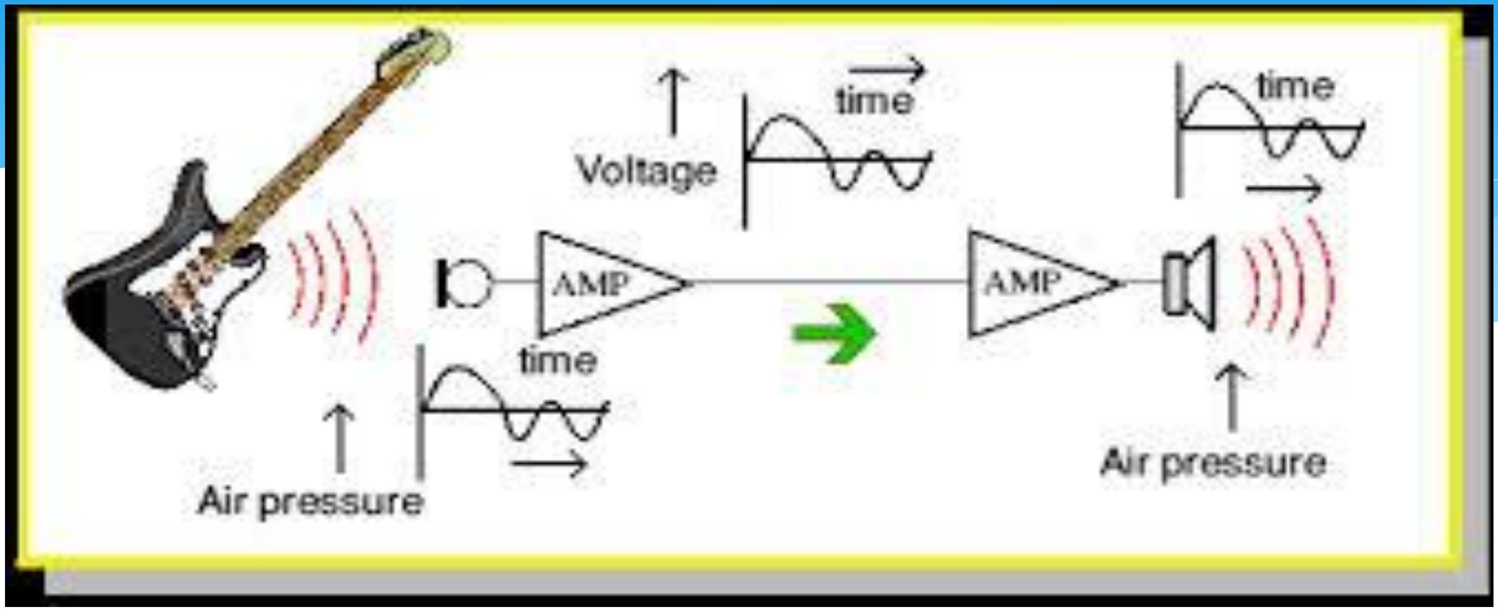
Every thing in this world is continuous.

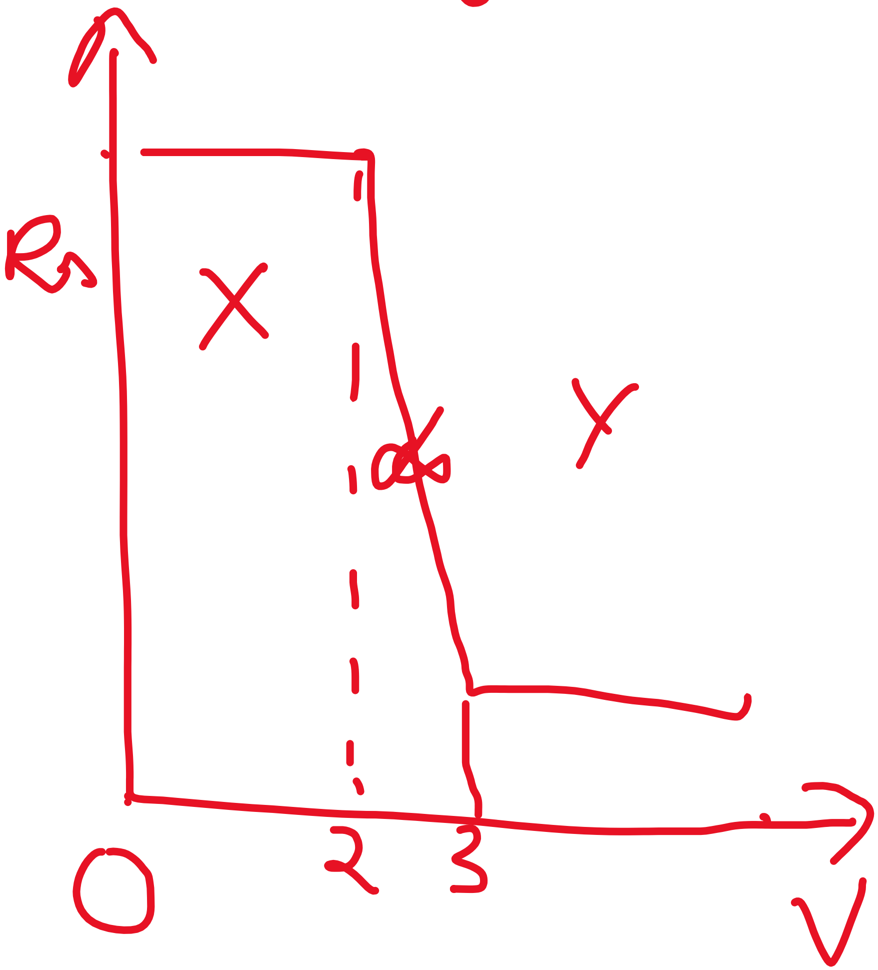
ANALOG



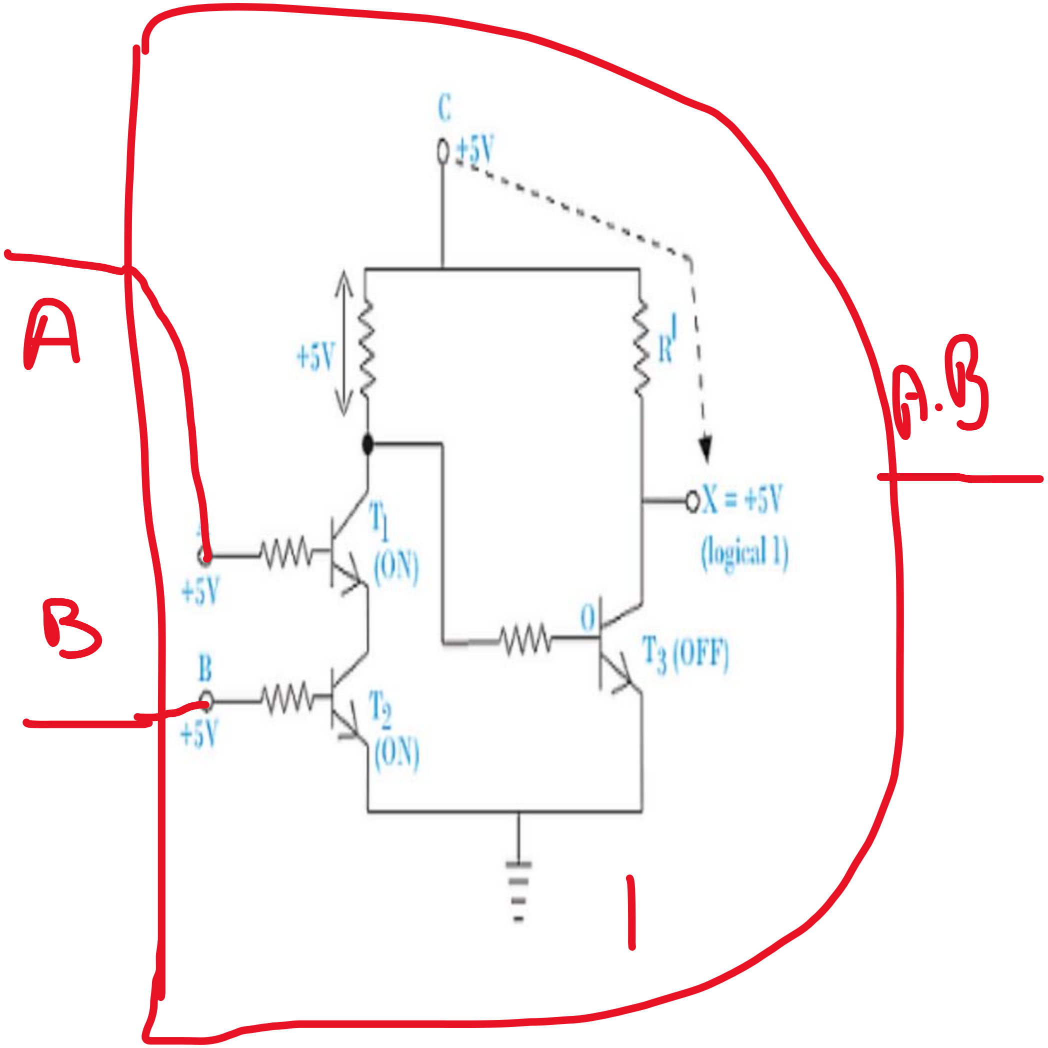
Analog information expresses the state of substances, systems, and so on through continuously varying physical values. Familiar things such as temperature, speed, pressure, flow, and human voices, are all analog.

