Project in Applied statistics
Determination of Earth's gravity

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Applied statistics 2022 - Project 1 - Group 24
(Dated: December 14, 2022)

This project aims to determine the gravitational acceleration, g, through two different experimental setups. The first experiment is a measurement of the simple pendulum while the other experiment is a measurement of a ball rolling down an incline. For each of the experiments, the measurements were conducted independently and include estimated uncertainties. Using statistical analysis, the value of g, is produced for each of the experiments. The pendulum value for g is shown to be $g = 9.831 \pm 0.004 m/s^2$. Two g values were obtained from the rolling ball on an incline experiment $g = 9.577 \pm 0.014 m/s^2$ for the big ball and $g = 9.410 \pm 0.014 m/s^2$ for the small ball. The results' deviation from the real value implies systematic errors in the ball on incline experiment. The pendulum experiment has therefore shown to be the most precise as it comes closest to the gravitational acceleration of Copenhagen being; $g = 9.815 m/s^2$ where said experiments were conducted.

INTRODUCTION

The purpose of this project is to determine the gravitational acceleration, g, in Copenhagen, Denmark with thoughtful considerations to achieve the highest precision and accuracy. To attempt this, two different experiments were conducted with multiple repeats. In the first experiment, g was found by measuring the lengths in multiple ways ($L_1 - L_4$, shorthanded as L_x) and the period (T) of a swinging pendulum (see Figure 1.A). The period for small angles is given in Eq. 1.1.

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{1.1}$$

While g is given in Eq. 1.2.

$$g = L\left(\frac{2\pi}{T}\right)^2 \tag{1.2}$$

The second experiment aimed to determine g using a ball on an incline. In this setup, g is calculated using the formula in Eq. 1.3

$$g = \frac{a}{\sin(\theta \pm \Delta\theta)}\left[1 + \frac{2}{5}\frac{R^2}{R^2 - d^2}\right] \tag{1.3}$$

Here a is the measured acceleration of the center of mass (measured using the distance and time between the light gates), θ is the angle of the incline with respect to the table, $\Delta\theta$ is the angle of the table to the earth, R is the radius of the ball and d is the width of the track it rolled on. The setup can be seen in Figure 1B and Figure 1C.

If we assume that there are no correlations between the measurements of the different parameters, the covariance matrix will be diagonal and the equation for error propagation can be reduced to Eq. 1.4

$$\sigma_f^2 = \sum_i^n \left[\frac{\partial f}{\partial x_i}\right]^2_{\bar{x}=\bar{y}} \sigma_i^2 \tag{1.4}$$

The angle $\Delta\theta$ in Eq. 1.3 can be calculated by measuring the angle of the incline one way, then turning the setup 180°, horizontally, redoing the measurement, and finding half the difference. We may also find a value from Eq. 1.3 by setting the terms with different signs for $\Delta\theta$ equal and solving for $\Delta\theta$. The expression is the following:

$$\Delta\theta = \frac{a_{norm} - a_{rev}}{a_{norm} + a_{rev}} \cdot \tan(\theta) \tag{1.5}$$

where a is the accelerations and θ is the angle of the setup assuming no table incline. This is found using trigonometric calculations.

METHODS

In both of the experiments, the measurements were performed individually by three members of the group. To minimize biasing and to create independent measurements each member's measurements were recorded before the results were disclosed. Each of the three group members also provided a subjective estimate of the uncertainty on every measurement they conducted.

Pendulum

The setup was a small weight attached to the ceiling of the stairway by a fishing line in building K at NBI. The lengths relevant to the experiment, see Figure 1A, were measured before the period. All measured values can be seen in the appendix: Table II and Table III. The lengths, L_x, of the pendulum setup were measured with both a laser and a measuring tape. The height of the pendulum was measured with a caliper. The center of mass of the pendulum weight was simply assumed to be at the center of the weight. The mass of the hook on the weight and the mass of the fishing line were assumed to have a negligible impact on the center of mass. The distance to the center of mass can be obtained by dividing the length of the weight by two. The period is obtained when the pendulum is freely swinging back and forth. The pendulum's swinging motion is initiated by displacing it by a small distance. By doing so, the small angle approximation applies and Eq. 1.1, can be used. The timing measurements are performed with the provided pyhton-script "stopwatch.py". The period was recorded 30-55 times per person.

