Chapter1-Notations

2023 年 2 月 24 日

1 Notations and Data

1.1 Notations

Several notations:

- $\mathbf{v}'$ and $\mathbf{M}'$ denote the transposes of $\mathbf{v}$ and $\mathbf{M}$, where $\mathbf{M} = [m]_{i,j}$, where i and j are row and column indices.
- Dependent variables: $\mathbf{y} = y_i$
- Features: $\mathbf{X}_i = (x_{i,1}, ..., x_{i,K})$, with dimenstion $I \times K$. This means there are I observatinons and each of them has K attributes.
- Discrete returns: $r_{t,n} = p_{t,n}/p_{t-1,n} - 1$ where t is time index and n is asset index
- Time series notations: $x_{t,n}^{(k)}$ is the time-t value of the k^{th} attribute of asset n
- $\mathbf{x}_{t,n}$ is a vector of all attributes for time-t and asset n
- $\mathbf{r}_t, \mathbf{r}_n$ and $r_{t,f}$ (riskless asset)
- $\mathbf{I}_N$ denotes the $(N \times N)$ identity matrix
- $\mathbb{E}[\cdot]$ and $\mathbb{E}_t[\cdot]$ are expectation and conditional expectation operators
- Filtration $\mathcal{F}_t$ is all information available at time t
- $\mathbb{V}[\cdot]$ is the variance operator
- Both P and $\mathbb{P}$ are probabilities, while f and F are p.d.f. and c.d.f. respectively
- $X \overset{d}{=} Y$ (equality in distribution) means $F_X(z) = F_Y(z)$ for all z
- Stationarity: a random process X_t satisfies $X_t \overset{d}{=} X_s$
- $x \propto y$ means x is proportional to y
- $1_{\{x\}} = 1$ when x is true else 0
- $\phi(\cdot)$ and $\Phi(\cdot)$ are standard Gaussian pdf and cdf.
- $card(\cdot) = \#(\cdot)$ are cardinal function which evaluates the number of elements in a given set
- $\lfloor x \rfloor$ is the integer part function
- $[x]^+ = \max(0, x)$
- $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$
- $ReLu(x) = \max(0, x)$

- $s(x)$ is the softmax function where $s(\mathbf{x})_i = \frac{e^{x_i}}{\sum_j e^{x_j}}$

1.2 Dataset

The original repository can be accessed here, while the .xlsx version of data set can be accessed here. This dataset comprises information on 1,207 stocks listed in the US (possibly originating from Canada or Mexico). The time range starts in November 1998 and ends in March 2019. For each point in time, 93 characteristics describe the firms in the sample. These attributes cover a wide range of topics including:

- **valuation** (earning yields, accounting ratios)
- **profitability** and quality (ROE, etc.)
- **momentum** and technical analysis (past returns, RSI, …)
- **risk** (volatilities)
- **estimates** (EPS)
- **volume and liquidity** (share turnover)

[ ]:
```
import pandas as pd
import pyreadr
# data = pd.read_excel("./data/data_ml.xlsx") # Not Recommended. Too Slow!
result = pyreadr.read_r('./data/data_ml.RData')
data = result['data_ml']
```