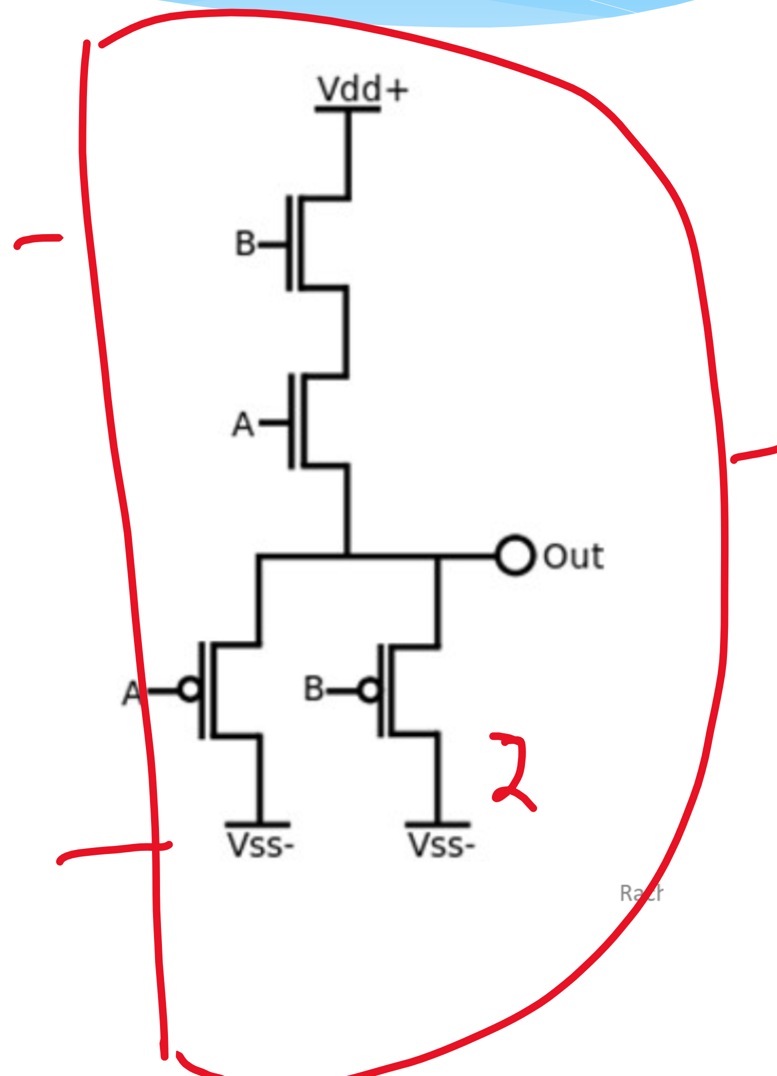
Dig

Analog



NOT

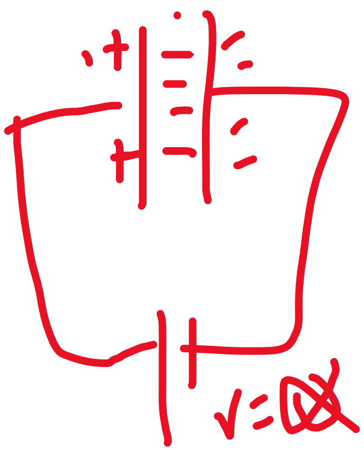

C
+5V
+5V
R
T1
(ON)
+5V
B
+5V
T2
(ON)
0
T3 (OFF)
X = +5V
(logical 1)
A
B
A.B
1

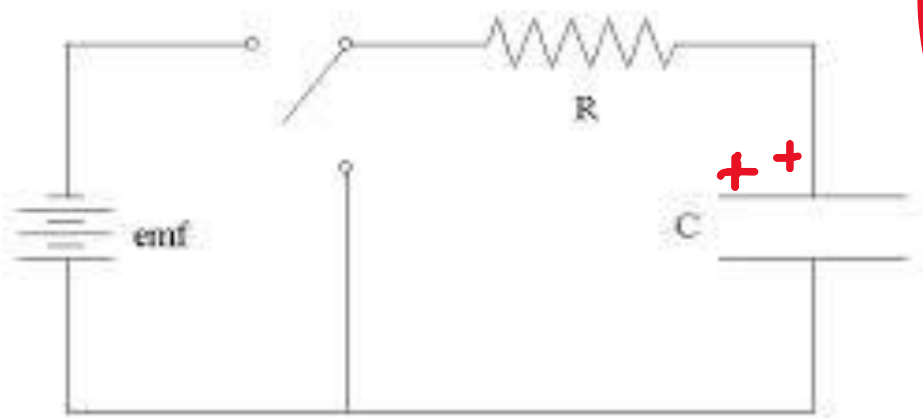

Vdd+
B
A
Out
A
B
Vss-
Vss-
2

RC Circuits

- Circuits that contain both resistance and capacitance are called *RC Circuits*

- RC Circuits are used in timing circuits, camera flashes, pacemakers

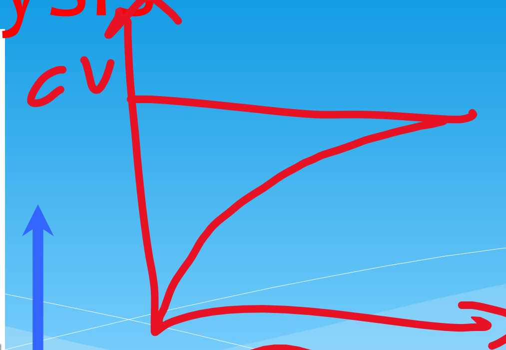


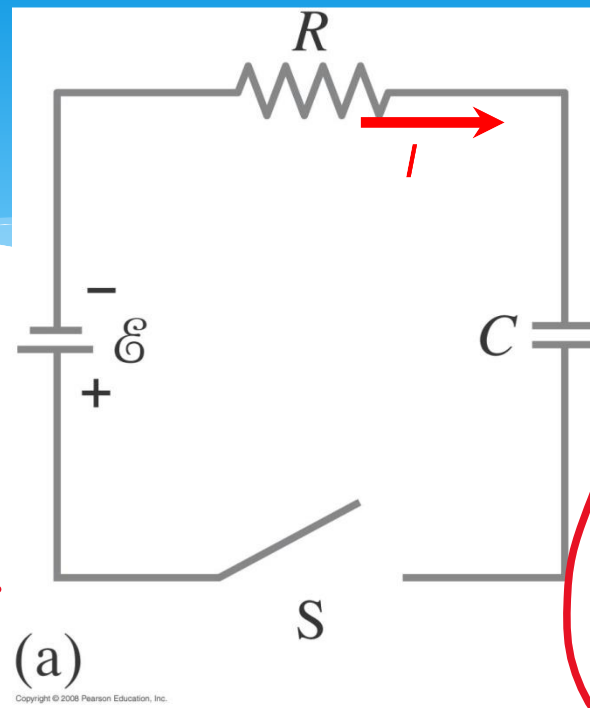


$C \to \Delta V$

RC Transient Analysis

$Q = CV$

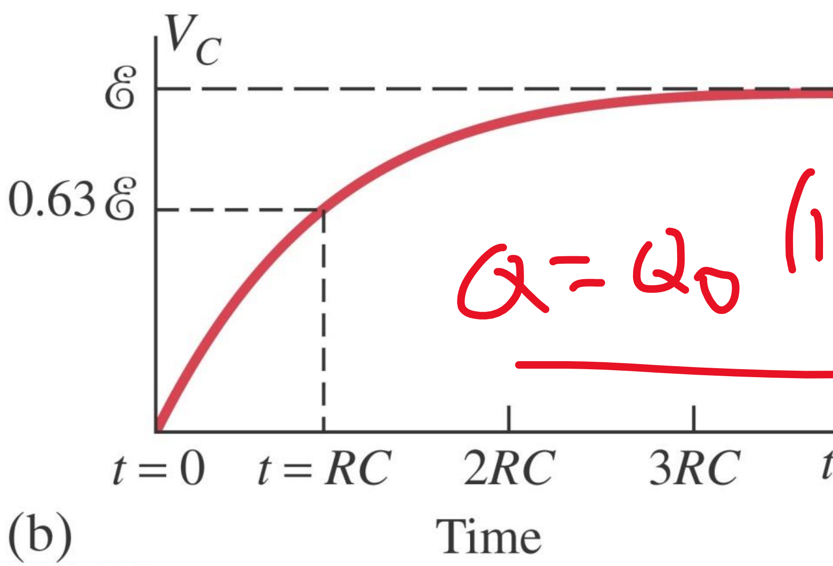




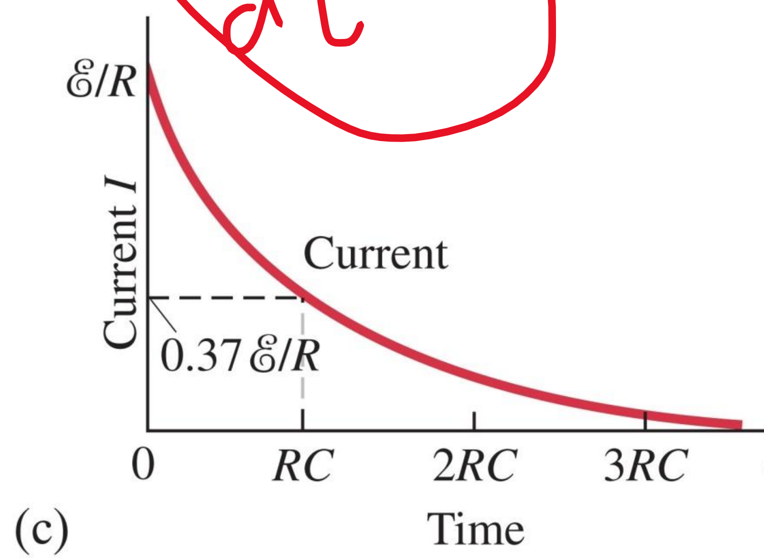
KVL

$$\Delta V(t) = I(t) \cdot R + \frac{Q(t)}{C}$$

$$\frac{dQ}{dt} = i$$



$Q = Q_0\left(1 - e^{-t/RC}\right)$



RC Circuit Transient Analysis

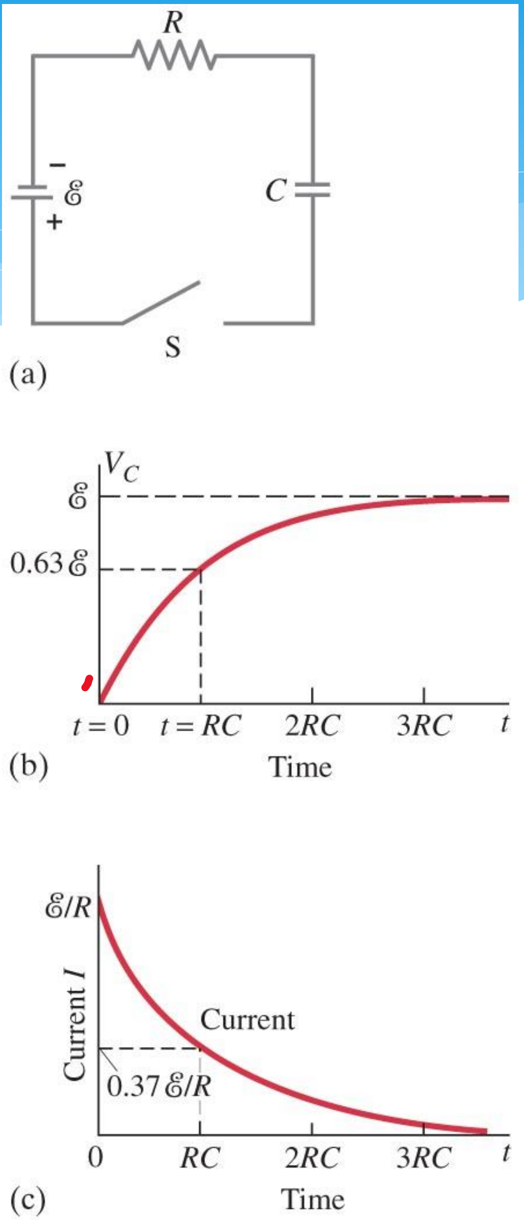
$$V_C = \xi\left(1 - e^{-t/RC}\right)$$

$$I = I_0 e^{-t/RC} \quad where \quad I_0 = \frac{\xi}{R}$$

$$and \quad let \quad \tau = RC$$

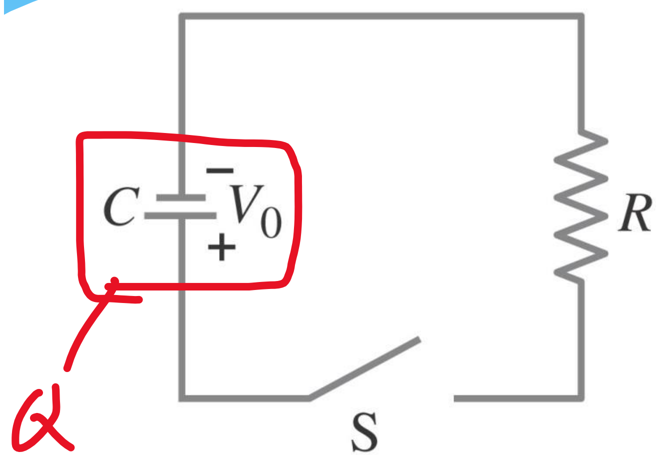
- τ is a time constant

- The transient response of the circuit is approximately 4τ - 5τ

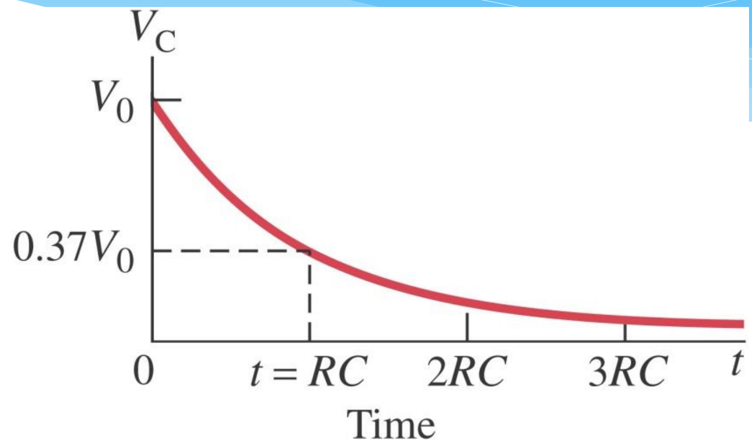


Discharging RC Circuit

- Suppose a capacitor is charged to a voltage V_0, and then connected across a resistor as shown below.



(a)

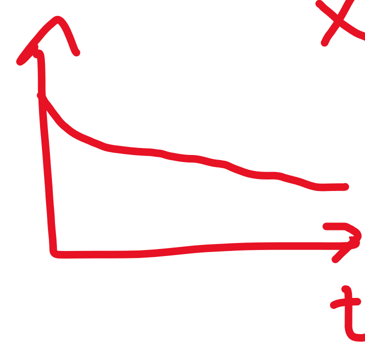


(b)

$\Delta Q \rightarrow i$

$$I = I_0 e^{-t/RC}$$

I



$\times e^{\int P\,dt}$

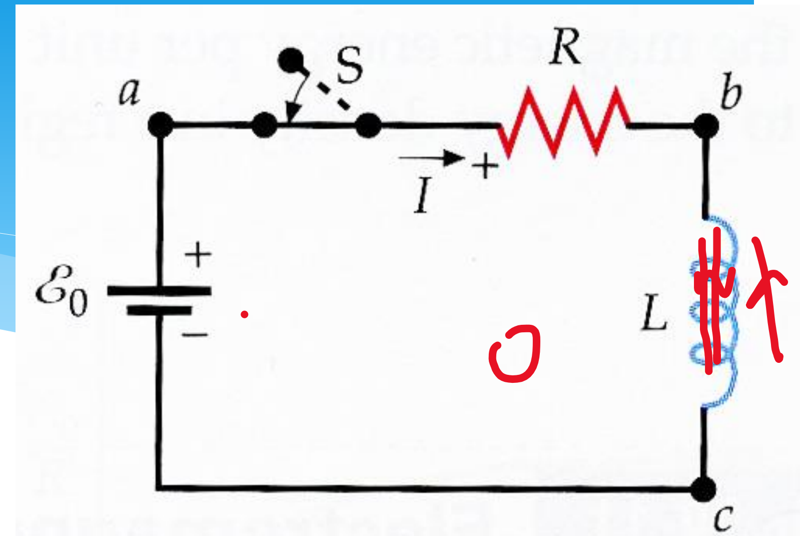
$y' + Py = Q(x)$

and

$$V_C = V_0 e^{-t/RC}$$

$\boxed{L \to \Delta Q \alpha}$

RL Circuit



* Consider a battery connected to a series RL circuit

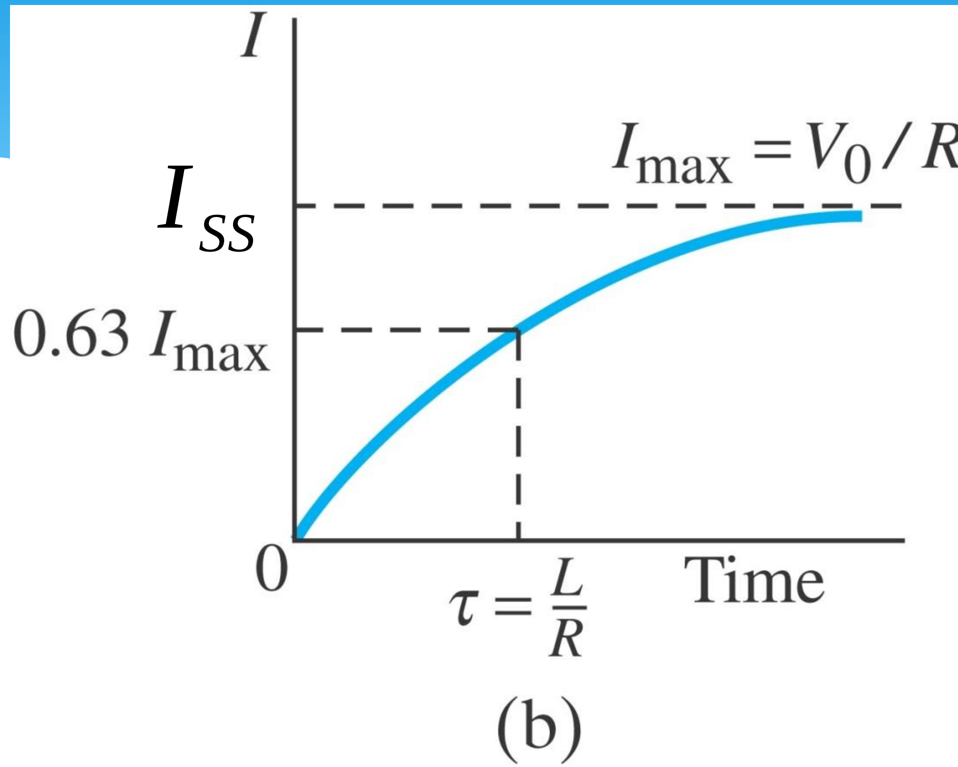
* The instant the switch is closed current begins to flow and it is opposed by the *EMF* induced in the inductor

$\mathcal{E} - iR - V_L = 0$

* Ass the current flows, there will also be a voltage drop across resistor *R*, $V_R = I \cdot R$ (Ohm's law), which will reduce the voltage drop across the coil

* The current will rise gradually as more voltage is dropped across the resistor and less across the inductor, until steady state is achieved, in which all of the voltage drop is across the resistor

Current Response of a Series RL-Circuit



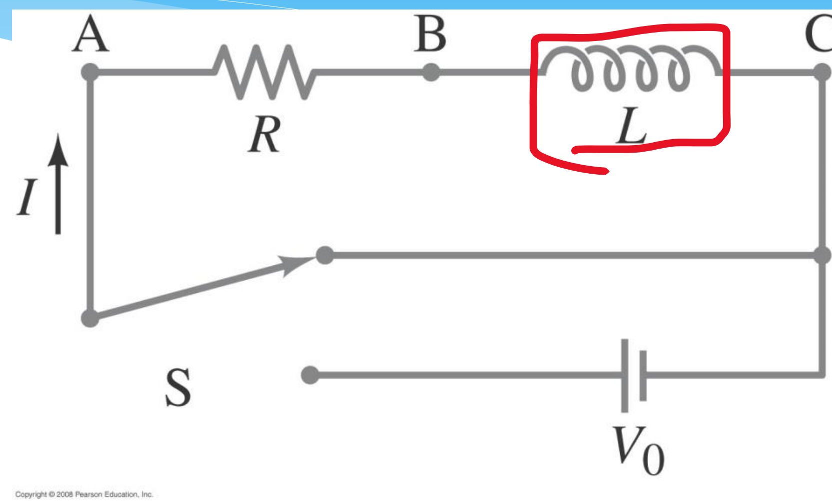
L·

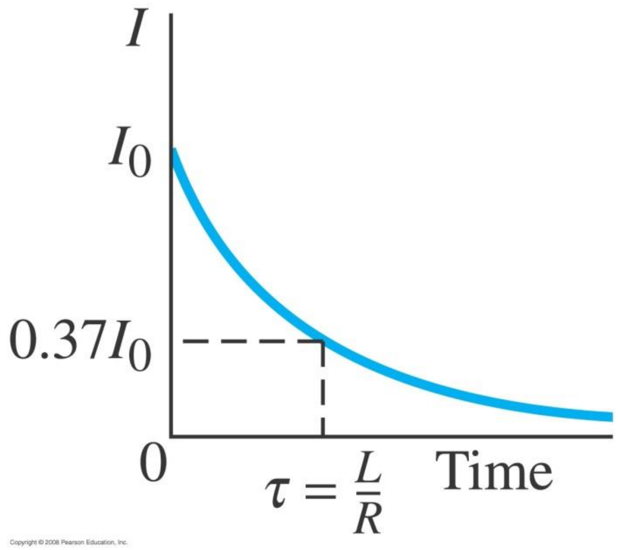
$$\boxed{I = I_{SS}\left(1 - e^{-\frac{t}{\tau}}\right) \quad where \quad \tau = \frac{L}{R}}$$

- where τ is called the ***time constant***

RL-Circuit, Steady State, Battery Removed

- Consider the same circuit, in steady state, with the battery removed.





$$\boxed{I = I_0 e^{-\frac{t}{\tau}}}$$

- The circuit reaches equilibrium again in approximately 4-5 τ

Draw the approximate voltage across capacitor vs time and current vs time graph for pulse input as source to an RC circuit.

AC Circuits

$$\boxed{V_0 \sin \omega t}$$

$$V_R = IR$$

R

$$V_C = IX_C$$

$$\boxed{X_C = 1/\omega C}$$

C

$$V_L = IX_L$$

$$\boxed{X_L = \omega L}$$

L

- An AC circuit is made up with components.
  - Power source
  - Resistors
  - Capacitor
  - Inductors

Resist

Reactance

RLC Circuits

A series RLC circuit: a resistor, inductor, and a capacitor

- An **RLC circuit** (or **LCR circuit** or **CRL circuit** or **RCL circuit**) is an electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series or in parallel.
- The circuit forms a harmonic oscillator for current and will resonate in a similar way as an LC circuit will. The main difference that the presence of the resistor makes is that any oscillation induced in the circuit will die away over time if it is not kept going by a source.
- This effect of the resistor is called damping. The presence of the resistance also reduces the peak resonant frequency somewhat.

