# RDD and diff-in-diff

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# Regression discontinuity design (RDD)

# Basic assumption

**RDD assumes a running variable ($x$) with a cut point ($c$) beyond which treatment is assigned ($D$).**

$$D_i = \begin{cases} 1 & if \quad x_i \quad \geqslant c \\ 0 & if \quad x_i \quad < c \end{cases} \tag{1}$$

# Distinction

**It has a flavor of logit or propensity scores, but there are some differences:**

- ~~**logit**~~ : $x$ is not latent and we know the cutpoint: Both are observed and included as a *predictors*.
- ~~**matching**~~ : we have no control/treatment group. However, we assume that units on either side of the treatment are increasingly similar as their $x$ is similar.

# Examples

**Administrative data are perfect:** You have some rule that kicks in at a specific threshold for otherwise almost identical observations.

- school test scores on school admission, restrictions on class size
- legal drinking age on alcohol related deaths
- election of candidates in close races

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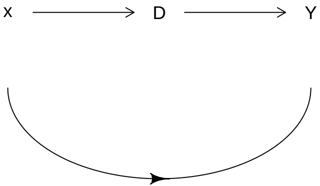
# Two ways of understanding RDD

**X is a confounder**
**...so we only control for X**

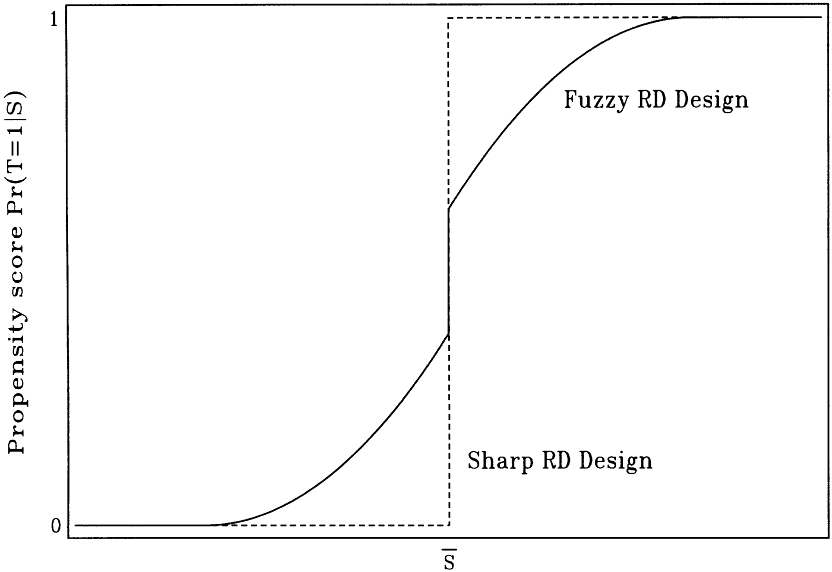


# Two designs

**We distinguish between two designs depending on how probable the treatment is:**

- **sharp** RD: assignment is deterministic
- **fuzzy** RD: assignment is probabilistic

# A visual representation



# The basic model

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# Do it in R

**Check out alcohol related deaths ($y$) as a function of legal drinking age ($D$)**

```
load("df.Rda")
mod1 <- lm(alcohol ~ over21 + age,
           df)
```

# Is that all?

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2. **no omited variable bias**:$x$ must capture all influence on $D$.

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. . . that's not a regression discontinuity.

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- recode the $x \rightarrow$ compare with the recoding of $y$ in GLMs.
- consider a sufficiently small window

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```
df$age2 <- df$age^2
mod2 <- lm(alcohol ~ over21 + age + age2,
           df)
```

# Recode the x: symmetric effect

**We can create a curvilinear effect of x using polynomials** (e.g.:)

$$y_i = \alpha + \rho D_i + \gamma_1 x_i + \gamma_2 x_i^2 \tag{4}$$

```
df$age2 <- df$age^2
mod2 <- lm(alcohol ~ over21 + age + age2,
           df)
```

⇒ *Here, $x$ has a symmetrical effect on both sides of the treatment.*

# Recode the x: assymetric effect

**We can assume x has different effects on each side of the treatment**

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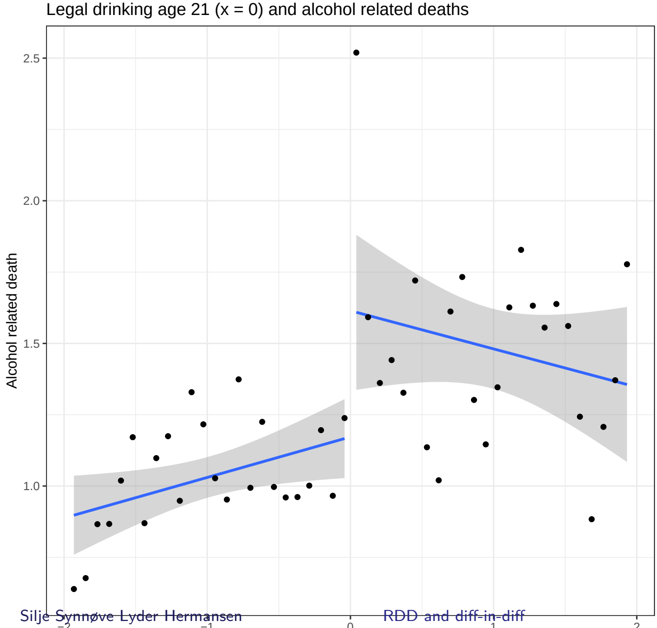
$\Rightarrow$ *we center the x on the cutpoint* $(x_i - c) \rightarrow \rho$ *still reports the change at the cutpoint.*

