Interpretation

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Where are we?

Where are we?

- ▶ week 1:
  - ▶ purpose of the course
  - ▶ R as an object oriented language
- ▶ week 2:
  - ▶ dialects in R
  - ▶ descriptive statistics:
    - ▶ measurement level and choice of descriptives
    - ▶ data exploration
- ▶ week 3: (this week)
  - ▶ linear regression (OLS)
  - ▶ interpretation
  - ▶ non-linear effects

Plan for the day

▶ lecture: uncertainty and interpretation of linear models
  ▶ substantive interest: the size of the effect
  ▶ statistical significance: sources of variation/uncertainty
  ▶ R notebook 1: interpretation (Gelman and Hill, King et al)
▶ study technique: how to use AI/LLMs in this class
▶ Thursday:
  ▶ implementation in R
  ▶ R notebook 2: non-linear effects (Berry et al)

Introduction

Today's example

What is the effect of electoral systems on parliamentarians resource allocation?

▶ Members of the European Parliament (MEPs) sit together in one institution, but run for election under different rules

▶ expectation: more local investment among MEPs in candidate-centered systems (compared to party-centered systems), because of their need for a personal brand

▶ variables:
  ▶ y: number of constituency-level assistants employed (metric)
  ▶ x : candidate vs. party-centered systems (binary)

Two views on linear regression

Two views on linear regression

Linear regression summarizes how the average values of a numerical outcome variable vary over subpopulations defined by linear functions of predictors. (Gelman and Hill, 2007, ch 3)

▶ **comparison of means:** descriptive approach to regression; makes sense for categorical predictors
▶ **relationship between variables:** their correlation; more causal, makes sense for numerical predictors

A comparison of means: group means

Most obvious when my predictor is categorical

```r
df %>%
  group_by(OpenList) %>%
  reframe("mean_y" = mean(LocalAssistants)) %>%
  ungroup %>%
  mutate(diff = mean_y - lag(mean_y))
```

```
## # A tibble: 2 x 3
##   OpenList mean_y  diff
##      <int>  <dbl> <dbl>
## 1        0   2.47    NA
## 2        1   3.42 0.949
```

- MEPs from *party-centered* systems employ on average 2.47 local assistants

- MEPs from *candidate-centered* systems employ on average 3.42 local assistants.

- The difference is 0.95

A comparison of means: regression

Most obvious when my predictor is categorical

```r
#Estimate the equation
mod <- lm(LocalAssistants ~ OpenList,
          df)
#Summarize the results
summary(mod)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList, data = df)
##
## Residuals:
##    Min    1Q Median    3Q    Max
## -3.42  -2.42  -0.47   1.53  36.08
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    2.468      0.161   15.35  < 2e-16 ***
## OpenList       0.949      0.234    4.05 5.7e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.2 on 737 degrees of freedom
## Multiple R-squared:  0.0218, Adjusted R-squared:  0.0204
## F-statistic: 16.4 on 1 and 737 DF,  p-value: 5.68e-05
```

- MEPs from *party-centered* systems employ on average 2.47 local assistants

- The *difference* is 0.95.

- MEPs from *candidate-centered* systems employ on average 2.47 $+ 0.95 = 3.42$ local assistants.

Relationship between variables: regression

More descriptive statistics

```
mod2 <- lm(LocalAssistants ~ OpenList + LaborCost,
          df)

summary(mod2)
```

- ▶ the relationship (correlation)
- ▶ net of other variable's influence (controlling for. . .)
- ▶ the precision (uncertainty)
- ▶ the shared variation (R^2)
- ▶ the remaining variation (residuals, σ^2)

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min    1Q Median    3Q    Max
## -4.49  -1.94  -0.41   1.08  35.00
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.1266     0.2861   14.42  < 2e-16 ***
## OpenList      0.8288     0.2278    3.64  0.00029 ***
## LaborCost    -0.0702     0.0102   -6.91    1e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.1 on 736 degrees of freedom
## Multiple R-squared:  0.0814, Adjusted R-squared:  0.0789
## F-statistic: 32.6 on 2 and 736 DF,  p-value: 2.69e-14
```

Interpretation

Stages of interpretaion

- **hypothesis testing:** direction and signficance
- **marginal effect:** the relative increase in your predictor wo/accounting for the value of other preditors.
- **prediction**: fill in the equation for all predictors and calculate the predicted effect
- **first difference**: fill in the equation for two *scenarios* and calculate the difference in y
- **effect plot**: fill in the equation for all scenarios relevant to your predictor

$\Rightarrow$ *as we move to GLMs, the importance of stages 3-6 becomes important*

Hypothesis testing

Hypothesis testing

Hypotheses are mostly about direction and significance

```
summary(mod2)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -4.49  -1.94  -0.41   1.08  35.00
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```

▶ **direction:** MEPs from candidate-centered systems have on average more local assistants on their payroll

▶ **significance:** this is unlikely to be random

⇒ ... but what is the substantive effect?

