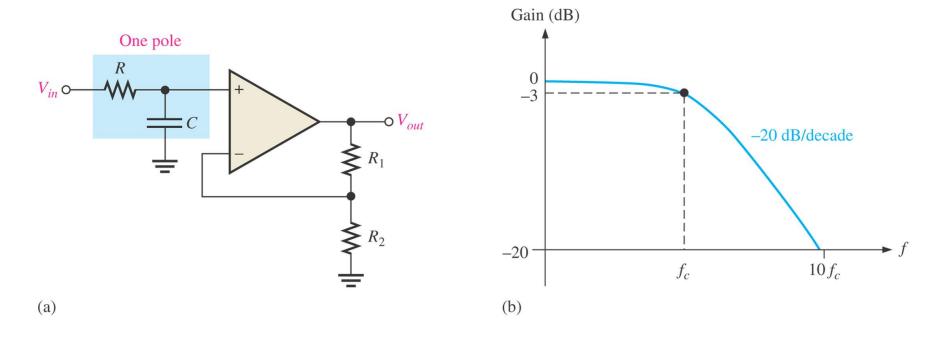
$$R = \frac{1}{\omega_c C}$$

$$R = \frac{1}{2\pi f_c C}$$

So, critical frequency;

$$f_c = \frac{1}{2\pi RC}$$

Sign-Political



Single-pole active low-pass filter and response curve.

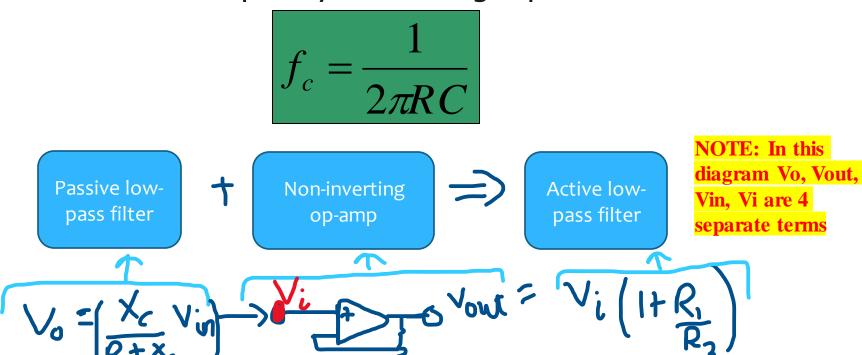
➤ This filter provides a roll-off rate of -20 dB/decade above the critical frequency.

 \triangleright The op-amp in single-pole filter is connected as a noninverting amplifier with the closed-loop voltage gain in the passband is set by the values of R₁ and R₂:

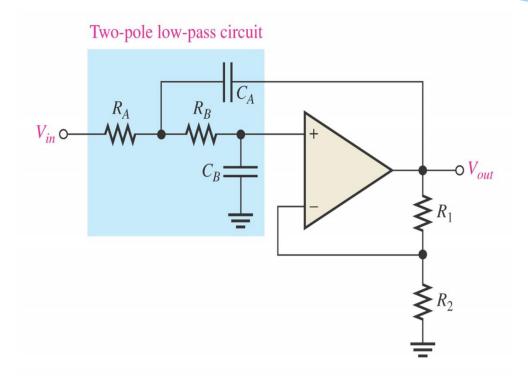
Closed loop gain in non-inverting configuration $A_{\text{cl(NI)}}$

$$A_{cl(NI)} = \frac{R_1}{R_2} + 1$$

> The critical frequency of the single-pole filter is:



Sallen-Key is one of the most common configurations for a second order (two-pole) filter.



- ➤ There are two low-pass RC circuits that provide a roll-off of -40 dB/decade above f_c (assuming a Butterworth characteristics).
- \triangleright One RC circuit consists of R_A and C_A , and the second circuit consists of R_B and C_B .

Basic Sallen-Key low-pass filter.