AC Source and Resistor Only

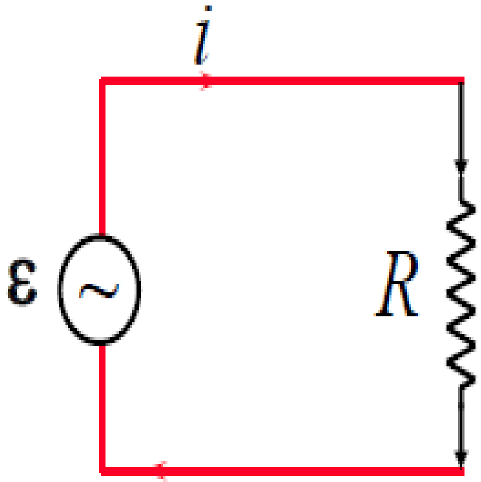
- Driving voltage is $\boxed{\varepsilon = \varepsilon_m \sin \omega t}$

- Relation of current and voltage

$$i = \varepsilon / R$$

$$i = I_m \sin \omega t \qquad I_m = \frac{\varepsilon_m}{R}$$

- Current is *in phase* with voltage (ϕ = 0)



AC Source and Capacitor Only

$i = \frac{dq}{dt}$

$Q \propto V$

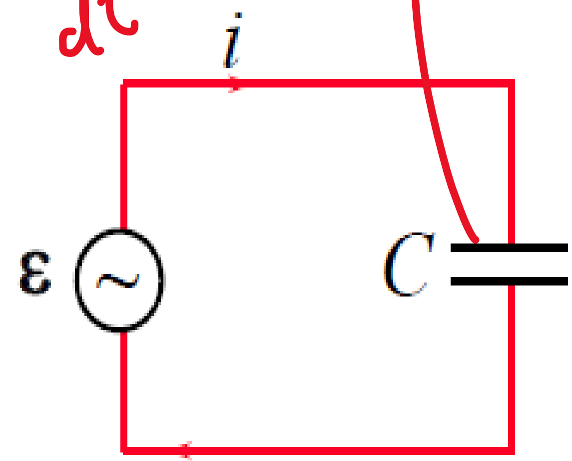
- Voltage is $\boxed{v_C = \frac{q}{C}} = \varepsilon_m \sin \omega t$
- Differentiate to find current

$$q = C\varepsilon_m \sin \omega t \rightarrow CV$$

$$\boxed{i = dq/dt} = \omega C V_C \cos \omega t$$

- Rewrite using phase (check this!)

$$\boxed{i = \omega C V_C \sin\left(\omega t + 90°\right)}$$



- Relation of current and voltage

$$i = I_m \sin\left(\omega t + 90°\right) \quad I_m = \frac{\varepsilon_m}{X_C} \quad \left(X_C = 1/\omega C\right)$$

- "Capacitive reactance": $X_C = 1/\omega C$
  - Current "leads" voltage by 90°

AC Source and Inductor Only

CIVIL

- Voltage is $v_L = L\,di/dt = \varepsilon_m \sin \omega t$
- Integrate di/dt to find current:

$$di/dt = \left(\varepsilon_m / L\right)\sin \omega t$$

$$i = -\left(\varepsilon_m / \omega L\right)\cos \omega t$$

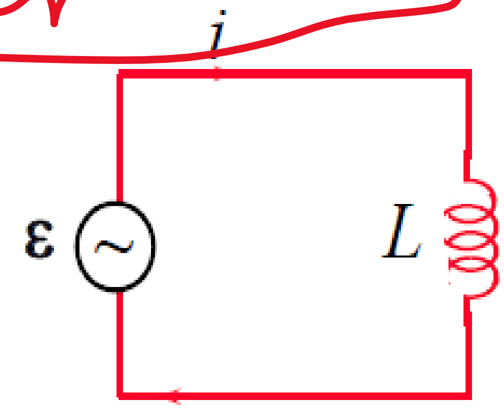
- Rewrite using phase (check this!)

$$i = \left(\varepsilon_m / \omega L\right)\sin\left(\omega t - 90^\circ\right)$$

- Relation of current and voltage

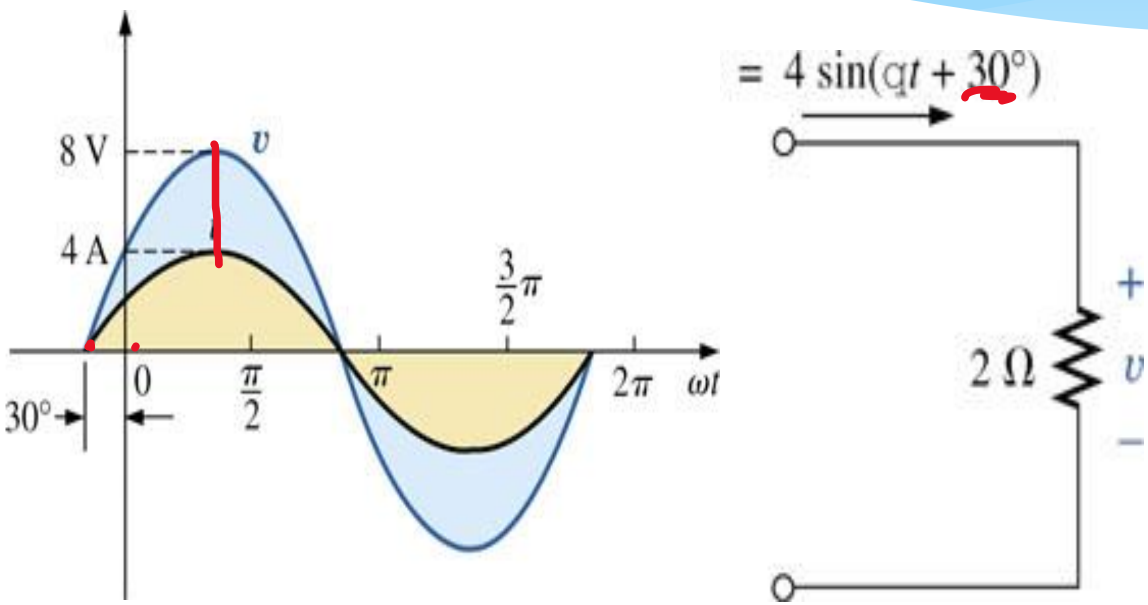
$$i = I_m \sin\left(\omega t - 90^\circ\right) \quad I_m = \frac{\varepsilon_m}{X_L} \quad \left(X_L = \omega L\right)$$

- "Inductive reactance": $X_L = \omega L$
  - Current "lags" voltage by 90°



IMPEDANCE AND THE PHASOR DIAGRAM

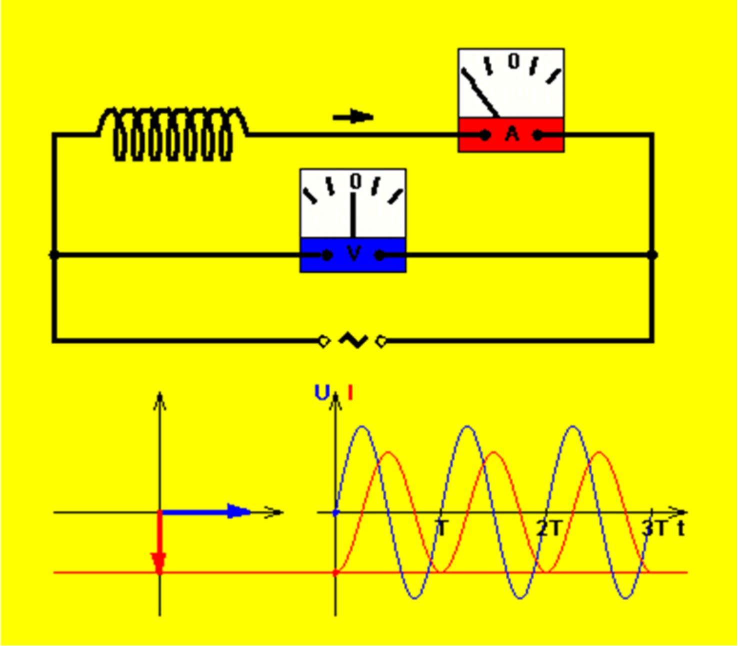
Resistive Elements

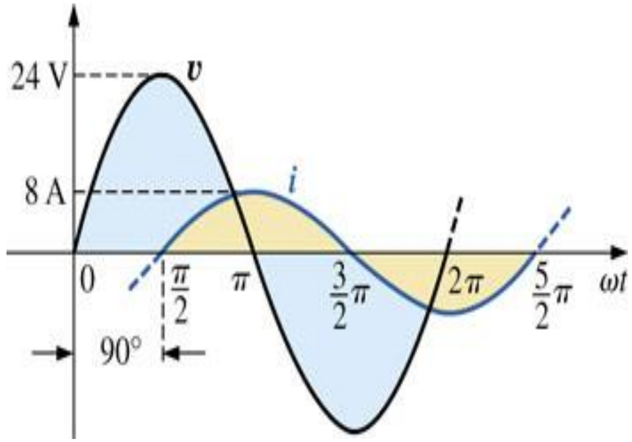


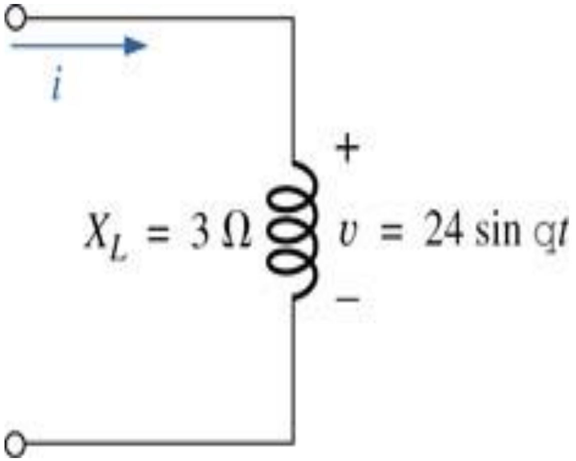
IMPEDANCE AND THE PHASOR DIAGRAM

Inductive Reactance

For the pure inductor, the voltage leads the current by 90° and that the reactance of the coil X_L is determined by ψ*L*.







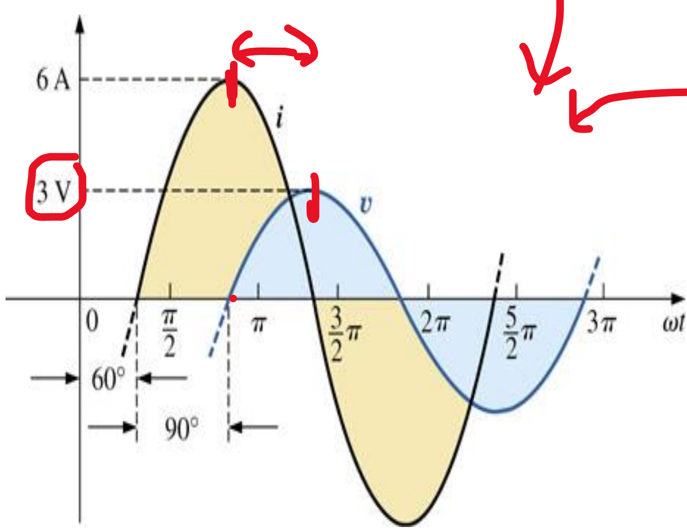
IMPEDANCE AND THE PHASOR DIAGRAM

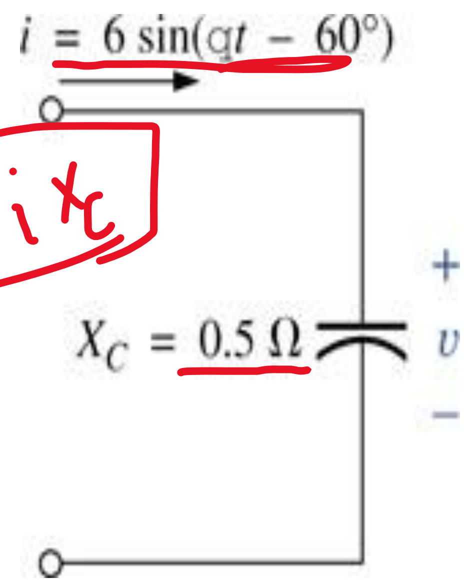
Capacitive Reactance

For the pure capacitor, the current leads the voltage by 90° and that the reactance of the capacitor X_C is determined by 1/ψ*C.*

CIVIL

$v = iX_c$



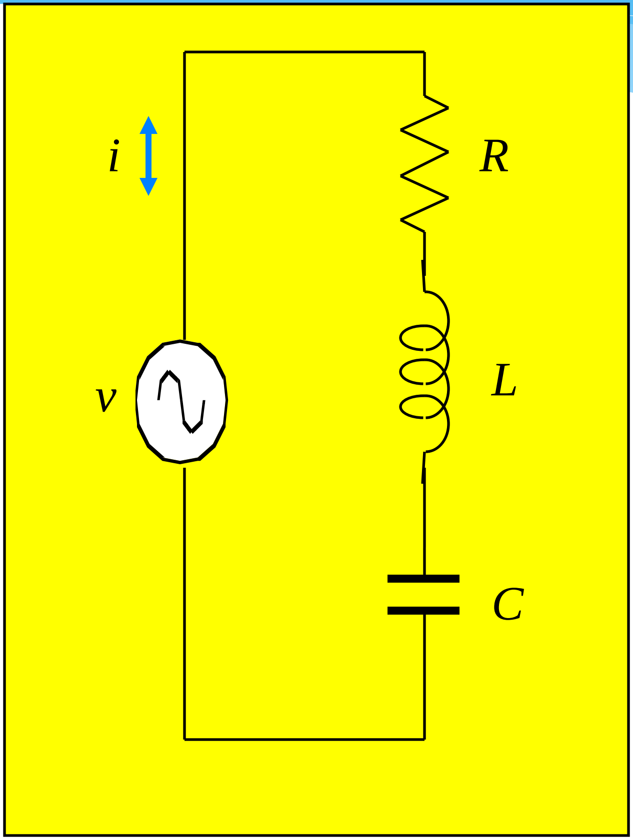


Why were phasors introduced ???

It becomes easier to deal with RLC circuits using phasors and the inferences that,
For Inductor -> current lags
For Capacitor -> current leads
It is a bit easier to derive signs/phase differences, etc. using phasors and deduction than solving D.E. in some cases

CIVIL

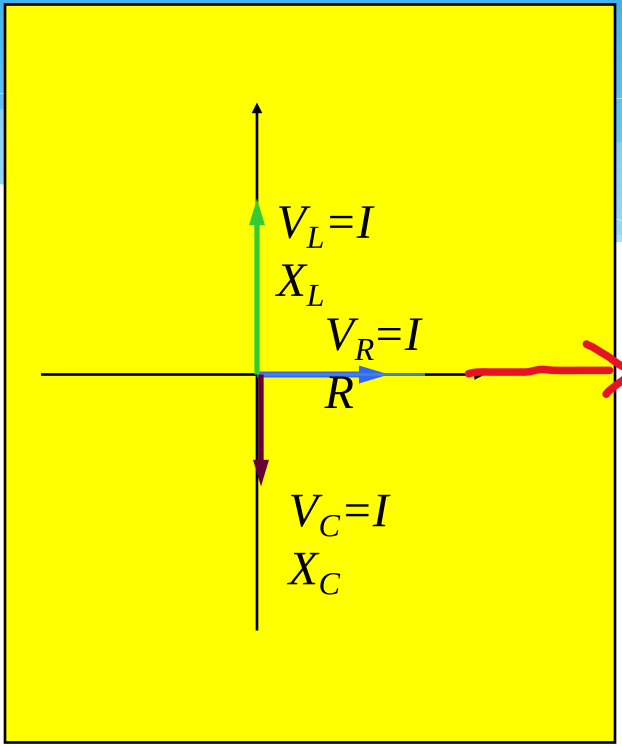
Series RLC



- A series RLC circuit can be made from each component.
  - <u>*One loop*</u>
  - <u>*Same current everywhere*</u>
- Reactances are used for the capacitors and inductors.
- The combination of resistances and reactances in a circuit is called <u>*impedance.*</u>

Vector Map

- Phase shifts are present in AC circuits.
  - +90° for inductors
  - -90° for capacitors
- These can be treated as if on the y-axis.
  - 2 D vector
  - Phasor diagram



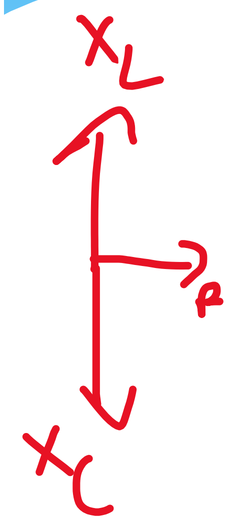
CIVIL

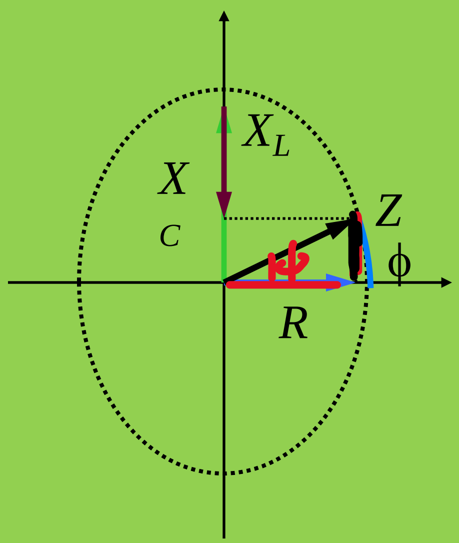
CRL

$V = 5$

$V_R =$

Vector Sum





CRL

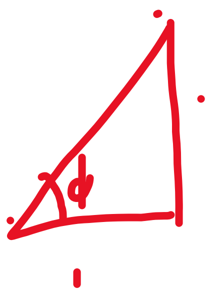
$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

* The total impedance is the magnitude of Z.
* The phase between the current and voltage is the angle ϕ between Z and the x-axis.