Marginal effect: change in x

Marginal effect: change in x

The relative (marginal) increase in your predictor (difference in means)

- ▶ without accounting for the value of other predictors
    - ▶ important once we move to GLMs
- ▶ regression is the estimation of an equation

  $y = \alpha + \beta x$

- ▶ marginal effects focus on βx
    - ▶ β: from the model (you estimated it)
    - ▶ x: from the data (you pick it)

Marginal effect: example of change in x

The relative (marginal) increase in your predictor (difference in means) witout accounting for the value of other predictors.

```r
summary(mod2)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
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```

▶ interpretation:

  ▶ β: "when x increases with one unit, y increases with β units"

  ▶ x: when labor cost increases with *one* unit (x, here 1000 euros), the average number of assistants decreases by 0.07

$\Rightarrow$ *but is this what we want to know?*

Partial scenario: set values for x

Find an increment (change) in x that makes sense for your story

```
##Summary of x
summary(df$LaborCost)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       4      10      26      23      31      41
```

```
##Find two typical values
summary(df$LaborCost)[c(4,5)]
```

```
##    Mean 3rd Qu.
##      23      31
```

```
## E.g. change from mean to 3rd quartile
summary(df$LaborCost)[c(4,5)] %>% diff
```

```
## 3rd Qu.
##     8.6
```

- ▶ univariate statistics / data exploration helps you find interesting changes in x

- ▶ calculate βx by filling in a realistic *change* in x.

- ▶ 8580 euro increase (increase by 8.58) corresponds to a 0.6 decrease in assistants ($\beta x = -0.07 \times 8.58$).

  $\Rightarrow$ *use the univariate statistics to find an interesting increments*

Prediction: fill in all x's

Prediction: fill in all x's

We estimated an equation with the help of our data

$$Y_i = \alpha + \beta_1 x_i + \beta_2 x_i$$

data (observed)

- ▶ variables: X and Y
- ▶ observations: i is a counter for the observations, refers to the ith observation. $i...N$

parameters (estimated)

- ▶ α intercept, the value of Y when $X == 0$
- ▶ β slope, the increase in Y when X increases by one unit

We make predictions by filling in data points for that equation

$Y_i = 4.13 + 0.83 \times OpenList + \text{-}0.07 \times LaborCost$

If all x's were 1:

$4.89 = 4.13 + 0.83 \times 1 + \text{-}0.07 \times 1$

Why prediction?

▶ data description: "out-of-sample"
  ▶ forecasting: e. g. election
  ▶ machine learning: e.g create a new variable
▶ model statistics: "in-sample"
  ▶ compare observed and predicted y
▶ interpretation:
  ▶ set scenarios (fill in x)
  ▶ predict outcomes (using β)

Creating one full scenario

You create a predicted scenario when you fill in values for *all* the predictors (x).

In R:

```
##Create variables
x1 = 1; x2 = 22

# or a data frame
scenario <- data.frame(
  OpenList = 1,
  LaborCost = 22)

# extract coefficients and apply to new data
predict(mod2, newdata = scenario)
```

```
##   1
## 3.4
```

By hand:

$Y_i = \alpha + \beta_1 OpenList + \beta_2 LaborCost$

$Y_i = \alpha + \beta_1 \times 1 + \beta_2 \times 22$

$3.41 = 4.13 + 0.83 \times 1 + -0.07 \times 22$

$\Rightarrow$ *MEPs from candidate-centered electoral systems with average labor cost, are predicted to have – on average – a local staff of 3.41 people.*

When would you be interested in full scenarios

When we use prediction for interpretation, we are interested in three metrics:

▶ two assymmetric scenarios: describe two typical value constellations (Ward and Ahlquist, ch 3)
▶ first difference: the difference in y between two predicted scenarios
▶ effect plots: the predicted y, as x increases, holding all other x constant.

First difference

First difference

First difference compares the predicted outcomes of two scenarios where one x changes, holding all other predictors constant

- ▶ first difference: difference between the two
- ▶ marginal effect vs first difference:
  - ▶ linear effects: marginal effect with partial scenario is the same as first difference
  - ▶ non-linear effects: the two are different

How to calculate a first difference

You create *two* scenarios and calculate the difference in y between the two

In R:

