Linear Regression

Silva, Eder H N

2022-12-27

Correspondence Between Force and Command Input

Experiments were carried out to characterize the relations between the force generated by the main motor and the command input. For this purpose, a wire rope was used to connect the beam to a load cell a device used to measure the force. The force exerted by the main motor, drove by u, actuated in the load cell through the wire rope the corresponding data were collected and stored.

```
data <- read_csv("CSV/motor0_data1.csv")
```

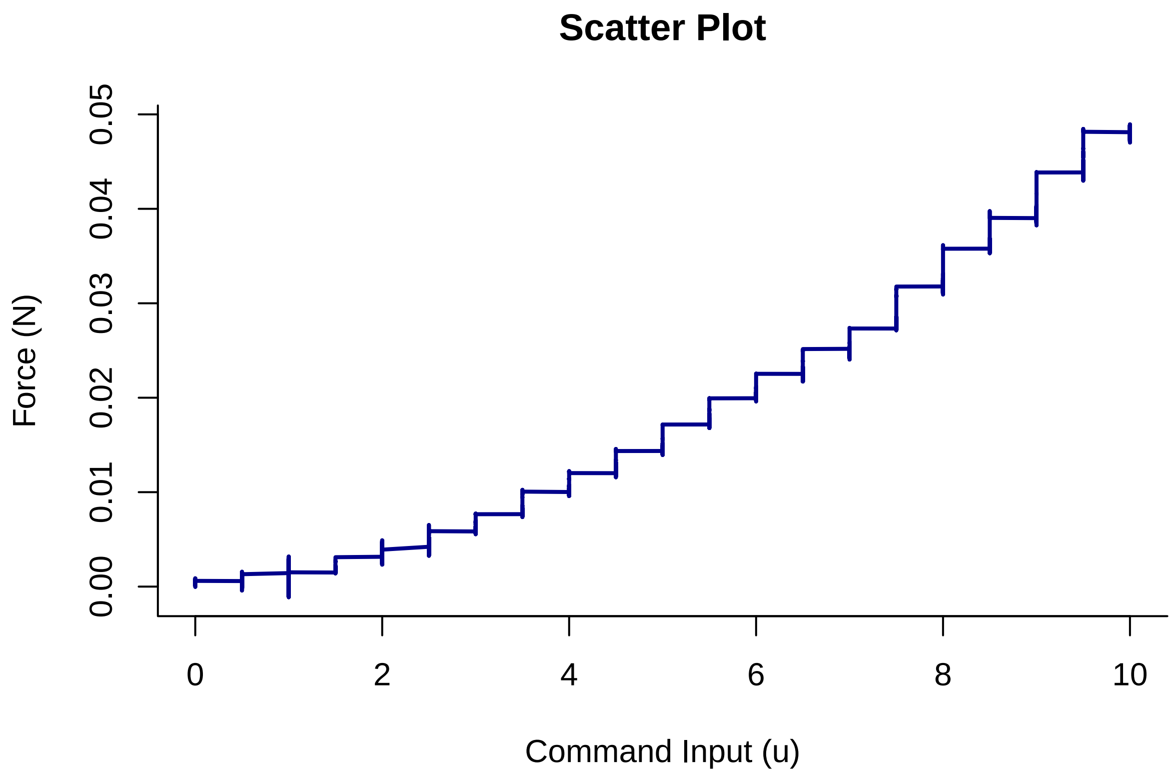
```
## New names:
## Rows: 38700 Columns: 3
## -- Column specification
## -------------------------------------------------------- Delimiter: "," dbl
## (3): -1...1, -1...2, -1...3
## i Use `spec()` to retrieve the full column specification for this data. i
## Specify the column types or set `show_col_types = FALSE` to quiet this message.
## * `-1` -> `-1...1`
## * `-1` -> `-1...2`
## * `-1` -> `-1...3`
```

```
data <- as.data.frame(data)
colnames(data)=c("sample","force","u")
head(data)
```

```
##   sample    force u
## 1 38.699 0.000327 0
## 2 38.698 0.000313 0
## 3 38.697 0.000300 0
## 4 38.696 0.000290 0
## 5 38.695 0.000286 0
## 6 38.694 0.000287 0
```

