Interpretation

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Where are we?

Where are we?

- week 1:
  - purpose of the course
  - R as an object oriented language
- week 2:
  - dialects in R
  - descriptive statistics:
    - measurement level and choice of descriptives
    - data exploration
- week 3: (this week)
  - linear regression (OLS)
  - interpretation
  - non-linear effects

Plan for the day

- lecture: uncertainty and interpretation of linear models
  - substantive interest: the size of the effect
  - statistical significance: sources of variation/uncertainty
  - R notebook 1: interpretation (Gelman and Hill, King et al)
- study technique: how to use AI/LLMs in this class
- Thursday:
  - implementation in R
  - R notebook 2: non-linear effects (Berry et al)

Introduction

Today's example

What is the effect of electoral systems on parliamentarians resource allocation?

- Members of the European Parliament (MEPs) sit together in one institution, but run for election under different rules
- expectation: more local investment among MEPs in candidate-centered systems (compared to party-centered systems), because of their need for a personal brand
- variables:
  - y: number of constituency-level assistants employed (metric)
  - x : candidate vs. party-centered systems (binary)

Two views on linear regression

Two views on linear regression

Linear regression summarizes how the average values of a numerical outcome variable vary over subpopulations defined by linear functions of predictors. (Gelman and Hill, 2007, ch 3)

- **comparison of means:** descriptive approach to regression; makes sense for categorical predictors
- **relationship between variables:** their correlation; more causal, makes sense for numerical predictors

A comparison of means: group means

Most obvious when my predictor is categorical

```
df %>%
  group_by(OpenList) %>%
  reframe("mean_y" = mean(LocalAssistants)) %>%
  ungroup %>%
  mutate(diff = mean_y - lag(mean_y))
```

```
## # A tibble: 2 x 3
##   OpenList mean_y   diff
##      <int>  <dbl>  <dbl>
## 1        0   2.47 NA
## 2        1   3.42  0.949
```

- MEPs from *party-centered* systems employ on average 2.47 local assistants
- MEPs from *candidate-centered* systems employ on average 3.42 local assistants.
- The difference is 0.95

A comparison of means: regression

Most obvious when my predictor is categorical

```
#Estimate the equation
mod <- lm(LocalAssistants ~ OpenList,
          df)
#Summarize the results
summary(mod)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -3.42  -2.42  -0.47   1.53  36.08
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    2.468      0.161   15.35  < 2e-16 ***
## OpenList       0.949      0.234    4.05  5.7e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.2 on 737 degrees of freedom
## Multiple R-squared:  0.0218, Adjusted R-squared:  0.0204
## F-statistic: 16.4 on 1 and 737 DF,  p-value: 5.68e-05
```

- MEPs from *party-centered* systems employ on average 2.47 local assistants
- The *difference* is 0.95.
- MEPs from *candidate-centered* systems employ on average 2.47 + 0.95 = 3.42 local assistants.

Relationship between variables: regression

```
mod2 <- lm(LocalAssistants ~ OpenList + LaborCost,
           df)

summary(mod2)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -4.49  -1.94  -0.41   1.08  35.00
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.1266     0.2861   14.42  < 2e-16 ***
## OpenList      0.8288     0.2278    3.64  0.00029 ***
## LaborCost    -0.0702     0.0102   -6.91    1e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.1 on 736 degrees of freedom
## Multiple R-squared:  0.0814, Adjusted R-squared:  0.0789
## F-statistic: 32.6 on 2 and 736 DF,  p-value: 2.69e-14
```

More descriptive statistics

- the relationship (correlation)
- net of other variable's influence (controlling for...)
- the precision (uncertainty)
- the shared variation (R^2)
- the remaining variation (residuals, σ^2)

Interpretation

Stages of interpretaion

- **hypothesis testing:** direction and signficance
- **marginal effect:** the relative increase in your predictor wo/accounting for the value of other preditors.
- **prediction**: fill in the equation for all predictors and calculate the predicted effect
- **first difference**: fill in the equation for two *scenarios* and calculate the difference in y
- **effect plot**: fill in the equation for all scenarios relevant to your predictor

$\Rightarrow$ *as we move to GLMs, the importance of stages 3-6 becomes important*

Hypothesis testing

Hypothesis testing

Hypotheses are mostly about direction and significance

```
summary(mod2)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -4.49  -1.94  -0.41   1.08  35.00
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```

- **direction:** MEPs from candidate-centered systems have on average more local assistants on their payroll
- **significance:** this is unlikely to be random

$\Rightarrow$ *... but what is the substantive effect?*

Marginal effect: change in x

Marginal effect: change in x

The relative (marginal) increase in your predictor (difference in means)

- without accounting for the value of other predictors
  - important once we move to GLMs
- regression is the estimation of an equation

  $y = \alpha + \beta x$

- marginal effects focus on βx
  - β: from the model (you estimated it)
  - x: from the data (you pick it)

Marginal effect: example of change in x

The relative (marginal) increase in your predictor (difference in means) witout accounting for the value of other predictors.