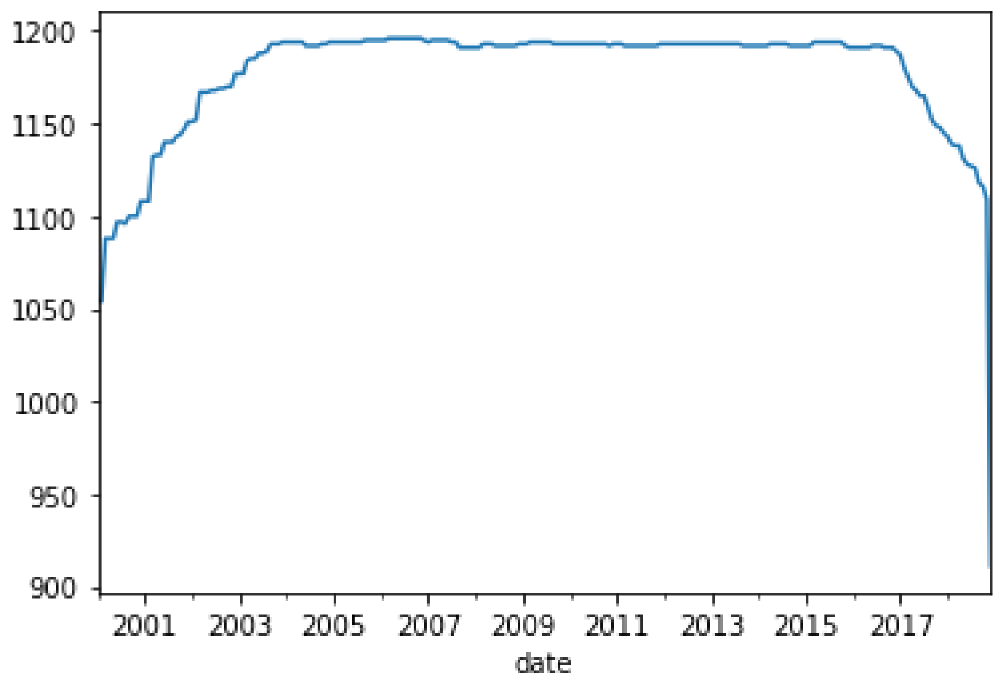
[ ]:
```
data['date'] = pd.to_datetime(data['date'])
data = data[(data['date'] > "1999-12-31") & (data['date'] < "2019-01-01")].
 ↪sort_values(by = ["stock_id", "date"]).reset_index().drop(columns = "index")
data.head()
```

[ ]:
```
   stock_id       date  Advt_12M_Usd  Advt_3M_Usd  Advt_6M_Usd  \
0         1 2000-01-31          0.41         0.39         0.42
1         1 2000-02-29          0.41         0.39         0.40
2         1 2000-03-31          0.40         0.37         0.37
3         1 2000-04-30          0.39         0.36         0.37
4         1 2000-05-31          0.40         0.42         0.40

   Asset_Turnover  Bb_Yld    Bv  Capex_Ps_Cf  Capex_Sales  …  Total_Capital  \
0            0.19    0.65  0.63          0.5          0.9  …           0.74
1            0.19    0.81  0.63          0.5          0.9  …           0.74
```

```
2             0.20     0.68  0.65          0.5          0.9  …            0.76
3             0.20     0.38  0.65          0.5          0.9  …            0.76
4             0.20     0.60  0.65          0.5          0.9  …            0.76

   Total_Debt  Total_Debt_Capital  Total_Liabilities_Total_Assets  Vol1Y_Usd  \
0        0.82                0.88                            0.74       0.77
1        0.82                0.88                            0.74       0.73
2        0.83                0.87                            0.75       0.67
3        0.83                0.87                            0.75       0.63
4        0.83                0.87                            0.75       0.54

   Vol3Y_Usd  R1M_Usd  R3M_Usd  R6M_Usd  R12M_Usd
0       0.68   -0.036    0.196    0.255     1.044
1       0.67    0.263    0.797    0.669     1.192
2       0.66    0.031    0.275    0.419     0.495
3       0.65    0.448    0.042    0.267     0.853
4       0.67   -0.097   -0.071    0.027     0.413

[5 rows x 99 columns]
```

The data has 99 columns and 268336 rows. The first two columns indicate the stock identifier and the date. The next 93 columns are the features. The last four columns are the labels. The points are sampled at the monthly frequency.

[ ]:
```python
import matplotlib.pyplot as plt
data.groupby('date')['stock_id'].count().plot()
# data.groupby('date')['stock_id'].count().plot.bar()
# plt.xticks(list(range(2000, 2019)))
plt.show()
```



There are four immediate labels in the dataset: R1M_Usd, R3M_Usd, R6M_Usd and R12M_Usd, which correspond to the 1-month, 3-month, 6-month and 12-month future/forward returns of the stocks. The returns are total returns, that is, they incorporate potential dividend payments over the considered periods. This is a better proxy of financial gain compared to price returns only. We refer to the analysis of Hartzmark and Solomon (2019) for a study on the impact of decoupling price returns and dividends. These labels are located in the last 4 columns of the dataset. We provide their descriptive statistics below.

