

Plan for the day

Introduction

Interpretation

Two sources of variation in the data

Study technique

Introduction to R

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11 februar 2026

Plan for the day

Plan for the day

- ▶ lecture: uncertainty and interpretation of linear models
 - ▶ substantive interest: the size of the effect
 - ▶ statistical significance: sources of variation/uncertainty
- ▶ chatGPT/chatTutor: how to use AI/LLMs in this class

Introduction

Today's example

What is the effect of electoral systems on parliamentarians resource allocation?

- ▶ Members of the European Parliament (MEPs) sit together in one institution, but run for election under different rules
- ▶ expectation: more local investment among MEPs in candidate-centered systems (compared to party-centered systems), because of their need for a personal brand
- ▶ variables:
 - ▶ y: number of constituency-level assistants employed
 - ▶ x : candidate vs. party-centered systems

Two views on linear regression

Two views on linear regression

Linear regression summarizes how the average values of a numerical outcome variable vary over subpopulations defined by linear functions of predictors. (Gelman and Hill, 2007, ch 3)

- ▶ **comparison of means:** descriptive approach to regression; makes sense for categorical predictors
- ▶ **relationship between variables:** their correlation; more causal, makes sense for numerical predictors

Regression as a comparison of means

```
df %>%
  group_by(OpenList) %>%
  reframe("mean_y" = mean(LocalAssistants)) %>%
  ungroup %>%
  mutate(diff = mean_y - lag(mean_y))

## # A tibble: 2 x 3
##   Openlist mean_y   diff
##       <int>  <dbl>  <dbl>
## 1        0    2.47 NA
## 2        1    3.42  0.949
```

- ▶ MEPs from *party-centered* systems employ on average 2.47 local assistants
- ▶ MEPs from *candidate-centered* systems employ on average 3.42 local assistants.
- ▶ The difference is 0.95

Relationship between variables

```
mod <- lm(LocalAssistants ~ OpenList,
           df)

summary(mod)

##
## Call:
## lm(formula = LocalAssistants ~ OpenList, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -3.42  -2.42  -0.47   1.53   36.08 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  2.468     0.161   15.35 < 2e-16 ***
## OpenList      0.949     0.234    4.05  5.7e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.2 on 737 degrees of freedom
## Multiple R-squared:  0.0218, Adjusted R-squared:  0.0204 
## F-statistic: 16.4 on 1 and 737 DF,  p-value: 5.68e-05
```

Relationship between variables

summary(mod)

```
##  
## Call:  
## lm(formula = LocalAssistants ~ OpenList, data = df)  
##  
## Residuals:  
##     Min      1Q Median      3Q     Max  
## -3.42  -2.42  -0.47   1.53  36.08  
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```

- ▶ MEPs from *party-centered* systems employ on average 2.47 local assistants
- ▶ The difference is 0.95.
- ▶ MEPs from *candidate-centered* systems employ on average $2.47 + 0.95 = 3.42$ local assistants.

Interpretation

Linear predictor

```
mod2 <- lm(LocalAssistants ~ OpenList + LaborCost,
            df)

summary(mod2)

##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -4.49   -1.94   -0.41    1.08   35.00 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  4.1266    0.2861   14.42 < 2e-16 ***
## OpenList      0.8288    0.2278    3.64  0.00029 ***
## LaborCost   -0.0702    0.0102   -6.91   1e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 3.1 on 736 degrees of freedom
## Multiple R-squared:  0.0814, Adjusted R-squared:  0.0789 
## F-statistic: 32.6 on 2 and 736 DF,  p-value: 2.69e-14
```

Stages of interpretation

- ▶ **hypothesis testing:** direction and significance
- ▶ **marginal effect:** the relative increase in your predictor w/o accounting for the value of other predictors.
- ▶ **prediction:** fill in the equation for all predictors and calculate the predicted effect
- ▶ **first difference:** fill in the equation for two *scenarios* and calculate the difference in y
- ▶ **effect plot:** fill in the equation for all scenarios relevant to your predictor

⇒ as we move to GLMs, the importance of stages 3-6 becomes important

Hypothesis testing

Hypothesis testing