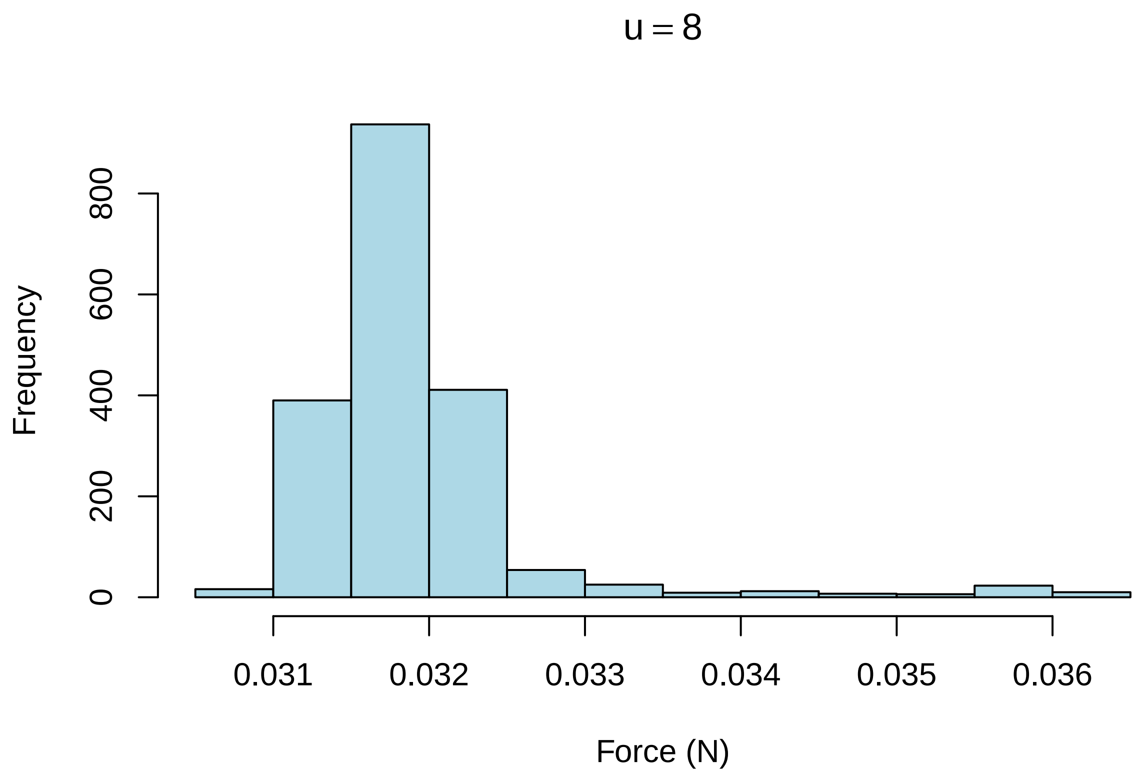
Standardized

```
plot(x=data[,3],y=data[,2],ylim=c(min(data[,2]),max(data[,2])),type="n",ylab="Force (N)",xlab="Command I
par(new=TRUE)
plot(x=data[,3],y=data[,2],type="l",ylim=c(min(data[,2]),max(data[,2])),axes=FALSE,ann=FALSE,col="darkbl
```

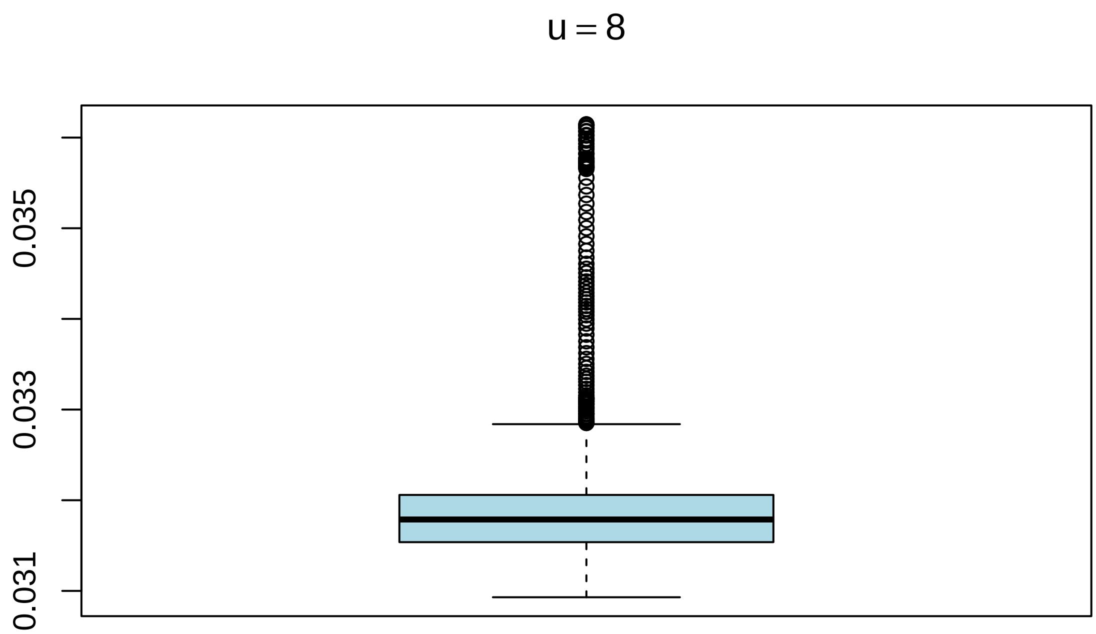



```r
filtered_rows <- data$u == 8
rows <- data[filtered_rows,]

hist(rows[,2],xlab="Force (N)",main=TeX('$u = 8$'),col="#ADD8E6")
```