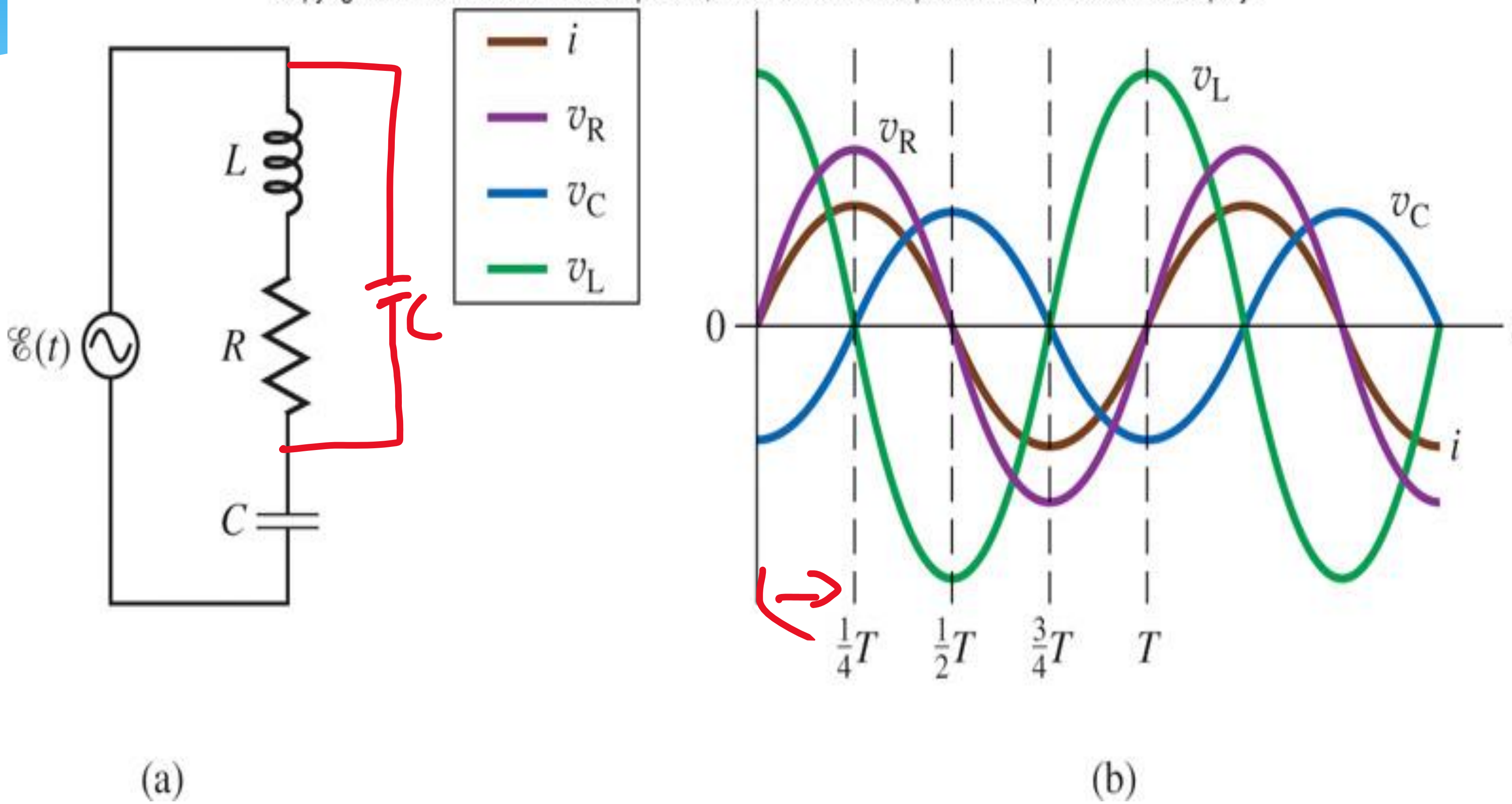
$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\phi = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$



Phase Changes

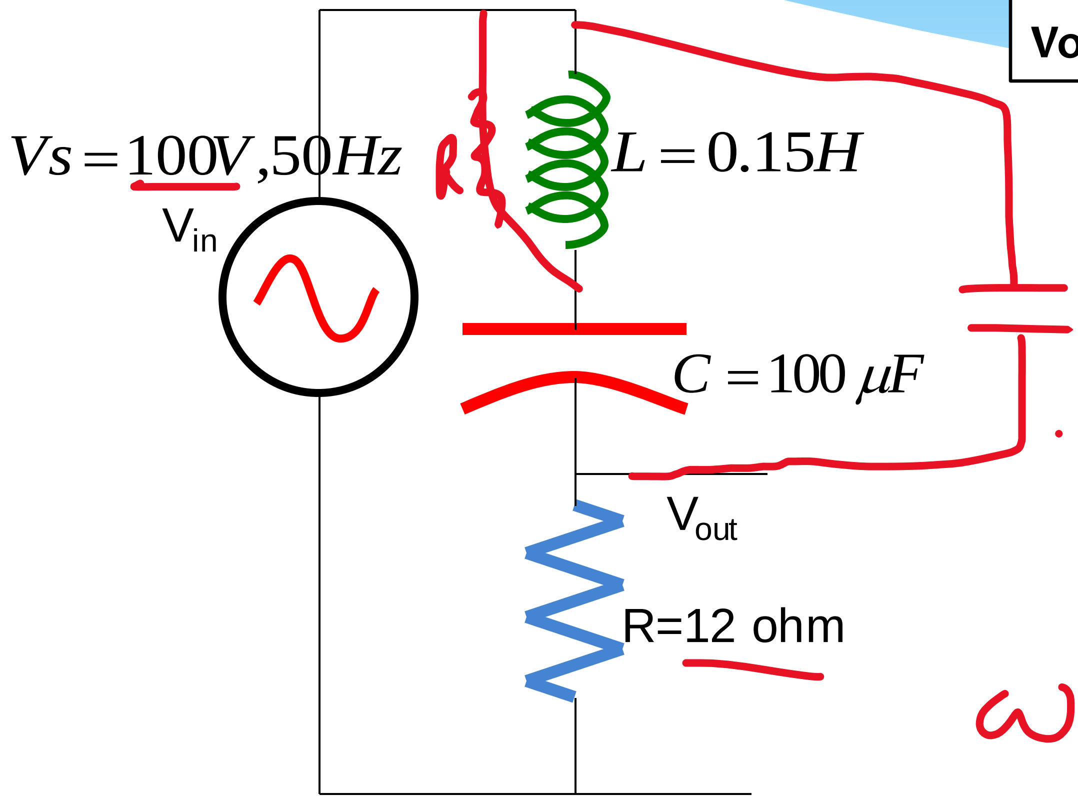
- The phase shift is different in each component



$$\mathcal{E}(t) = \mathcal{E}_m \sin(\omega t + \phi) = V_L \sin\left(\omega t + \frac{\pi}{2}\right) + V_R \sin \omega t + V_C \sin\left(\omega t - \frac{\pi}{2}\right)$$

Mathematical analysis of a series LRC circuit

Find the phase shift and Vout/Vin



$\omega = 2\pi f$

Inductive Reactance, X_L.

ωL

1.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

Capacitive Reactance, X_C.

$1/\omega C$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

Circuit Impedance, Z.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Circuits Current, I.

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14\,\text{Amps}$$

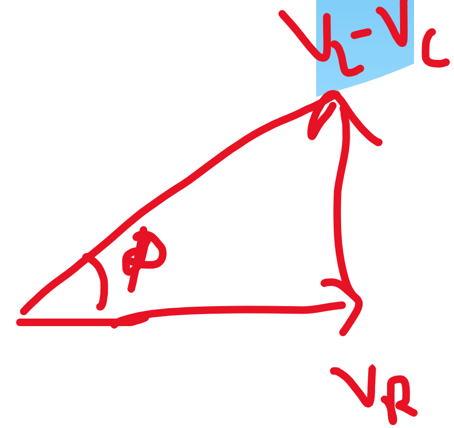
Voltages across the Series RLC Circuit, V_R, V_L, V_C.

2

$$V_R = I \times R = 5.14 \times 12 = 61.7 \text{ volts}$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2 \text{ volts}$$

$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5 \text{ volts}$$



Circuits Power factor and Phase Angle, θ.

1 D.E.

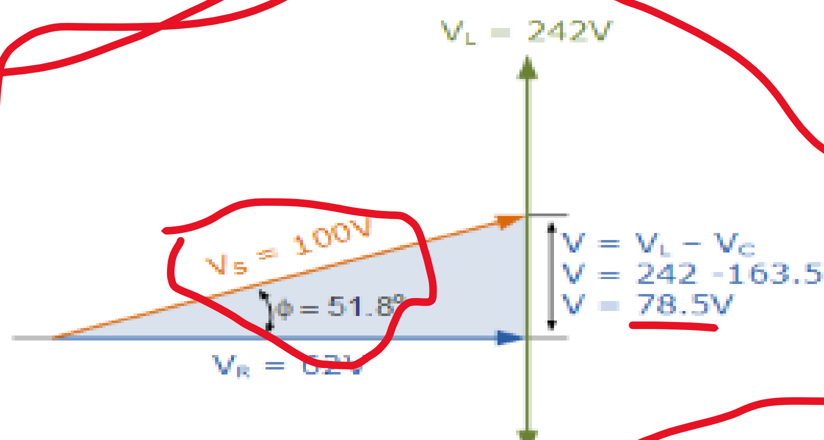
3

$$\cos\phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619$$

$$\therefore \cos^{-1} 0.619 = 51.8^{\circ} \text{ lagging}$$

Phasor Diagram.

4 -



Solving circuits

✓ 1-100

✓ $6 \times 8 = 48$

$6 + 6 +$

* Traditional Methods

* D.E. solving
* Phasors
* Partial Fraction Expansion

* Using Laplace

* Laplace Transform Pair
* P.F.E.

$r = \sqrt{x^2+y^2+z^2}$

The Laplace Transform

S-dom

To construct a Laplace transformation for a given function of time f(t) , we first multiply f(t) by e^{-st} , where is a complex number,given as s=$\sigma + jw$. This product is integrated with respect to time from zero to infinity. The result is the Laplace transform of f(t) which is designated F(s). It is denoted by the script letter '£' , mathematically.

y

x

$$£[f(t)]=F(s)=\int_0^{\infty} f(t)\ e^{-st}\ dt$$

* What actually does Laplace formula do?

* Why is it called a transform and not an operator or function or something else ?

* How is it better ?

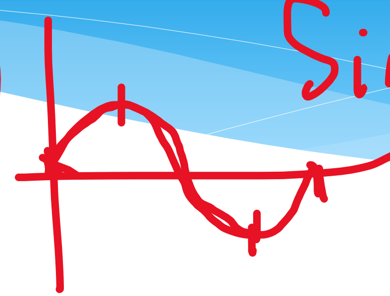
x

sin(t)

Wave

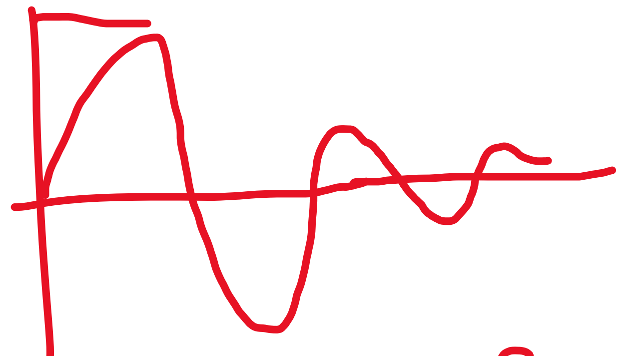
AC

1 Sinusoid

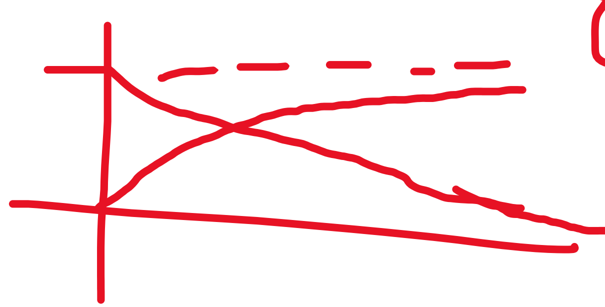


Damp

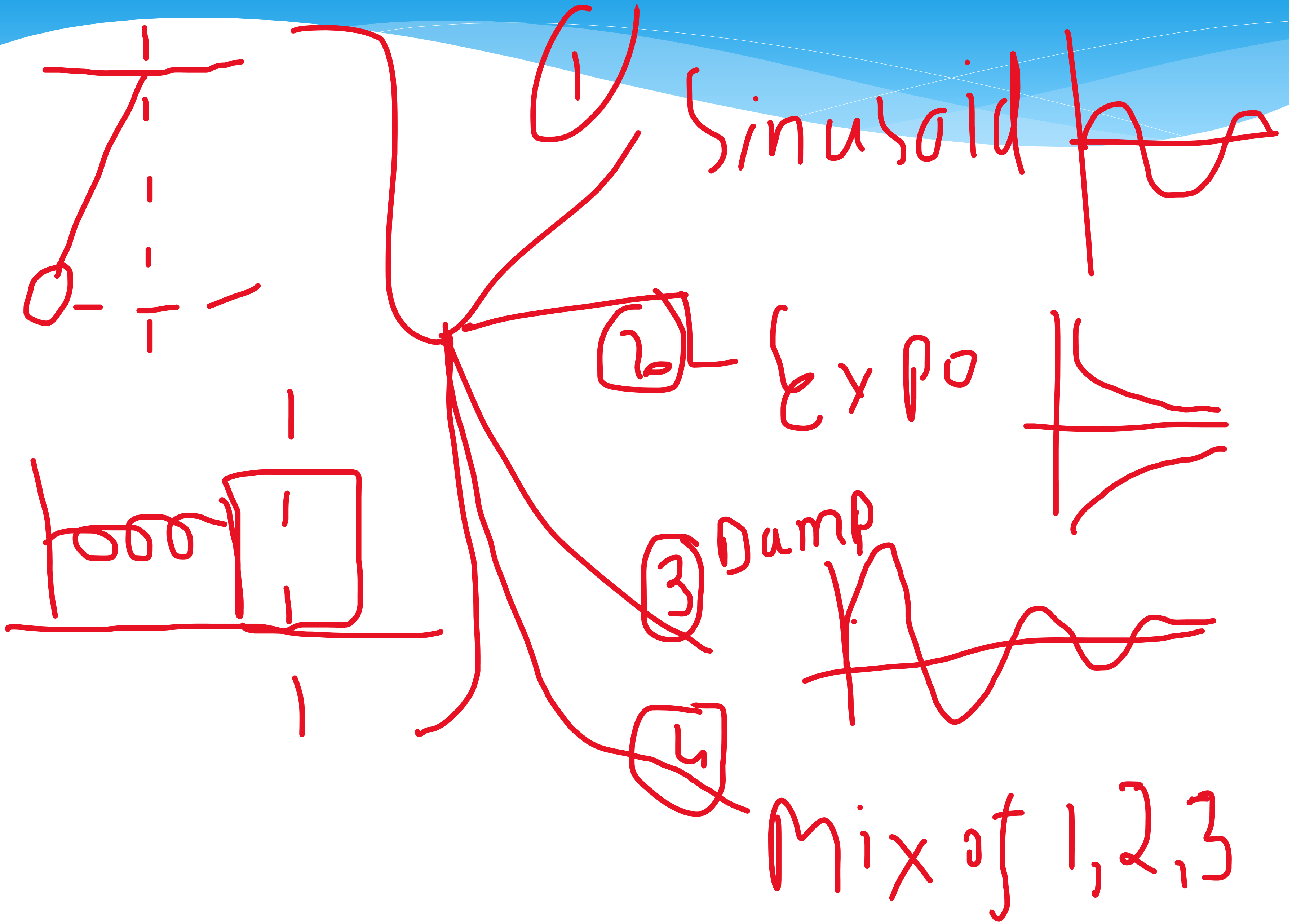
2



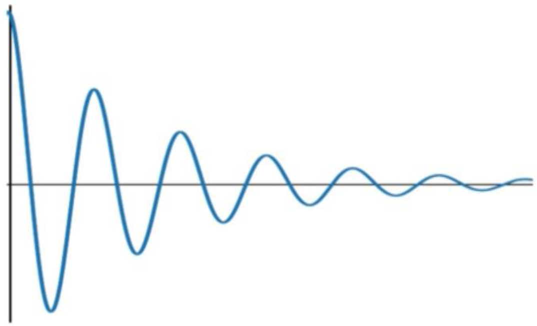
3 Expo

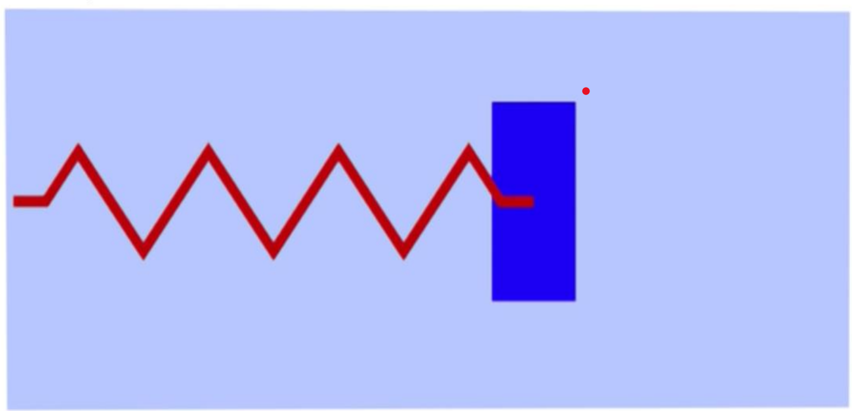


4 Any 1+2+3



1 Sinusoid
2 Expo
3 Damp
4 Mix of 1,2,3





1) *Sinusoidal*

2) *Exponential Decay*
"Overdamped"