[ ]:
```
data[['R12M_Usd', 'R1M_Usd', 'R3M_Usd', 'R6M_Usd']].describe().T
```

[ ]:

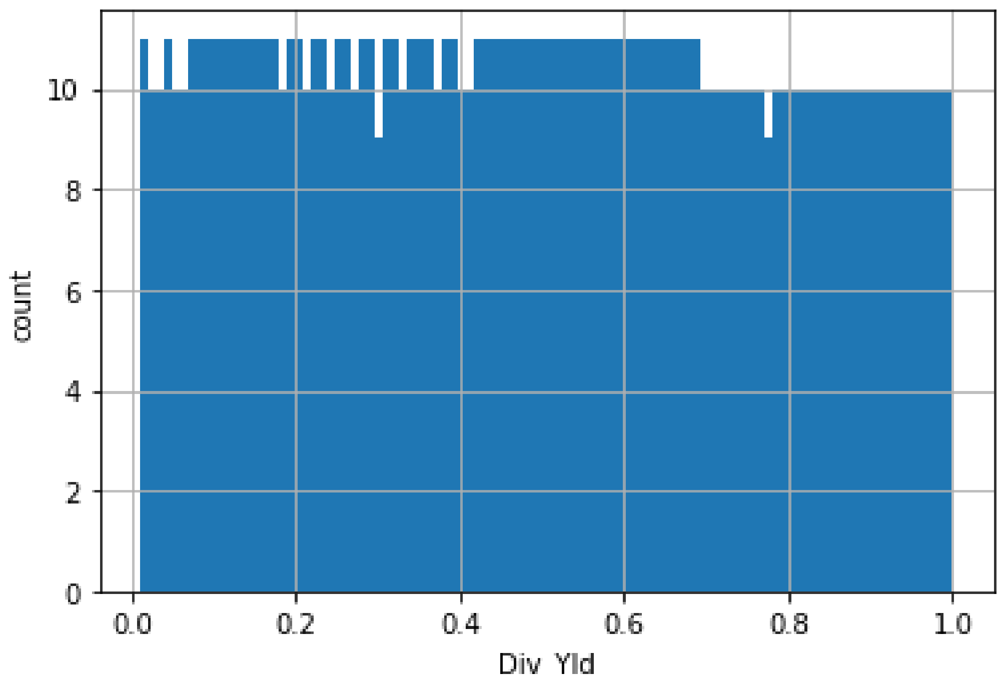
	count	mean	std	min	25%	50%	75%	max
R12M_Usd	268336.0	0.136865	0.737872	-0.991	-0.090	0.080	0.285	95.972
R1M_Usd	268336.0	0.012732	0.176431	-0.922	-0.042	0.011	0.064	30.176
R3M_Usd	268336.0	0.036901	0.328289	-0.929	-0.063	0.030	0.123	39.389
R6M_Usd	268336.0	0.072339	0.527420	-0.980	-0.079	0.050	0.186	106.929

We keep some shorter lists of predictors.

[ ]:
```
features = data.columns[2:]
```

```
features_short = ["Div_Yld", "Eps", "Mkt_Cap_12M_Usd", "Mom_11M_Usd", "Ocf",␣
  ↪"Pb", "Vol1Y_Usd"]
```

[ ]:
```
data[data['date'] == "2000-02-29"]["Div_Yld"].hist(bins = 100)
plt.ylabel('count')
plt.xlabel('Div_Yld')
plt.show()
```



The original labels (future returns) are numerical and will be used for regression exercises, that is, when the objective is to predict a scalar real number. Sometimes, the exercises can be different and the purpose may be to forecast categories (also called classes), like "buy", "hold" or "sell". In order to be able to perform this type of classification analysis, we create additional labels that are categorical.

