Mixture of Gaussians and the EM Algorithm

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Point 1

Implement the EM algorithm to estimate the parameters of a mixture of Gaussians when we have data of any dimension and any number of classes.

Solution:

```
source(file = "EM_function.R")
```

```
## Warning: package 'birdring' was built under R version 4.4.3
```

```
library(ggplot2)
library(mvtnorm)
```

```
## Warning: package 'mvtnorm' was built under R version 4.4.2
```

```
##
## Attaching package: 'mvtnorm'
```

```
## The following object is masked from 'package:birdring':
##
##     dmvnorm
```

```
library(reshape)
```

```
## Warning: package 'reshape' was built under R version 4.4.3
```

```
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.4     v readr     2.1.5
## v forcats   1.0.0     v stringr   1.5.1
## v lubridate 1.9.3     v tibble    3.2.1
## v purrr     1.0.2     v tidyr     1.3.1
```

```
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x tidyr::expand()    masks reshape::expand()
## x dplyr::filter()    masks stats::filter()
## x dplyr::lag()       masks stats::lag()
## x dplyr::rename()    masks reshape::rename()
## x lubridate::stamp() masks reshape::stamp()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become error
```

```
#Example usage:
library(MASS)  # For generating multivariate normal data
```

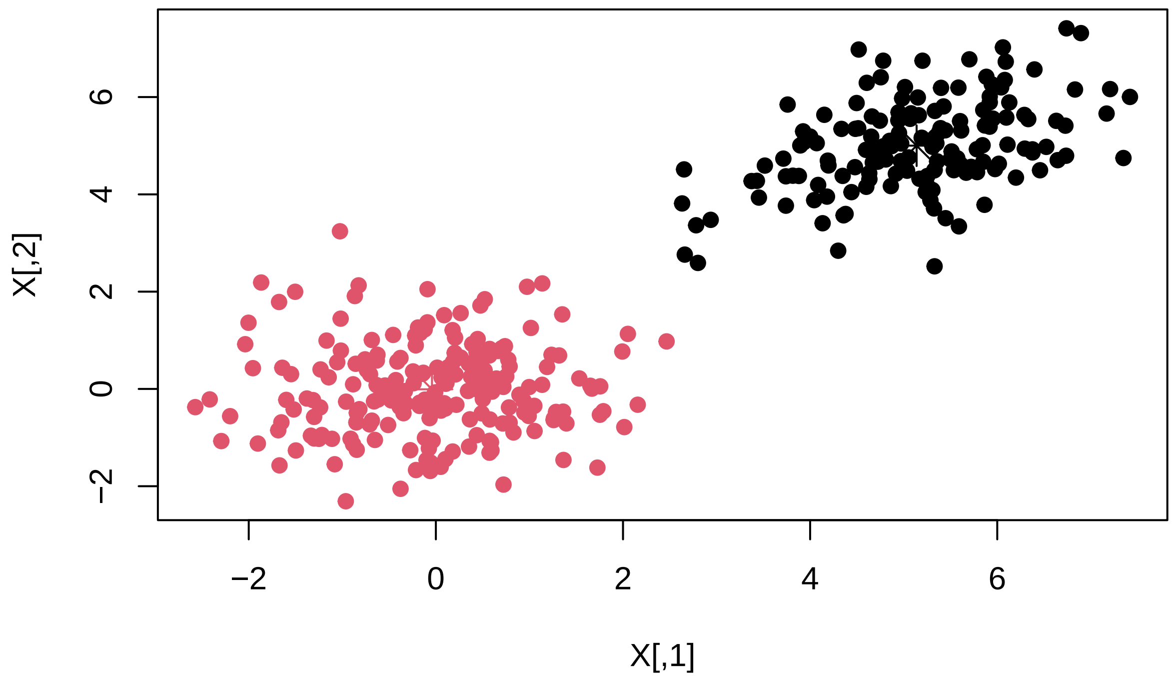
```
##
## Attaching package: 'MASS'

## The following object is masked from 'package:dplyr':
##
##     select
```

```
# Generate some example data
set.seed(123)
n1 <- 200
n2 <- 150
X1 <- mvrnorm(n1, mu = c(0, 0), Sigma = matrix(c(1, 0, 0, 1), 2, 2))
X2 <- mvrnorm(n2, mu = c(5, 5), Sigma = matrix(c(1, 0.5, 0.5, 1), 2, 2))
X <- rbind(X1, X2)

# Set initial parameters
K <- 2
D <- 2

# Run EM algorithm with initial parameters
model <- em_gaussian_mixture(X, K, max_iter = 50)
```

