RDD and diff-in-diff

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Regression discontinuity design (RDD)

Basic assumption

RDD assumes a running variable (x) with a cut point (c) beyond which treatment is assigned (D).

$$D_i = \begin{cases} 1 & if \quad x_i \geqslant c \\ 0 & if \quad x_i < c \end{cases} \tag{1}$$

Distinction

It has a flavor of logit or propensity scores, but there are some differences:

- ~~**logit**~~ : x (not y) is not latent and we know the cutpoint: Both are observed and included as a *predictors*.
- ~~**matching**~~ : we have no control/treatment group. However, we assume that units on either side of the treatment are increasingly similar as their x is similar.

$\Rightarrow$ *Supposes clear rules with little administrative discretion.*

Examples

Administrative data are perfect: You have some rule that kicks in at a specific threshold for otherwise almost identical observations.

- school test scores on school admission, restrictions on class size
- legal drinking age on alcohol related deaths
- election of candidates in close races

Two ways of understanding RDD

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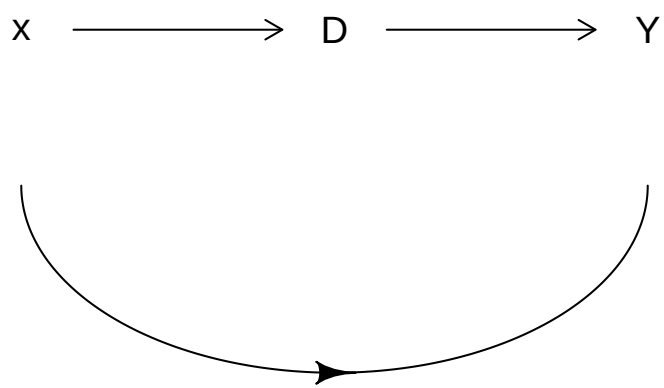
X is a confounder
...so we only control for X