```
summary(mod2)
```

```
##  
## Call:  
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)  
##  
## Residuals:  
##      Min      1Q Median      3Q     Max  
## -4.49 -1.94 -0.41   1.08  35.00  
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```

- ▶ MEPs from candidate-centered systems have on average more local assistants on their payroll

Marginal effect

Marginal effect

The relative increase in your predictor w/o accounting for the value of other predictors.

```
summary(mod2)
```

```
##  
## Call:  
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)  
##  
## Residuals:  
##      Min    1Q Median    3Q   Max  
## -4.49 -1.94 -0.41  1.08 35.00  
##  
## Coefficients:  
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```

- ▶ when labor cost increases with 1 unit (here 1000 euros), the average number of assistants decreases by 0.07
- ▶ 10.000 euro increase (increase by 10) corresponds to a

Prediction

Prediction

We make predictions by filling in the equation

$$Y_i = \alpha + \beta X_i$$

$$Y_i = 4.13 + 0.95 \times X_i$$

data (observed)

- ▶ variables: X and Y
- ▶ observations: i is a counter for the observations, refers to the i^{th} observation. $i \dots N$

parameters (estimated)

- ▶ α intercept, the value of Y when X == 0
- ▶ β slope, the increase in Y when X increases by one unit

Creating scenarios

You create a scenario when you fill in values in all the predictors (x).

$$Y_i = \alpha + \beta X_i$$

$$5.08 = 4.13 + 0.95 \times 1$$

In R:

```
x = 1  
  
# or  
  
scenario <- data.frame(OpenList = 1)
```

First difference

You create two scenarios and calculate the difference in y

$$Y_i = \alpha + \beta X_i$$

$$\text{scenario 1: } 4.13 = 4.13 + 0.95 \times 0 \quad \text{scenario 2: } 5.08 = 4.13 + 0.95 \times 1$$

In R:

```
x = c(0, 1)
```

```
# or
```

```
scenario <- data.frame(OpenList = c(0, 1))
```

⇒ The first difference is 0.95.

Effect plot

Prediction

You create a bunch of scenarios covering the entire range of the variable

In R:

```
#Scenario
scenario <- data.frame(OpenList = c(0,
                                     LaborCost = min(df$LaborCost): max(df$LaborCost))

scenario[1:3,]

##   OpenList LaborCost
## 1         0      3.8
## 2         0      4.8
## 3         0      5.8

#Predict
scenario <- scenario %>% mutate(preds = predict(mod2, newdata = scenario))
scenario$preds[1:3]

