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CogSci Methods 2 – F24

Solutions to Lecture 4 exercises

Exercises

From slides from lecture 4:

Find the value of y

a. $\log_8 \frac{1}{64} = y$ **b.** $\log_6 36 = y$ **c.** $\log_7 1 = y$ **d.** $\log_4 \frac{1}{4} = y$

Write the following expressions in terms of logs of x, y and z.

e. $\log(xy)^{\frac{1}{3}}$ **f.** $\log x\sqrt{z}$ **g.** $\log \frac{x^3y^2}{z}$ **h.** $\log \frac{\sqrt{x}\cdot\sqrt[3]{y^2}}{z^4}$

True or False?

i. $\frac{\log a}{\log b} = \log(a-b)$ **j.** $\log(a-b) = \log a - \log b$ **k.** $-\ln\left(\frac{1}{x}\right) = \ln x$

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Find the value of y:

a. $\log_8 \frac{1}{64} = y$

$$\log_8 \frac{1}{64}$$

$$= \log_8 \frac{1}{8^2}$$

$$= \log_8 8^{-2}$$

$$= -2$$

RULES USED

(Approximately on the line where the given rule has been applied)

Using that $\frac{1}{64} = 8^2$

$$\frac{1}{a^n} = a^{-n}$$

$$\log_b(b^x) = x$$

it has been applied)

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b. $\log_6 36 = y$

$$\log_6 36$$

$$= \log_6 6^2$$

Using that $36 = 6^2$

$$= 2$$

$\log_b(b^x) = x$

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c. $\log_7 1 = y$

$$\log_7 1$$

$$= 0$$

$\log_b(1) = 0$

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d. $\log_4 \frac{1}{4} = y$

$$\log_4 \frac{1}{4}$$

$$= \log_4 4^{-1}$$

$a^{-n} = \frac{1}{a^n}$ gets us 4^{-1} from $\frac{1}{4}$

$$= -1$$

$\log_b(b^x) = x$

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Write the following expressions in terms of logs of x, y and z: (i.e. expression must have separate log terms of the individual variables)

e. $\log((xy)^{\frac{1}{3}})$ (I'm assuming this is what's meant on the slide – have added an extra pair of '()')

We only have x and y (not z) in this one, so we'll only express in terms of logs of x and y. We will use these rules:

- **The Power Rule**: $\log(a^b) = b \cdot \log(a)$
- **The Product Rule**: $\log(ab) = \log(a) + \log(b)$

Let's go:

$$\log((xy)^{\frac{1}{3}})$$

$$= \frac{1}{3}\log(xy)$$ $\log(a^b) = b \cdot \log(a)$ **(The Power Rule)**

$$= \frac{1}{3}(\log(x) + \log(y))$$ $\log(ab) = \log(a) + \log(b)$ **(The Product Rule)**

$$= \frac{1}{3}\log(x) + \frac{1}{3}\log(y)$$

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f. $\log x\sqrt{z}$

We'll express in terms of logs of x and z (not y cause it's not there).

First, let's ✨think creatively✨ and see that $\sqrt{z} = z^{\frac{1}{2}}$.

Other than that we'll use **The Power Rule** and **The Product Rule.** Let's go:

$$\log(x\sqrt{z})$$

RULES USED

(Approximately on the line where the given rule has been applied)

$$= \log\left(x \cdot z^{\frac{1}{2}}\right)$$ Using that $\sqrt{z} = z^{\frac{1}{2}}$

$$= \log(x) + \log\left(z^{\frac{1}{2}}\right)$$ $\log(ab) = \log(a) + \log(b)$ **(The Product Rule)**

$$= \log(x) + \frac{1}{2}\log(z)$$ $\log(a^b) = b \cdot \log(a)$ **(The Power Rule)**

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g. $\log\frac{x^3y^2}{z}$

We'll use

- **The Quotient Rule:** $log\left(\frac{a}{b}\right) = log(a) - log(b)$

And **The Power Rule**. Let's go:

$$\log\left(\frac{x^3y^2}{z}\right)$$

$$= \log(x^3y^2) - \log(z)$$ $log\left(\frac{a}{b}\right) = log(a) - log(b)$ **(The Quotient Rule)**

$$= \log(x^3) + \log(y^2) - \log(z)$$ $\log(ab) = \log(a) + \log(b)$ **(The Product Rule)**

$$= 3 \cdot \log(x) + 2 \cdot \log(y) - \log(z)$$ $\log(a^b) = b \cdot \log(a)$ **(The Power Rule)**

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h. $\log\frac{\sqrt[3]{x}\sqrt{y^2}}{z^4}$

We'll use **The Quotient Rule, The Product Rule, and The Power Rule.** Let's go:

$$\log\left(\frac{\sqrt[3]{x}\sqrt{y^2}}{z^4}\right)$$

$$= \log\left(\sqrt[3]{x}\sqrt{y^2}\right) - \log(z^4)$$

$log\left(\frac{a}{b}\right) = log(a) - log(b)$ **(The Quotient Rule)**

$$= \log(\sqrt[3]{x}) + \log\left(\sqrt{y^2}\right) - \log(z^4)$$

$\log(ab) = \log(a) + \log(b)$ **(The Product Rule)**

From here, I can rewrite the roots a bit and apply **The Power Rule** to each term:

INTERMEDIATE CALCULATION

$\log(\sqrt[3]{x}) = \log\left(x^{\frac{1}{3}}\right) = \frac{1}{3}\log(x)$

$\log\left(\sqrt{y^2}\right) = \log(y)$ (gonna assume a ≥ 0 so I can do this 💀 don't @ me, mathematicians)

$-\log(z^4) = -4\log(z)$

$\log(a^b) = b \cdot \log(a)$ **(The Power Rule)**

Now we can continue:

$$= \frac{1}{3}\log(x) + \log(y) - 4\log(z)$$

Substituting with terms from intermediate calculation

True or False?

i. $\frac{\log a}{\log b} = \log(a - b)$

False. There is no (to my knowledge) logarithm property that equates to the division of two logarithms resulting in the logarithm of their difference.

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j. $\log(a - b) = \log a - \log b$

False. The logarithm of a difference does not equal the difference of logarithms. There's no logarithmic property that supports this operation. We also have The Quotient Rule which states

$$log\left(\frac{a}{b}\right) = log(a) - log(b)$$

Where the right hand side is what we indeed have on the right side too, but the left does not equate to the left side in The Quotient Rule.

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k. $-\ln\left(\frac{1}{x}\right) = \ln x$

It's ✨💅🏻true✨💅🏻. (Btw if you're thinking what the heck is 'ln', when we've been working with 'log' all this time: log is defined for base 10, ln is defined for base e. It means that $\ln = \log_e$. So laws that apply to log also apply to ln.)

It's actually a logarithmic identity which makes it true. But we can start from the given equation to see it. We have, on the left side:

$$-ln\left(\frac{1}{x}\right)$$

The negative sign in front of the logarithm indicates the reciprocal relationship. The logarithm of a reciprocal equals the negative logarithm of the base number. This means that we can write:

$$-ln\left(\frac{1}{x}\right) = ln(x)$$

And we can make that equation and it's valid because of The Power Rule, actually (= NICE) (with a bit of rewriting), and because of the mentioned $\frac{1}{x} = x^{-1}$:

$$-ln\left(\frac{1}{x}\right) = -(\ln(x^{-1})) = -(-1\ln(x)) = \ln(x)$$