```
summary(mod2)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -4.49  -1.94  -0.41   1.08  35.00
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```

- interpretation:
  - β: "when x increases with one unit, y increases with β units"
  - x: when labor cost increases with *one* unit (x, here 1000 euros), the average number of assistants decreases by 0.07

$\Rightarrow$ *but is this what we want to know?*

Partial scenario: set values for x

Find an increment (change) in x that makes sense for your story

```
##Summary of x
summary(df$LaborCost)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       4      10      26      23      31      41
```

```
##Find two typical values
summary(df$LaborCost)[c(4,5)]
```

```
##    Mean 3rd Qu.
##      23      31
```

```
## E.g. change from mean to 3rd quartile
summary(df$LaborCost)[c(4,5)] %>% diff
```

```
## 3rd Qu.
##     8.6
```

- univariate statistics / data exploration helps you find interesting changes in x
- calculate βx by filling in a realistic *change* in x.
- 8580 euro increase (increase by 8.58) corresponds to a 0.6 decrease in assistants ($\beta x = -0.07 \times 8.58$).

$\Rightarrow$ *use the univariate statistics to find an interesting increments*

Prediction: fill in all x's

Prediction: fill in all x's

We estimated an equation with the help of our data

$$Y_i = \alpha + \beta_1 x_i + \beta_2 x_i$$

data (observed)

- variables: X and Y
- observations: i is a counter for the observations, refers to the i^{th} observation. $i...N$

parameters (estimated)

- α intercept, the value of Y when X $==$ 0
- β slope, the increase in Y when X increases by one unit

We make predictions by filling in data points for that equation

$$Y_i = 4.13 + 0.83 \times OpenList + -0.07 \times LaborCost$$

If all x's were 1:

$$4.89 = 4.13 + 0.83 \times 1 + -0.07 \times 1$$

Why prediction?

- data description: "out-of-sample"
  - forecasting: e. g. election
  - machine learning: e.g create a new variable
- model statistics: "in-sample"
  - compare observed and predicted y
- interpretation:
  - set scenarios (fill in x)
  - predict outcomes (using β)

Creating one full scenario

You create a predicted scenario when you fill in values for *all* the predictors (x).

In R:

```
##Create variables
x1 = 1; x2 = 22

# or a data frame
scenario <- data.frame(
  OpenList = 1,
  LaborCost = 22)

# extract coefficients and apply to new data
predict(mod2, newdata = scenario)
```

```
##   1
## 3.4
```

By hand:

$$Y_i = \alpha + \beta_1 OpenList + \beta_2 LaborCost$$

$$Y_i = \alpha + \beta_1 \times 1 + \beta_2 \times 22$$

$$3.41 = 4.13 + 0.83 \times 1 + -0.07 \times 22$$

$\Rightarrow$ *MEPs from candidate-centered electoral systems with average labor cost, are predicted to have – on average – a local staff of 3.41 people.*

When would you be interested in full scenarios

When we use prediction for interpretation, we are interested in three metrics:

- two assymmetric scenarios: describe two typical value constellations (Ward and Ahlquist, ch 3)
- first difference: the difference in y between two predicted scenarios
- effect plots: the predicted y, as x increases, holding all other x constant.