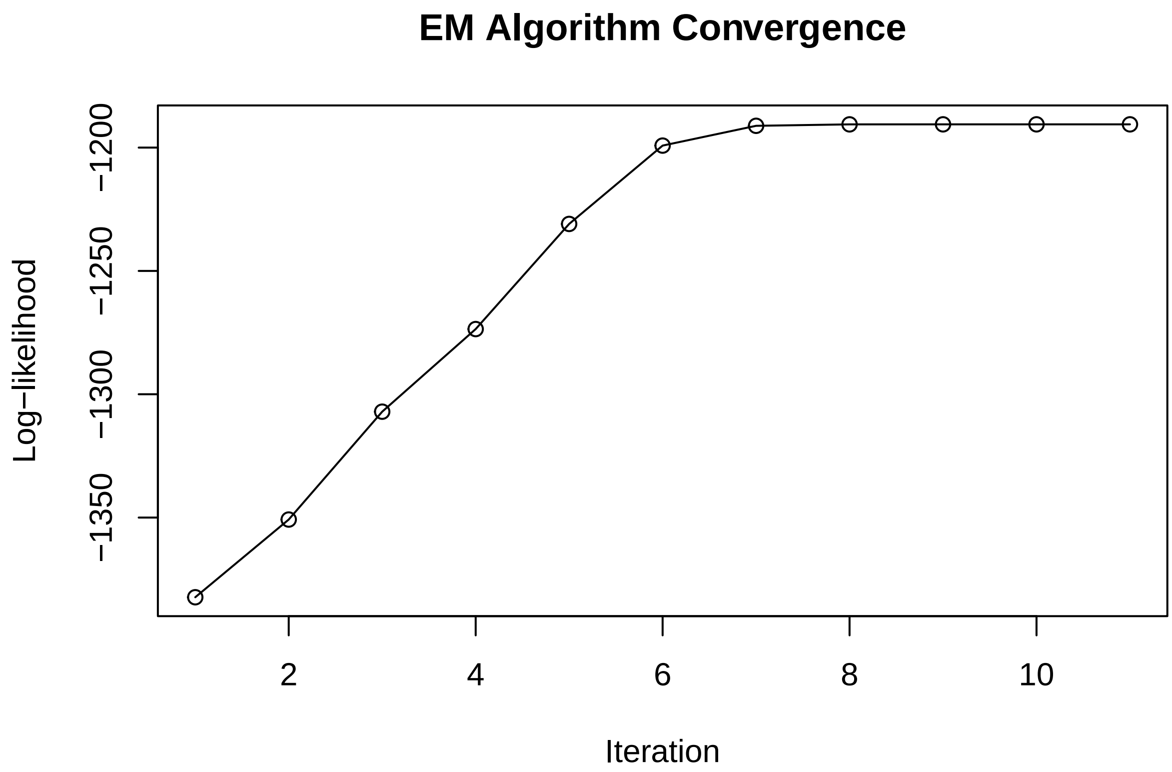
```
## Iteration: 1
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1382.304 Improvement: Inf
## Iteration: 2
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1350.8 Improvement: 31.50387
## Iteration: 3
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1307.065 Improvement: 43.73533
## Iteration: 4
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1273.576 Improvement: 33.48868
## Iteration: 5
##   E-step: Computing responsibilities
##   M-step: Updating parameters
```

```
##   Log-likelihood: -1230.952 Improvement: 42.62426
## Iteration: 6
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1199.193 Improvement: 31.7585
## Iteration: 7
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1191.172 Improvement: 8.021464
## Iteration: 8
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1190.582 Improvement: 0.5896712
## Iteration: 9
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1190.577 Improvement: 0.005030017
## Iteration: 10
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1190.577 Improvement: 1.902908e-05
## Iteration: 11
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1190.577 Improvement: 6.8791e-08
## Converged after 11 iterations
```

```r
# Plot results
plot(X, col = predict_cluster(model, X), pch = 19)
points(model$mu, col = 1:model$K, pch = 8, cex = 2)
```




```r
plot_convergence(model)
```