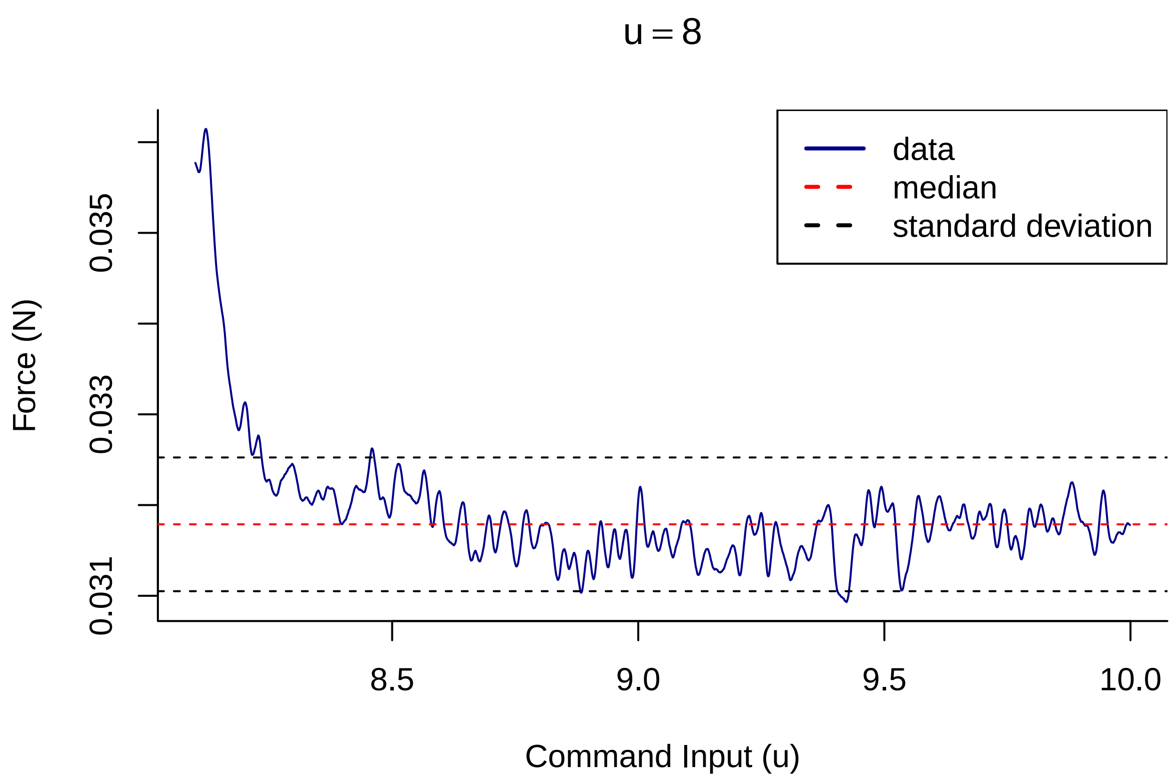

```
boxplot(rows[,2],main=TeX('$u = 8$'),col="#ADD8E6")
```




```
shapiro.test(rows[,2])
```

```
##
##  Shapiro-Wilk normality test
##
## data:  rows[, 2]
## W = 0.65565, p-value < 2.2e-16
```

```
plot(rows[,1],rows[,2],type="l",col="darkblue",lwd=1,ylab="Force (N)",xlab="Command Input (u)",main=TeX
abline(h=median(rows[,2]),lty=2,col="red")
abline(h=(median(rows[,2])+sd(rows[,2])),lty=2,col="black")
abline(h=(median(rows[,2])-sd(rows[,2])),lty=2,col="black")
legend("topright",legend = c('data','median','standard deviation'),col = c("darkblue","red","black"),lty
```




```r
uni <- unique(data[,3])
modelagem = matrix(data=0, ncol=4, nrow=length(uni))
colnames(modelagem)=c("u","Mean","Median","Length")

for (i in 1:length(uni)) {

  #Seleciona as linhas com tensões iguais ao valor do vetor uni
  filtered_rows <- data$u == uni[i]
  modelagem[i,1] <- uni[i]
  #Guarda as linhas com tensões iguais no vetor rows
  rows <- data[filtered_rows,]

  #Guarda a média dos valores de força
  modelagem[i,2]  <- mean(rows[,2])

  #Guarda a mediana dos valores de força
  modelagem[i,3] <- median(rows[,2])

  #Guarda a moda dos valores de força
  modelagem[i,4] <- length(rows[,2])
}
head(modelagem)
```

```
##          u         Mean    Median Length
## [1,] 0.0 0.0004251545 0.0004280   1800
## [2,] 0.5 0.0004659592 0.0004700   1800
```

