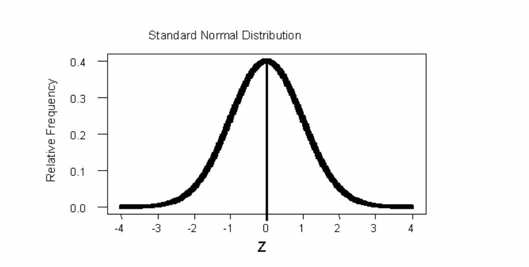
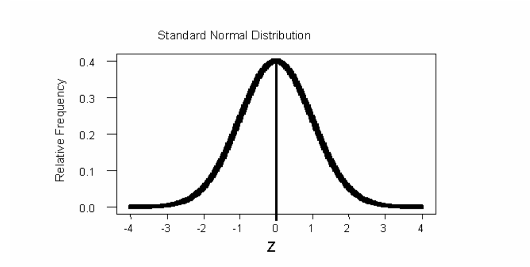
The Normal Distribution

- The normal distribution is the “bell curve”
- It is a distribution that is often used to model responses from quantitative continuous data.



Standard Normal Distribution

- When the horizontal axis has standard Z units the mean is 0 and a standard deviation distance is 1.
- This called the standard normal distribution



Nonstandard Normal Populations

- It’s easy to compute probabilities for populations that are normally distributed; but, perhaps, not distributed as a standard normal

Strategy for Solving Nonstandard Normal Problems

$$x \xrightarrow{z=\frac{x-\mu}{\sigma}} z \xrightarrow{Table\ \ A1} \Pr\{\, z < a\}$$

When presented with a normal probability problem which is not a standard normal probability, the first thing to do is to convert the values of interest into z scores.

Strategy for solving nonstandard normal problems – Type 1

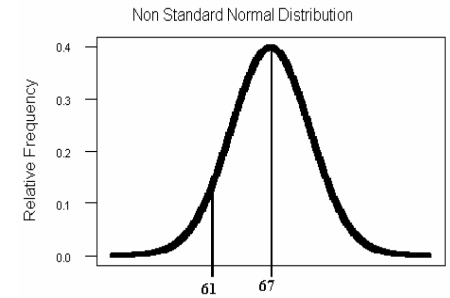
$$x \xrightarrow{z=\frac{x-\mu}{\sigma}} z \xrightarrow{Table\ A1} \Pr\{z < a\}$$

After the values have been standardized the problem will fit into 1 of 3 types.

Nonstandard Normal Type I

Example: Suppose that the heights of men are normally distributed with a mean of 67 inches and a std. dev. of 3 inches. What proportion of this population will have heights less than 61 inches?

Pr{x < 61}

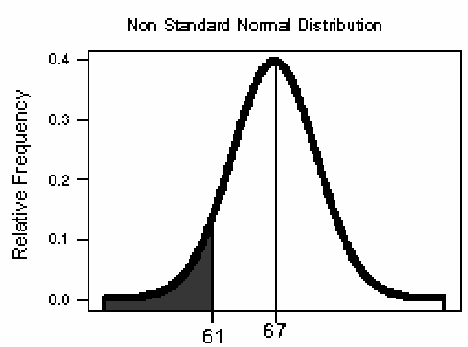


Nonstandard Normal Type I

Shade the area of interest.

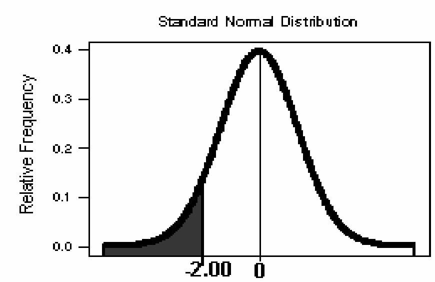
This is a "left tail" problem

You are finding the **<u>cumulative probability</u>** and these are the easiest problems to answer



Nonstandard Normal: Type I

1) Standardize the problem by converting to z-scores
   - $z = \dfrac{61 - 67}{3} = -2.00$
2) Now look the z-score up in Table A1 and read the cumulative probability
   - Pr{x < 61} = Pr{x < -2.00}
     = 0.028



Strategy for solving nonstandard normal problems – Type 2

$$x \xrightarrow{z=\frac{x-\mu}{\sigma}} z \xrightarrow{\text{Table } A1} \Pr\{z < a\}$$

After the values have been standardized the problem will fit into 1 of 3 types.