Point 2

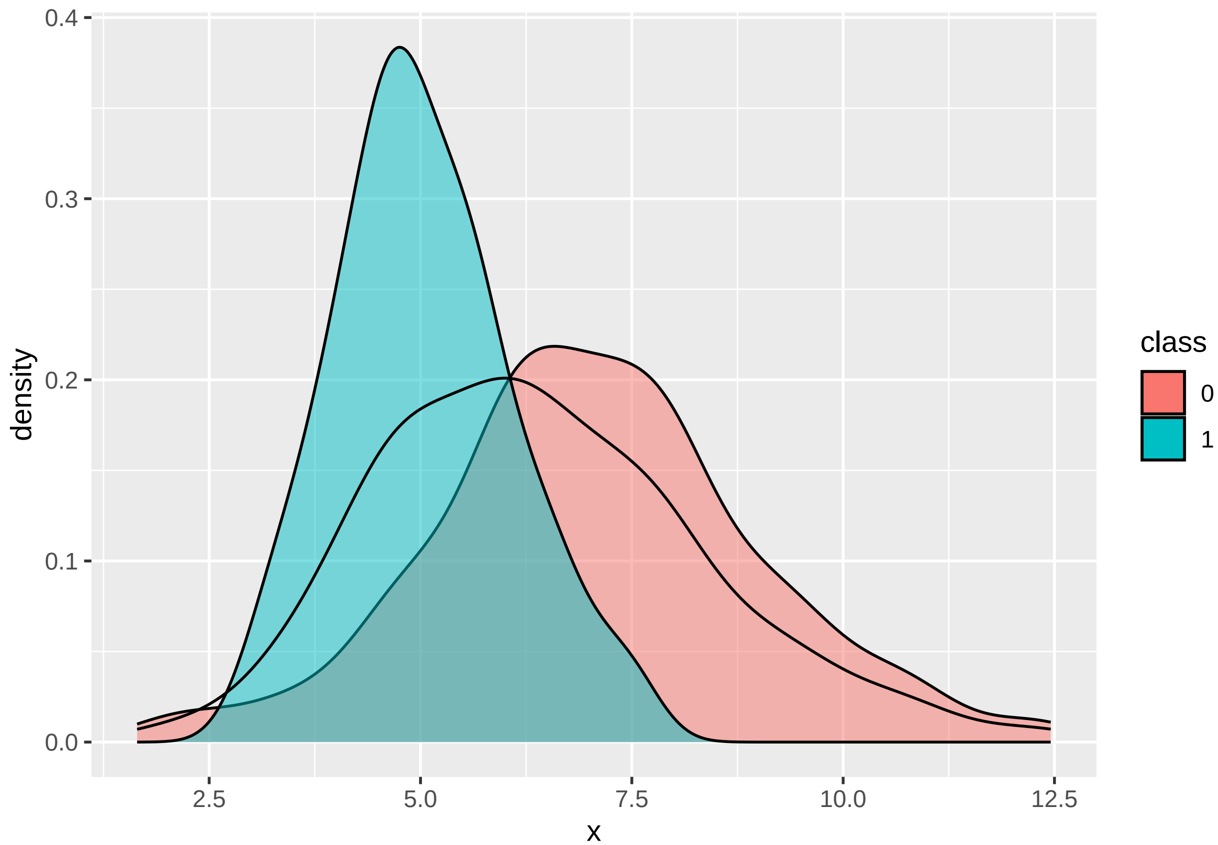
Check that it works for synthetic data generated according to a mixture of Gaussians in:

a) 1 dimensions

Solution:

```
Mu1 = 5
Mu2 = 7
S1 = 1
S2 = 2

pi = 1/3
n = 300

set.seed(100)
data = matrix(0, n, 1)
z = rep(0,n)
for (i in 1:n){
  z[i] = rbinom(1,1,pi)
  if (z[i] ==1){
    data[i,] = rnorm(1, Mu1,S1)
```

```r
  }else{
    data[i,] = rnorm(1, Mu2,S2)
  }
}

to.plot = data.frame(x = data[,1],
                     class =  as.factor(z))
ggplot(to.plot) + geom_density(aes(x=x,fill=class), alpha=.5) + geom_density(aes(x=x))
```



The black line through the two distributions shows the mixed distribution curve.

```r
X=data
K=2
# Run EM algorithm with initial parameters
model_1d <- em_gaussian_mixture(X, K, max_iter = 50)
```

```
## Iteration: 1
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.8413 Improvement: Inf
## Iteration: 2
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.4404 Improvement: 0.4009375
## Iteration: 3
```