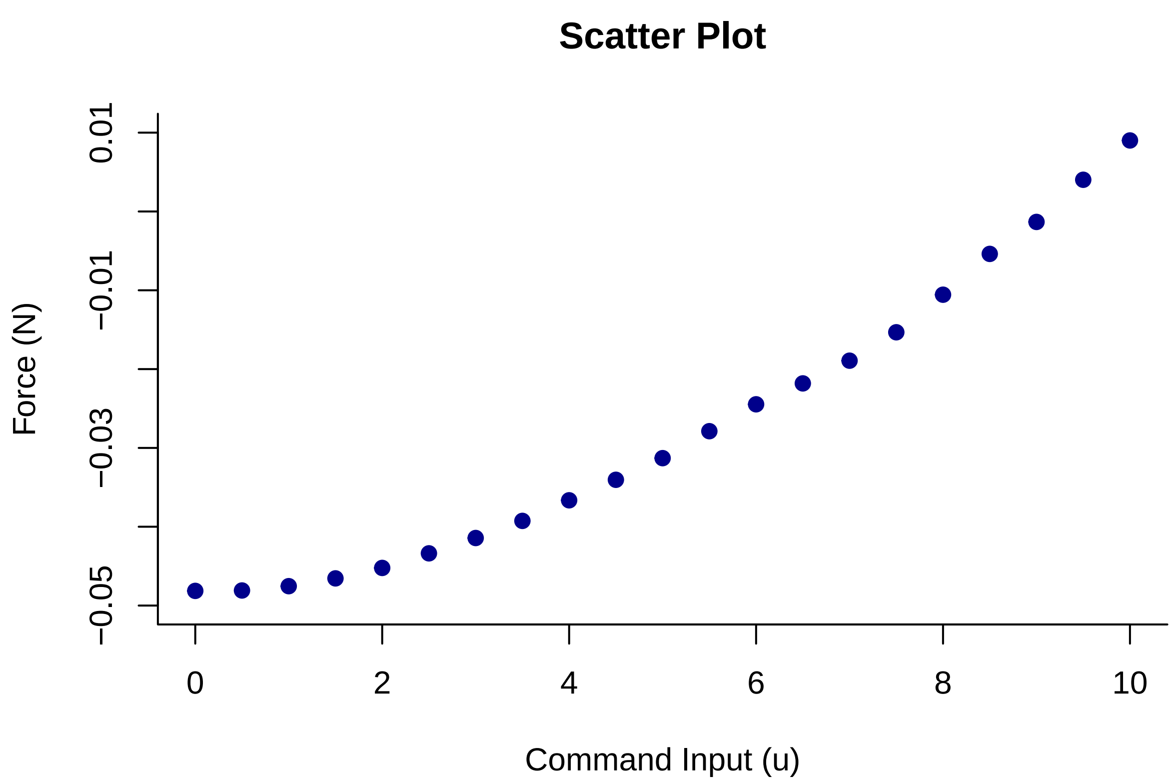
```
## [3,] 1.0 0.0009030337 0.0009270   1700
## [4,] 1.5 0.0017865271 0.0017500   1700
## [5,] 2.0 0.0029095326 0.0028575   1900
## [6,] 2.5 0.0043809567 0.0044010   1800
```

Using scatter plots to explore relationships

Before looking ahead to predicting a value of force by using a value of u, first establish the legitimate reason to using a linear function to make that prediction will actually work well.

In order to achieve both of these important steps, first plot the data in a pairwise fashion so you can visually look for a relationship; then need to somehow quantify that relationship in terms of how well those points follow a linear function.

```
plot(x=data[,3],y=data[,2],ylim=c(-.05,.01),type="n",ylab="Force (N)",xlab="Command Input (u)",main="Sca
par(new=TRUE)
plot(x=modelagem[,1],y=modelagem[,3],pch=19,ylim=c(min(data[,2]),max(data[,2])),axes=FALSE,ann=FALSE,col
```



After displaying the data using a scatter plot, the next step is to find a statistic that quantifies the relationship somehow. The correlation coefficient (also known as Pearson's correlation coefficient) measures the strength and direction of the linear relationship between two quantitative variables u and F.

```
cor.test(x=modelagem[,1],y=modelagem[,3])
```

```
##
##  Pearson's product-moment correlation
##
## data:  modelagem[, 1] and modelagem[, 3]
```

```
## t = 19.897, df = 19, p-value = 3.497e-14
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.9426521 0.9907399
## sample estimates:
##      cor
## 0.976835
```