3) *Sinusoidal + Exponential Decay*
"Underdamped"

4) *Anything Else*

$e^{-\alpha t}$

Any oscillation/ac source = Sinusoid + Exponential / wave function

Fourier

Fourier $X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-i\omega t}\,dt$

Laplace

Laplace $X(s)=\int_{0}^{\infty} x(t)e^{-(\alpha+i\omega)}\,dt \quad s=\alpha+i\omega$

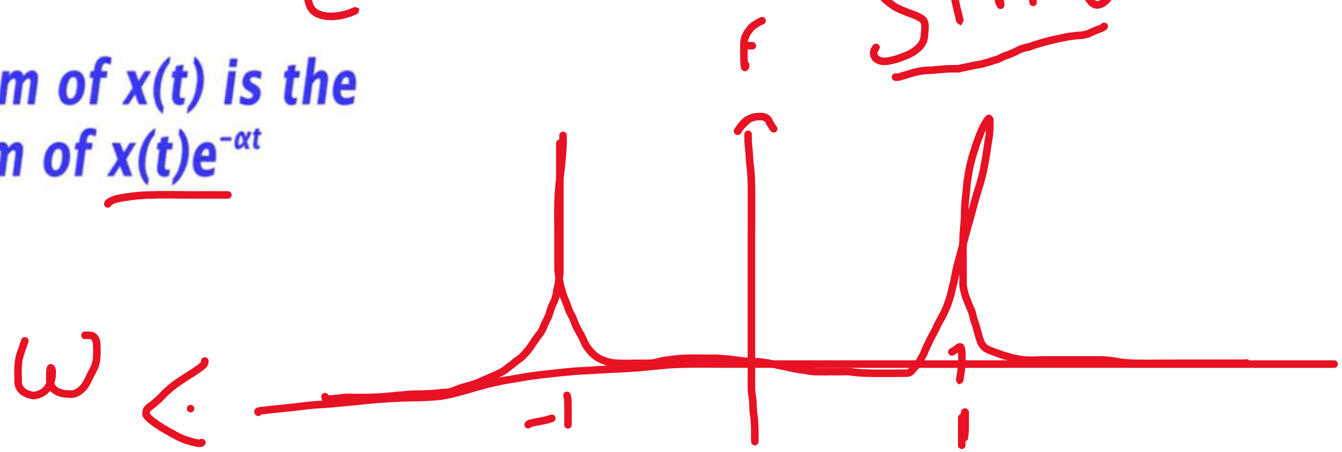
65

$X(s)=\int_{0}^{\infty} \boxed{x(t)e^{-\alpha t}}\, e^{-i\omega t}\,dt$

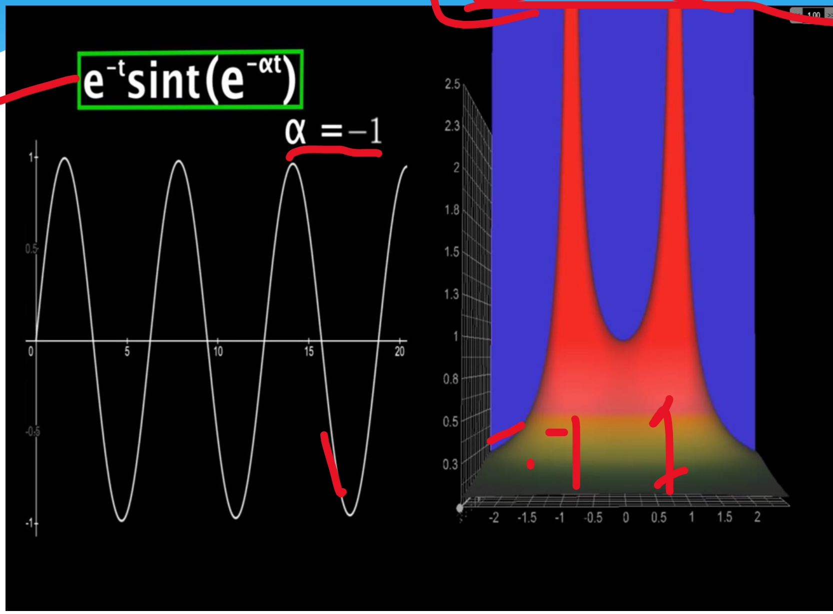
e^{-2t}

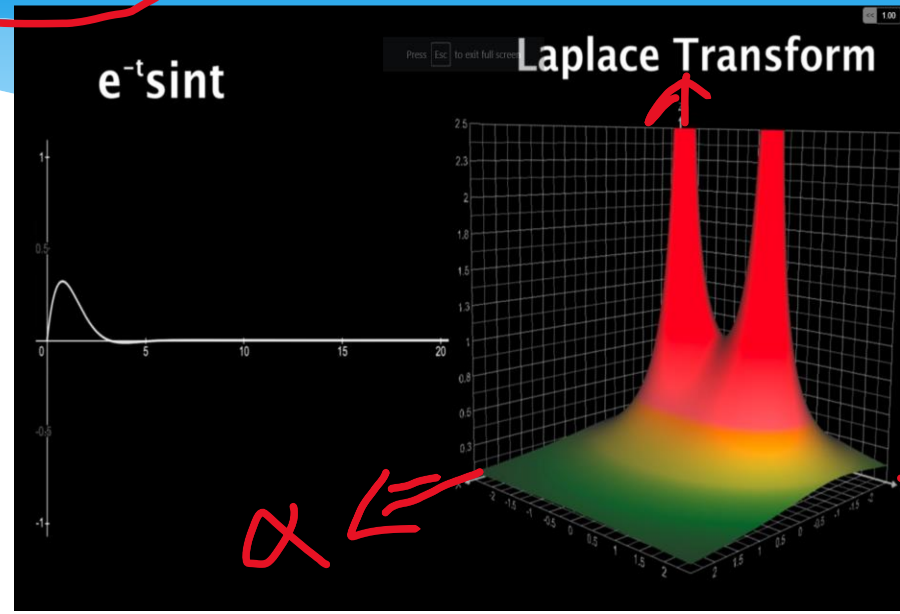
$\sin t$

Laplace transform of x(t) is the fourier transform of $\boldsymbol{x(t)e^{-\alpha t}}$

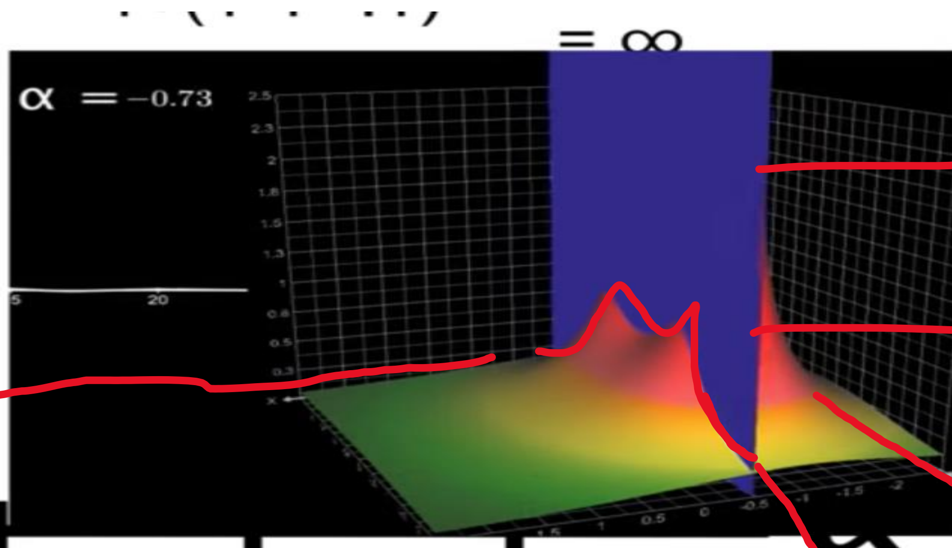


$\alpha = -1$ $e^{-t} \cdot \sin t$





α $i\omega$



α

$-\alpha$

α

$\alpha =$

$i\omega$

$\alpha:$

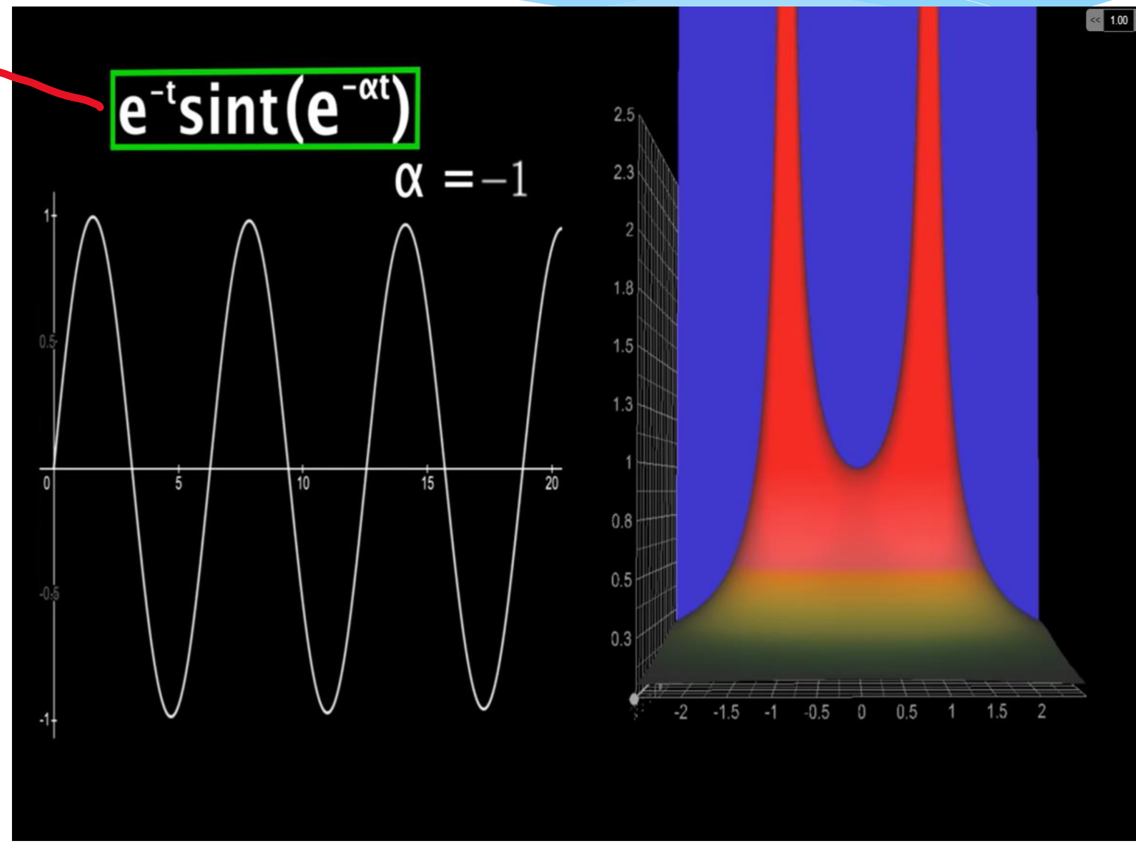
$e^{-t}\sin t$ has 2 functions in it:

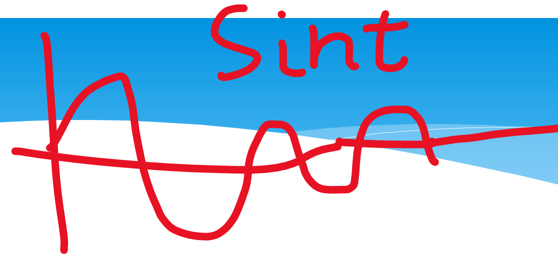
has:

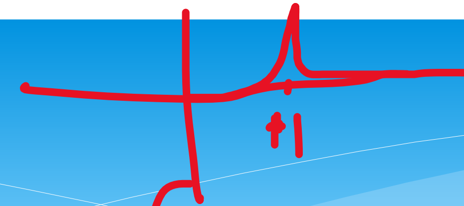
① expo: $\alpha = -1$

① sinusoid

$\boxed{\omega = \pm 1}$



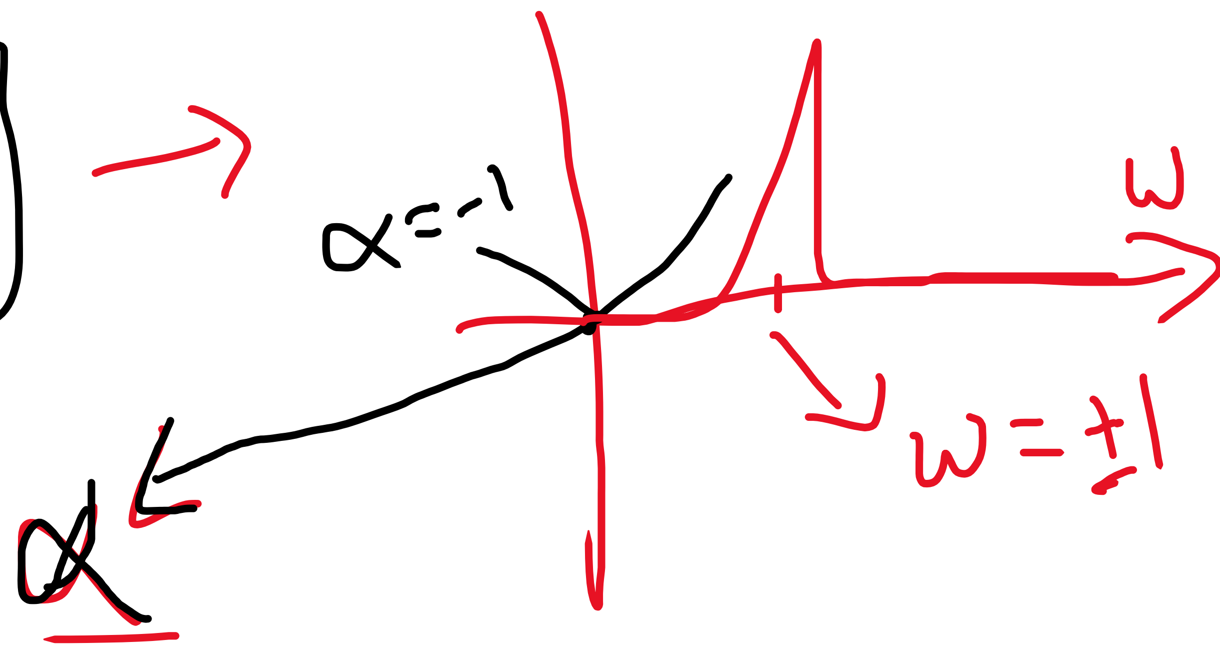




Fourier → t → freq

Laplace → t → s-domain = 2D domain

$\boxed{e^{-t}\sin t}$ →



What does the Laplace Transform really tell us? A visual explanation

https://youtu.be/n2y7n6jw5d0

Transform Pairs:

1 - 10

f(t)	F(s)
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
df/dt	$sF(s) - f(0^-)$
$\int f(t)dt$	$\frac{F(s)}{s}$

The Laplace Transform

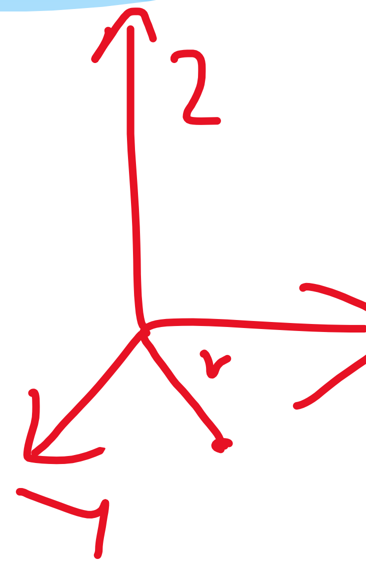
Transform Pairs:

f(t)	F(s)
$t^n f(t)$	$n!(F(s))^{n+1}$
te^{-at}	$\dfrac{1}{(s+a)^2}$
$t^n e^{-at}$	$\dfrac{n!}{(s+a)^{n+1}}$
$\sin(wt)$	$\dfrac{w}{s^2+w^2}$
$\cos(wt)$	$\dfrac{s}{s^2+w^2}$

The Laplace Transform

Transform Pairs:

f(t)	V(s) F(s)
$\boxed{e^{-at}.f(t)}$ V(t)	$\boxed{F(s+a)}$
$e^{-at}\sin(wt)$	$\dfrac{w}{(s+a)^2+w^2}$
$e^{-at}\cos(wt)$	$\dfrac{s+a}{(s+a)^2+w^2}$
$\sin(wt+\theta)$	$\dfrac{s\sin\theta+w\cos\theta}{s^2+w^2}$
$\cos(wt+\theta)$	$\dfrac{s\cos\theta-w\sin\theta}{s^2+w^2}$



$r = \sqrt{x^2}$

Differential Equation to Algebraic Expression

$\int x'(t)$

$$X(t) \xrightarrow{L.T.} X(S)$$

$$X'(t) \xrightarrow{L.T.} SX(S) - \cancel{X(0^-)}$$

$$X''(t) \xrightarrow{L.T.} S^2X(S) - \cancel{SX(0^-)} - \cancel{X'(0^-)}$$

1 $\boxed{\mathcal{L}(my''+by'+ky) = \mathcal{L}(x(t))}$

$$\mathcal{L}(my'') + \mathcal{L}(by') + \mathcal{L}(ky) = \mathcal{L}(x(t))$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$ms^2y(s) + bsy(s) + ky(s) = x(s)$$

$$\underline{y(s)}\,\underline{(ms^2 + bs + k)} = x(\underline{s})$$

$\boxed{V = IR}$

CL

RLC

RC

AC

Now you have your answers

* What actually does Laplace formula do?
- Calculus --> Algebra, time domain --> s-domain

* Why is it called a transform and not an operator or function or something else ?
- Because it "converts/transforms" time domain equations into another domain (s-domain)

* How is it better ?
- reduces load of solving Differential equations.
- circuits in time domain sometimes cannot be simplified so we may need to deal with multiple differential equations, whereas, in s-domain they are simplifiable and usually the final equation is of the form $Y(s) = (as^n + \ldots .. + bs + c)\, X(s)$, $n = 1,2,3,4.....$

The Laplace Transform

Theorem:

Initial Value Theorem:

If the function f(t) and its first derivative are Laplace transformable and f(t) Has the Laplace transform F(s),& the $\lim_{s \to \infty} sF(s)$ exists, then

$$\lim_{s \to \infty} sF(s) = \lim_{t \to 0} f(t) = f(0)$$

Initial Value Theorem

The utility of this theorem lies in not having to take the inverse of F(s) in order to find out the initial condition in the time domain. This is particularly useful in circuits and systems.