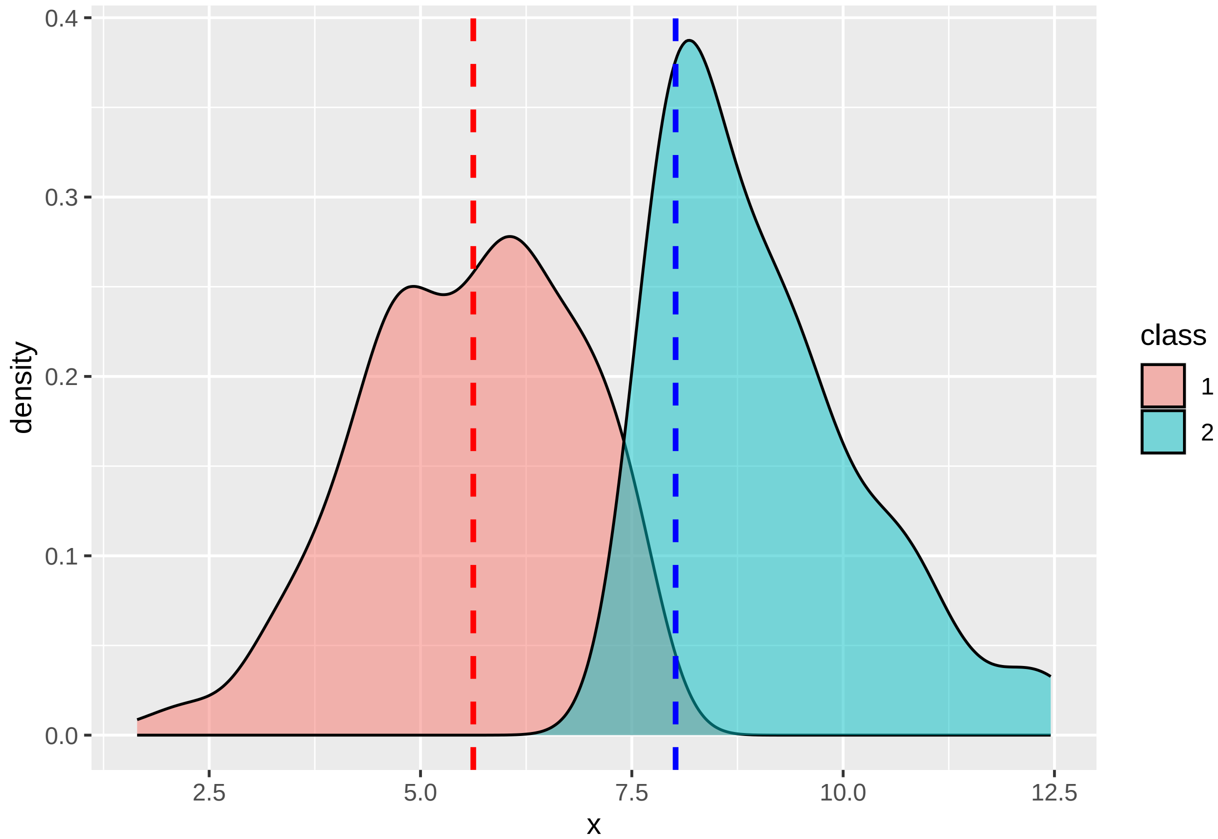
```
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3937 Improvement: 0.04666365
## Iteration: 4
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3862 Improvement: 0.007539204
## Iteration: 5
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3833 Improvement: 0.00289946
## Iteration: 6
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3812 Improvement: 0.002020967
## Iteration: 7
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3796 Improvement: 0.001608739
## Iteration: 8
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3783 Improvement: 0.001303829
## Iteration: 9
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3773 Improvement: 0.001058715
## Iteration: 10
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3764 Improvement: 0.0008598799
## Iteration: 11
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3757 Improvement: 0.0006987098
## Iteration: 12
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3751 Improvement: 0.0005683067
## Iteration: 13
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3747 Improvement: 0.0004629834
## Iteration: 14
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3743 Improvement: 0.0003780514
## Iteration: 15
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.374 Improvement: 0.0003096591
## Iteration: 16
##   E-step: Computing responsibilities
##   M-step: Updating parameters
```

```
##   Log-likelihood: -618.3737 Improvement: 0.0002546534
## Iteration: 17
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3735 Improvement: 0.000210461
## Iteration: 18
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3733 Improvement: 0.0001749882
## Iteration: 19
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3732 Improvement: 0.0001465356
## Iteration: 20
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3731 Improvement: 0.000123727
## Iteration: 21
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.373 Improvement: 0.0001054505
## Iteration: 22
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3729 Improvement: 9.080918e-05
## Iteration: 23
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3728 Improvement: 7.908075e-05
## Iteration: 24
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3727 Improvement: 6.968443e-05
## Iteration: 25
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3727 Improvement: 6.215381e-05
## Iteration: 26
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3726 Improvement: 5.611478e-05
## Iteration: 27
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3726 Improvement: 5.126752e-05
## Iteration: 28
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3725 Improvement: 4.737204e-05
## Iteration: 29
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3725 Improvement: 4.423633e-05
## Iteration: 30
```