Building a Simple Linear Regression Model

$$F_i = \beta_0 + \beta_1 u_i + \beta_2 u_i^2 + \beta_3 u_i^3 + \varepsilon_i$$

```
x <- modelagem[,1]
y <- modelagem[,3]

X <- x
XSQ <- x**2
XCUB <- x**3

model=lm(y~1+X+XSQ+XCUB)
anova(model)
```

```
## Analysis of Variance Table
##
## Response: y
##           Df    Sum Sq   Mean Sq    F value Pr(>F)
## X          1 0.0044551 0.0044551 22676.9971 <2e-16 ***
## XSQ        1 0.0002105 0.0002105  1071.2191 <2e-16 ***
## XCUB       1 0.0000000 0.0000000     0.0748 0.7877
## Residuals 17 0.0000033 0.0000002
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(model)
```

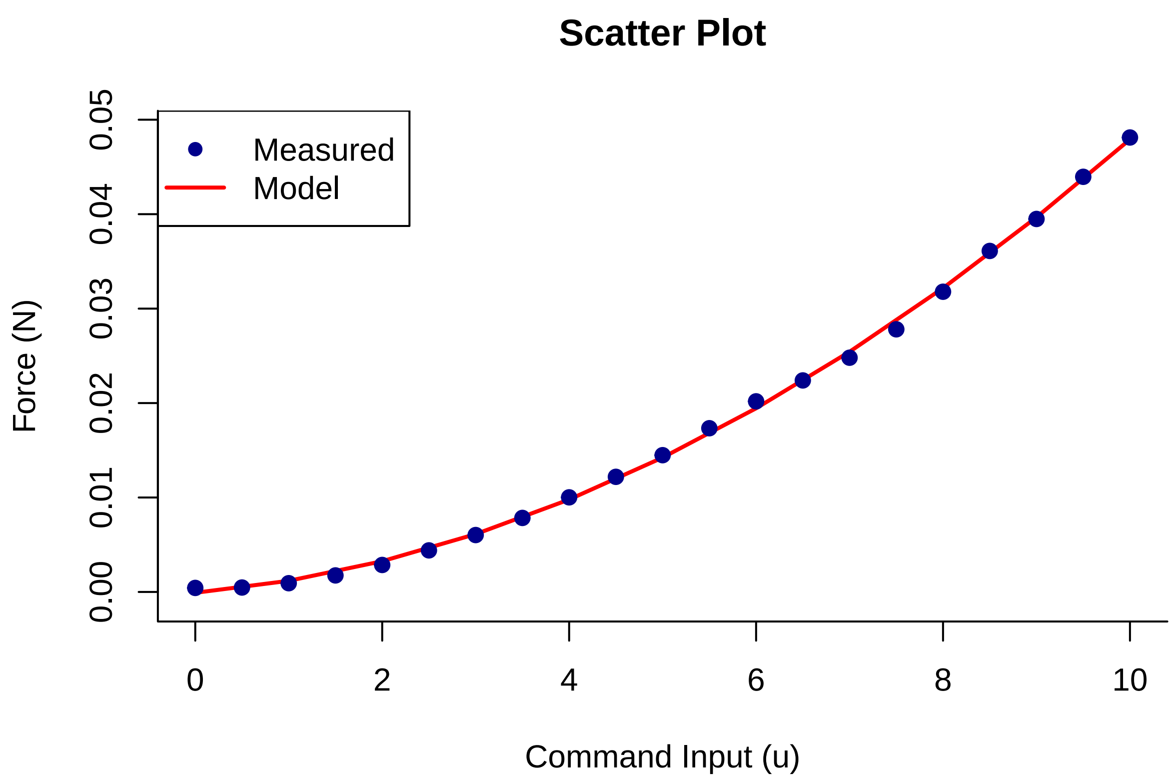
```
##
## Call:
## lm(formula = y ~ 1 + X + XSQ + XCUB)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -8.994e-04 -2.544e-04  2.414e-05  2.534e-04  7.376e-04
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -8.770e-05  3.269e-04  -0.268  0.79173
## X            8.645e-04  2.902e-04   2.979  0.00843 **
## XSQ          4.059e-04  6.842e-05   5.932 1.64e-05 ***
## XCUB        -1.229e-06  4.492e-06  -0.274  0.78773
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0004432 on 17 degrees of freedom
## Multiple R-squared:  0.9993, Adjusted R-squared:  0.9992
## F-statistic:  7916 on 3 and 17 DF,  p-value: < 2.2e-16
```

```
func <- model$coefficients
print(func)
```

```
##   (Intercept)             X           XSQ          XCUB
## -8.770158e-05  8.644903e-04  4.058642e-04 -1.228902e-06
```

```
x=0:10
y=as.numeric(func[1])+as.numeric(func[2])*x+as.numeric(func[3])*x**2+as.numeric(func[4])*x**3

plot(x=data[,3],y=data[,2],ylim=c(min(data[,2]),max(data[,2])),type="n",ylab="Force (N)",xlab="Command 
par(new=TRUE)
plot(x,y,type="l",ylim=c(min(data[,2]),max(data[,2])),axes=FALSE,ann=FALSE,col="red",lwd=2)
par(new=TRUE)
plot(x=modelagem[,1],y=modelagem[,3],pch=19,ylim=c(min(data[,2]),max(data[,2])),axes=FALSE,ann=FALSE,co
legend("topleft",legend = c('Measured','Model'),col = c("darkblue","red"),lty = c(0, 1), lwd=2, pch =c(
```