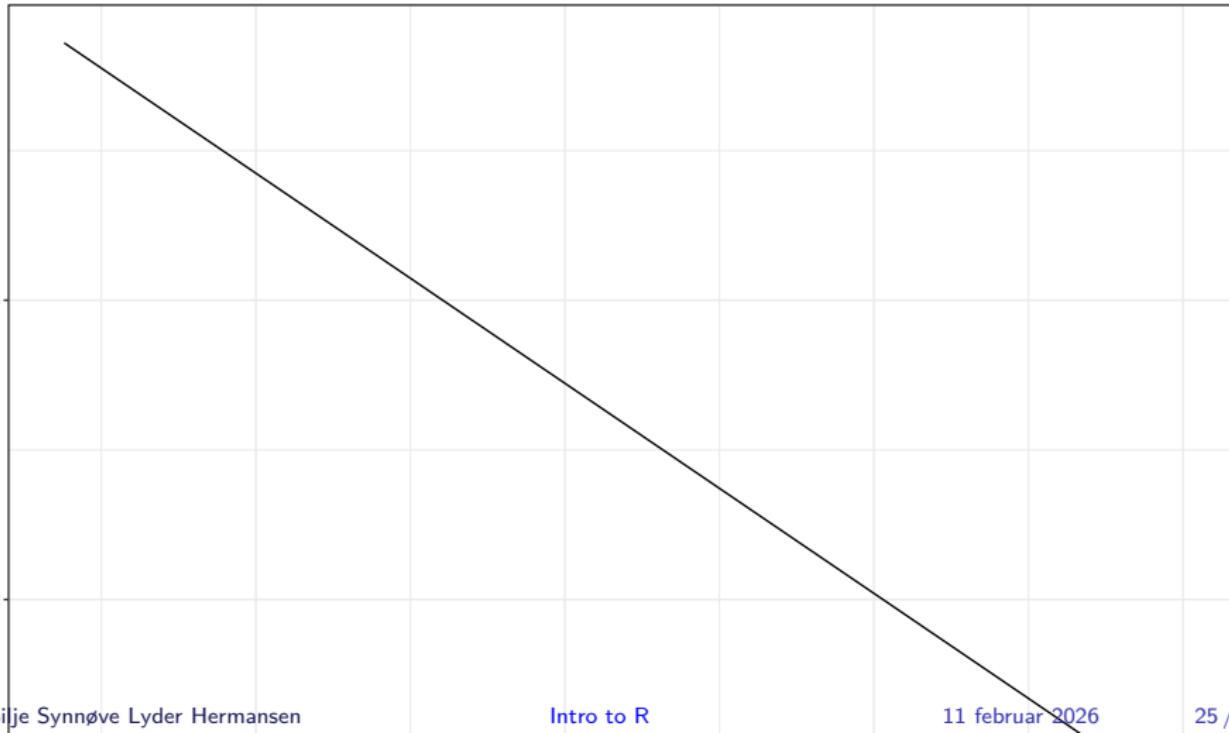
##   1   2   3
## 3.9 3.8 3.7
```

- ▶ The first difference is 0.95

⇒ The first difference can be calculated for any two scenarios of your choice!

Plot

```
scenario %>%
  ggplot +
  geom_line(aes(x = LaborCost,
                 y = preds))
```



Two sources of variation in the data

Two sources of variation in the data

But are these effects statistically significant?

- ▶ **Fundamental uncertainty:** The natural randomness in outcomes, even if the true parameters were known (Captured by residual variance).
- ▶ **Estimation uncertainty:** How precisely are the coefficients estimated? (Captured by the variance-covariance matrix)

⇒ *the uncertainty of your predictions depend on both*

Fundamental uncertainty

Fundamental uncertainty

$$Y_i = \alpha + \beta X1_i + \beta X2_i + \sigma^2$$

data (observed)

- ▶ variables: X and Y
- ▶ observations: i is a counter for the observations, refers to the i^{th} observation. $i \dots N$

parameters (estimated)

- ▶ α intercept, the value of Y when X == 0
- ▶ β slope, the increase in Y when X increases by one unit
- ▶ σ^2 variance in the error term; $\sqrt{\sigma^2}$ = standard deviation

Let's rewrite

$$Y \sim g(\theta, \sigma^2)$$

$$\theta = \alpha + \beta X_i + \sigma^2$$

- ▶ θ : the average value of y
- ▶ $g()$: the link function

The normal model

$$Y_i \sim N(\mu_i, \sigma^2)$$

$$\mu_i = \alpha + \beta X_i + \sigma^2$$

- ▶ μ : mean predicted value
- ▶ $N()$: the normal distribution

What are the residuals?

We are always wrong in our predictions, but how wrong are we (in-sample)?

```
df <- df %>% mutate(  
  #Predict in sample  
  preds = predict(mod2, newdata = .),  
  #Calculate the difference between expected and observed  
  residuals = LocalAssistants - preds  
)
```

How to describe the residuals?

We describe the residuals by their spread (standard deviation/residual standard error)

```
mean(df$residuals)
```

```
## [1] -9.8e-15
```

- ▶ mean: with an unbiased estimator, their average is 0

```
sd(df$residuals)
```

```
## [1] 3.1
```

- ▶ standard deviation: but their spread can be more or less high
- ▶ here, the average distance from their mean is a staff size of 3.08 local assistants.

⇒ *residual standard error*

Where is it reported?