```
mod3 <- lm(alcohol ~ over21 * age,
          df)
```

# Do it in R


Legal drinking age 21 (x = 0) and alcohol related deaths
Alcohol related death
2.5
2.0
1.5
1.0
0
1
2

# Extrapolation

**We do this to estimate the effect at the cutpoint ($\rho$)**

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- but we can also extrapolate $y$ beyond the cutpoint with $x$: $\rho + \delta(x - c)$

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- subsetting the data to tweak the window around the cutpoint is a **non-parametric approach.**

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- There's a tradeoff between linearity and statistical power (we need sufficient N).

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```
mod5 <- lm(alcohol ~ over21 * age,
           df[df$agecell > 20 & df$agecell < 22,])
```

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```
mod5 <- lm(alcohol ~ over21 * age,
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```

$\Rightarrow$ *When you narrow down, do you get a weaker or stronger effect?*

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- We can do it by hand
- . . . or we can make an algorithm do it:
  - run a local weighted regression line
  - bandwidth is estimated accordingly

$\Rightarrow$ *the point is to show robustness, not p-hack!*

# Omited variable bias

# Fuzzy RD

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**Often the $D$ increases the probability of a treatment, but we don't know!**

$\Rightarrow$ *This is a Instrumental Variable approach (more on Thursday)*

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$\Rightarrow \gamma$ is the causal effect of D in a fuzzy design.

# Differences-in-differences

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$\Rightarrow$ *Treatment effect is that difference*

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$\Rightarrow$ *based on panel data (units are observed several times).*

# Example: States' monetary policy and number of banks

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$\Rightarrow$ *What was the treatment effect?*

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$\Rightarrow$ *Basically a 2-by-2 table*

# How to do it?

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- $T$ represents the treatment group: differences *between* units
- $\beta_3$ is the causal effect

# Data

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- Requires panel data → which means correcting the standard errors.
- Common panel types: state-year/administrative unit-time period; people over time ...

⇒ *we want to know the trend before and after the break*

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**Does the legal drinking age has an effect on death rates among the young?**

- $y$ is number of deaths per 100 000
- $P$ is post-treatment dummy
- $T$ is dummies for states
- *trend* is year dummies

# Another example: step 1 → calculate differences

The authors have two tricks:

- Hardcode the interaction effect (dummy before/after treatment)
- They remove the intercept to retain all dummies

```
load("df2.Rda")
##with intercept
mod <- lm(mrate ~ legal +
            state +
            year_fct,
          df)
##without intercept; with all dummies
mod <- lm(mrate ~ 0 +
            legal +
            state +
            year_fct,
          df)
```

# Another example: step 2 $\rightarrow$ calculate errors

Calculate robust standard errors:

```
library(clubSandwich)
```

```
## Registered S3 method overwritten by 'clubSandwich':
##   method    from
##   bread.mlm sandwich
```

```
vcov <- vcovCR(mod, cluster = df[["state"]],
               type = "CR2")
robust <- coef_test(mod, vcov = vcov)$SE
```

# Another example: step 3 → interpretation

Display the results and interpret:

```r
library(stargazer)
stargazer(mod, se = robust,
                    omit = "state|year",
                    type = "html",
                    out = "regtable.html")
```

# Another example: step 3 $\rightarrow$ interpretation

Table 1: Death rates among young as a function of legal drinking age

| | *Dependent variable:* |
|---|---|
| | mrate |
| Legal drinking age (causal effect) | 10.804** |
| | (4.479) |
| Observations | 714 |
| $R^2$ | 0.986 |
| Adjusted $R^2$ | 0.985 |
| Residual Std. Error | 17.339 (df = 649) |
| F Statistic | 726.005*** (df = 65; 649) |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01 |

$\Rightarrow$ *What did we find?*

# The parallel trends assumption

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- The regression assumes a counterfactual $\rightarrow$ remember the extrapolation.

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⇒ *We do that with an interaction effect!*

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⇒ *There's a tradeoff: treatment is at the unit level, statistical power at the subunit level.*