```
x1 = c(0, 1); x2 = 22

# or data frame
scenario <- data.frame(OpenList = c(0, 1),
                       LaborCost = 22)
#Predict both
predict(mod2, scenario)
```

```
##   1   2
## 2.6 3.4
```

```
#Take the difference
predict(mod2, scenario) %>% diff
```

```
##    2
## 0.83
```

By hand:

$Y_i = \alpha + \beta OpenList_{1:2} + \beta_2 LaborCost$

scenario 1: $2.58 = 4.13 + 0.83 \times 0 + -0.07 \times 22$

scenario 1: $3.41 = 4.13 + 0.83 \times 1 + -0.07 \times 22$

First difference: $0.83 = 2.58 - 3.41$

$\Rightarrow$ *The first difference can be calculated for any two scenarios of your choice!*

Effect plot

Effect plot

Effect plots allow us to visualize our effects

- ▶ choice depends on the measurement level of x
  - ▶ categorical predictor: a titlted coefficient plot
  - ▶ numeric predictor: effectplot

Prediction using many scenarios

You create a bunch of scenarios covering the entire range of x

In R:

```
#Scenario
scenario <- data.frame(OpenList = c(0),
                       LaborCost = min(df$LaborCost): max(df$LaborCost))
#Inspect the first three scenarios
scenario[1:3,]
```

```
##   OpenList LaborCost
## 1        0       3.8
## 2        0       4.8
## 3        0       5.8
```
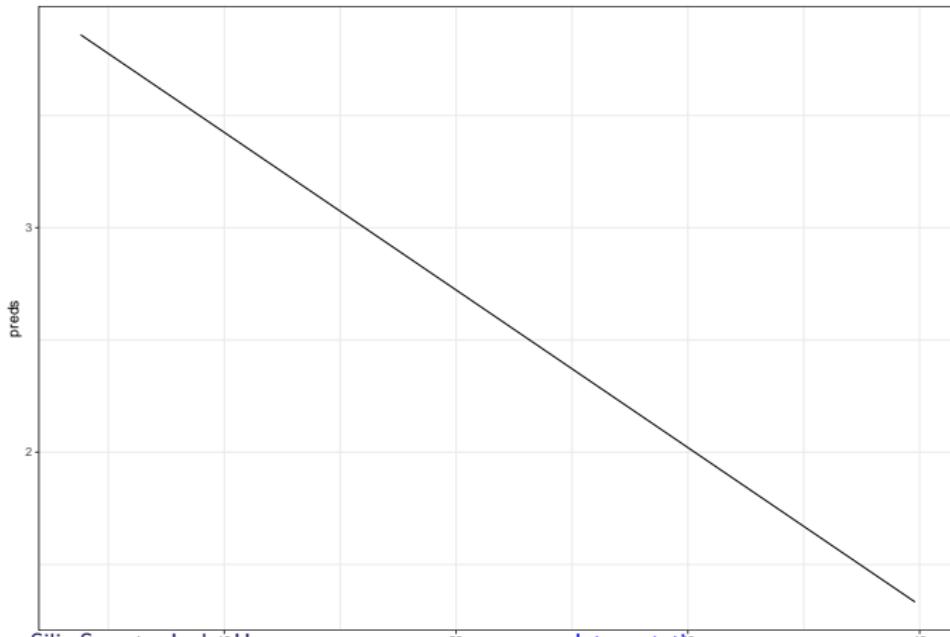
```
#Predict
scenario <- scenario %>% mutate(preds = predict(mod2, newdata = scenario))
scenario[1:3, ]
```