NonStandard Normal: Type II

- The non-standard normal type II problems involve right tail areas rather than left tail areas.
  - This area is the **complement** of the cumulative probability

NonStandard Normal: Type II

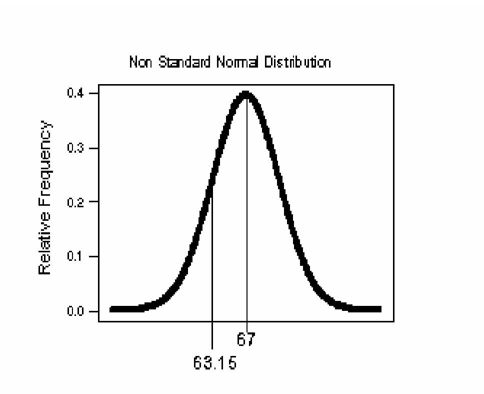
- The standard normal is a **Probability Density Function** **(PDF)**
  - This means that the total area under the curve equals 100%

Type II Example

Suppose that the heights of men are normally distributed with a mean of 67 inches and a standard deviation of 3 inches. What proportion of this population would you expect to have heights that are **more** than 63.15 inches?

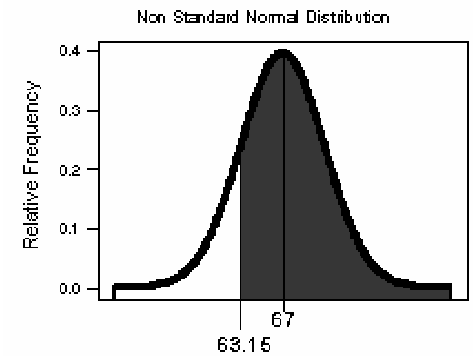
Type II Example

- Since the "heights" are normally distributed we draw a bell curve and locate the mean.
- Then we locate the "cutoff" of interest.



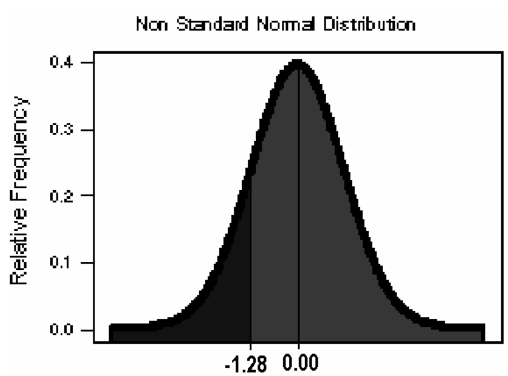
Type II Example

Now shade the area of interest. Note that we are interested in the proportion of height values that are greater than 63.15.



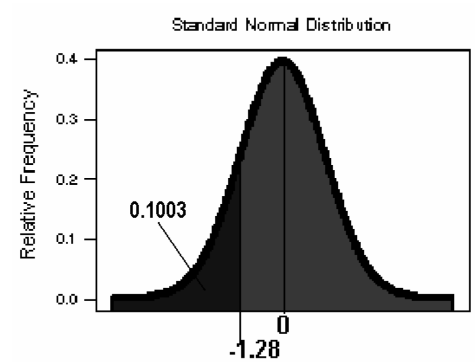
NonStandard Normal: Type II

- Standardize the problem by converting to Z-scores.
- $Z = \dfrac{63.15 - 67}{3} = -1.28$



NonStandard Normal: Type II

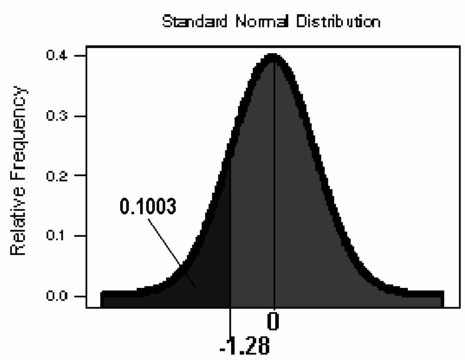
- Look up the z-value in table A1.
- Remember that this is a left tail area.
- $Pr\{Z < -1.28\} = 0.1003$



NonStandard Normal: Type II

- Now use the fact that the area under the whole curve is 1 (100%) and get the right tail area by subtraction.

$Pr\{z > -1.28\} = 1 - 0.1003$
$= 0.8997$



NonStandard Normal: Type III

- The non-standard normal type III problems involve central areas rather than left or right tail areas.
- We will also solve these “type III” problems by exploiting the fact area at the left-most tail is “0”.