Using r^2 to measure model fit

One important way to assess how well the model fits is to measure the value of r^2, where r is the correlation coefficient. Statisticians measure how well a model fits by looking at what percentage of the variability in F is explained by the model.

Finding and exploring the residuals

After you've established a relationship between u and F and have come up with an equation of a linear that represents that relationship, the job is not done. (Many researchers erringly stop here, so I'm depending on you to break the cycle on this!)

But the most important job remains to be completed: checking to be sure that the conditions of the model are truly met and that the model fits well in more specific ways than the scatter plot and correlation measure. This section presents methods for defining and assessing the fit of a simple linear regression model.

Two major conditions must be met before you apply a simple linear regression model to a data set:

- The y's have to have a normal distribution for each value of x.
- The y's have to have a constant amount of spread (standard deviation) for each value of x.

Finding the residuals

A residual is the difference between the observed value of force and the predicted value of F. Specifically, for any data point, takes observed F-value (from the data) and subtract the expected F-value (from the line). If the residual is large, the linear function doesn't fit well in that spot. If the residual is small, the line fits well in that spot.

```
error=resid(model)
```

To check to see whether the F-values come from a normal distribution. The residuals are a data set just like any other data set, so can find their mean and standard deviation.

Checking normality

```
summary(error)
```

```
##       Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
## -8.994e-04 -2.544e-04  2.414e-05  0.000e+00  2.534e-04  7.376e-04
```

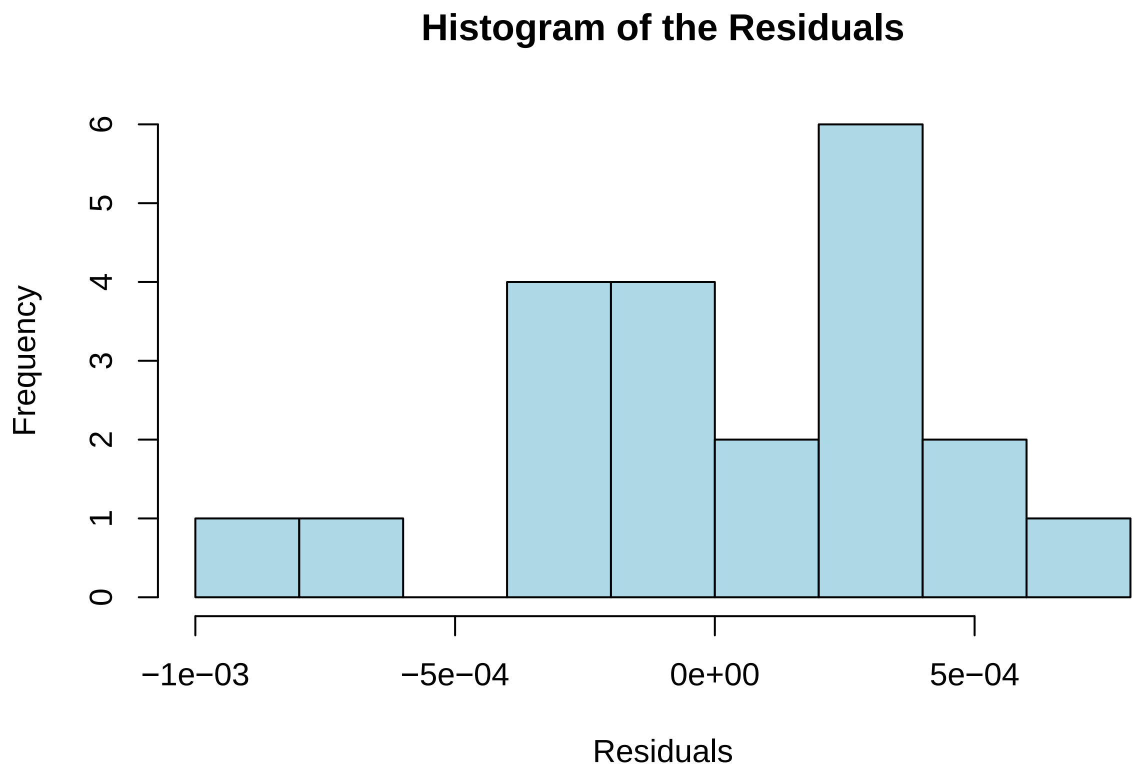
```
sd(error)
```

```
## [1] 0.0004086437
```