> The critical frequency for the Sallen-Key filter is:

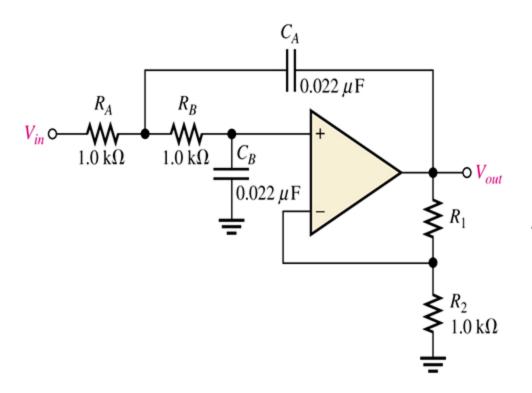
$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

For $R_A = R_B = R$ and $C_A = C_B = C$, thus the critical frequency :

$$f_c = \frac{1}{2\pi RC}$$

Bemple:

- Determine critical frequency
- Set the value of R₁ for Butterworth response. Given that Butterworth response for second order is 0.586

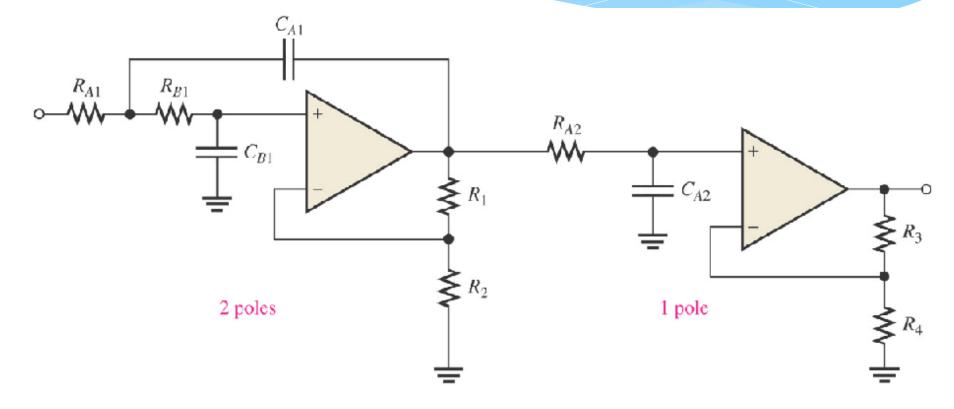


Critical frequency

Butterworth response

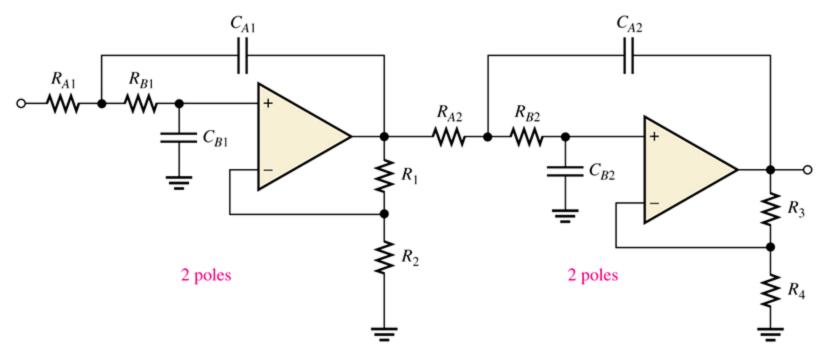
Butterworth => flat amplitude response => Same gain And, Here we have gain = 1+ (r1/r2) => throughout response is same = r1/r2

➤ A three-pole filter is required to provide a roll-off rate of -60 dB/decade. This is done by cascading a two-pole Sallen-Key low-pass filter and a single-pole low-pass filter.



Cascaded low-pass filter: third-order configuration.

➤ A four-pole filter is required to provide a roll-off rate of -80 dB/decade. This is done by cascading a two-pole Sallen-Key low-pass filter and a two-pole Sallen-Key low-pass filter.

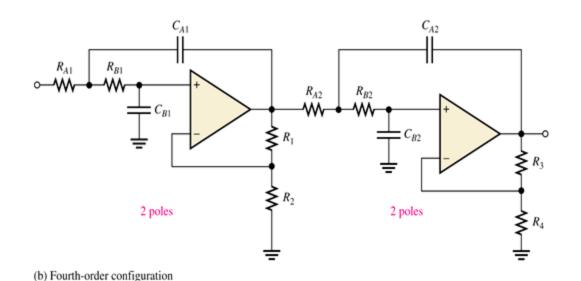


(b) Fourth-order configuration

Cascaded low-pass filter: fourth-order configuration.