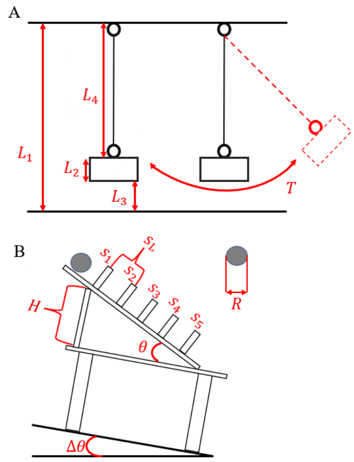

Figure 1: Schematic of the two experimental setups, with their respective variables measured. **A:** The pendulum schematic. **B:** The ball on incline schematic. **C:** The the ball on incline schematic seen from above.

Ball on an incline

The ball on incline experiment requires several measurements. Firstly, the angle of the inclined track was measured in two different ways: with trigonometry by measuring the length and the height of the track or with a clinometer. Accounting for an imperfect clinometer, the angle is first measured one way after which the clinometer is flipped 180 deg and the angles is measured again. Afterwards, the ramp is turned 180 deg to be able to account for possible inclination of the table. The setup can be seen in Figure 1B.

The ball is released from somewhere at the top of the track and is allowed to roll freely through 5 voltage-sensors (s_x) measuring at 5 kHz and the data is collected with the Waveform software. and the sensor positions are measured with a long ruler and is kept on the track in order to get independent results. The position of the sensor is defined to be at the start of the sensor as it is not possible to determine the position of the light sensor with the used ruler. The width of the track, d, is measured along with the diameter of the two different ball sizes (R), as seen in Figure 1C. The experiment is repeated five times with each ball.

RESULTS

Pendulum

There is one measurement of L_4 that is quite different from the others, see table II, and correction for possible systematic errors (i.e. the length of the pendulum added to the measurement) does not correct for this. Removing this data would give way too low statistics, therefore we will not continue using L_4 and not do any calculations for the direct measurement of the fishing line length. In table III the χ^2 value is very high, and the χ^2 probability is zero. This indicates that the estimated uncertainties are too low, and therefore they need to be corrected. This is done by finding the standard deviation, std, of the three measurements and using this as uncertainty for all three measurements.

The complete length L is determined from three different length measurements: From floor to ceiling, L1, the width of the weight, L2, and from the floor to the bottom of the weight, L3. A weighted average for each of the three lengths is determined and used to calculate the full length of the pendulum with 1.6:

$$L = L_1 - L_3 - \frac{L_2}{2} \tag{1.6}$$

The estimated uncertainties for these measurements were used to propagate the errors onto the complete L using equation 1.7:

$$\sigma_L = \sqrt{\sigma_{L_1}^2 + \sigma_{L_3}^2 + \frac{1}{4}\cdot\sigma_{L_2}^2} \tag{1.7}$$

Combining these lengths derived from the measuring tape and the laser using Eq.1.6, a weighted mean between these gives a final length L:

$$L = 18.450m \pm 0.008m$$

In Figure 2, the time measurements from one dataset are shown. Here the number of measurements is plotted as a function of time elapsed. To determine the period, T, a linear function is fitted to the data and from this, residuals can be extracted. The residuals are fitted with an unbinned likelihood fit to a Gaussian pdf from which a standard deviation, σ, can be determined. This is shown