```
##   OpenList LaborCost preds
## 1        0       3.8   3.9
## 2        0       4.8   3.8
## 3        0       5.8   3.7
```

Visualize

Visualize by plotting your x against your predicted y
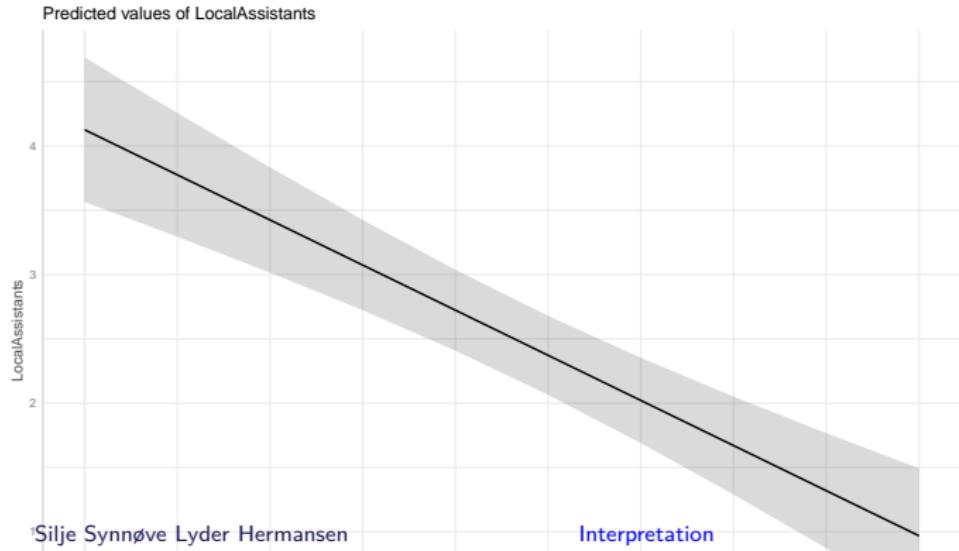
```
scenario %>%
ggplot +
  geom_line(aes(x = LaborCost,
                y = preds))
```

Visualization for metric predictors

Metric predictors are best visualized with an effect plot (a line).
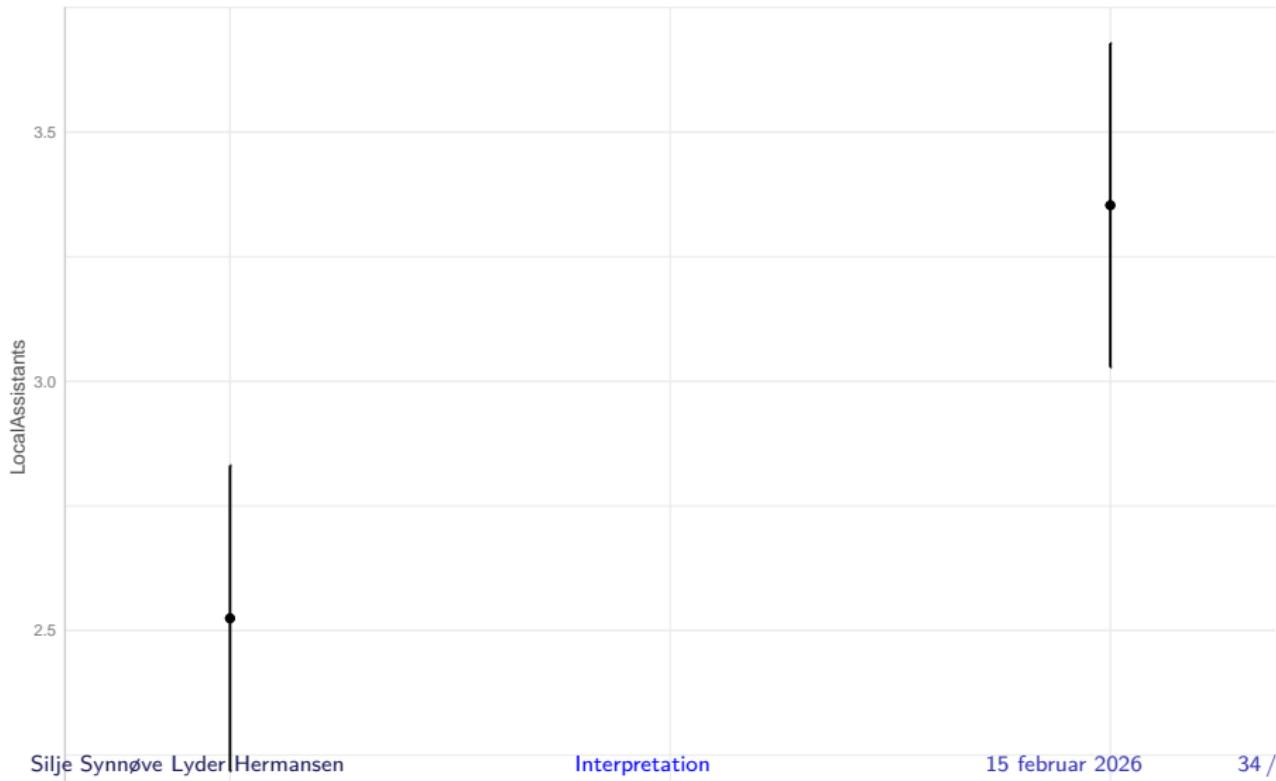
```
eff <- ggpredict(mod2,
                 terms = "LaborCost")