Bample:

• Determine the capacitance values required to produce a critical frequency of 2680 Hz if all resistors in RC low pass circuit is $1.8k\Omega$



$$f_c = \frac{1}{2\pi RC}$$

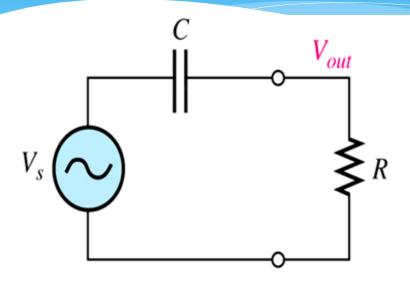
$$C = \frac{1}{2\pi f_c R} = 0.033 \,\mu F$$

$$C_{A1} = C_{B1} = C_{A2} = C_{B2} = 0.033 \mu f$$

• Both stages must have the same f_c . Assume equal-value of capacitor

ACTIVE HIGH-PASS FILTERS

> Figure below shows the basic High-Pass filter circuit :



(b) Basic high-pass circuit

At critical frequency,

Resistance = Capacitance

$$R = X_c$$

$$R = \frac{1}{\omega_c C}$$

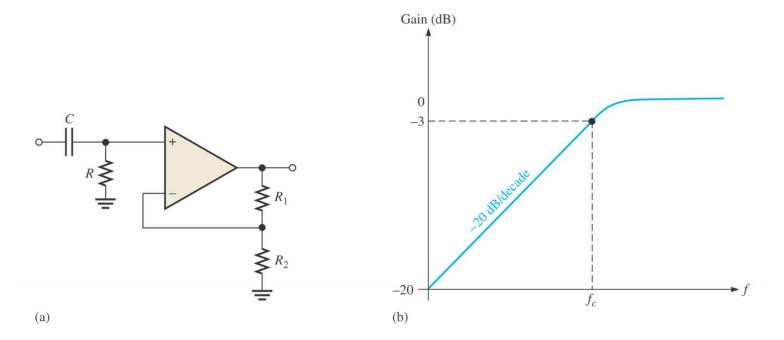
$$R = \frac{1}{2\pi f_c C}$$

So, critical frequency;

$$f_c = \frac{1}{2\pi RC}$$

Sign-Politici

- In high-pass filters, the roles of the capacitor and resistor are reversed in the RC circuits as shown from Figure (a). The negative feedback circuit is the same as for the low-pass filters.
- > Figure (b) shows a high-pass active filter with a -20dB/decade roll-off



Single-pole active high-pass filter and response curve.

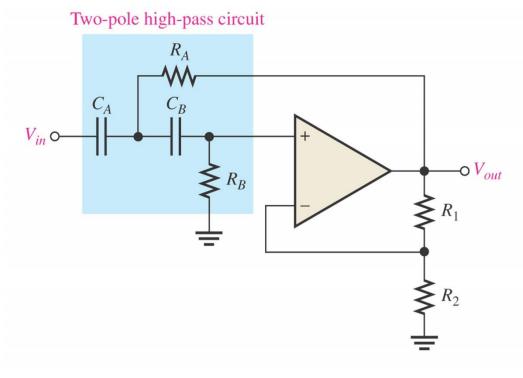
 \triangleright The op-amp in single-pole filter is connected as a noninverting amplifier with the closed-loop voltage gain in the passband is set by the values of R_1 and R_2 :

$$A_{cl(NI)} = \frac{R_1}{R_2} + 1$$

> The critical frequency of the single-pole filter is:

$$f_c = \frac{1}{2\pi RC}$$

- ➤ Components R_A, C_A, R_B, and C_B form the second order (two-pole) frequency-selective circuit.
- ➤ The position of the resistors and capacitors in the frequencyselective circuit are opposite in low pass configuration.
- ➤ There are two high-pass RC circuits that provide a roll-off of -40 dB/decade above fc
- ➤ The response characteristics can be optimized by proper selection of the feedback resistors, R₁ and R₂.



Basic Sallen-Key high-pass filter.

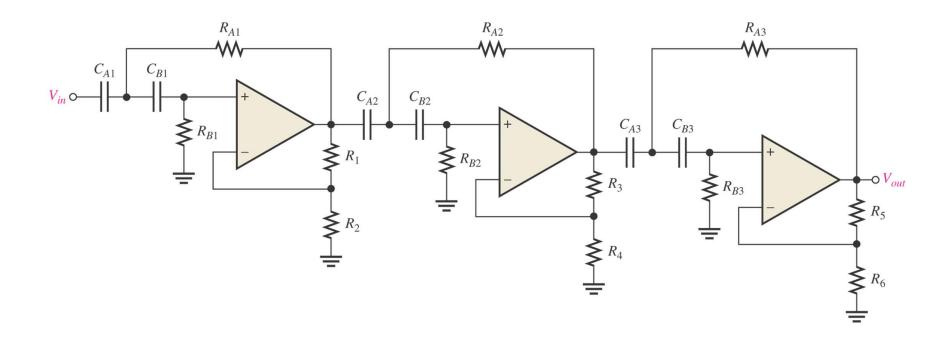
> The critical frequency for the Sallen-Key filter is:

$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

For $R_A = R_B = R$ and $C_A = C_B = C$, thus the critical frequency :

$$f_c = \frac{1}{2\pi RC}$$

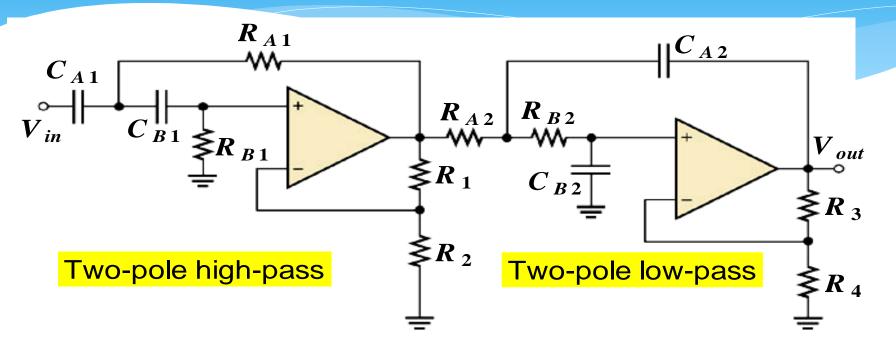
- As with the low-pass filter, first- and second-order high-pass filters can be cascaded to provide three or more poles and thereby create faster roll-off rates.
- ➤ A six-pole high-pass filter consisting of three Sallen-Key two-pole stages with the roll-off rate of -120 dB/decade.



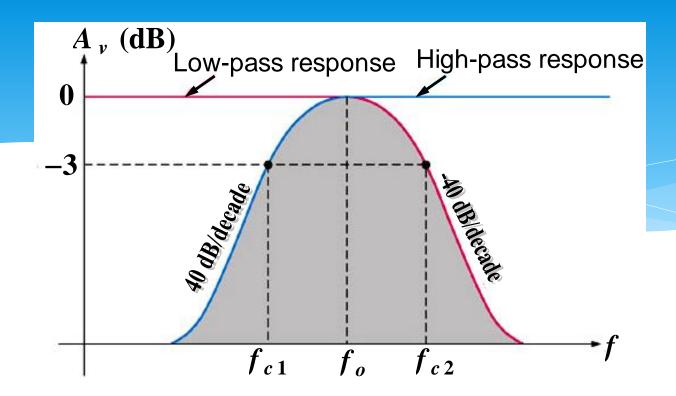
Sixth-order high-pass filter

ACTIVE BANDERASS FILTERS

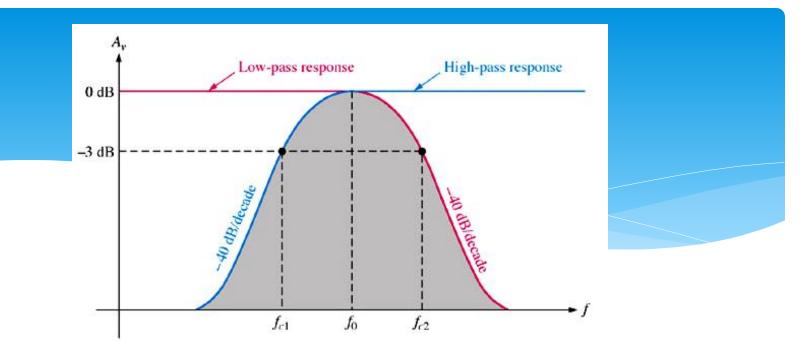
FASCHILL PASSALL LIGHT PASSALL



- > Band-pass filter is formed by cascading a two-pole high-pass and two pole low-pass filter.
- ➤ Each of the filters shown is Sallen-Key Butterworth configuration, so that the roll-off rate are -40dB/decade.



- \succ The lower frequency f_{c1} of the passband is the critical frequency of the high-pass filter.
- \succ The upper frequency f_{c2} of the passband is the critical frequency of the low-pass filter.



> The following formulas express the three frequencies of the band-pass filter.