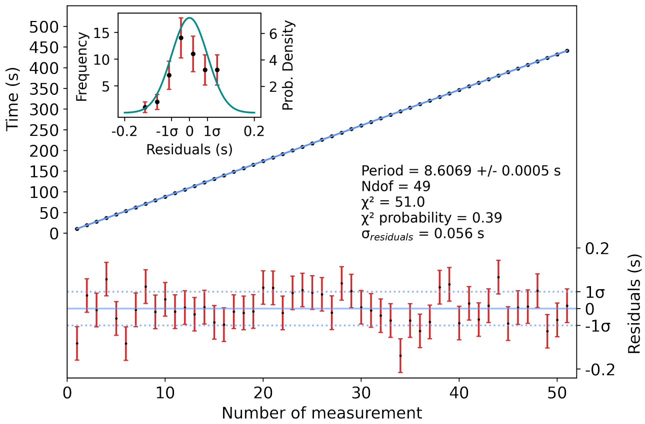

Figure 2: Plot of timing measurements with a linear fit, where the slope of the fit is the period, T. Errors on the data points are the std of the residuals. The residuals are plotted in the bottom compared to $\pm 1\sigma$. The insert shows the distribution of the residuals and a Gaussian pdf obtained from an unbinned likelihood fit.

in the corner of Figure 2 where the residuals are plotted in bins. The newly derived σ is used as errors on the data points, and the data is fitted with a linear fit again and χ^2 value and χ^2 probability is derived. This is to get an accurate error on the period. The slope of the newly fitted line to the data is equal to the period, T. The same procedure is used on the other data-sets and a weighted mean of each individual calculated T yields the final:

$$T = 8.6077s \pm 0.0002s$$
$$\chi^2\text{-value} = 13.6, \chi^2\text{-probability} = 0.0011$$

Now with Eq.1.2, g can be calculated and the error can be calculated with Eq.1.8.

$$\sigma_g = \sqrt{\frac{64\pi^4 L^2}{T^6}\sigma_T^2 + \frac{16\pi^4}{T^4}\sigma_L^2} \tag{1.8}$$

This is done in two different ways. First, the g of the fishing line measured with laser and the g of the fishing line measured with measuring tape is calculated into a weighted mean g. Second, the two L's are calculated into a final L and a g from this is derived.

$$\text{Weighted mean of the } g\text{'s} = 9.831 \pm 0.004 m/s^2$$
$$g \text{ with weighted mean of L} = 9.831 \pm 0.004 m/s^2$$

Ball on an incline

In the ball on incline experiment, quite a few parameters were calculated along with their errors. Not every parameter was equally easy to calculate and measures had to be put in place to get results with correct errors.

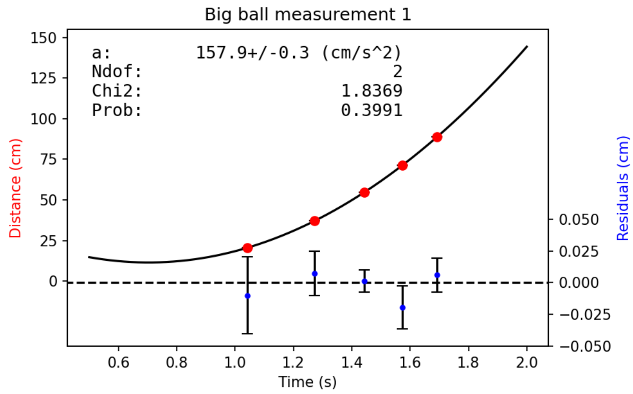

Figure 3: Plot showing the polynomial fit of time and position for the first experiment with the big ball. Values show the acceleration, number of degrees of freedom, χ^2 and p-value.

For the radius for each of the two balls, it was calculated to be the weighted mean of each of the three measurements. Likewise, the rail width is equal to the weighted mean of the measured data. The errors on all three quantities were the weighted errors calculated from the estimated errors (see Table IV in appendix).

The positions of the sensors were measured two times. One time by each member before the ball was dropped down the rail and one time afterward. While expected to be fully correlated (after all, the measured values should be the same), a χ^2 test revealed that they were not mutually consistent and could not be considered to be from the same setup. While several methods of combining the two length measurements were tried, each combination yielded bad fits for the acceleration with χ^2 probabilities less than 0.01 for all runs. Even when abandoning the errors assigned by the members and using standard deviations this still did not resolve the fact that the χ^2 probabilities were too low. Therefore, it was decided that the second length measurements were abandoned. This was done with a heavy heart, but a systematic error must have happened doing the second measuring of the rail. This can be confirmed as the first measurement of the rail resulted in fits of the acceleration with reasonable χ^2 probabilities. See Figure 3 and Table V and VI.