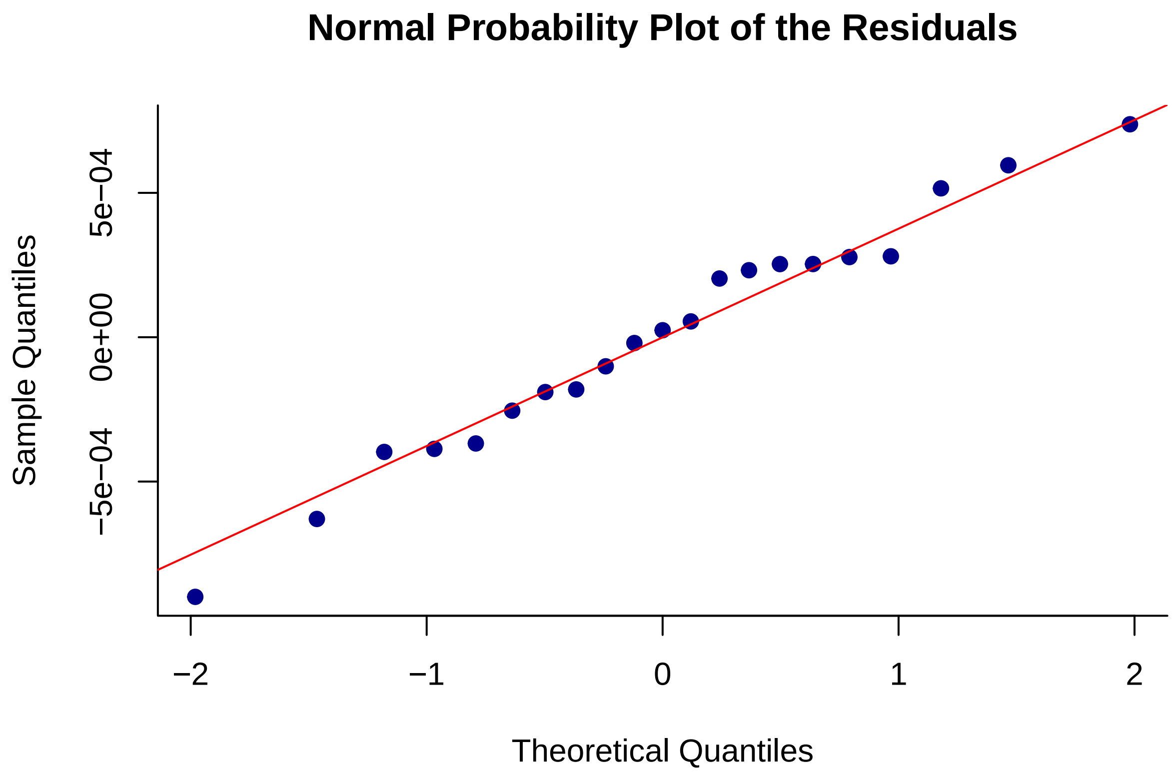
```
shapiro.test(error)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  error
## W = 0.98078, p-value = 0.936
```

```
hist(error,col="#ADD8E6",main="Histogram of the Residuals",xlab="Residuals")
```