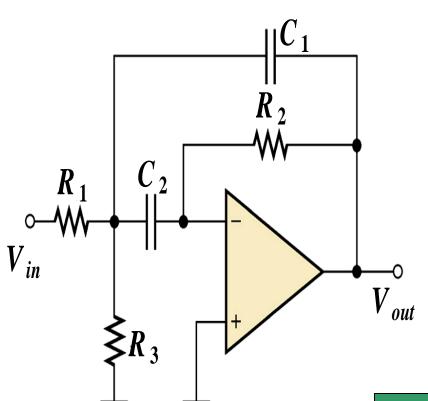
$$f_{c1} = \frac{1}{2\pi\sqrt{R_{A1}R_{B1}C_{A1}C_{B1}}}$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_{A2}R_{B2}C_{A2}C_{B2}}}$$

$$f_0 = \sqrt{f_{c1}f_{c2}}$$

> If equal-value components are used in implementing each filter,

$$f_c = \frac{1}{2\pi RC}$$



- The low-pass circuit consists of R_1 and C_1 .
- > The high-pass circuit consists of R_2 and C_2 .
- \succ The feedback paths are through C_1 and R_2 .
- > Center frequency;

$$f_0 = \frac{1}{2\pi\sqrt{(R_1//R_3)R_2C_1C_2}}$$

 \triangleright By making C1 = C2 = C, yields

$$f_0 = \frac{1}{2\pi C} \sqrt{\frac{R_1 + R_3}{R_1 R_2 R_3}}$$

> The resistor values can be found by using following formula

$$R_1 = \frac{Q}{2\pi f_o CA_o}$$

$$R_2 = \frac{Q}{\pi f_o C}$$

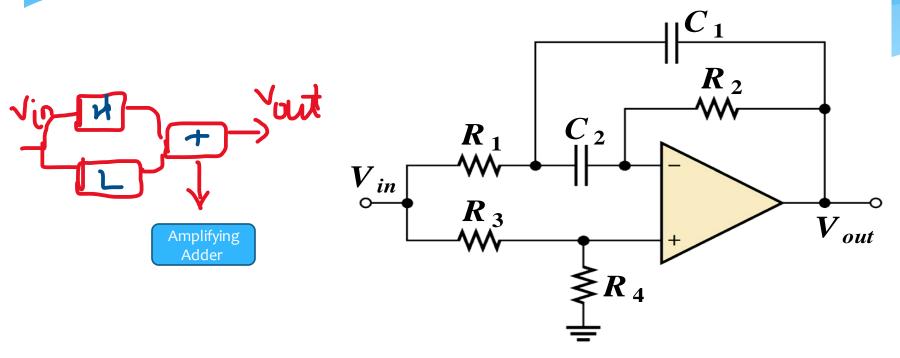
$$R_3 = \frac{Q}{2\pi f_o C(2Q^2 - A_o)}$$

 \triangleright The maximum gain, A_o occurs at the center frequency.

$$A_o = \frac{R_2}{2R_1}$$

ACTIVE BAND-STOP FILTERS

Milliple-feediachBand-Supfilier



- \succ The configuration is similar to the band-pass version BUT R₃ has been moved and R₄ has been added.
- ➤ The BSF is opposite of BPF in that it blocks a specific band of frequencies

- Measuring frequency response can be performed with typical bench-type equipment.
- ➤ It is a process of setting and measuring frequencies both outside and inside the known cutoff points in predetermined steps.
- > Use the output measurements to plot a graph.
- ➤ More accurate measurements can be performed with sweep generators along with an oscilloscope, a spectrum analyzer, or a scalar analyzer.

- The bandwidth of a low-pass filter is the same as the upper critical frequency.
- ➤ The bandwidth of a high-pass filter extends from the lower critical frequency up to the inherent limits of the circuit.
- The band-pass passes frequencies between the lower critical frequency and the upper critical frequency.

- A band-stop filter rejects frequencies within the upper critical frequency and upper critical frequency.
- ➤ The Butterworth filter response is very flat and has a roll-off rate of -20 B
- ➤ The Chebyshev filter response has ripples and overshoot in the passband but can have roll-off rates greater than -20 dB

- The Bessel response exhibits a linear phase characteristic, and filters with the Bessel response are better for filtering pulse waveforms.
- ➤ A filter pole consists of one RC circuit. Each pole doubles the roll-off rate. The Q of a filter indicates a band-pass filter's selectivity. The higher the Q the narrower the bandwidth.
- ➤ The damping factor determines the filter response characteristic.