The time measurements, which were used in the fitting of acceleration, were recorded as voltage against time, such that each run presents five distinct peaks corresponding to the ball passing through each sensor. While the peaks are relatively localized, a choice of algorithm to find a time value and uncertainty has to be made. A Full Width Half Max (FWHM) approach was chosen, in which each peak position was calculated as the mean of all values larger than half of the peak's maximum voltage with an error equal to the standard deviation of the peak divided by the square root of the number of measurements. This error turned out to be negligible compared to the errors in the position of the sensors and was therefore ignored when fitting for the acceleration. For each ball and direction, this resulted in five polynomial fits of position against time using iMinuit fitting to Eq.1.9, with each point having an error equal to the error in the position. The weighted mean of the accelerations from these fits was used as a in Eq. 1.3 as well as in the formula for error propagation as seen in Eq. 1.10 in the appendix. Two fits for the small ball had a χ^2 probability of 0 and were excluded from the further calculation. See Figure 3 and Table V and VI.

$$x = \frac{1}{2}at^2 + vt + x_0 \tag{1.9}$$

The angle θ was found from measurements using the clinometer and from trigonometric calculations.

For the clinometer, due to very low uncertainties assigned by the group members on three of the four angle measurements which resulted in χ^2 probabilities on the weighted mean to be zero for all but the first measurement (See appendix Table VII), we used the weighted mean for the first measurements, but the naive mean for the remaining three. The same applies to the errors assigned to each angle. For the first measurement, we used the weighted error and for the remaining three we assigned the error on the angle as $\sigma/\sqrt{N}$. The clinometer measured the angle directly between the rail and the earth which results in no $\Delta\theta$. Instead, an angle is obtained for each direction of the rail. Each direction of the rail had two angle measurements by the clinometer as it is imperfect. Therefore, the true angle was assumed to be right between the two angle measurement, as the systematic error created by the clinometer should be equally large in each direction yielding: $\theta = (\theta_{front}+\theta_{back})/2$ and $\sigma_\theta = \sqrt{\frac{1}{4}\sigma^2_{\theta_{front}} + \frac{1}{4}\sigma^2_{\theta_{back}}}$. More details are mentioned in the Method section and resulting angles in Appendix: Table VIII.

For the trigonometric calculations, θ is found using the weighted mean of the length and height measurements, as well as the measured "right" angle as seen in Table IX. From the trigonometric calculations and the weighted accelerations, $\Delta\theta$ for both balls were calculated using Eq. 1.5. The final angles and their errors can be seen in Table X.

The resulting combined g's for the two balls obtained from the ball on incline are:

$$\begin{aligned} g_{big\ ball} &= 9.577 \pm 0.014 \\ g_{small\ ball} &= 9.410 \pm 0.014 \end{aligned}$$

All the obtained g-values, the χ^2 value, and χ^2-p values can be seen in Table XIV and Table XV

DISCUSSION

The gravitational acceleration in Copenhagen is reported to be g = 9.815 m/s^2 [1]. For the pendulum experiment g = 9.831 $\pm 0.004 m/s^2$. This is 3.55σ away from the reference. The reason for this, is likely due to an underestimation of the errors on the measurements, despite the p-values being reasonable. Systematic errors such as the angle of the ceiling, an assumption of zero mass for the fishing line and hook of the weight, zero friction and air drag, the elasticity of the string etc. might also be part of this deviation. In combination with low statistics on L, this could be an explanation. When looking at the variance values for the g calculated in table I, it shows that L has the largest contribution to the error on g, as seen from the variance contribution to g. Meaning the precision of g is dependent on the measurements and errors on L, which could be improved. Generally, the experiment could be improved by getting more statistics. This would be done by having more measurements of the different lengths and the oscillations with more people and possible with more sophisticated measuring tools that do not require human estimations (since we tend to be imprecise and inconsistent).

Table I: Different variables and their uncertainty from the pendulum experiment used to determine g, and their impact on g.