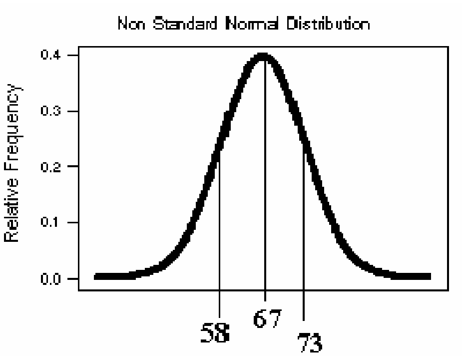
Rigorously this last statement is incorrect. However, it’s a close enough approximation and gets us the right answers without a bunch of mathematical technicalities.

Type III Example

Suppose that the heights of men are normally distributed with a mean of 67 inches and a standard deviation of 3 inches. What proportion of this population would you expect to have heights between 58 and 73 inches?

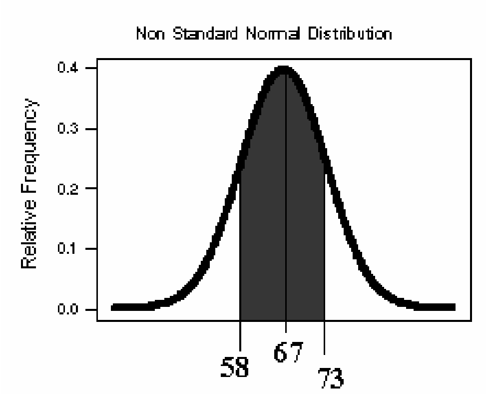
Type III Example

- Since the “heights” are normally distributed we draw a bell curve and locate the mean.
- Then we locate the two “cutoff” values of interest.



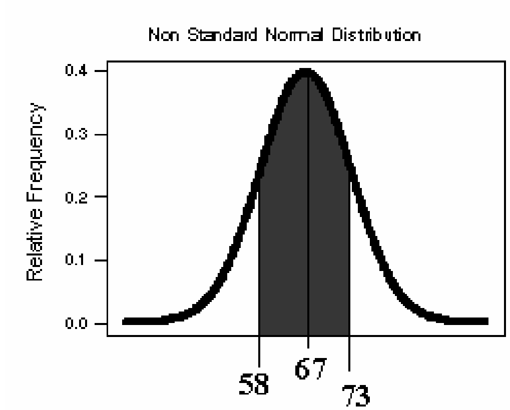
Type III Example

Now shade the area of interest. Note that we are interested in the proportion of height values that are greater than 58 inches and less than 73 inches.



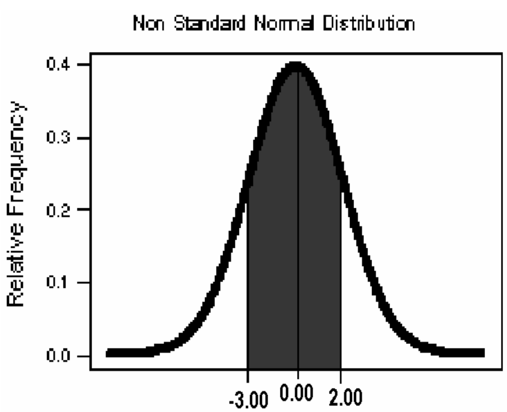
NonStandard Normal: Type III

- Standardize the problem by converting to z-scores.
- $z = \dfrac{58 - 67}{3} = -3.000$
- $z = \dfrac{73 - 67}{3} = 2.000$



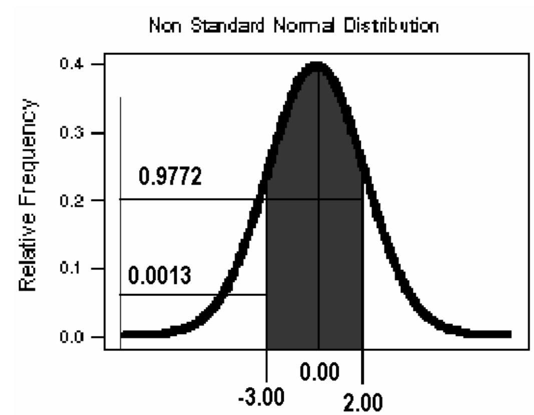
NonStandard Normal: Type III

- What this step does is to convert from a non-standard normally distributed (bell shaped) variable to a standard normal variable



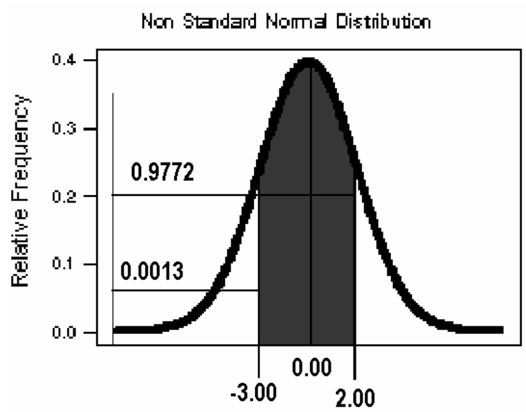
NonStandard Normal: Type III

- Look up the corresponding left tail probabilities in Table A1.
- To find the central proportion of interest simply subtract.