[ ]:
```
sub_data = data[['date', 'R1M_Usd', 'R12M_Usd']]
data_median = sub_data.groupby('date').median()
data_median.columns = ['R1M_Usd_M', 'R12M_Usd_M']
data_ml = pd.merge(data, data_median, how = "left", on = "date")
data_ml['R1M_Usd_C'] = 1 * (data_ml['R1M_Usd'] > data_ml['R1M_Usd_M'])
```

```python
data_ml['R12M_Usd_C'] = 1 * (data_ml['R12M_Usd'] > data_ml['R12M_Usd_M'])
data_ml
```

[ ]:

```
        stock_id        date  Advt_12M_Usd  Advt_3M_Usd  Advt_6M_Usd  \
0              1  2000-01-31          0.41         0.39         0.42
1              1  2000-02-29          0.41         0.39         0.40
2              1  2000-03-31          0.40         0.37         0.37
3              1  2000-04-30          0.39         0.36         0.37
4              1  2000-05-31          0.40         0.42         0.40
…            …           …             …            …            …
268331      1212  2018-08-31          0.85         0.84         0.85
268332      1212  2018-09-30          0.85         0.84         0.85
268333      1212  2018-10-31          0.86         0.87         0.86
268334      1212  2018-11-30          0.86         0.88         0.86
268335      1212  2018-12-31          0.86         0.87         0.86

        Asset_Turnover  Bb_Yld    Bv  Capex_Ps_Cf  Capex_Sales  …  \
0                 0.19    0.65  0.63          0.5         0.90  …
1                 0.19    0.81  0.63          0.5         0.90  …
2                 0.20    0.68  0.65          0.5         0.90  …
3                 0.20    0.38  0.65          0.5         0.90  …
4                 0.20    0.60  0.65          0.5         0.90  …
…                  …      …    …            …            …  …
268331            0.43    0.93  0.66          1.0         0.93  …
268332            0.41    0.87  0.66          1.0         0.94  …
268333            0.41    0.91  0.66          1.0         0.94  …
268334            0.41    0.91  0.66          1.0         0.94  …
268335            0.40    0.94  0.67          1.0         0.94  …

        Vol1Y_Usd  Vol3Y_Usd  R1M_Usd  R3M_Usd  R6M_Usd  R12M_Usd  R1M_Usd_M  \
0            0.77       0.68   -0.036    0.196    0.255     1.044     -0.032
1            0.73       0.67    0.263    0.797    0.669     1.192      0.060
2            0.67       0.66    0.031    0.275    0.419     0.495      0.021
3            0.63       0.65    0.448    0.042    0.267     0.853     -0.006
4            0.54       0.67   -0.097   -0.071    0.027     0.413     -0.003
…             …         …        …       …        …        …         …
```

```
268331          0.69        0.85   0.044  -0.252  -0.144    0.000   -0.011
268332          0.70        0.84  -0.266  -0.373   0.000    0.000   -0.079
268333          0.87        0.87   0.013   0.043   0.000    0.000    0.036
268334          0.88        0.87  -0.125   0.157   0.000    0.000   -0.105
268335          0.88        0.87   0.222   0.000   0.000    0.000    0.100

        R12M_Usd_M  R1M_Usd_C  R12M_Usd_C
0           0.1440          0           1
1           0.1900          1           1
2           0.0790          1           1
3           0.1315          1           1
4           0.1750          0           1
…              …         …         …
268331      0.0000          1           0
268332      0.0000          0           0
268333      0.0000          0           0
268334      0.0000          0           0
268335      0.0000          1           0

[268336 rows x 103 columns]
```

The new labels are binary: they are equal to 1 (true) if the original return is above that of the median return over the considered period and to 0 (false) if not. Hence, at each point in time, half of the sample has a label equal to zero and the other half to one: some stocks overperform and others underperform.

In machine learning, models are estimated on one portion of data (**training set**) and then tested on another portion of the data (**testing set**) to assess their quality. We split our sample accordingly.