Variable	Value	Error	Variance
g_{final}	9.831 m/s^2	0.004 m/s^2	
Period T	8.6077s	0.0002s	$2.4 \cdot 10^{-7} m^2/s^4$
Final L	8.434m	0.015m	$1.8 \cdot 10^{-5} m^2/s^4$

For the ball on incline experiment, the value obtained for the big ball was $g = 9.577 \pm 0.014 m/s^2$ while the small ball yielded a value of $g = 9.410 \pm 0.014 m/s^2$. This amounts to being 17.0σ and 29.0σ away from the mean respectively. Clearly, this massive deviation is not simply unlucky and must have roots of systematic nature. Some of this comes from the theoretical approach in which no friction or air resistance was assumed. These effects systematically lowers the acceleration which in turn lowers g, offering explanations as to why the values are so low. For the clinometer calculations, the biggest contribution to the error was from θ, as seen in Table XVI. Not surprising as clinometers tend to not be very precise. For the Trig calculation, the acceleration contributes the most to the error on g, as seen in Table XVII.

CONCLUSION

This project determines the gravitational acceleration from two different experiments. From the simple pendulum experiment the $g = 9.831 \pm 0.004 m/s^2$ and from the ball on an incline $g = 9.577 \pm 0.014 m/s^2$ for the big ball and $g = 9.410 \pm 0.014 m/s^2$ for the small ball. Comparing with the actual value for Copenhagen $g = 9.815 m/s^2$ [1]. The rolling ball experiment points to a systematic error as the results are far from the real value, while the pendulum experiment is both more accurate and precise.

[1] Danish National Space Center, Gravity measurements in Denmark in 2005, (Technical Report no. 6, 2006, May 2006)

[2] Barlow, Barlow, R. (Roger J.). (1999). Statistics: a guide to the use of statistical methods in the physical sciences (Repr.). John Wiley Sons.

APPENDIX

$$\begin{aligned}
\sigma_g^2 = {} & \frac{1}{\sin^2(\theta \pm \Delta\theta)} \left(1 + \frac{2D^2}{5(D^2 - d^2)}\right)^2 \sigma_g^2 \\
& + a^2 \left(1 + \frac{2D^2}{5(D^2 - d^2)}\right)^2 \cdot \frac{\cos^2(\theta \pm \Delta\theta)}{\sin^4(\theta \pm \Delta\theta)} \sigma_\theta^2 \\
& + a^2 \left(1 + \frac{2D^2}{5(D^2 - d^2)}\right)^2 \cdot \frac{\cos^2(\theta \pm \Delta\theta)}{\sin^4(\theta \pm \Delta\theta)} \sigma_{\Delta\theta}^2 \\
& + 16 \frac{a^2 D^2 d^4}{25 \sin^2(\theta \pm \Delta\theta)(D^2 - d^2)^4} \sigma_D^2 \\
& + 16 \frac{a^2 D^2 d^4}{25 \sin^2(\theta \pm \Delta\theta)(D^2 - d^2)^4} \sigma_d^2
\end{aligned} \tag{1.10}$$

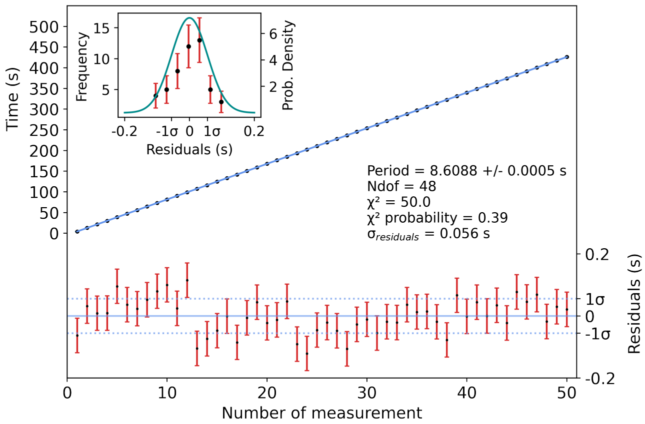

Figure 4: Plot of timing measurements with a linear fit, where the slope of the fit is the period, T. Errors on the data points are the std of the residuals. The residuals are plotted in the bottom compared to $\pm 1\sigma$. The insert shows the distribution of the residuals and a Gaussian PDF obtained from an unbinned likelihood fit.