The Laplace Transform

Theorem:

Final Value Theorem:

If the function f(t) and its first derivative are Laplace transformable and f(t) has the Laplace transform F(s), and the $\lim_{s \to \infty} sF(s)$ exists, then

$$\lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t) = f(\infty)$$

Final Value Theorem

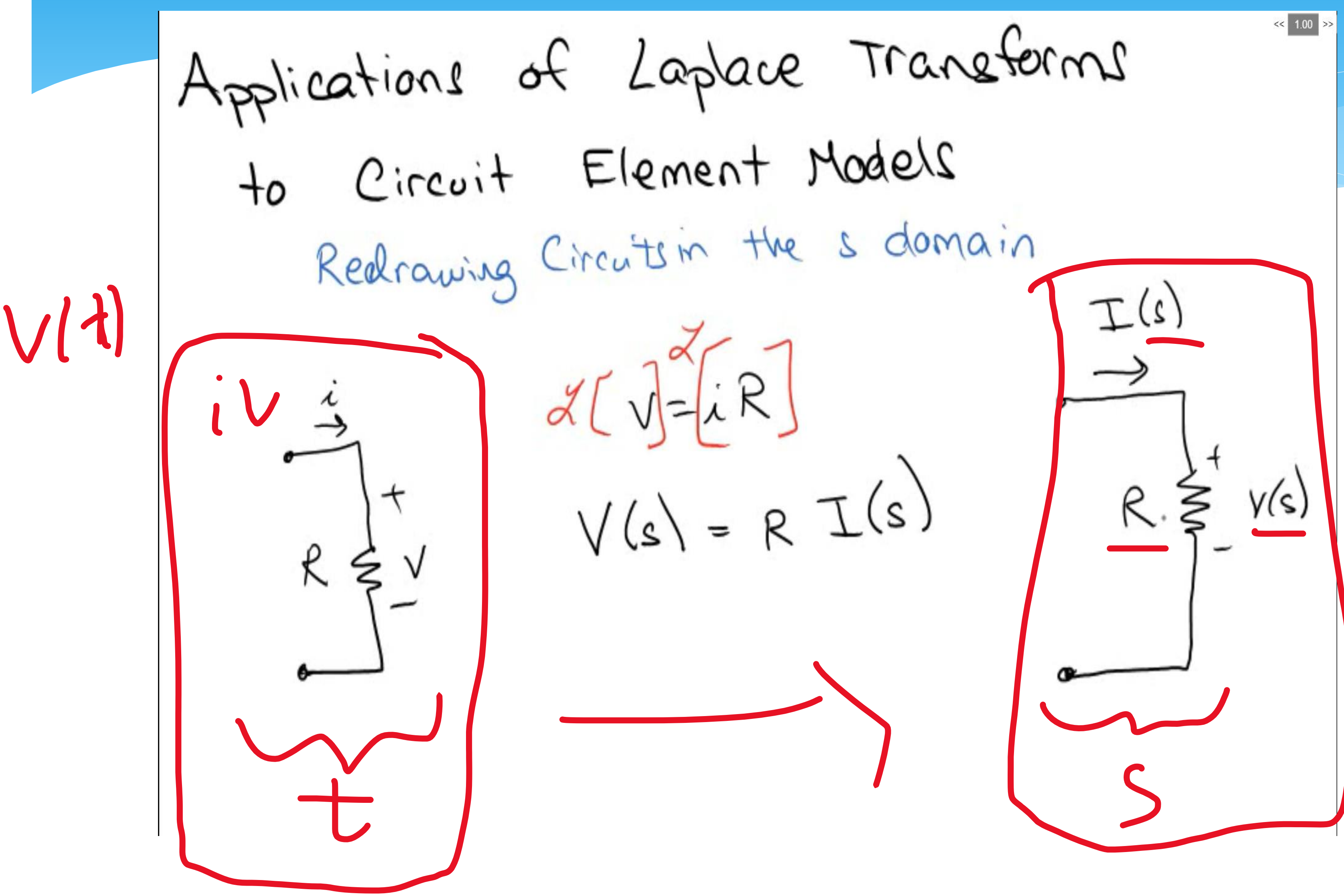
L(s)

X(s)

a, m, n

Again, the utility of this theorem lies in not having to take the inverse of F(s) in order to find out the final value of f(t) in the time domain. This is particularly useful in circuits and systems.

RESISTOR



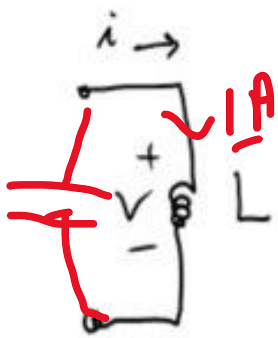
Applications of Laplace Transforms
to Circuit Element Models
Redrawing Circuits in the s domain
v(t)
i v
i
+
R
v
−
t
ℒ[v] = ℒ[iR]
V(s) = R I(s)
I(s)
+
R
V(s)
−
s

D.E. → Alg

INDUCTOR

$i = \frac{dq}{dt}$ $Q = CV$

L, C



$$\mathcal{L}[v] = \mathcal{L}\left[L \frac{di}{dt}\right]$$

$$V(s) = L\left[sI(s) - i(0^-)\right]$$

$$V(s) = sL\, I(s) - L\,i(0^-)$$

$$\boxed{V = R_L I - C}$$

$i(0) = i_0$

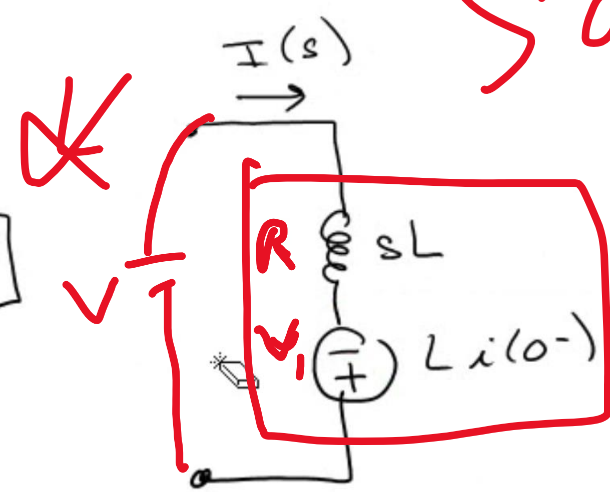
We can separate the terms into two parts:
One a (resitance x current) and the constant as a volatge source
And note the sign of constant terms ans notice where the higher and lower potential of Li(0-) are marked.

S-domain

$$\mathcal{L}[v] = \mathcal{L}\left[L \frac{di}{dt}\right]$$

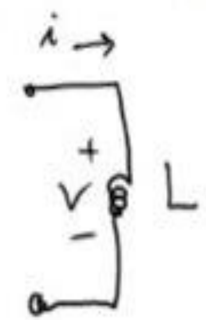
$$V(s) = L\left[sI(s) - i(0^-)\right]$$

$$V(s) = sL\,I(s) \underbrace{- Li(0^-)}_{\text{voltage}}$$



D.E. $t \} \downarrow i_{0^-}$ $\longrightarrow$ $V = IR$

$V = isL + V_1$

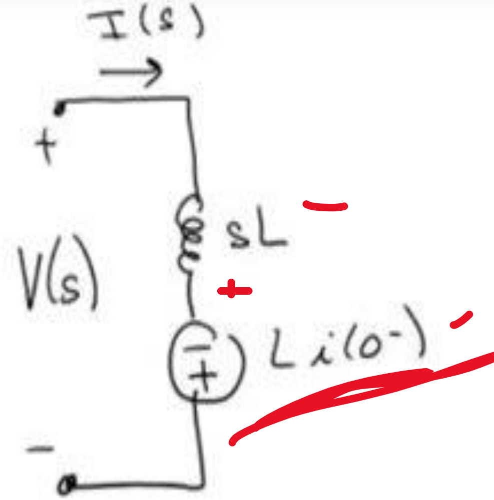


$i(0) = i_0$

$$\mathcal{L}[v] = \mathcal{L}\left[L \frac{di}{dt}\right]$$

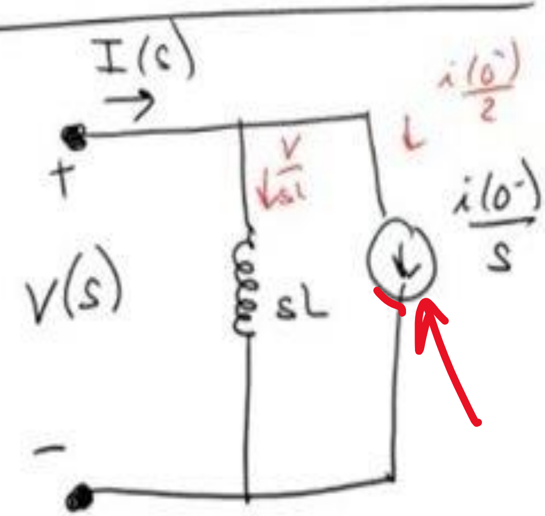
$$V(s) = L\left[s I(s) - i(0^-)\right]$$

$$V(s) = sL\, I(s) - \underbrace{L i(0^-)}_{\text{voltage}}$$



$$I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}$$

$\frac{V}{R}$



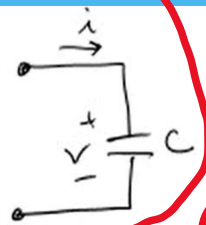
$$V(s) = sL\, I(s)$$

$$I(s) = \frac{V(s)}{sL}$$

$$X_C = \frac{1}{\omega C}$$

$$X_L = \omega L$$

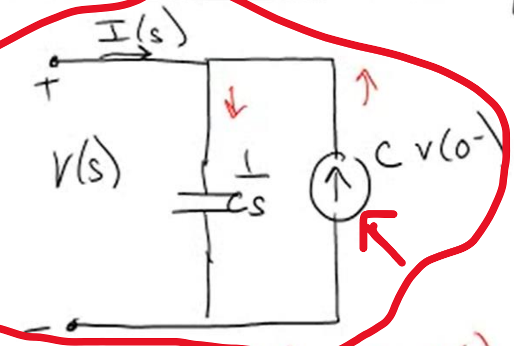
CAPACITOR



$$\mathcal{L}[i] = \mathcal{L}\left[C \frac{dv}{dt}\right]$$

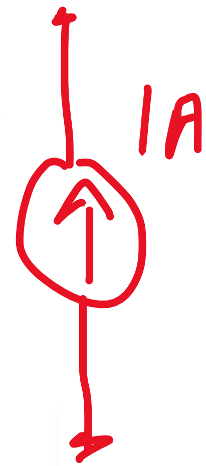
$$I(s) = C\left[sV(s) - v(0^-)\right]$$

$$\boxed{I(s) = CsV(s) - Cv(0^-)}$$

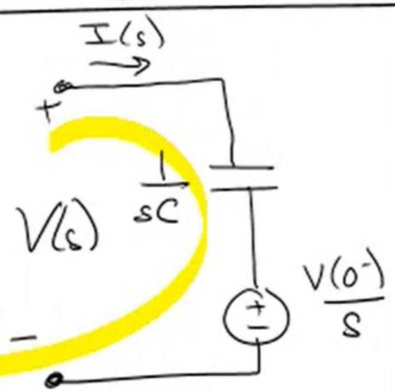


$$i = C\, dV/dt$$

$$I(s) + Cv(0^-) = CsV(s)$$



$$V(s) = \frac{1}{sC} I(s) + \frac{v(0^-)}{s}$$



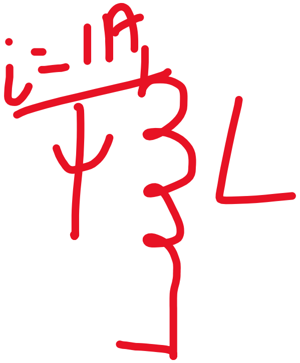
$$-V(s) + I(s)\left(\frac{1}{sC}\right) + \frac{v(0^-)}{s} = 0$$

$$V(s) = \frac{I(s)}{sC} + \frac{v(0^-)}{s}$$

Lapla AC

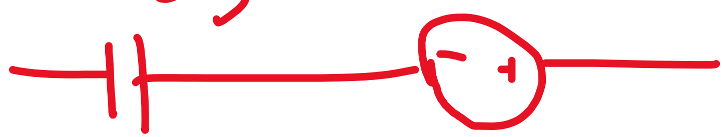
R → R

$i = 1A$



L → sL $Li(0^-)$

C → $\frac{1}{Cs}$ $v(0^-)/s$



Derivation YT video

* Laplace Transforms of Circuit Elements-

https://youtu.be/QlM1dC2gBLM

RESISTOR

* Consider the Ohm's Law in time domain

$$\boxed{v_R(t) = i_R(t)R}$$

* Apply the Laplace transform

$$\boxed{V_R(s) = I_R(s)R}$$

INDUCTOR

* Inductor's voltage
  * In the time domain:

$$v_L(t) = L\frac{di}{dt}$$

  * In the s-domain:

$$\boxed{V_L(s) = L[sI_L(s) - i_L(0^-)]}$$

CAPACITOR

* Capacitor's current
  * In the time domain:

$$i_c(t) = C\frac{dv}{dt}$$

  * In the s-domain:

$$I_c(s) = C[sV_c(s) - v_c(0^-)]$$

IMPEDANCE & ADMITANCE

T exp

$$Z(s) = V(s) / I(s)$$

jωL

→ AC

* The impedances in the s-domain are

1/jωC

$$Z_R(s) = R$$

$$Z_L(s) = sL$$

$$Z_C(s) = \frac{1}{sC}$$

R

ωL

$\frac{1}{\omega C}$

1/Z

* The admittance is defined as:

l

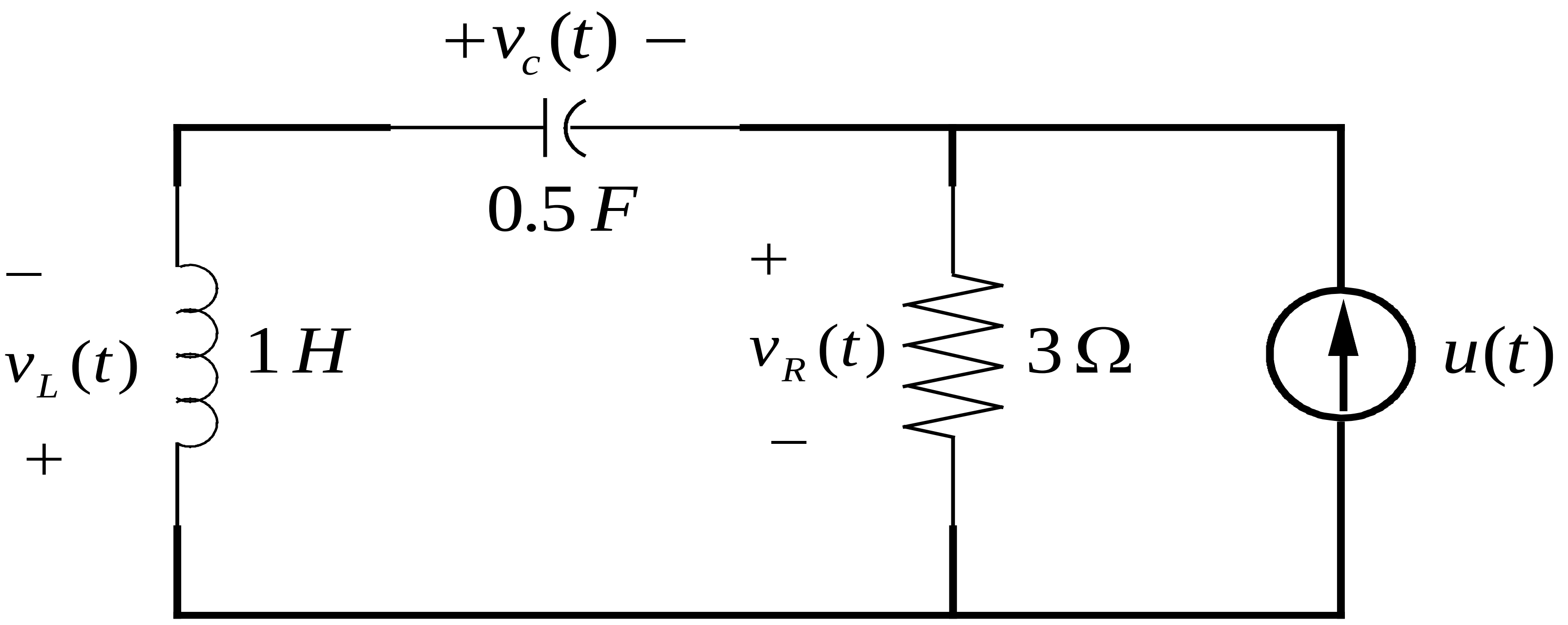
Conductance

$$Y_R(s) = \frac{1}{R}$$

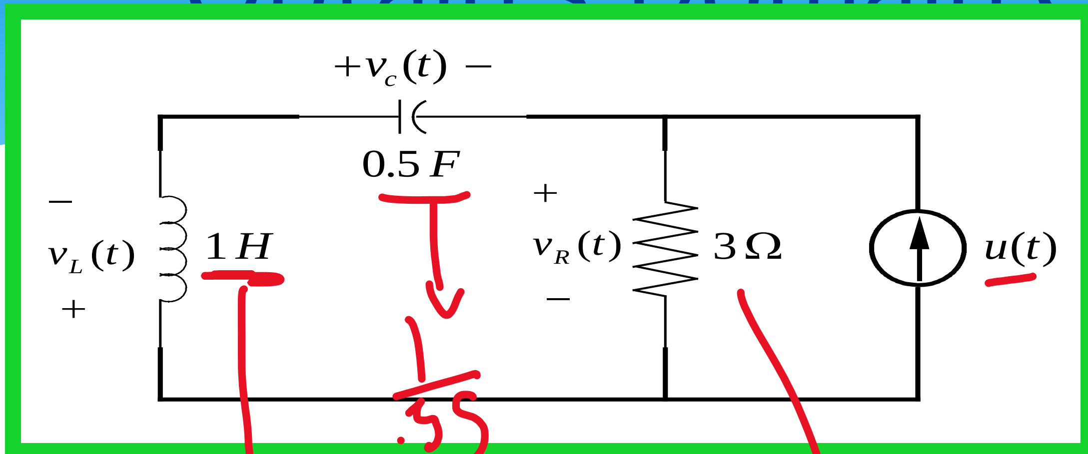
$$Y_L(s) = \frac{1}{sL}$$

$$Y_C(s) = sC$$

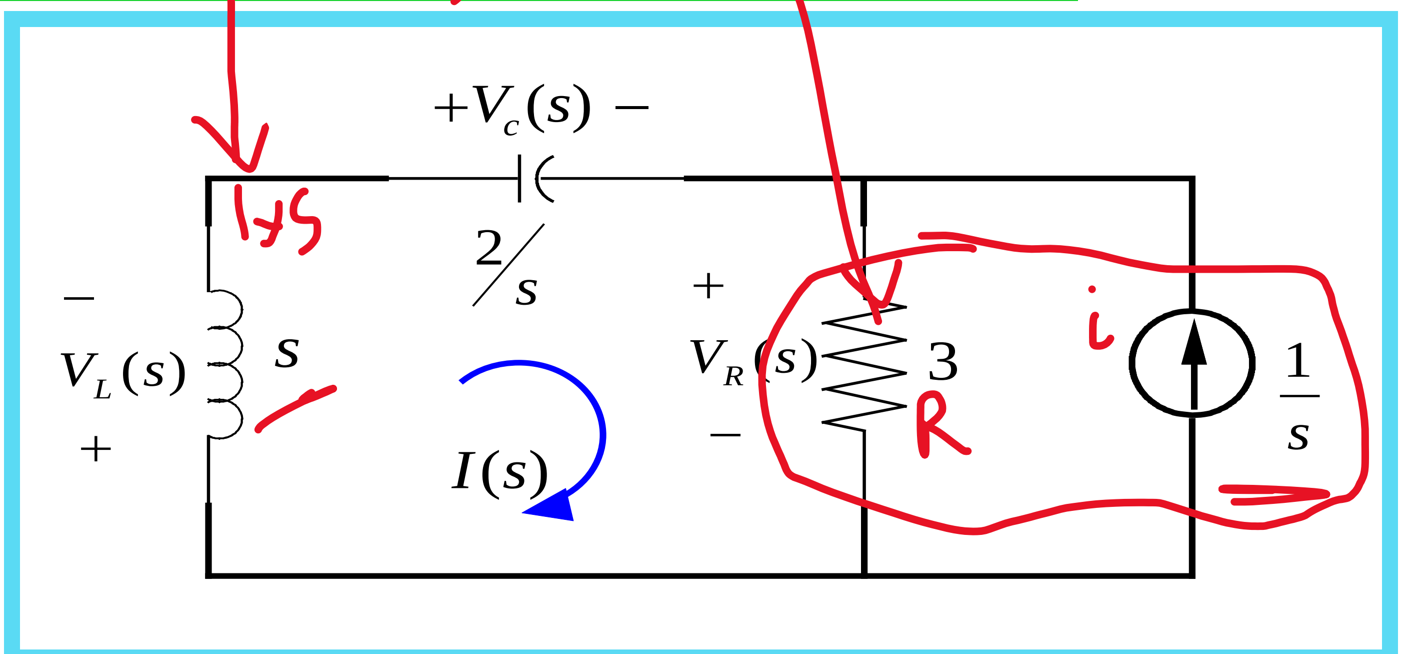
Ex. 1



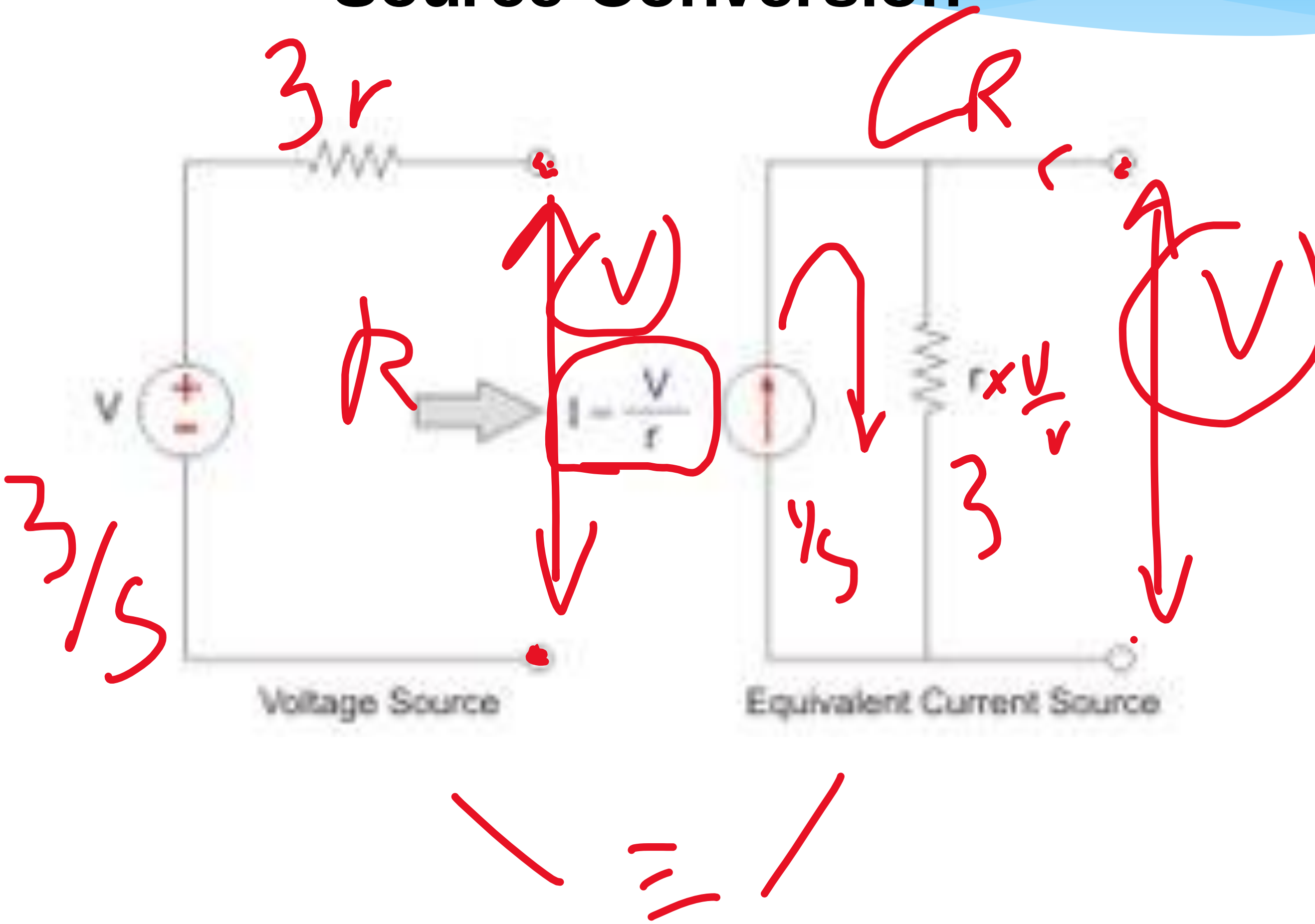
Obtain s-Domain Circuit



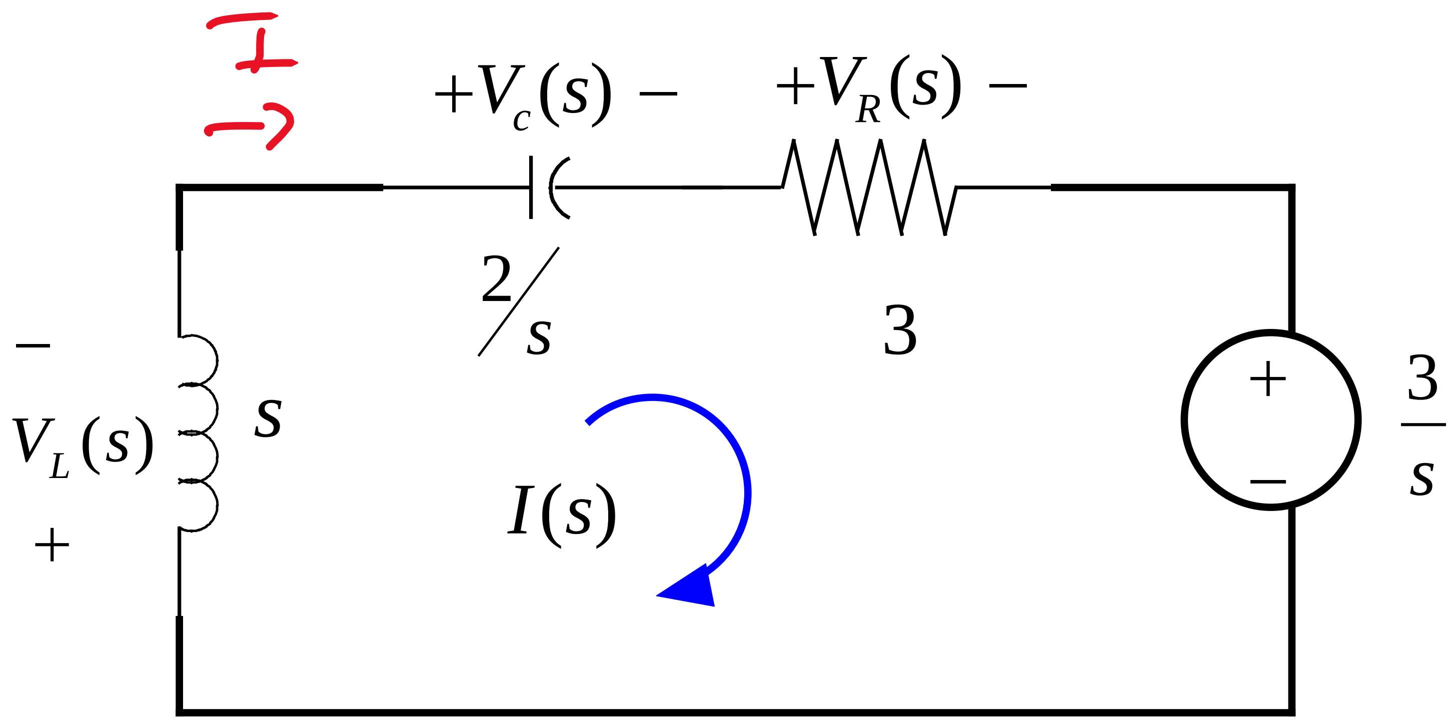
e for t<0



Source Conversion



Convert to voltage sourced s-Domain Circuit



KVL

Find I(s)

$$\text{By KVL}: \left(s + \frac{2}{s} + 3 \right) I(s) + \frac{3}{s} = 0$$

$$\Rightarrow I(s) = \frac{-3}{s^2 + 3s + 2}$$

PFE

t

Find Capacitor’s Voltage

* The capacitor’s voltage:

$$V_c(s) = \frac{2}{s} \cdot I(s) = \frac{-6}{s(s^2+3s+2)}$$

IR

* Rewritten:

$$V_c(s) = \frac{-6}{s(s^2+3s+2)} = \frac{-6}{s(s+1)(s+2)}$$

Using PFE

* Expanding $V_c(s)$ using PFE:

$$V_c(s) = \frac{-6}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} - \frac{K_3}{s+2}$$

* Solved for K_1, K_2, and K_3:

$$V_c(s) = \frac{-6}{s(s+1)(s+2)} = \frac{-3}{s} + \frac{6}{s+1} - \frac{3}{s+2}$$

Find v(t)

$$V_c(s) = \frac{-6}{s(s+1)(s+2)} = \frac{-3}{s} + \frac{6}{s+1} - \frac{3}{s+2}$$

$t \times 3$

* Using look up table:

$$v_c(t) = \left(-3 + 6e^{-t} - 3e^{-2t}\right)u(t)$$

Summary

Fourier $X(\omega)=\int_{-\infty}^{\infty} \boxed{x(t)} e^{-i\omega t}\,dt$

Laplace $X(s)=\int_{0}^{\infty} x(t)e^{-(\alpha+i\omega)}\,dt \quad s=\alpha+i\omega$

$$X(s)=\int_{0}^{\infty} \boxed{x(t)e^{-\alpha t}}\, e^{-i\omega t}\,dt$$

Laplace transform of x(t) is the fourier transform of x(t)e $^{-\alpha t}$

$$V_R(s)=I_R(s)R$$

$$V_L(s)=L[sI_L(s)-i_L(0^-)]$$

$$I_c(s)=C[sV_c(s)-v_c(0^-)]$$

$$\lim_{s\to\infty} sF(s)=\lim_{t\to 0} f(t)=f(0)$$

$$\lim_{s\to 0} sF(s)=\lim_{t\to\infty} f(t)=f(\infty)$$

$$Z_R(s)=R$$

$$Z_L(s)=sL$$

$$Z_C(s)=\frac{1}{sC}$$

SOME ADDITIONAL NETWORK ANALYSIS TERMINOLOGIES

Transfer Function

$$\text{Transfer function} = G(s) = \left.\frac{\mathscr{L}[\text{output}]}{\mathscr{L}[\text{input}]}\right|_{\text{zero initial conditions}}$$

$$= \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

- When order of the denominator polynomial is greater than the numerator polynomial the transfer function is said to be ‘proper’.

- Otherwise ‘improper’

Transfer Function

- Transfer function can be used to check
  - The stability of the system
  - Time domain and frequency domain characteristics of the system
  - Response of the system for any given input

Stability of Control System

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

* Roots of denominator polynomial of a transfer function are called ‘poles’.

* The roots of numerator polynomials of a transfer function are called ‘zeros’.

Stability of Control System

- Poles of the system are represented by ‘x’ and zeros of the system are represented by ‘o’.
- System order is always equal to number of poles of the transfer function.
- Following transfer function represents n^{th} order plant (i.e., any physical object).

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Stability of Control System

* Poles is also defined as "it is the frequency at which system becomes infinite". Hence the name pole where field is infinite.

