BST 140.752
Problem Set 5

1 LMM

1. Let $Y_{ij} = \mu + u_i + \epsilon_{ij}$ for $u_i \sim N(0, \sigma_u^2)$ and $\epsilon_{ij} \sim N(0, \sigma^2)$. Calculate the BLUP for u_i
2. Let $Y = X\beta + Zu + \epsilon$ for $u \sim N(0, \Sigma_u)$ and $\epsilon \sim N(0, \sigma^2 I)$. Calculate the BLUP for u.
3. Load the Rail data set in R. Fit a mixed model of the form from question 1. Compare a the estimates of the mean for each rail with the empirical mean.
4. Load the pixel data set in R. Fit a linear mixed effect model where you have $Y_{ijk} = \beta_0 + \beta_1 x_k + u_i + u_{ij} + \epsilon_{ijk}$ where Y_{ijk} is pixel, i is dog, j is side and k is day index and x_k is day. Fit the model and interpret the results.
5. Consider the model $Y_i = \mu + \epsilon_{ij}$. Consider putting a so-called "flat" prior on μ. That is acting like a distribution that is 1 from $-\infty$ to $+\infty$ is a valid density. Calculate the distribution marginalized over μ and show that it is the same likelihood used to obtain the REML estimates.
6. Derive the BLUP estimates for a linear mixed effect model of the form $Y = X\beta + ZU + \epsilon$ where $U \sim N(0, \Sigma_u)$ and $\epsilon \sim N(0, \sigma^2 I)$ and assuming known variance components.
7. Find the BLUPs for the Rail data set in R. Compare the BLUP estimates with those obtained using a fixed effect estimate for the U_i.
8. Consider the model from Question 1. Show that the estimate for σ^2 is not consistent as $i \to \infty$ for equal numbers of observations per cluster.