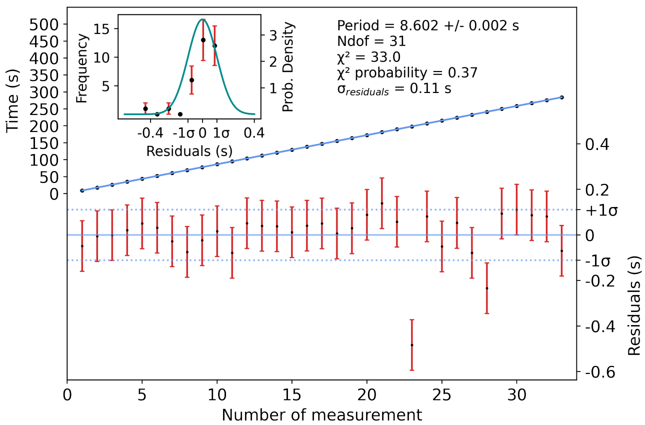

Figure 5: Plot of timing measurements with a linear fit, where the slope of the fit is the period, T. Errors on the data points are the std of the residuals. The residuals are plotted in the bottom compared to $\pm 1\sigma$. The insert shows the distribution of the residuals and a Gaussian PDF obtained from an unbinned likelihood fit.

Table II: Measurements of the line with measuring tape (M.T.) and the weight with a caliper.

	Pendulum to ceiling M.T. [m]		Floor to ceiling M.T. [m]		Floor to pendulum M.T. [cm]		Pendulum height caliper [cm]	
	Measurement	Error	Measurement	Error	Measurement	Error	Measurement	Error
Exp. 1	18.10	0.01	18.760	0.015	31.1	0.1	3.01	0.05
Exp. 2	18.15	0.05	18.60	0.50	31.0	0.1	3.01	0.02
Exp. 3	18.42	0.2	18.74	0.20	31.1	0.1	3.01	0.03
Weighted mean	18.103	0.006	18.760	0.009	31.067	0.033	3.016	0.009
χ^2 value	3.49		0.11		0.67		0.094	
χ^2 probability	0.18		0.95		0.72		0.95	

Table III: Measurements of the line with a laser.

	Floor to ceiling Laser [m]		Corrected floor to pendulum Laser [m]		Floor to pendulum Laser [m]	
	Measurement	Error	Measurement	Error	Measurement	Error
Exp. 1	18.775	0.001	18.78	0.03	0.310	0.001
Exp. 2	18.784	0.001	18.78	0.03	0.311	0.001
Exp. 3	18.830	0.001	18.83	0.03	0.310	0.001
Weighted mean	18.80	0	18.796	0.010	0.310	0
χ^2 value	1740.67		1.93		0.67	
χ^2 probability	0		0.38		0.72	

Table IV: Measured and calculated values for the width of the rail, the big ball diameter and the small ball diameter.

	Width rail [mm]		Big ball diameter [mm]		Small ball diameter [mm]	
	Measurement	Error	Measurement	Error	Measurement	Error
Exp. 1	6.2	0.2	14.95	0.5	12.66	0.012
Exp. 2	6.1	0.1	14.95	0.5	12.65	0.012
Exp. 3	6.1	0.2	14.93	0.05	12.65	0.012
Weighted mean	6.11	0.08	14.93	0.05	12.65	0.01
χ^2 value	0.142		0.00		0.01	
χ^2 probability	0.93		0.99		0.98	

Table V: Big ball on incline experiment calculated Acceleration, χ^2 and χ^2-probability values and the for the 180 degrees turned setup values

Big ball exp.	Acceleration $[cm/s^2]$	χ^2	χ^2-probability
Exp. 1	157.9 ± 0.3	1.84	0.40
Exp. 2	158.4 ± 0.3	3.16	0.21
Exp. 3	158.8 ± 0.3	2.84	0.24
Exp. 4	158.4 ± 0.3	3.65	0.16
Exp. 5	158.6 ± 0.3	2.54	0.28
Weighted average	158.47 $\pm$0.19		
Big Ball turned exp.			
Exp. 1	150.2 ± 0.3	3.65	0.16
Exp. 2	150.2 ± 0.3	2.63	0.27
Exp. 3	150.6 ± 0.3	4.19	0.12
Exp. 4	150.6 ± 0.3	1.96	0.37
Exp. 5	150.5 ± 0.3	3.07	0.22
Weighted average	150.42 $\pm$0.18		

