Event count models

Silje Synnøve Lyder Hermansen

2020-11-19

The dependent variable

Count data

Count data is common in political science

- Discrete: consists only in integers (0, 1, 2, ... no digits)
- Bounded at zero, often long tail upwards.

Count models: What are they good for?

When do we use count models?

The data generating process allows us to

- observe and count a number of events and
- define a time frame or geograhical space for the occurence(s)

$\Rightarrow$ *e.g. number of meetings between decision makers, violent events, legislative proposals, etc.*

Why not a binomial logistic regression?

These are indeed binary outcomes but we don't have information on the event level

$\Rightarrow$ *Variables are on the exposure level; related to when (where) the events took place.*

Why not OLS?

The variable could be approximated to a continuous measure but

- it is bounded at zero, so predictions would be wrong $\rightarrow$ *same problems as logit*
- it is scewed. Some people add a constant and logtransform: $log(y+0.1) \rightarrow$ *but heteroskedasticity and non normal errors remain*

$\Rightarrow$ *We replace the normal distribution with another probability distribution*

The generalized linear model strategy

There are many count models

- Poisson model: the base-line
- Other models: to address problems with the poisson

The Poisson model

Poisson process

The poisson distribution maps probabilities of events within a window to outcomes

- **Exposure $(t, t+h)$:** A window of opportunity between two bounaries (geographical or spacial)
- **Probability of event (λ):** Simply the logtransformed mean of events within that window
  - Probability of event $= h\lambda$
  - Probability of no event $=$ 1 - $h\lambda$

Formula

The equation the model estimates:

$$E(y_i) \equiv h\lambda_i = h \times exp(\alpha + \beta \times x_i) \tag{1}$$

Estimation of the exposure

What to do with the exposure parameter?

$$E(y_i) \equiv h\lambda_i = h \times exp(\alpha + \beta \times x_i) \tag{2}$$

Two strategies :

- **Offset:** Move it into the equation but constrain parameter: $exp(\alpha + \beta \times x_i + 1 \times log(h_i)) \rightarrow$ *we don't see it in the BUTON*
- **Estimate a parameter:** $exp(\alpha + \beta_1 \times x_i + \beta_2 \times log(h_i))$

$\Rightarrow$ *If the exposure is the same for all units, we set it to 1 and ignore it.*

Interpretation: back and forth

Interpretation is relatively easy with all count models

- Recoding (for estimation): we logtransform the mean of the y (within x-values)
- We back-transform (for interpretation): $exp(\lambda)$ is simply an approximation (with digits) of our counts!

Interpretation: effects

Interpretation is relatively easy

- Recoding (for estimation): we logtransform the mean of the y (within x-values)
- We back-transform (for interpretation):
  - Predicted value: $exp(\hat{\lambda})$ is simply an approximation (with digits) of our counts
  - Effect of β: $exp(\beta)$ is multiplicative of predicted $\hat{\lambda} \rightarrow$ easy!

$\Rightarrow$ *Make scenarios, predict, knock yourself out*

Dispersion

The main assumption of the Poisson model

The model assumes equidispersion: The spread equals the mean

- The y can be overdispersed, but not the $\hat{\lambda}$ $\rightarrow$ *as in OLS*

$\Rightarrow$ *The standerd errors will be too small*

Identifying overdispersion

- Poissonness plot
- Rootograms
- Formal tests: Using residuals and significance tests.

Reasons for overdispersion

- Lack of exposure time
- Poor choice of variables (include more, also random intercepts)
- Too many zeros
- Events are related

Adressing overdispersion

The quasi-poisson model

- Adds an additional parameter, ϕ, to the variance estimation $\rightarrow$ *similar to robust standard errors*

$\Rightarrow$ β *remains the same, standard errors are larger*

The negative binomial model

The event is in fact generated by two processes

- $\lambda_i = exp(\beta \times x_i + 1 \times u_i)$
- $v = exp(u_i)$ is in itself generated by a gamma distribution $v_i \sim f\Gamma(\alpha)$
- The latent variable is manipulated directly: the rate increases over y

Exess zeros

Substantially that two data generating processes are at work.

- One producing zeros
- One producing (at least some) positive counts

$\Rightarrow$ *We can model this in two parallel regressions with possibly different* x *or just an additional intercept.*

Hurdle models

Observations have a higher hurdle/threshold/distance to pass in order to obtain a positive count (from 0 to 1) than between positive counts (1 to 2, 2 to 3, etc)

- Hurdle part: A binomial logit where success is $y > 0$
- Count model: A zero-truncated poisson (or negative binomial) on all the positive counts.

$\Rightarrow$ *Can accomodate under-dispersion too.*

Zero-inflated models

There are two sources of zeros, but only one of positive counts.

- Zero-inflated part: A binomial logit where success is the “always zeros”.
- Count model: A poisson or negative binomial that is not truncated.

$\Rightarrow$ *functions as a switch that is turned on/off after a threshold. The observation is then passed to the count-model group.*

Recap on GLMs

What are the criteria for model selection?

You can think of model selection as a set of criteria that should be met

Try out the model selection decision tree to see my mental map!

https://siljehermansen.github.io/teaching/model_choice