NonStandard Normal: Type III

- The proportion of men with heights between 58 and 73 inches is the same as the proportion of z-scores between -3.00 and +2.00.
- Pr = 0.9772 - 0.0013 = 0.9759



Reverse Standard Normal Problems

$$x \xleftarrow{x=\mu+z\sigma} z \xleftarrow{Table\ A1} \Pr\{z < a\}$$

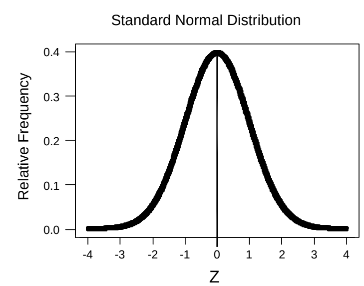
- Given a quantile value or, simply, a specified probability work backwards to obtain a z score.

Determine a z value

- What value separates the bottom 10% from the remaining 90% of the standard normal distribution.
- What is the z value for the 10'th percentile of the standard normal distribution
- Find "a" such that Pr{z < a} = 0.100

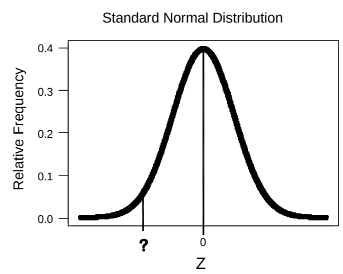
Solution Strategy

1) A) Draw a bell curve and drop a mean.
Since this is a standard normal problem we draw a standard normal curve.



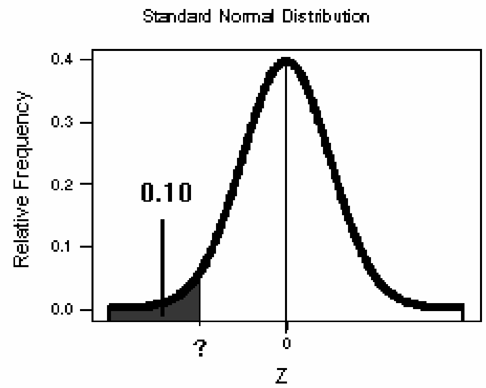
Solution Strategy

2) Start at the tail that's indicated in the problem. A "less than" problem begins in the left tail. Move into the distribution until you've reached the specified area. Then drop another line at this cutoff.



Solution Strategy

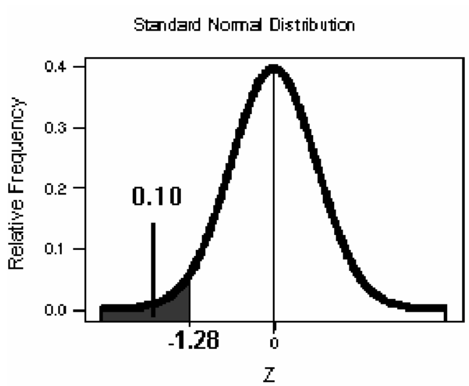
3) Shade the area of interest



Solution Strategy

4) Since this is already a left tail area we simply go to the z - table and look down the probability column until we come to 0.100.
5. Find the row header and the column header to get the z-value:

   z = -1.28



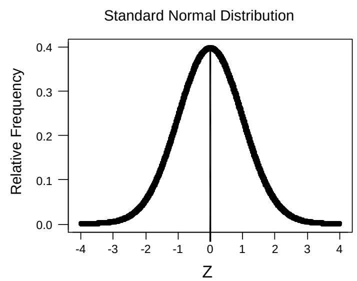
Reverse Std. Normal: Type II

- The type II reverse standard normal problem requires that you work backwards from a right tail probability.
- Find "a" such that Pr{z > a} = 0.841
- Find a value "a" that separates the top 84.1% of the standard normal distribution from the bottom 15.9%
- In terms of percentiles this problem is asking you to find the 15.9'th percentile

Type II Solution Strategy

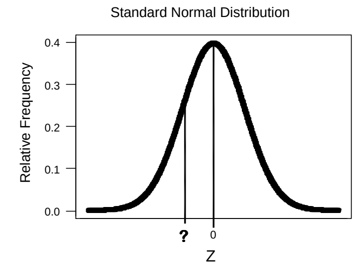
Find "a" such that
$Pr\{z > a\} = 0.841$

1) Draw a bell curve and drop a mean.