Table VI: Small ball on incline experiment calculated Acceleration, χ^2 and χ^2-probability values and the for the 180 degrees turned setup values

Small ball exp.	Acceleration [cm/s^2]	χ^2	χ^2-probability
Exp. 1	152.9 ± 0.3	27.06	0
Exp. 2	151.7 ± 0.3	2.43	0.30
Exp. 3	153.7 ± 0.3	56.31	0
Exp. 4	150.8 ± 0.3	5.00	0.08
Exp. 5	151.4 ± 0.3	4.90	0.08
Weighted average	151.26 $\pm$0.19		
Small Ball turned exp.			
Exp. 1	144.3 ± 0.3	1.52	0.46
Exp. 2	143.6 ± 0.3	3.39	0.18
Exp. 3	143.6 ± 0.3	0.94	0.63
Exp. 4	144.0 ± 0.3	1.78	0.41
Exp. 5	144.7 ± 0.3	4.20	0.12
Weighted average	144.10 $\pm$0.18		

Table VII: The angle measured and calculated using a clinometer first front and back and then turned and measured front and back again. Due to very small chi square probabilities when taking the weighted mean only the front measurement has weighted mean and error on weighted mean (as indicated by *)

	Front [degree]		Back [degree]		Turned front [degree]		Turned Back [degree]	
	Measurement	Error	Measurement	Error	Measurement	Error	Measurement	Error
Exp. 1	14.50	0.05	14.00	0.01	13.10	0.01	13.90	0.01
Exp. 2	14.50	0.01	14.10	0.01	13.00	0.02	13.50	0.02
Exp. 3	14.50	0.04	13.75	0.02	12.80	0.03	14.00	0.02
Mean	14.50*	0.01*	13.95	0.10	12.97	0.09	13.80	0.15
χ^2 value	0*		2.00		2.00		2.00	
χ^2 probability	1.00*		0.37		0.37		0.37	

Table VIII: The weighted mean and weighted errors for the angle θ on the clinometer from before and after turning the setup. All the angles are in degrees.

	θ	θ_{turned}
Weighted mean	14.23	13.38
Weighted error	0.05	0.09

Table IX: The measured and calculated values of the setup used to make a trigonometry cross-check with the clinometer.

	Length Left [cm]		Height [cm]		Angle [degree]	
	Measurement	Error	Measurement	Error	Measurement	Error
Exp. 1	89.7	0.01	22.1	0.01	89	0.01
Exp. 2	90	0.09	22.15	0.2	89.9	0.1
Exp. 3	89.95	0.05	22.1	0.1	90	0.05
Weighted mean	89.92	0.04	22.10	0.01	89.95	0.01
χ^2 value	5.99		0.06		1.23	
χ^2 probability	0.05		0.97		0.54	

Table X: Calculated values and errors for the angle θ from trigonometry and $\Delta\theta$ for both balls. All the angles are in degrees.

	θ_{trig}	$\Delta\theta_{Big}$	$\Delta\theta_{Small}$
Calculated value	13.808	0.367	0.342
Error	0.008	0.012	0.012

Table XI: The period, T, from three different data-set.

	T [s]	χ^2	χ^2-p
Exp. 1	8.6069 ± 0.0005	51	0.39
Exp. 2	8.602 ± 0.002	33	0.37
Exp. 3	8.6088 ± 0.0005	50	0.39
Weighted mean	8.608 ± 0.0002	13.6	0.0011

Table XII: Gravitational acceleration, g, from different measurements in the pendulum and ball on incline experiment

	Gravitational acceleration $[m/s^2]$	Deviation away from table value $[\sigma]$
Measuring tape (g)	9.822 ± 0.008	0.85
Laser (g)	9.842 ± 0.009	2.87
Weighted mean of the g's	9.831 ± 0.004	3.55
g with weighted mean of L	9.831 ± 0.004	3.57
Big ball (g)	9.577 ± 0.014	17
Small ball (g)	9.410 ± 0.014	29
Gravitational acceleration Copenhagen	9.815	0