eff %>% plot
```


Predicted values of LocalAssistants

Visualization: categorical predictor

Categorical predictors are best visoualized with a titled coefficient plot

Predicted values of LocalAssistants

Two sources of variation in the data

Two sources of variation in the data

But are these effects statistically significant?

▶ **Fundamental uncertainty:** The natural randomness in outcomes, even if the true parameters were known (Captured by residual variance).

▶ **Estimation uncertainty:** How precisely are the coefficients estimated? (Captured by the variance-covariance matrix)

⇒ *the uncertainty of your predictions depend on both*

Fundamental uncertainty

Fundamental uncertainty

$$Y_i = \alpha + \beta X1_i + \beta X2_i + \sigma^2$$

data (observed)

- ▶ variables: X and Y
- ▶ observations: i is a counter for the observations, refers to the ith observation. $i...N$

parameters (estimated)

- ▶ α intercept, the value of Y when $X == 0$
- ▶ β slope, the increase in Y when X increases by one unit
- ▶ σ^2 variance in the error term; $\sqrt{\sigma^2} =$ standard deviation

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Let's rewrite

Any model in this class

$$Y \sim g(\theta, \sigma^2)$$
$$\theta = \alpha + \beta X_i$$

▶ $g()$: probability distribution (the link between parameters and outcomes)
▶ θ: central tendency (systematic component)
▶ σ^2: spread around that systematic component

The normal (linear) model

$$Y_i \sim N(\mu_i, \sigma^2)$$
$$\mu_i = \alpha + \beta X_i$$

▶ $N()$: the normal distribution (identity link; linear; OLS)
▶ μ: mean predicted value
▶ σ^2: residual variance (variation of Y around μ)
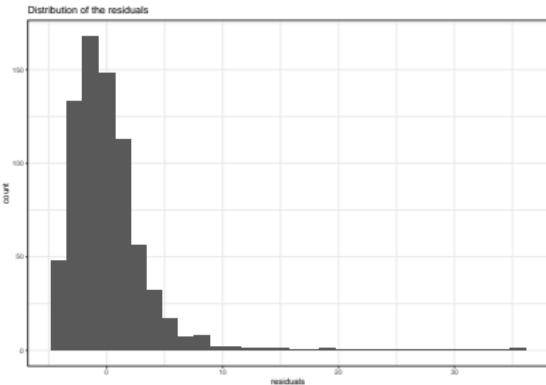
$\Rightarrow$ *focus on the distribution / link function*

What are the residuals?

We are always wrong in our predictions, but how wrong are we (in-sample)?

- ▶ **residuals:** difference between expected and observed outcomes ($Y_i - \mu_i$)
- ▶ a variable: one error / distance per observation in the data

```r
df <- df %>% mutate(
  #Predict in sample
  preds = predict(mod2, newdata = .),
  #Difference between expected and observed
  residuals = LocalAssistants - preds
  )
```


Distribution of the residuals

How to describe the residuals?

We describe the residuals by their spread (standard deviation/residual standard error)

```
mean(df$residuals)
```

[1] -9.8e-15

▶ mean: with an unbiased estimator, their average is 0

```
sd(df$residuals)
```

[1] 3.1

▶ standard deviation: but their spread can be more or less high
▶ here, the average distance from their mean is is a staff size of 3.08 local assistants.

$\Rightarrow$ *residual standard error*

Where is it reported?