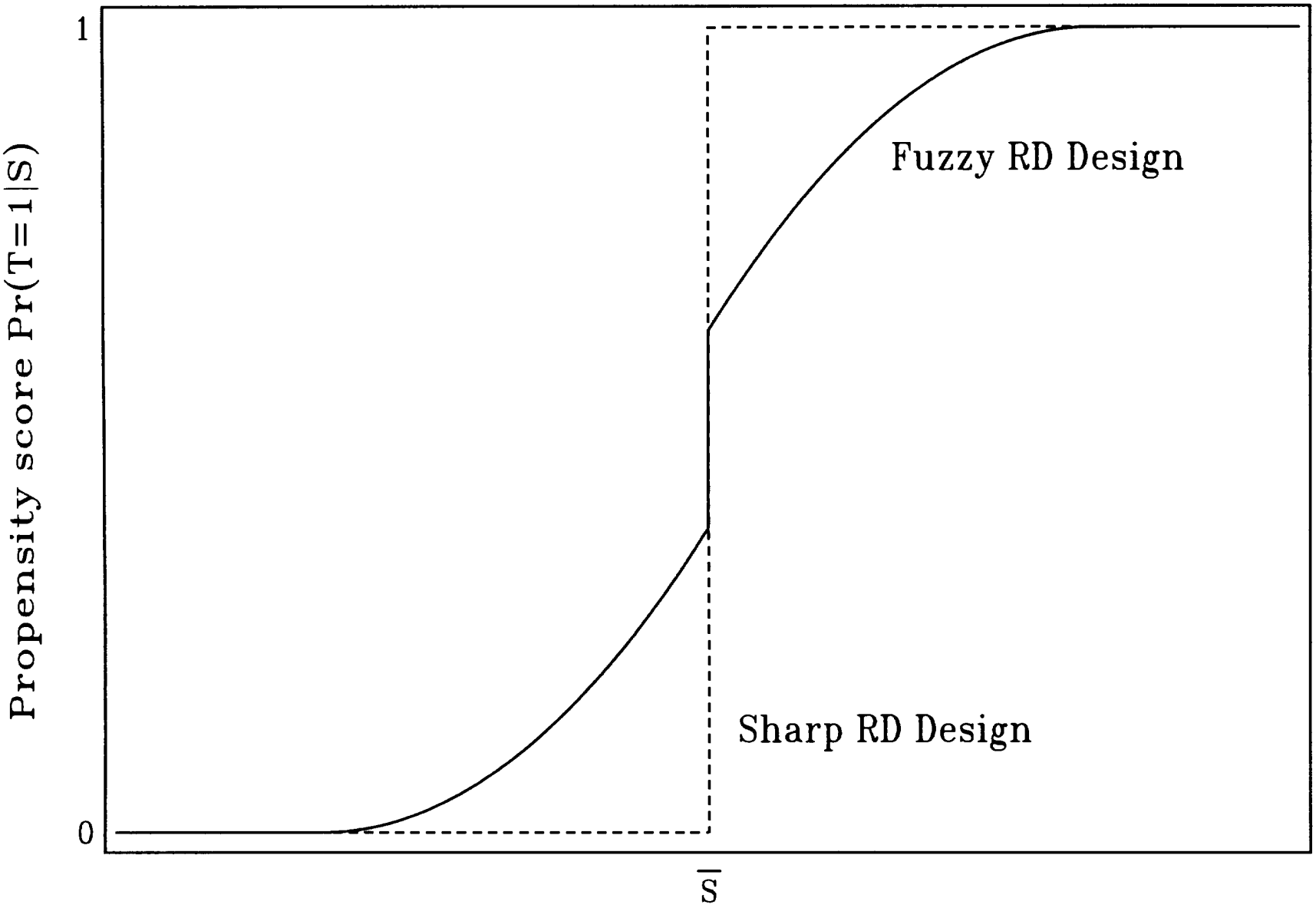


Two designs

We distinguish between two designs depending on how probable the treatment is:

- **sharp** RD: assignment is *deterministic*
- **fuzzy** RD: assignment is *probabilistic*

A visual representation



The basic model

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Do it in R

Check out alcohol related deaths (y) as a function of legal drinking age (D)

```
load("df_ch4.Rda")
mod1 <- lm(alcohol ~ over21 + age,
           df)
```

Is that all?

This is true on two conditions

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2. **no omited variable bias**:x must capture all influence on D.

The continuity assumption

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. . . that's not a regression discontinuity.

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We can obtain a linear effect in two ways:

- recode the $x \rightarrow$ compare with the recoding of y in GLMs.
- consider a sufficiently small window

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```
df$age2 <- df$age^2
mod2 <- lm(alcohol ~ over21 + age + age2,
           df)
```

Recode the x: symmetric effect

We can create a curvilinear effect of x using polynomials (e.g.:)

$$y_i = \alpha + \rho D_i + \gamma_1 x_i + \gamma_2 x_i^2 \tag{4}$$

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df$age2 <- df$age^2
mod2 <- lm(alcohol ~ over21 + age + age2,
           df)
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⇒ *Here, x has a symmetrical effect on both sides of the treatment.*

Recode the x: assymetric effect

We can assume x has different effects on each side of the treatment

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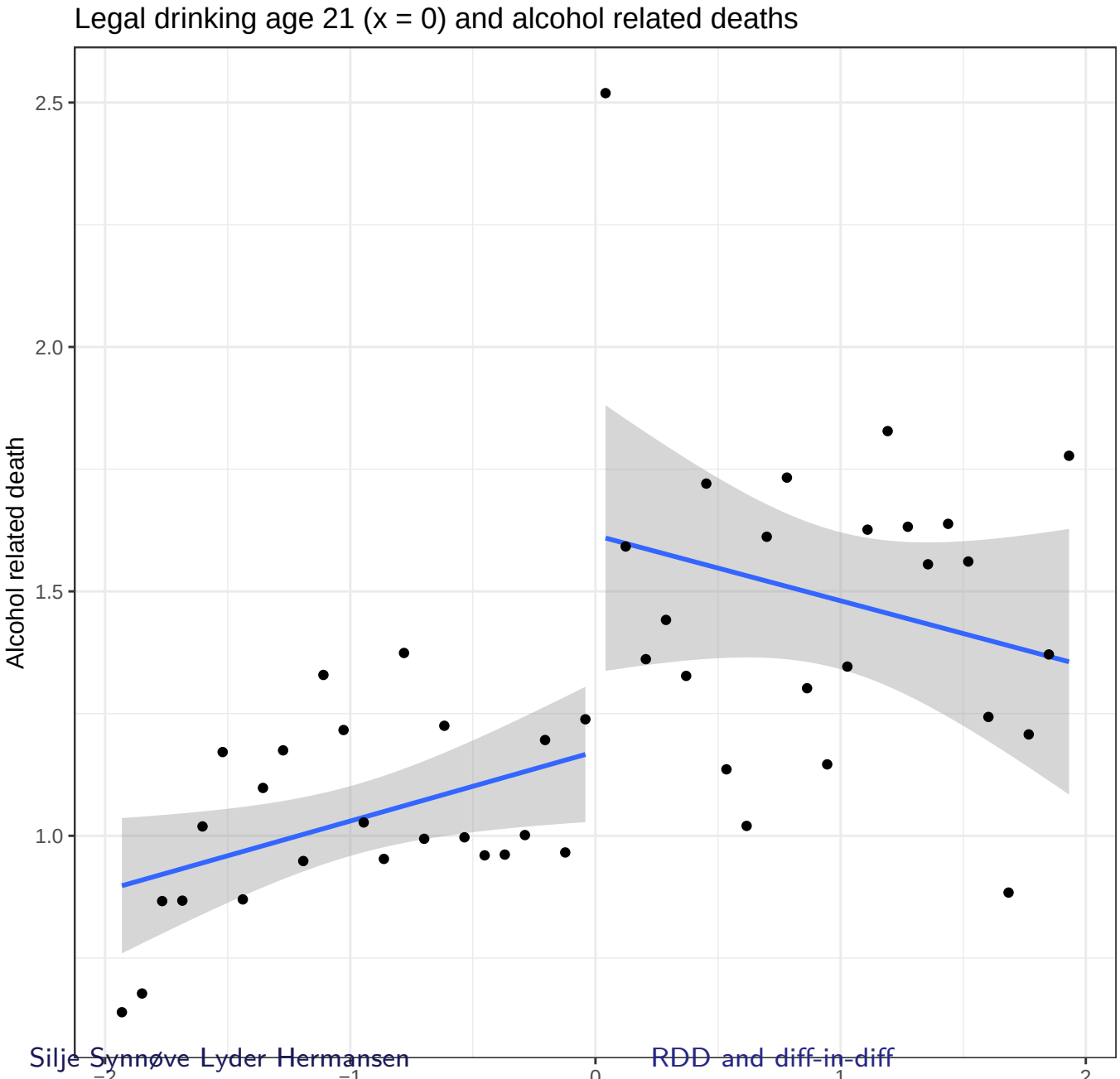
$\Rightarrow$ *we center the x on the cutpoint* $(x_i - c) \rightarrow \rho$ *still reports the change at the cutpoint.*