First difference

First difference

First difference compares the predicted outcomes of two scenarios where one x changes, holding all other predictors constant

- first difference: difference between the two
- marginal effect vs first difference:
  - linear effects: marginal effect with partial scenario is the same as first difference
  - non-linear effects: the two are different

How to calculate a first difference

You create *two* scenarios and calculate the difference in y between the two

In R:

```
x1 = c(0, 1); x2 = 22

# or data frame
scenario <- data.frame(OpenList = c(0, 1),
                       LaborCost = 22)
#Predict both
predict(mod2, scenario)
```

```
##   1   2
## 2.6 3.4
```

```
#Take the difference
predict(mod2, scenario) %>% diff
```

```
##    2
## 0.83
```

By hand:

$Y_i = \alpha + \beta OpenList_{1:2} + \beta_2 LaborCost$

scenario 1: $2.58 = 4.13 + 0.83 \times 0 + -0.07 \times 22$

scenario 1: $3.41 = 4.13 + 0.83 \times 1 + -0.07 \times 22$

First difference: $0.83 = 2.58 - 3.41$

$\Rightarrow$ *The first difference can be calculated for any two scenarios of your choice!*

Effect plot

Effect plot

Effect plots allow us to visualize our effects

- choice depends on the measurement level of x

Prediction

You create a bunch of scenarios covering the entire range of the variable

In R:

```
#Scenario
scenario <- data.frame(OpenList = c(0),
                       LaborCost = min(df$LaborCost): max(df$LaborCost))
#Inspect the first three scenarios
scenario[1:3,]
```

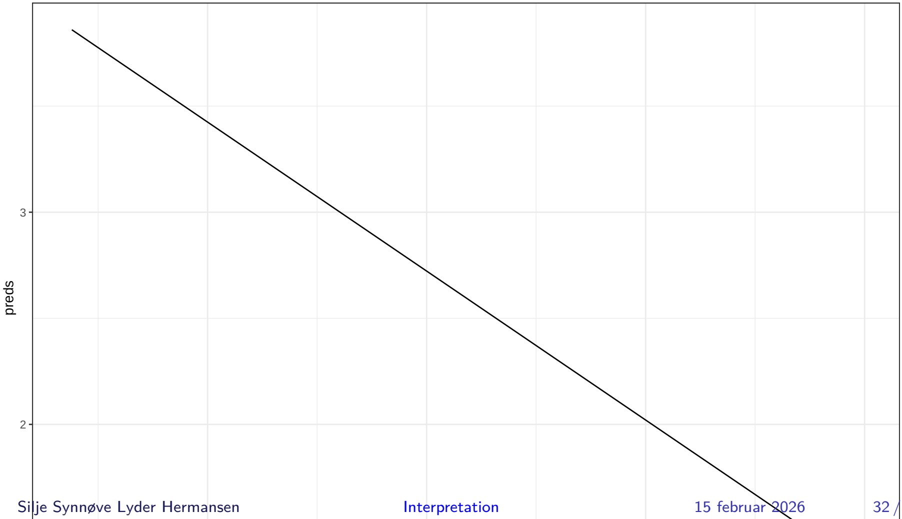
```
##   OpenList LaborCost
## 1        0       3.8
## 2        0       4.8
## 3        0       5.8
```

```
#Predict
scenario <- scenario %>% mutate(preds = predict(mod2, newdata = scenario))
scenario[1:3, ]
```

```
##   OpenList LaborCost preds
## 1        0       3.8   3.9
## 2        0       4.8   3.8
## 3        0       5.8   3.7
```

Plot

```
scenario %>%
ggplot +
  geom_line(aes(x = LaborCost,
                y = preds))
```



Two sources of variation in the data

Two sources of variation in the data

But are these effects statistically significant?

- **Fundamental uncertainty:** The natural randomness in outcomes, even if the true parameters were known (Captured by residual variance).
- **Estimation uncertainty:** How precisely are the coefficients estimated? (Captured by the variance-covariance matrix)

$\Rightarrow$ *the uncertainty of your predictions depend on both*

Fundamental uncertainty

Fundamental uncertainty

$$Y_i = \alpha + \beta X1_i + \beta X2_i + \sigma^2$$

data (observed)

- variables: X and Y
- observations: i is a counter for the observations, refers to the i^{th} observation. $i ... N$

parameters (estimated)

- α intercept, the value of Y when X == 0
- β slope, the increase in Y when X increases by one unit
- σ^2 variance in the error term; $\sqrt{\sigma^2}$ = standard deviation

Let's rewrite

$$Y \sim g(\theta, \sigma^2)$$
$$\theta = \alpha + \beta X_i + \sigma^2$$

- θ: the average value of y
- $g()$: the link function

The normal model

$$Y_i \sim N(\mu_i, \sigma^2)$$
$$\mu_i = \alpha + \beta X_i + \sigma^2$$

- μ: mean predicted value
- $N()$: the normal distribution

What are the residuals?