```
summary(mod2)
```

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min     1Q  Median     3Q    Max
## -4.49  -1.94   -0.41   1.08  35.00
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.1266     0.2861   14.42  < 2e-16 ***
## OpenList     0.8288     0.2278    3.64  0.00029 ***
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## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 3.1 on 736 degrees of freedom
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```

```
summary(mod2)$sigma
```

```
## [1] 3.1
```

► Residual standard error: 3.08

► we may have predicted an average staff size of 3.86 per MEP in candidate-centered systems

► ... but individual MEPs typically deviate by 3.08 staffers

► from the variance, we can calculate the standard deviation, and thus prediction intervals around the mean prediction.

Conclusion: fundamental error

▶ important for predictions and model statistics
▶ not really for the uncertainty of the estimation of our effect

Estimation uncertainty

Estimation uncertainty

- ▶ most research is about the *effect of x* on y
- ▶ so, we're interested in the uncertainty of β

The central limit theorem and sampling

A fiction: the assumptions underpinning the uncertainty of the parameters

- ▶ assumption that data is a sample from a population
- ▶ we *could* sample many times
- ▶ we calculate the same parameter (e.g. mean, differences in means. . .) in each sample
- ▶ they will vary, but will follow a *normal distribution*

$\Rightarrow$ *each parameter is a distribution with a mean and a standard deviation*

Standard errors

`summary(mod2)`

```
##
## Call:
## lm(formula = LocalAssistants ~ OpenList + LaborCost, data = df)
##
## Residuals:
##    Min    1Q Median    3Q    Max
## -4.49  -1.94  -0.41   1.08  35.00
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```

▶ mean: average of all the differences in means between the two groups of MEPs: 0.95

▶ spread: the standard deviation of this distribution is 0.23

⇒ *a standard error is the standard deviation of a hypothetical distribution (parameters)*

Colinearities

Colinearities

Regression parameters may be correlated

```
mat <-vcov(mod2)
mat
```

```
##            (Intercept) OpenList LaborCost
## (Intercept)    0.0819 -0.02846  -0.00244
## OpenList      -0.0285  0.05191   0.00018
## LaborCost     -0.0024  0.00018   0.00010
```

- ▶ reported in the *variance-covariance matrix*
- ▶ diagonal: the variance of the parameter.
    - ▶ variance in effect of electoral system: $\sigma^2 = 0.05$
    - ▶ standard error in effect of electoral system: $\sqrt{\sigma^2} = 0.23$
- ▶ off-diagonal: the covariance of the parameters
    - ▶ low correlation between labor cost and electoral system

Estimate

King et al. (2000) make two points

- ▶ find interesting scenarios when you interpret
- ▶ estimate the uncertainty for the scenarios including
    - ▶ standard error (diagonal)
    - ▶ covariance (off-diagonal)

⇒ *the correlation between variables may mean higher or lower uncertainty than only using the standard error*

Simulation

They do this using simulation

- ▶ set scenario for all predictors
- ▶ draw from the distribution of parameters
- ▶ make prediction
- ▶ repeat many times
- ▶ extract the information and report
    - ▶ mean
    - ▶ median
    - ▶ mode
    - ▶ standard deviation
    - ▶ plot the distribution!

Our class

We will see two ways of doing this in R

▶ ggeffects package: simulates scenarios for us and can be plotted seamlessly → *effect plots, coefplots and point predictions*

▶ MASS package: the "manual" simulation from a multivariate normal distribution using the variance-covariance matrix. → *entire vector of simulations; for other plots/purposes*

Study technique

For this class

- ▶ learn by doing!
  - ▶ all readings include R examples; code along!
  - ▶ my R notebooks
  - ▶ then play around with the concepts; also with your own data/former exams
- ▶ dialogue with AI (ChatGPT, Claude)

What to ask and not to ask chat for?

R codes

- ▶ dont ask for complex codes
    - ▶ requires quirey competence on your end
    - ▶ you don't learn
- ▶ ask it to annotate your scripts
    - ▶ explain what each line means
    - ▶ dissect all code chunks you find and ask

What to ask and not to ask chat for?

Statistics

- ▶ don't ask for a summary of the reading
    - ▶ it's not necessarily what we will focus on
    - ▶ you don't learn
- ▶ ask for definitions
    - ▶ ask it to define key concepts you don't understand while you read
    - ▶ rephrase definitions and ask it this is a good understanding
- ▶ match with your readings
    - ▶ upload the PDF and ask specific questions
    - ▶ ask for examples, possibly with R codes
- ▶ interpretation
    - ▶ copy-paste your model output and ask for an explainer
    - ▶ use descriptive statistics to find interesting scenarios, ask it to help you find a plain English intuitive sentence