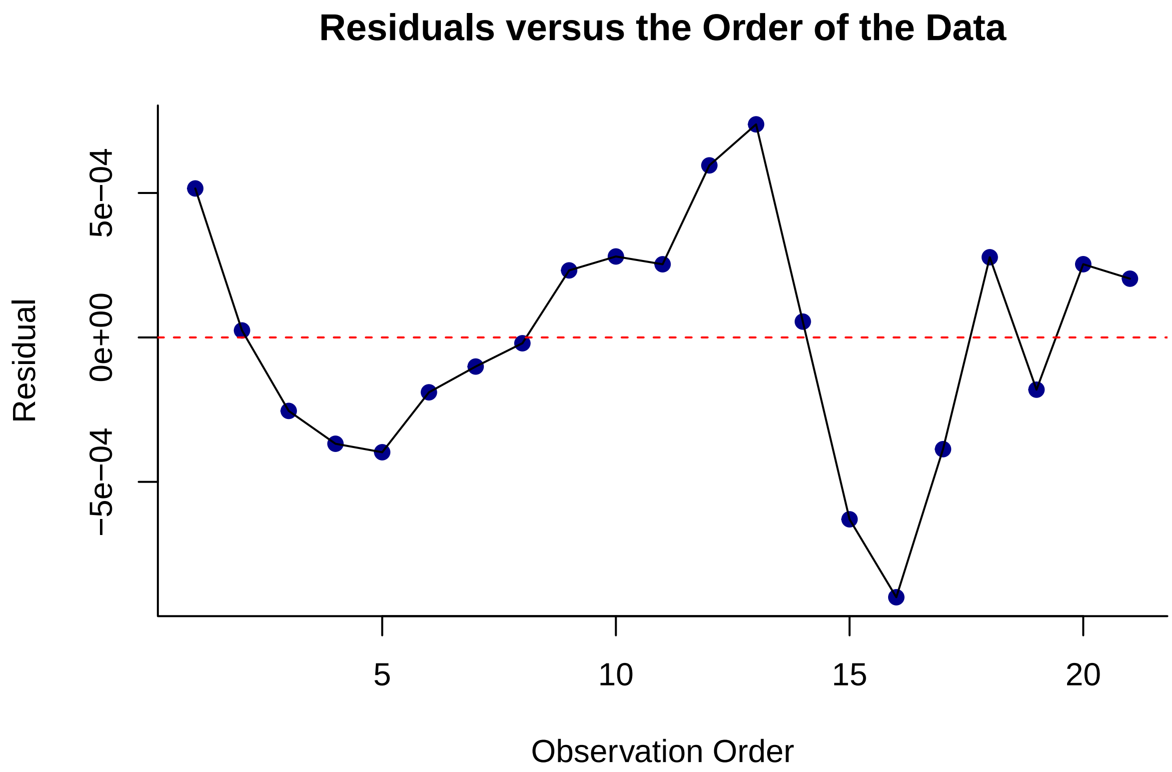



```
qqnorm(error,pch=19,col="darkblue",main="Normal Probability Plot of the Residuals",bty = "l")
qqline(error,col="red")
```



As the condition of normality is met, then residual plot lots of residuals close to zero. The residuals also occur at random some above the line, some below the line.

```
plot(error,pch=19,col="darkblue",main="Residuals versus the Order of the Data",xlab="Observation Order"
par(new=TRUE)
plot(error,type="l",axes=FALSE,ann=FALSE)
abline(h=0,lty=2,col="red")
```



Reference

```r
#Citing
citation()
```

```
##
## To cite R in publications use:
##
##   R Core Team (2021). R: A language and environment for statistical
##   computing. R Foundation for Statistical Computing, Vienna, Austria.
##   URL https://www.R-project.org/.
##
## A BibTeX entry for LaTeX users is
##
##   @Manual{,
##     title = {R: A Language and Environment for Statistical Computing},
##     author = {{R Core Team}},
##     organization = {R Foundation for Statistical Computing},
##     address = {Vienna, Austria},
##     year = {2021},
##     url = {https://www.R-project.org/},
##   }
##
## We have invested a lot of time and effort in creating R, please cite it
## when using it for data analysis. See also 'citation("pkgname")' for
```

```
## citing R packages.
```

```r
citation("readr")
```

```
##
## To cite package 'readr' in publications use:
##
##   Hadley Wickham, Jim Hester and Jennifer Bryan (2022). readr: Read
##   Rectangular Text Data. R package version 2.1.3.
##   https://CRAN.R-project.org/package=readr
##
## A BibTeX entry for LaTeX users is
##
##   @Manual{,
##     title = {readr: Read Rectangular Text Data},
##     author = {Hadley Wickham and Jim Hester and Jennifer Bryan},
##     year = {2022},
##     note = {R package version 2.1.3},
##     url = {https://CRAN.R-project.org/package=readr},
##   }
```

```r
citation("rjson")
```

```
##
## To cite package 'rjson' in publications use:
##
##   Alex Couture-Beil (2022). rjson: JSON for R. R package version
##   0.2.21. https://CRAN.R-project.org/package=rjson
##
## A BibTeX entry for LaTeX users is
##
##   @Manual{,
##     title = {rjson: JSON for R},
##     author = {Alex Couture-Beil},
##     year = {2022},
##     note = {R package version 0.2.21},
##     url = {https://CRAN.R-project.org/package=rjson},
##   }
##
## ATTENTION: This citation information has been auto-generated from the
## package DESCRIPTION file and may need manual editing, see
## 'help("citation")'.
```

```r
citation("latex2exp")
```

```
##
## To cite package 'latex2exp' in publications use:
##
##   Stefano Meschiari (2022). latex2exp: Use LaTeX Expressions in Plots.
##   R package version 0.9.6. https://CRAN.R-project.org/package=latex2exp
##
## A BibTeX entry for LaTeX users is
##
##   @Manual{,
##     title = {latex2exp: Use LaTeX Expressions in Plots},
##     author = {Stefano Meschiari},
```

```
##     year = {2022},
##     note = {R package version 0.9.6},
##     url = {https://CRAN.R-project.org/package=latex2exp},
##   }
```