

Count models

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The dependent variable

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Count models: What are they good for?

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⇒ *e.g. number of meetings between decision makers, violent events, legislative proposals, etc.*

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⇒ Variables are on the exposure level; related to when (where) the events took place.

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⇒ *We replace the normal distribution with another probability distribution*

The generalized linear model strategy

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Formula

The equation the model estimates:

$$E(y_i) \equiv h\lambda_i = h \times \exp(\alpha + \beta \times x_i) \quad (1)$$

Estimation of the exposure

What to do with the exposure parameter?

$$E(y_i) \equiv h\lambda_i = h \times \exp(\alpha + \beta \times x_i) \quad (2)$$

Two strategies :

- **Offset:** Move it into the equation but constrain parameter:
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\Rightarrow *If the exposure is the same for all units, we set it to 1 and ignore it.*

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\Rightarrow *Make scenarios, predict, knock yourself out*

Dispersion

The main assumption of the Poisson model

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► The y can be overdispersed, but not the $\hat{\lambda} \rightarrow$ as in OLS

\Rightarrow *The standard errors will be too small*

Identifying overdispersion

- ▶ Poissonness plot

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- ▶ Events are related

Addressing overdispersion

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$\Rightarrow \beta$ *remains the same, standard errors are larger*

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- ▶ One producing (at least some) positive counts \Rightarrow *We can model this in two parallel regressions*

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⇒ *Can accomodate under-dispersion too.*

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