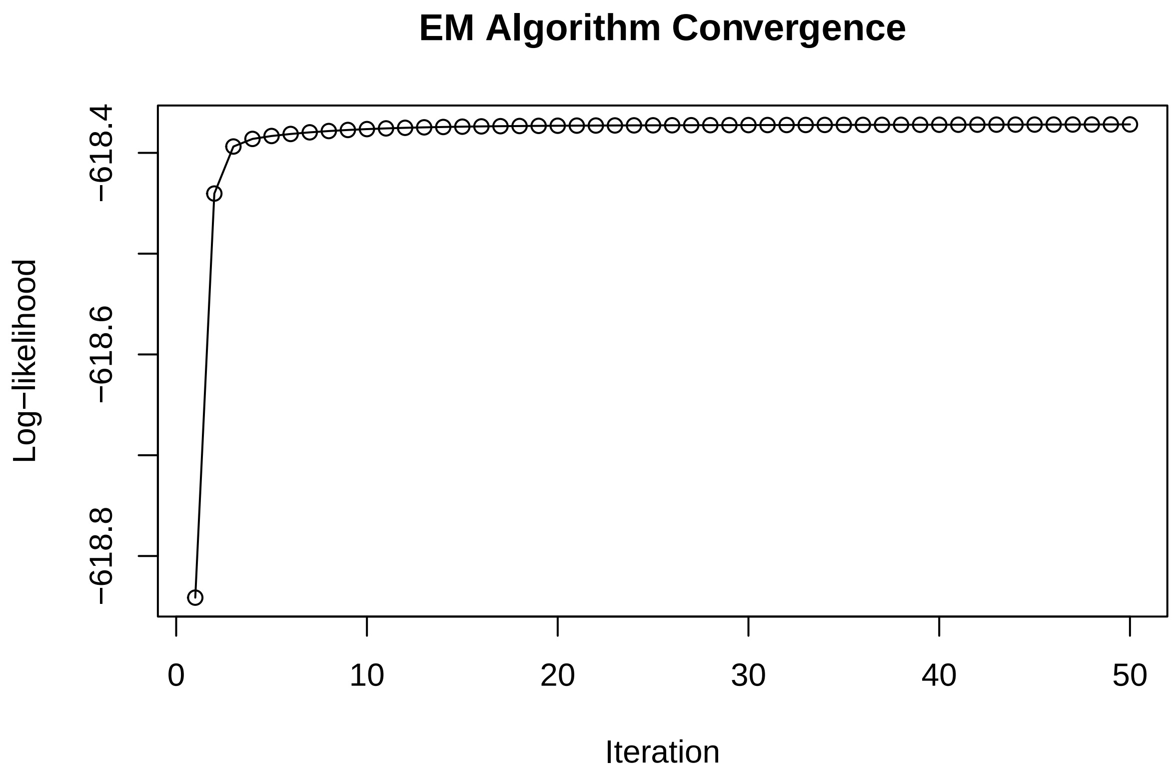
```
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3724 Improvement: 4.170689e-05
## Iteration: 31
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3724 Improvement: 3.96611e-05
## Iteration: 32
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3724 Improvement: 3.800104e-05
## Iteration: 33
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3723 Improvement: 3.664857e-05
## Iteration: 34
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3723 Improvement: 3.554134e-05
## Iteration: 35
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3722 Improvement: 3.462964e-05
## Iteration: 36
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3722 Improvement: 3.387381e-05
## Iteration: 37
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3722 Improvement: 3.324227e-05
## Iteration: 38
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3721 Improvement: 3.270983e-05
## Iteration: 39
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3721 Improvement: 3.225644e-05
## Iteration: 40
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3721 Improvement: 3.186612e-05
## Iteration: 41
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3721 Improvement: 3.152614e-05
## Iteration: 42
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.372 Improvement: 3.122636e-05
## Iteration: 43
##   E-step: Computing responsibilities
##   M-step: Updating parameters
```

```
##   Log-likelihood: -618.372 Improvement: 3.09587e-05
## Iteration: 44
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.372 Improvement: 3.071675e-05
## Iteration: 45
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3719 Improvement: 3.049537e-05
## Iteration: 46
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3719 Improvement: 3.029049e-05
## Iteration: 47
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3719 Improvement: 3.009887e-05
## Iteration: 48
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3718 Improvement: 2.991792e-05
## Iteration: 49
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3718 Improvement: 2.974558e-05
## Iteration: 50
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -618.3718 Improvement: 2.95802e-05
```

```r
# Plot results
to.plot = data.frame(x = data[,1],
                     class = as.factor(predict_cluster(model_1d, X)))

# hist(X, col = predict_cluster(model_1d, X), pch = 19)
ggplot(to.plot) + geom_density(aes(x=x,fill=class), alpha=.5)+
  geom_vline(aes(xintercept = model_1d$mu[1]), color = "red", linetype = "dashed", size = 1)+
  geom_vline(aes(xintercept = model_1d$mu[2]), color = "blue", linetype = "dashed", size = 1)
```