We are always wrong in our predictions, but how wrong are we (in-sample)?

```
df <- df %>% mutate(
  #Predict in sample
  preds = predict(mod2, newdata = .),
  #Calculate the difference between expected and observed
  residuals = LocalAssistants - preds
  )
```

How to describe the residuals?

We describe the residuals by their spread (standard deviation/residual standard error)

```
mean(df$residuals)
```

```
## [1] -9.8e-15
```

- mean: with an unbiased estimator, their average is 0

```
sd(df$residuals)
```

```
## [1] 3.1
```

- standard deviation: but their spread can be more or less high
- here, the average distance from their mean is is a staff size of 3.08 local assistants.

⇒ *residual standard error*

Where is it reported?

```r
summary(mod2)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -4.49  -1.94  -0.41   1.08  35.00
##
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```

```r
summary(mod2)$sigma
```

```
## [1] 3.1
```

$\Rightarrow$ *residual standard error is 3.08*

Conclusion: fundamental error

- important for predictions and model statistics
- not really for the uncertainty of the estimation of our effect

Estimation uncertainty

Estimation uncertainty

- most research is about the *effect of x* on y
- so, we're interested in the uncertainty of β

The central limit theorem and sampling

A fiction: the assumptions underpinning the uncertainty of the parameters

- assumption that data is a sample from a population
- we *could* sample many times
- we calculate the same parameter (e.g. mean, differences in means. . .) in each sample
- they will vary, but will follow a *normal distribution*

$\Rightarrow$ *each parameter is a distribution with a mean and a standard deviation*

Standard errors

```
summary(mod2)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -4.49  -1.94  -0.41   1.08  35.00
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```

- mean: average of all the differences in means between the two groups of MEPs: 0.95
- spread: the standard deviation of this distribution is 0.23

$\Rightarrow$ *a standard error is the standard deviation of a hypothetical distribution (parameters)*

Colinearities

Colinearities

Regression parameters may be correlated

```
mat <-vcov(mod2)
mat
```

```
##             (Intercept) OpenList LaborCost
## (Intercept)      0.0819 -0.02846  -0.00244
## OpenList        -0.0285  0.05191   0.00018
## LaborCost       -0.0024  0.00018   0.00010
```

- reported in the *variance-covariance matrix*
- diagonal: the variance of the parameter.
  - variance in effect of electoral system: $\sigma^2 = 0.05$
  - standard error in effect of electoral system: $\sqrt{\sigma^2} = 0.23$
- off-diagonal: the covariance of the parameters
  - low correlation between labor cost and electoral system

Estimate

King et al. (2000) make two points

- find interesting scenarios when you interpret
- estimate the uncertainty for the scenarios including
  - standard error (diagonal)
  - covariance (off-diagonal)

$\Rightarrow$ *the correlation between variables may mean higher or lower uncertainty than only using the standard error*

Simulation

They do this using simulation

- set scenario for all predictors
- draw from the distribution of parameters
- make prediction
- repeat many times
- extract the information and report
    - mean
    - median
    - mode
    - standard deviation
    - plot the distribution!

Our class

We will see two ways of doing this in R

- `ggeffects` package: simulates scenarios for us and can be plotted seamlessly $\rightarrow$ *effect plots, coefplots and point predictions*
- `MASS` package: the "manual" simulation from a multivariate normal distribution using the variance-covariance matrix. $\rightarrow$ *entire vector of simulations; for other plots/purposes*

Study technique

For this class

- learn by doing!
    - all readings include R examples; code along!
    - my R notebooks
    - then play around with the concepts; also with your own data/former exams
- dialogue with AI (ChatGPT, Claude)

What to ask and not to ask chat for?

R codes

- dont ask for complex codes
  - requires quirey competence on your end
  - you don't learn
- ask it to annotate your scripts
  - explain what each line means
  - dissect all code chunks you find and ask

What to ask and not to ask chat for?

Statistics

- don't ask for a summary of the reading
  - it's not necessarily what we will focus on
  - you don't learn
- ask for definitions
  - ask it to define key concepts you don't understand while you read
  - rephrase definitions and ask it this is a good understanding
- match with your readings
  - upload the PDF and ask specific questions
  - ask for examples, possibly with R codes
- interpretation
  - copy-paste your model output and ask for an explainer
  - use descriptive statistics to find interesting scenarios, ask it to help you find a plain English intuitive sentence