```
mod3 <- lm(alcohol ~ over21 * age,
           df)
```

Do it in R


Legal drinking age 21 (x = 0) and alcohol related deaths
Alcohol related death
2.5
2.0
1.5
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Extrapolation

We do this to estimate the effect at the cutpoint (ρ)

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- but we can also extrapolate y beyond the cutpoint with x: $\rho + \delta(x - c)$

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- subsetting the data to tweak the window around the cutpoint is a **non-parametric approach.**

Bandwidth : the idea

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- There's a tradeoff between linearity and statistical power (we need sufficient N).

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```
mod5 <- lm(alcohol ~ over21 * age,
           df[df$agecell > 20 & df$agecell < 22,])
```

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mod5 <- lm(alcohol ~ over21 * age,
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⇒ *When you narrow down, do you get a weaker or stronger effect?*

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- We can do it by hand
- . . . or we can make an algorithm do it:
  - run a local weighted regression line
  - bandwidth is estimated accordingly

$\Rightarrow$ *the point is to show robustness, not p-hack!*

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Often the D increases the probability of a treatment, but we don't know!

$\Rightarrow$ *This is a Instrumental Variable approach (more on Thursday)*

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$\Rightarrow \gamma$ is the causal effect of D in a fuzzy design.

Differences-in-differences

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$\Rightarrow$ *Treatment effect is that difference*

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$\Rightarrow$ *based on panel data (units are observed several times).*

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$\Rightarrow$ *What was the treatment effect?*

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$\Rightarrow$ *Basically a 2-by-2 table*

How to do it?

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- T represents the treatment group: differences *between* units
- β_3 is the causal effect

Data

Data requirements

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- Requires panel data $\rightarrow$ which means correcting the standard errors.
- Common panel types: state-year/administrative unit-time period; people over time ...

$\Rightarrow$ *we want to know the trend before and after the break*

Another example: drinking age and death

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Does the legal drinking age has an effect on death rates among the young?

- y is number of deaths per 100 000
- P is post-treatment dummy
- T is dummies for states
- *trend* is year dummies

Another example: step 1 → calculate differences

The authors have two tricks:

- Hardcode the interaction effect (dummy before/after treatment)
- They remove the intercept to retain all dummies

```
load("df2.Rda")
##with intercept
mod <- lm(mrate ~ legal +
            state +
            year_fct,
          df)
##without intercept; with all dummies
mod <- lm(mrate ~ 0 +
            legal +
            state +
            year_fct,
          df)
```

Another example: step 2 → calculate errors

Calculate robust standard errors:

```
library(clubSandwich)
```

```
## Warning: package 'clubSandwich' was built under R version 4
```

```
## Registered S3 method overwritten by 'clubSandwich':
##   method    from
##   bread.mlm sandwich
```

```
vcov <- vcovCR(mod, cluster = df[["state"]],
               type = "CR2")
robust <- coef_test(mod, vcov = vcov)$SE
```

Another example: step 3 → interpretation

Display the results and interpret:

```
library(stargazer)
stargazer(mod, se = robust,
                    omit = "state|year",
                    type = "html",
                    out = "regtable.html")
```

Another example: step 3 → interpretation

Table 1: Death rates among young as a function of legal drinking age

	Dependent variable:
	mrate
Legal drinking age (causal effect)	10.804**
	(4.479)
Observations	714
R^2	0.986
Adjusted R^2	0.985
Residual Std. Error	17.339 (df = 649)
F Statistic	726.005*** (df = 65; 649)
Note:	*p<0.1; **p<0.05; ***p<0.01

⇒ *What did we find?*

The parallel trends assumption

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Units can be different, but – absent treatment – they must follow the same trend (hence the panel data).

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- The regression assumes a counterfactual $\rightarrow$ remember the extrapolation.

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$\Rightarrow$ *We do that with an interaction effect!*

```
mod <- lm(mrate ~ 0 +
            legal +
            state *
            year_fct,
          df)
```

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$\Rightarrow$ *There's a tradeoff: treatment is at the unit level, statistical power at the subunit level.*