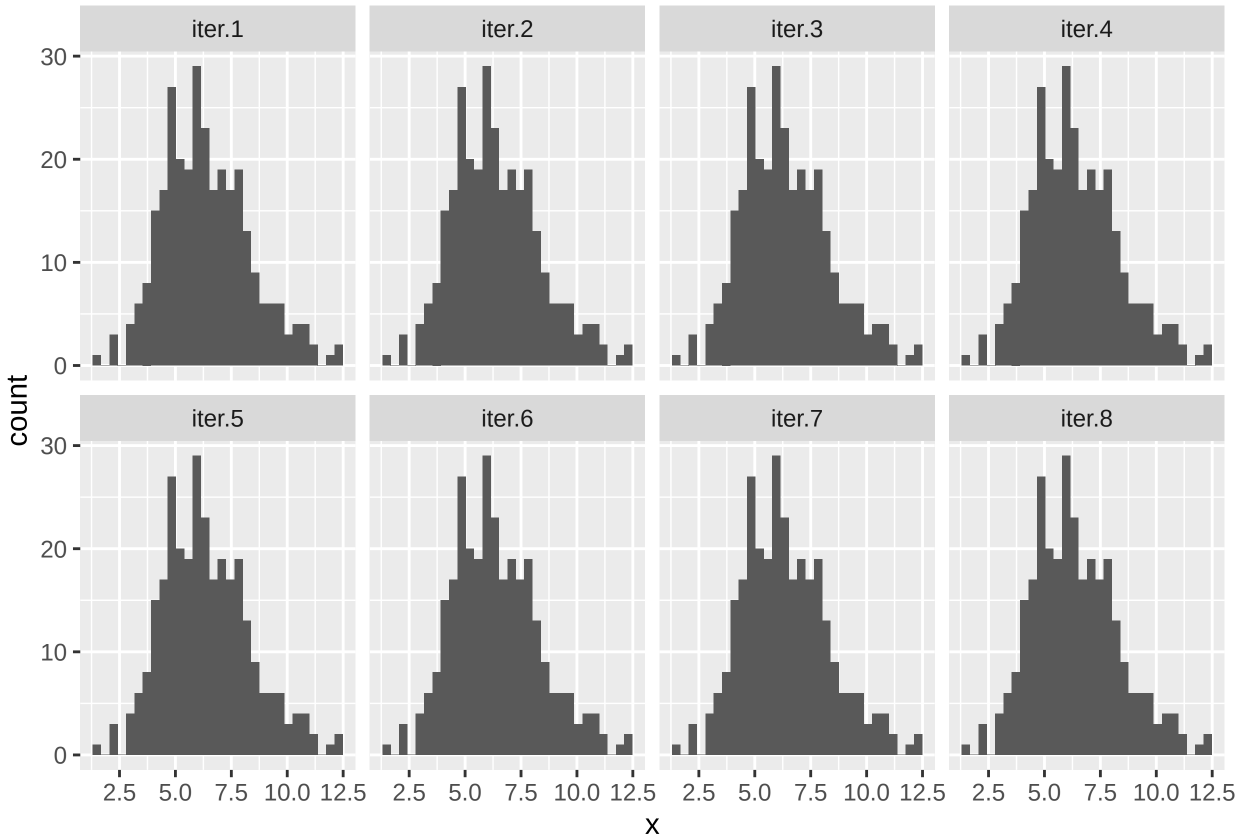



```
plot_convergence(model_1d)
```




```
to.plot = data.frame(x = data[,1],
                     iter = model_1d$Alpha[,1:8])
to.plot <- pivot_longer(to.plot, cols = -c("x"))