Table XIII: Ball on incline rail parameters and their respective weighted mean, χ^2 and χ^2-probability values

	Length rail 1 $[cm]$	Length rail 2 $[cm]$	Length rail 3 $[cm]$	Length rail 4 $[cm]$	Length rail 5 $[cm]$
Exp. 1	20.55 ± 0.05	37.1 ± 0.02	54.75 ± 0.01	71.20 ± 0.05	88.80 ± 0.02
Exp. 2	20.55 ± 0.06	37.15 ± 0.05	54.75 ± 0.05	71.25 ± 0.04	88.85 ± 0.04
Exp. 3	20.57 ± 0.05	37.13 ± 0.05	54.76 ± 0.02	71.22 ± 0.02	88.81 ± 0.02
Weighted mean	20.55 ± 0.03	37.11 ± 0.02	54.75 ± 0.009	71.22 ± 0.02	88.81 ± 0.01
χ^2 value	0.10	1.05	0.20	0.69	1.25
χ^2 probability	0.95	0.59	0.90	0.71	0.54

Table XIV: Gravitational accelerations derived trigonometric and clinometric. The values for big ball and small ball turned are left out for the trigonometric due to them being used as control measurements.

	Trigonometric $[m/s^2]$		clinometeric $[m/s^2]$	
	Measurement	Error	Measurement	Error
Big Ball Original	9.580	0.016	9.55	0.03
Small ball Original	9.416	0.017	9.37	0.03
Big ball turned	-	-	9.62	0.06
Small ball turned	-	-	9.47	0.06

Table XV: Weighted mean, χ^2 and χ^2-probability of the final gravitational accelerations for the incline experiment

	Weighted mean $[m/s^2]$	χ^2	χ^2-probability
Big ball combined	9.577 ± 0.014	1.36	0.51
Small ball combined	9.410 ± 0.014	3.16	0.21

Table XVI: Contributions to variance in g for each parameter for each g made from clinometer angles

Clinometer	Acceleration	θ	Ball Diameter	Rail width
Bigball g	$1.1 \cdot 10^{-4}$	$9.7 \cdot 10^{-4}$	$4.4 \cdot 10^{-6}$	$7.6 \cdot 10^{-5}$
Bigball turned g	$1.1 \cdot 10^{-4}$	$3.2 \cdot 10^{-3}$	$4.5 \cdot 10^{-6}$	$7.8 \cdot 10^{-5}$
Smallball g	$9.9 \cdot 10^{-5}$	$8.6 \cdot 10^{-4}$	$3.3 \cdot 10^{-7}$	$1.1 \cdot 10^{-4}$
Smallball turned g	$1.0 \cdot 10^{-4}$	$2.8 \cdot 10^{-3}$	$3.8 \cdot 10^{-8}$	$1.1 \cdot 10^{-4}$

Table XVII: Contributions to variance in g for each parameter for each g made trigonometric calculations

Clinometer	Acceleration	θ	Ball Diameter	Rail width	$\Delta\theta$
Bigball g	$1.1 \cdot 10^{-4}$	$2.5 \cdot 10^{-5}$	$4.5 \cdot 10^{-6}$	$7.7 \cdot 10^{-5}$	$5.3 \cdot 10^{-5}$
Bigball turned g	$1.2 \cdot 10^{-4}$	$2.8 \cdot 10^{-5}$	$4.5 \cdot 10^{-6}$	$7.7 \cdot 10^{-5}$	$5.9 \cdot 10^{-5}$
Smallball g	$1.0 \cdot 10^{-4}$	$2.3 \cdot 10^{-5}$	$3.3 \cdot 10^{-7}$	$1.1 \cdot 10^{-4}$	$4.8 \cdot 10^{-5}$
Smallball turned g	$1.0 \cdot 10^{-4}$	$2.5 \cdot 10^{-5}$	$3.3 \cdot 10^{-7}$	$1.1 \cdot 10^{-4}$	$5.3 \cdot 10^{-5}$