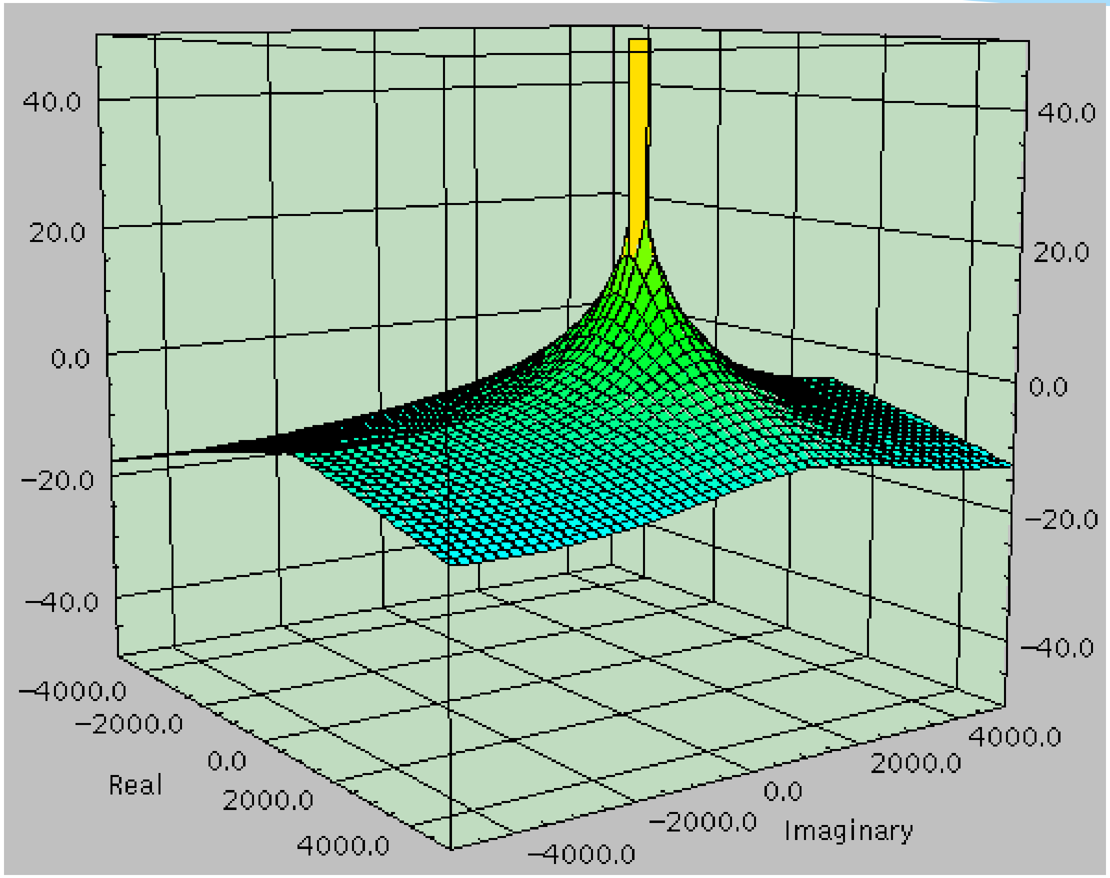
$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

* Zero is the frequency at which system becomes 0.

Relation b/w poles and zeros and frequency response of the system

* The relationship between poles and zeros and the frequency response of a system comes alive with this 3D pole-zero plot.

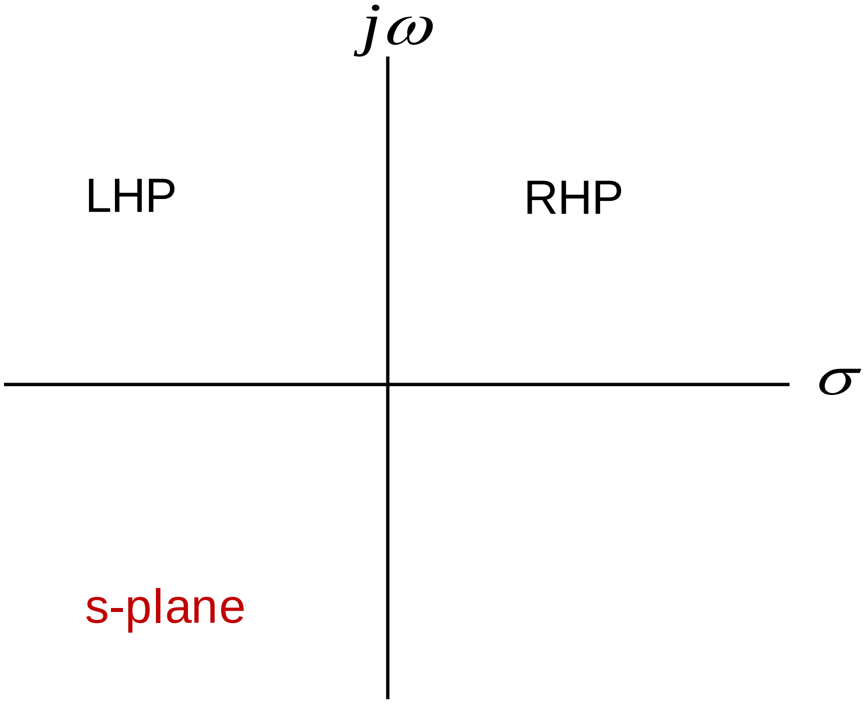
Single pole system



Stability of Control Systems

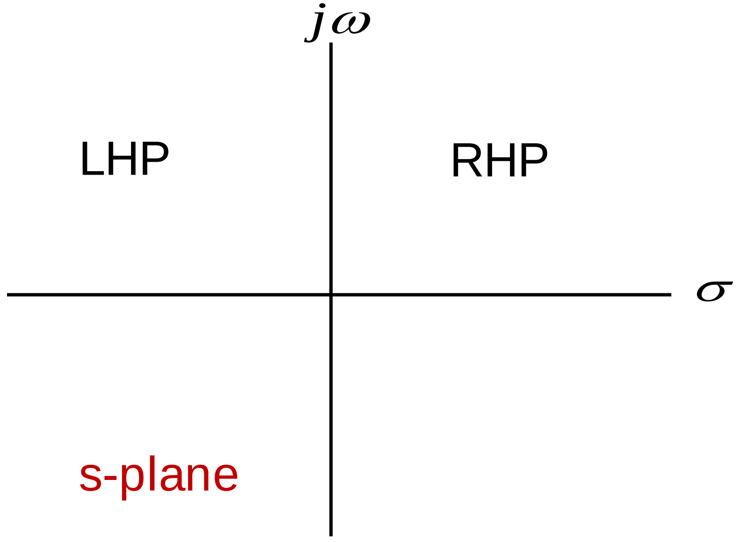
* The poles and zeros of the system are plotted in s-plane to check the stability of the system.

Recall $s = \sigma + j\omega$



Stability of Control Systems

- If all the poles of the system lie in left half plane the system is said to be Stable.
- If any of the poles lie in right half plane the system is said to be unstable.
- If pole(s) lie on imaginary axis the system is said to be marginally stable.



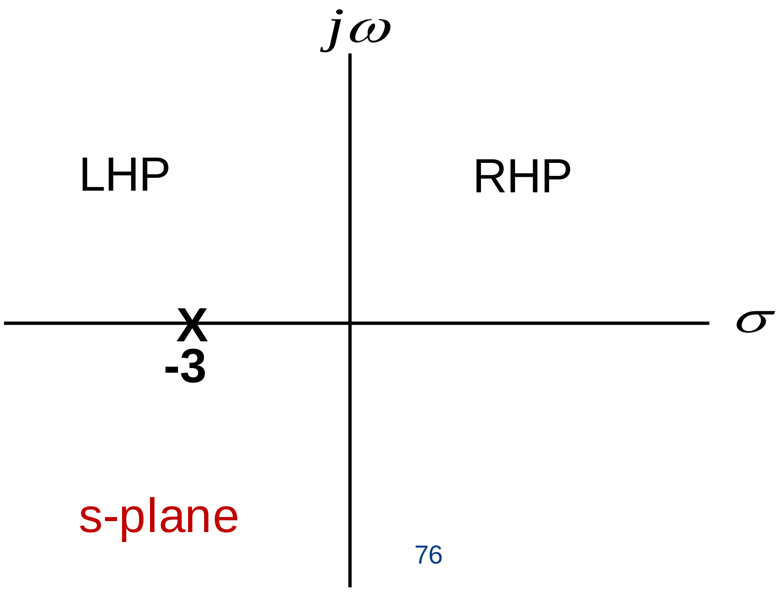
Stability of Control Systems

* For example

$$G(s) = \frac{C}{As + B}, \qquad \text{if } A = 1, B = 3 \; and \; C = 10$$

* Then the only pole of the system lie at

$$pole = -3$$



Examples

* Consider the following transfer functions.
  * Determine
    * Whether the transfer function is proper or improper
    * Poles of the system
    * zeros of the system
    * Order of the system

i) $$G(s) = \frac{s+3}{s(s+2)}$$

ii) $$G(s) = \frac{s}{(s+1)(s+2)(s+3)}$$

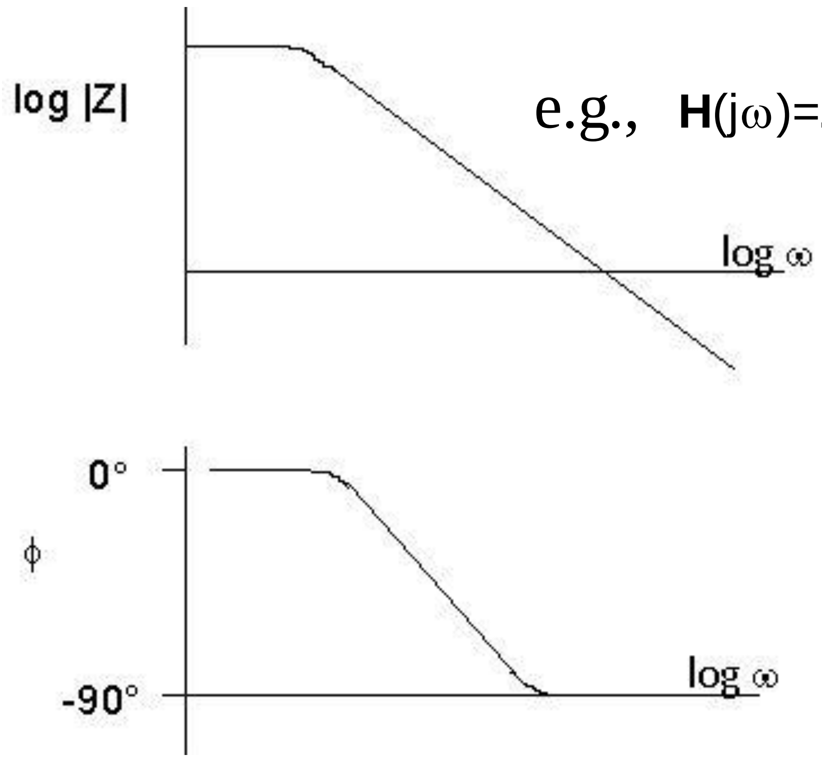
iii) $$G(s) = \frac{(s+3)^2}{s(s^2+10)}$$

iv) $$G(s) = \frac{s^2(s+1)}{s(s+10)}$$

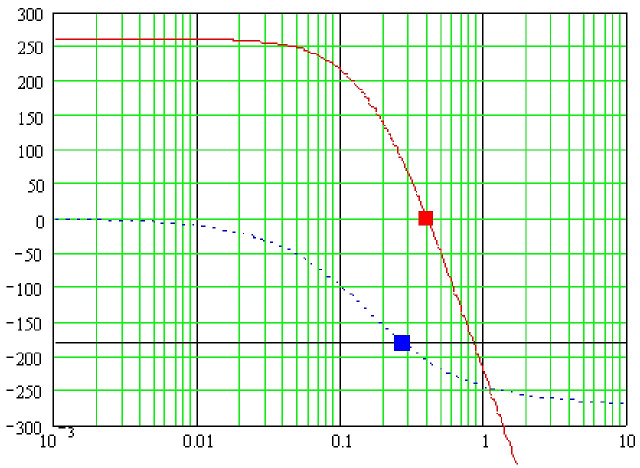
Frequency Response

* The transfer function can be separated into magnitude and phase angle information

$$H(j\omega) = |H(j\omega)| \angle \Phi(j\omega)$$



e.g., $H(j\omega)=Z(j\omega)$



Bode Plots

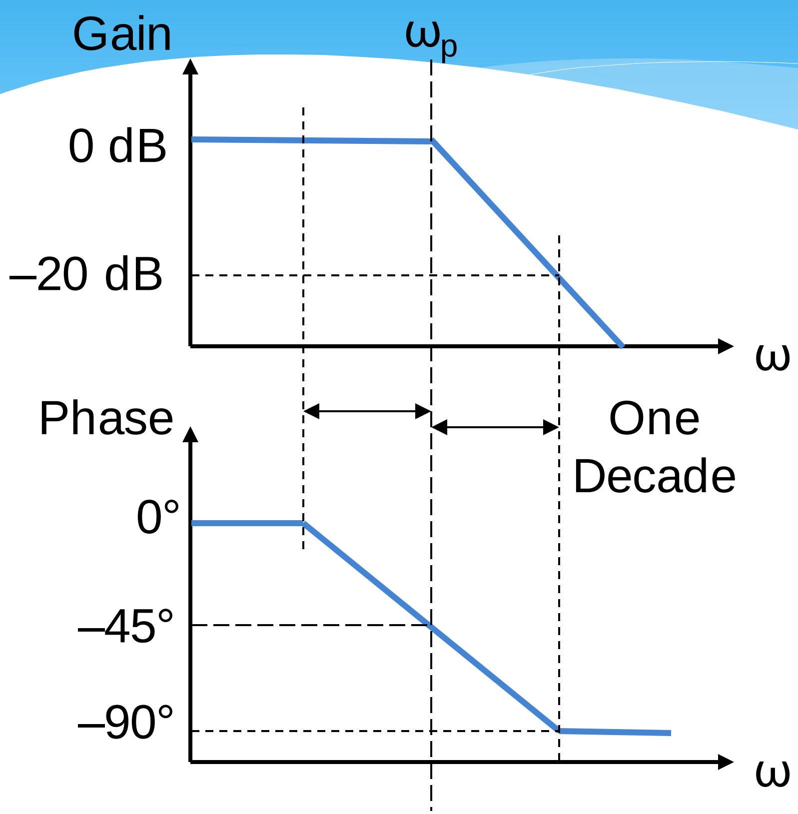
- A **Bode plot** is a (semilog) plot of the transfer function magnitude and phase angle as a function of frequency
- The gain magnitude is many times expressed in terms of decibels (dB)

$$dB = 20 \log_{10} A$$

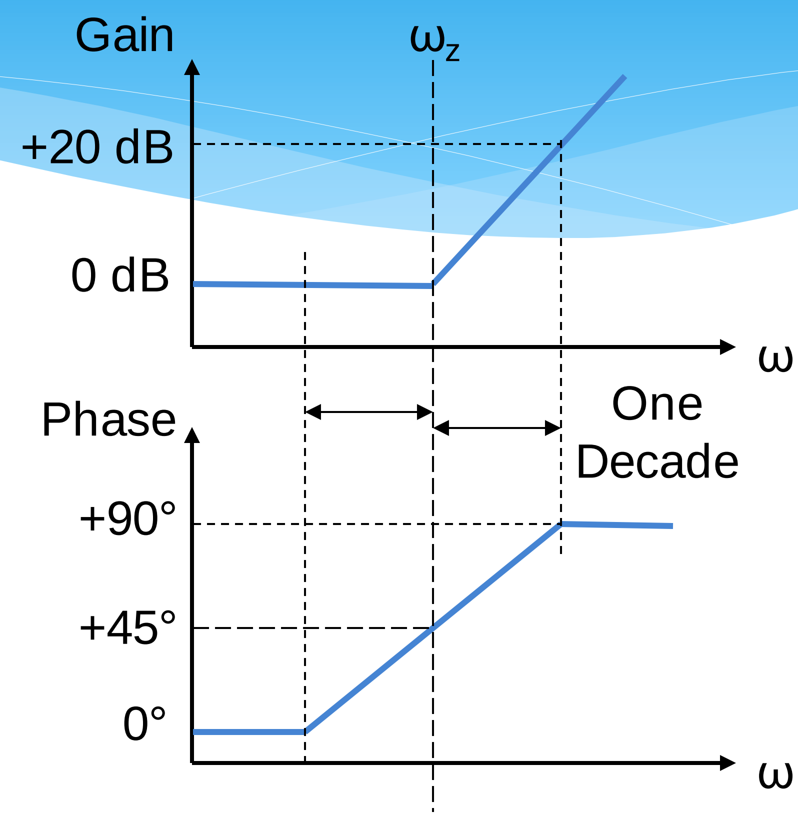
  where A is the amplitude or gain
  - a *decade* is defined as any 10-to-1 frequency range
  - an *octave* is any 2-to-1 frequency range

$$20 \text{ dB/decade} = 6 \text{ dB/octave}$$

Single Pole & Zero Bode Plots



Pole at
$\omega_p = 1/\tau$



Zero at
$\omega_z = 1/\tau$

THANK YOU

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