Type II Solution Strategy

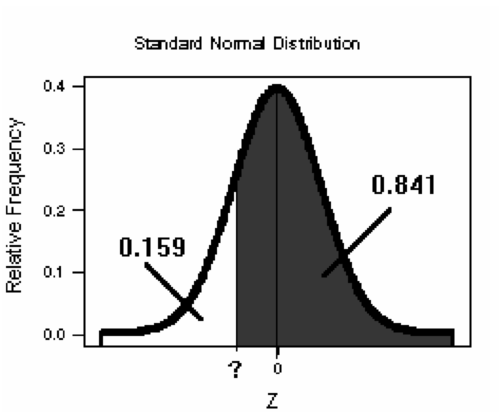
2) Start at the tail that's indicated in the problem. Since this is a "greater than" problem you begin in the right tail and move into the distribution until you've reached the specified area. Then drop another line at this cutoff.



◆Find "a" such that
$Pr\{z > a\} = 0.841$

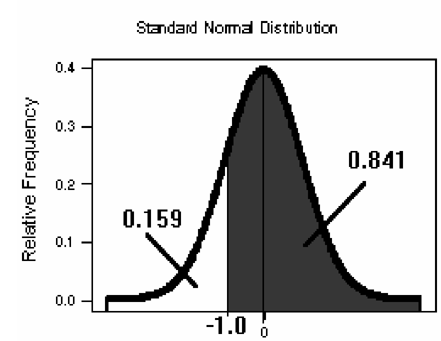
Type II Solution Strategy

3) Shade the area of interest



Type II Solution Strategy

- The given probability is a right tail probability. Since the table is set up to give left tail probabilities we need to get this value
- 1 - 0.841 = 0.159
- The corresponding z-value is -1.00



Reverse Nonstandard Normal Problems

- In order to become a member of MENSA a person must have an IQ that is in the top 5% of the population. If IQ scores are normally distributed with a mean of 100 and a standard deviation of 15, what is the minimum IQ score that would qualify for admittance to this organization?

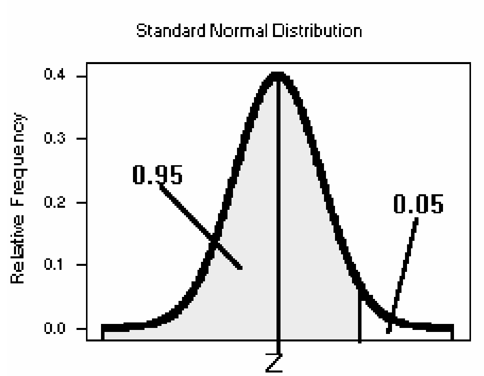
Solution Strategy

Work the flow chart from right to left. You're given a probability. You need to determine a value for x

$$x \xleftarrow{x=\mu+z\sigma} z \xleftarrow{Table\ A1} \Pr\{z<a\}$$

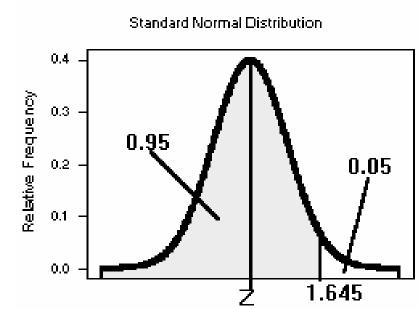
Strategy: Step 1

- Write a mathematical statement that corresponds to the problem.
- Find a value "a" such that **Pr{x < a} = 0.95**
- Or: Find a value "a" so that **Pr{x > a} = 0.05**
- Draw a normal curve, drop a mean and a cutoff.



Strategy: Step II

- We're on the right side of the flow chart.
- Use a Z-table in reverse to find the z value.



$$x \xleftarrow{x=\mu+z\sigma} z \xleftarrow{Table\ A1} \Pr\{z<a\}$$

Strategy: Step III

- Now plug the values into the formula that leads from z to the raw score, x.
- $x = \mu + Z\sigma$

$x = 100 + 1.645*15$
$= 100 + 24.675$
$= 124.675$

◆In order to be eligible to participate in MENSA a person must have an IQ of at least 124.675