[ ]:
```
separation_date = pd.to_datetime('2014-01-15')
training_sample = data_ml[data_ml['date'] < separation_date]
testing_sample = data_ml[data_ml['date'] >= separation_date]
```

We also keep in memory a few key variables, like the list of asset identifiers and a rectangular version of returns. For simplicity, in the computation of the latter, we shrink the investment universe to keep only the stocks for which we have the maximum number of points.

[ ]:

```python
stock_days = data_ml[['date', 'stock_id']].groupby('stock_id').count()
stock_ids = stock_days[stock_days['date'] == max(stock_days['date'])].index
returns = data_ml[data_ml['stock_id'].isin(stock_ids)]
returns.pivot(index = "date", columns = "stock_id", values = "R1M_Usd")
```

[ ]:

```
stock_id       1      3      4      7      9     11     12     16     17  \
date
2000-01-31 -0.036  0.077 -0.016 -0.009  0.032  0.144 -0.110 -0.191 -0.103
2000-02-29  0.263 -0.024  0.000  0.027  0.076  0.258  0.110  0.197 -0.053
2000-03-31  0.031  0.018  0.153  0.000 -0.025  0.049  0.134 -0.030  0.074
2000-04-30  0.448  0.027 -0.011 -0.017 -0.022  0.014  0.022  0.161  0.051
2000-05-31 -0.097  0.050  0.014  0.018 -0.121 -0.116 -0.038 -0.020  0.038
…              …      …      …      …      …      …      …      …      …
2018-08-31 -0.002 -0.049  0.036 -0.068 -0.028  0.146  0.040  0.016  0.297
2018-09-30 -0.155 -0.070 -0.025 -0.052 -0.098 -0.291 -0.044 -0.050 -0.150
2018-10-31  0.014  0.036  0.000  0.007 -0.018 -0.100  0.074  0.094 -0.081
2018-11-30 -0.110 -0.070 -0.088 -0.066 -0.099 -0.251 -0.105 -0.059 -0.171
2018-12-31  0.082  0.030  0.043  0.131  0.112  0.158  0.067  0.039  0.402

stock_id       18  …   1199   1200   1201   1202   1203   1204   1208  \
date               …
2000-01-31  0.098  … -0.509  0.039 -0.011 -0.019  0.168  0.080 -0.372
2000-02-29 -0.001  … -0.147  0.017 -0.148  0.118  0.177  0.161  0.143
2000-03-31 -0.002  …  0.083  0.006 -0.025 -0.009  0.237 -0.008  0.057
2000-04-30 -0.037  … -0.192  0.003 -0.061  0.053 -0.097 -0.114  0.070
2000-05-31 -0.131  … -0.279  0.014  0.118  0.089 -0.023 -0.089 -0.137
…              …  …      …      …      …      …      …      …      …
2018-08-31  0.025  … -0.006 -0.005 -0.048 -0.010  0.037  0.005  0.126
2018-09-30 -0.135  … -0.064 -0.013 -0.070 -0.024 -0.030 -0.161 -0.119
2018-10-31  0.074  … -0.075  0.068  0.051  0.078 -0.005  0.107  0.205
2018-11-30 -0.215  … -0.066 -0.054 -0.197 -0.149 -0.079 -0.050 -0.189
2018-12-31  0.247  … -0.005  0.071  0.184  0.095  0.061  0.147  0.227

stock_id     1209   1210   1212
date
2000-01-31 -0.560 -0.006 -0.186
2000-02-29 -0.102 -0.057  0.104
```

```
2000-03-31  0.144  0.085 -0.143
2000-04-30 -0.103  0.073  0.204
2000-05-31  0.113  0.016  0.042
…           …      …      …
2018-08-31 -0.009 -0.011  0.044
2018-09-30  0.000  0.028 -0.266
2018-10-31  0.076  0.063  0.013
2018-11-30 -0.147 -0.064 -0.125
2018-12-31  0.092  0.108  0.222

[228 rows x 793 columns]
```