```
summary(mod2)
```

```
##  
## Call:  
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max  
## -4.49 -1.94 -0.41  1.08  35.00  
##  
## Coefficients:  
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```

```
summary(mod2)$sigma
```

```
## [1] 3.1
```

⇒ residual standard error is 3.08

Conclusion: fundamental error

- ▶ important for predictions and model statistics
- ▶ not really for the uncertainty of our estimation of our effect

Estimation uncertainty

Estimation uncertainty

- ▶ most research is about the *effect of x on y*
- ▶ so, we're interested in the uncertainty of β

The central limit theorem and sampling

A fiction: the assumptions underpinning the uncertainty of the parameters

- ▶ assumption that data is a sample from a population
- ▶ we *could* sample many times
- ▶ we calculate the same parameter (e.g. mean, differences in means...) in each sample
- ▶ they will vary, but will follow a *normal distribution*

⇒ *each parameter is a distribution with a mean and a standard deviation*

Standard errors

```
summary(mod2)
```

```
##  
## Call:  
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##  
## Residuals:  
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```

- ▶ mean: average of all the differences in means between the two groups of MEPs: 0.95
 - ▶ spread: the standard deviation of this distribution is 0.23
- ⇒ a *standard error* is the standard deviation of a hypothetical distribution (parameters)

Colinearities

Colinearities

Regression parameters may be correlated

```
mat <-vcov(mod2)
mat
```

```
##           (Intercept) OpenList LaborCost
## (Intercept)    0.0819 -0.02846 -0.00244
## OpenList      -0.0285  0.05191  0.00018
## LaborCost     -0.0024  0.00018  0.00010
```

- ▶ reported in the *variance-covariance matrix*
- ▶ diagonal: the variance of the parameter.
 - ▶ variance in effect of electoral system: $\sigma^2 = 0.05$
 - ▶ standard error in effect of electoral system: $\sqrt{\sigma^2} = 0.23$
- ▶ off-diagonal: the covariance of the parameters
 - ▶ low correlation between labor cost and electoral system

Estimate

King et al. (2000) make two points

- ▶ find interesting scenarios when you interpret
- ▶ estimate the uncertainty for the scenarios including
 - ▶ standard error (diagonal)
 - ▶ covariance (off-diagonal)

⇒ *the correlation between variables may mean higher or lower uncertainty than only using the standard error*

Simulation

They do this using simulation

- ▶ set scenario for all predictors
- ▶ draw from the distribution of parameters
- ▶ make prediction
- ▶ repeat many times
- ▶ extract the information and report
 - ▶ mean
 - ▶ median
 - ▶ mode
 - ▶ standard deviation
 - ▶ plot the distribution!

Our class

We will see two ways of doing this in R

- ▶ ggeffects package: simulates scenarios for us and can be plotted seamlessly → *effect plots, coefplots and point predictions*
- ▶ MASS package: the “manual” simulation from a multivariate normal distribution using the variance-covariance matrix. → *entire vector of simulations; for other plots/purposes*

Study technique

For this class

- ▶ learn by doing!
 - ▶ all readings include R examples; code along!
 - ▶ my R notebooks
 - ▶ then play around with the concepts; also with your own data/former exams
- ▶ dialogue with AI (ChatGPT, Claude)

What to ask and not to ask chat for?

R codes

- ▶ dont ask for complex codes
 - ▶ requires quirey competence on your end
 - ▶ you don't learn
- ▶ ask it to annotate your scripts
 - ▶ explain what each line means
 - ▶ dissect all code chunks you find and ask

What to ask and not to ask chat for?

Statistics

- ▶ don't ask for a summary of the reading
 - ▶ it's not necessarily what we will focus on
 - ▶ you don't learn
- ▶ ask for definitions
 - ▶ ask it to define key concepts you don't understand while you read
 - ▶ rephrase definitions and ask it this is a good understanding
- ▶ match with your readings
 - ▶ upload the PDF and ask specific questions
 - ▶ ask for examples, possibly with R codes
- ▶ interpretation
 - ▶ copy-paste your model output and ask for an explainer
 - ▶ use descriptive statistics to find interesting scenarios, ask it to help you find a plain English intuitive sentence