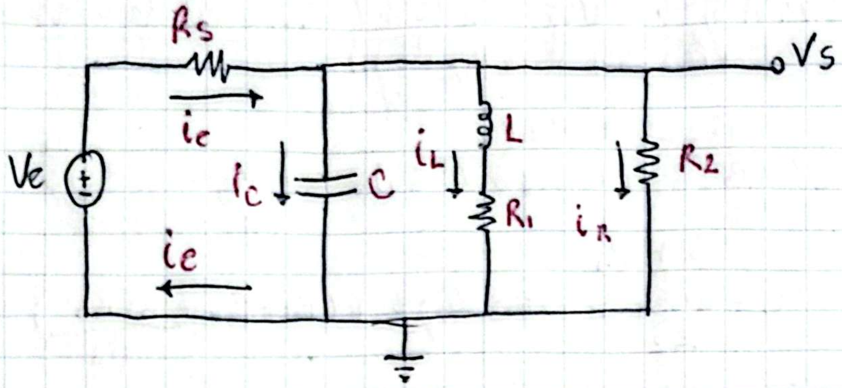
Proyecto final: Membrana neuronal

Circuito eléctrico



Funcion de transferencia

$$i_e(t) = i_c(t) + i_L(t) + i_R(t) = \frac{V_e(t) - V_s(t)}{R_s}$$

$$i_c = C\frac{dV_s(t)}{dt}$$

$$i_L = \frac{V_s(t)}{R_1 + Z_L}$$

$$i_R = \frac{V_s(t)}{R_2}$$

Procedimiento algebraico

$$C\frac{dV_s(t)}{dt} + \frac{V_s(t)}{R_1 + Z_L} + \frac{V_s(t)}{R_2} = \frac{V_e(t) - V_s(t)}{R_s}$$

$$C_s\, V_s(s) + \frac{V_s(t)}{R_1 + LS} + \frac{V_s(S)}{R_2} = \frac{V_e(s) - V_s(s)}{R_s}$$

$$C s\, V_s(s) + \frac{1}{R_1 + LS} V_s(t) + \frac{V_s(S)}{R_2} + \frac{V_s(S)}{R_s} = \frac{V_e(t)}{R_s}$$

$$\left(CS + \frac{1}{R_1 + LS} + \frac{1}{R_2} + \frac{1}{R_s}\right) V_s(s) = \left(\frac{1}{R_s}\right) V_e(s)$$

$$\left(\frac{(CsR_1 + CLS^2)R_s + R_2R_s + (R_1 + LS)R_s + (R_1 + LS)R_2}{(R_1 + LS)R_2R_s}\right) V_s(s) = \ldots$$

$$\ldots \left(\frac{1}{R_s}\right) V_e(s)$$

$$\left(\frac{(CR_1R_2)s + (CLR_s)s^2 + R_2R_s + R_1R_s + (LR_s)s + R_1R_2 + (2R_2)s}{R_1R_2R_s + (R_2R_sL)s}\right)$$

$$* \quad V_s(S) = \left(\frac{1}{R_s}\right) V_e(s)$$

$$\frac{V_s}{V_e}=\frac{\left(\frac{1}{R_s}\right)}{\left(\frac{(CR_1R_2)s+(CLR_s)s^2+R_2R_s+R_1R_s+(LR_s)s+R_1R_2+(LR_2)s}{R_1R_2R_s+(R_2R_sL)s}\right)}$$

$$=\frac{R_2Ls+R_2R_1}{(CR_1R_2)s+(CLR_s)s^2+R_2R_s+R_1R_s+(LR_s)s+R_1R_2+(LR_2)s}\quad\frac{\left(\frac{1}{R_s}\right)}{\left(\frac{1}{R_s}\right)}$$

$$\frac{V_s}{V_e}=\frac{Ls+R_1}{(LCR_s)s^2+\left(L+\frac{LR_s}{R_2}+CR_sR_1\right)s+\left(R_1+R_s+\frac{R_1R_s}{R_2}\right)}$$

Función de transferencia

Ecuaciones integro-diferenciales

$$i_{Rs}(t)=\frac{V_e(t)-V_s(t)}{R_s}$$

$$i_c(t)=C\frac{dV_s(t)}{dt}$$

$$V_s(t)=i_L(t)R_1+L\frac{di_L(t)}{dt}\rightarrow\frac{di_L(t)}{dt}=\frac{V_s(t)-i_L(t)R_1}{L}$$

$$i_{R2}(t)=\frac{V_s(t)}{R_2}$$

$$i_c(t)+i_L(t)+i_{R2}(t)=i_{Rs}(t)$$

$$C\frac{dV_s(t)}{dt}+i_L(t)+\frac{V_s(t)}{R_2}=\frac{V_e(t)-V_s(t)}{R_s}$$

$$i_L(t)=\frac{V_e(t)}{R_s}-\frac{V_s(t)}{R_s}-\frac{V_s(t)}{R_2}-C\frac{dV_s(t)}{dt}$$

$$\frac{d}{dt}\left[C\frac{dV_s(t)}{dt}+i_L(t)+\frac{V_s(t)}{R_2}\right]=\frac{d}{dt}\left[\frac{V_e(t)-V_s(t)}{R_s}\right]$$

$$C\frac{d^2V_s(t)}{dt^2}+\frac{di_L(t)}{dt}+\frac{1}{R_2}\frac{dV_s(t)}{dt}=\frac{1}{R_s}\frac{dV_e(t)}{dt}-\frac{1}{R_s}\frac{dV_s(t)}{dt}$$

$$C\frac{d^2V_s(t)}{dt^2}+\left(\frac{R_1C}{L}+\frac{1}{R_2}+\frac{1}{R_s}\right)\frac{dV_s(t)}{dt}+\left(\frac{1}{L}+\frac{R_1}{LR_s}+\frac{R_1}{LR_2}\right)$$

$$V_s(t)=\frac{1}{R_s}\frac{dV_e(t)}{dt}+\frac{R_1}{LR_s}V_e(t)$$

Error en estado estacionario

$$e(s) = \lim_{s \to 0} s \; V_e(s) \left[1 - \frac{V_s(s)}{V_e(s)} \right]$$

$$= \lim_{s \to 0} s \cdot \frac{1}{s} \left[1 - \frac{\cancel{Ls} + R_1}{(LCR_s)s^2 + (L + \frac{LR_s}{R_2} + CR_sR_1)s + (R_1 + R_s + \frac{R_1R_s}{R_2})} \right]$$

$$= \left[1 - \frac{R_1}{R_s + \frac{R_1R_s}{R_2}} \right] = \left[1 - \frac{(200k)}{(1M) + \frac{(200k)(1M)}{1M}} \right]$$

$$= \frac{5}{6} V = 0.83\bar{3} V$$

Estabilidad en lazo abierto

$a = LCR_s = 1000$

$b = L + \frac{LR_s}{R_2} + CR_2R_1 = 40000$

$c = R_1 + R_s + \frac{R_1R_s}{R_2} = 1400000$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\lambda_1 = -20 + 10\sqrt{10}\,i$

$\lambda_2 = -20 - 10\sqrt{10}\,i$

Control

$L = 10kH$

$C = 100nH$

$R_s = 1M$

$R_1 = 200k\Omega$

$R_2 = 1M\Omega$

Sistema estable con respuesta subamortiguada