ggplot(to.plot)+aes(x, color = value)+
  geom_histogram()+facet_wrap(~name, nrow = 2)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

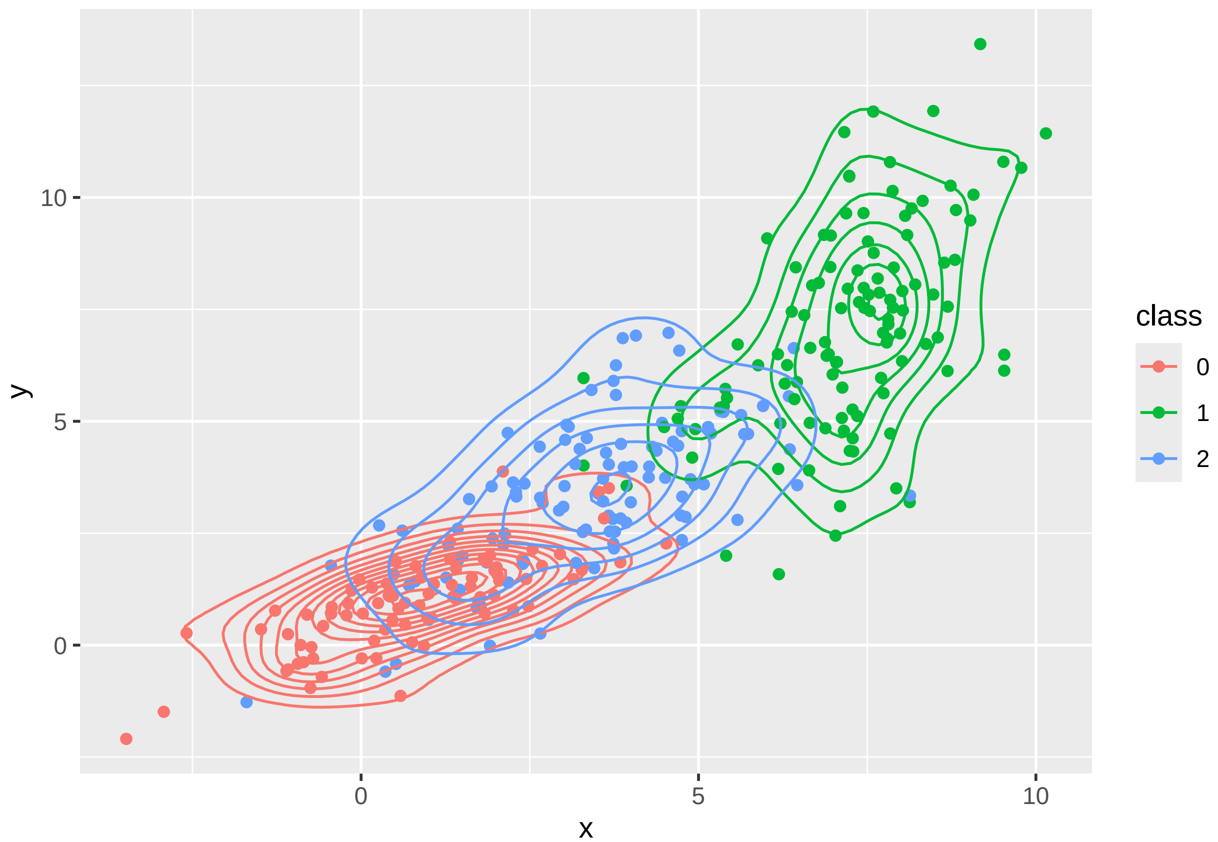


b) 2 dimensions

Solution:

```
library(ggplot2)
library(mvtnorm)
Mu1 = c(1,1)
Mu2 = c(7,7)
Mu3 = c(3,3)
Sigma1 = matrix(c(2, 1, 1, 1), 2,2)
Sigma2 = matrix(c(2, 2, 2, 5), 2,2)
Sigma3 = matrix(c(3, 2, 2, 3), 2,2)

pi = 1/3
n = 300

set.seed(100)
data = matrix(0, n, 2)
z = rep(0,n)
for (i in 1:n){
  z[i] = runif(1) #generate a random val to determine from what dist it comes
  if (z[i] <= 0.33){
    data[i,] = rmvnorm(1, Mu1,Sigma1)
    z[i] = 0
  }else if (z[i] >= 0.67){
```

```
    data[i,] = rmvnorm(1, Mu2,Sigma2)
    z[i] = 1
  }else {
    data[i,] = rmvnorm(1, Mu3,Sigma3)
    z[i] = 2
  }
}

to.plot = data.frame(x = data[,1],
                     y = data[,2],
                     class =  as.factor(z))
ggplot(to.plot)+ aes(x, y, color = class)+
  geom_point()+geom_density_2d()
```




```
X=data
K=3
# Run EM algorithm with initial parameters
model_2d <- em_gaussian_mixture(X, K, max_iter = 50)
```

```
## Iteration: 1
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1244.113 Improvement: Inf
## Iteration: 2
##   E-step: Computing responsibilities
```

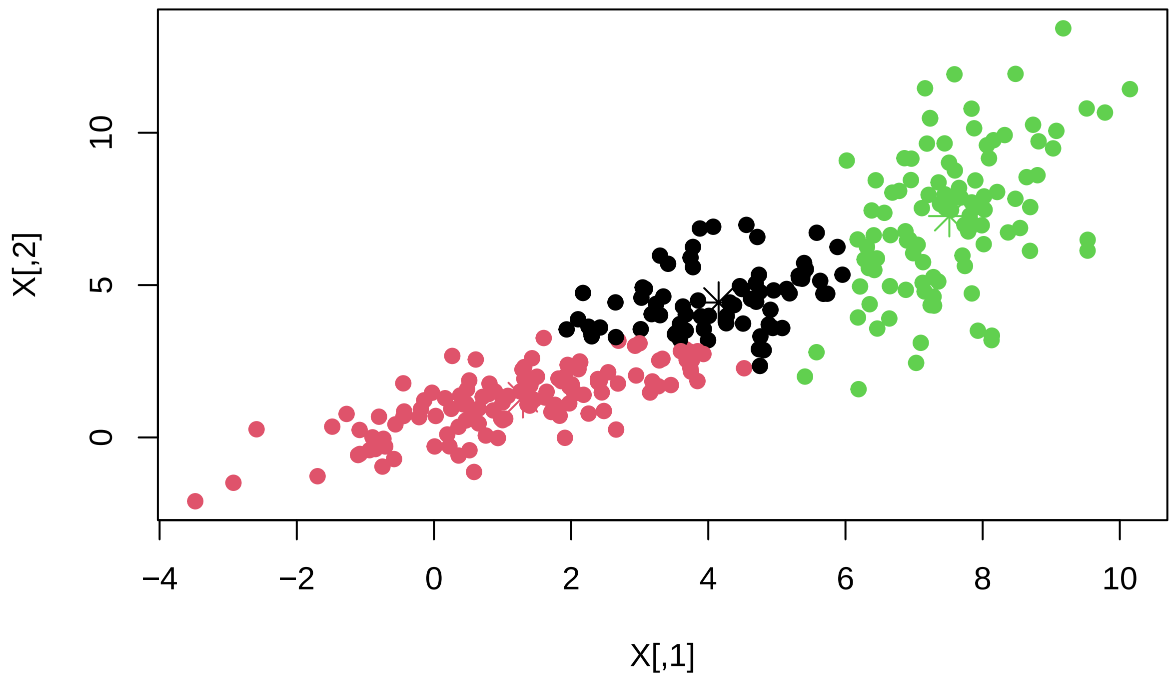
```
##   M-step: Updating parameters
##   Log-likelihood: -1237.93 Improvement: 6.183235
## Iteration: 3
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1236.04 Improvement: 1.889729
## Iteration: 4
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1235.109 Improvement: 0.9305814
## Iteration: 5
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1234.501 Improvement: 0.608576
## Iteration: 6
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1234.043 Improvement: 0.4578222
## Iteration: 7
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1233.676 Improvement: 0.3671716
## Iteration: 8
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1233.372 Improvement: 0.3035804
## Iteration: 9
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1233.117 Improvement: 0.2553471
## Iteration: 10
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1232.899 Improvement: 0.2174851
## Iteration: 11
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1232.712 Improvement: 0.1873507
## Iteration: 12
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1232.549 Improvement: 0.1632364
## Iteration: 13
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1232.405 Improvement: 0.1438995
## Iteration: 14
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1232.277 Improvement: 0.128392
## Iteration: 15
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1232.161 Improvement: 0.115986
```

```
## Iteration: 16
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1232.054 Improvement: 0.1061303
## Iteration: 17
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.956 Improvement: 0.09841874
## Iteration: 18
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.863 Improvement: 0.09256498
## Iteration: 19
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.775 Improvement: 0.08838473
## Iteration: 20
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.689 Improvement: 0.08578386
## Iteration: 21
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.605 Improvement: 0.08475349
## Iteration: 22
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.519 Improvement: 0.08537231
## Iteration: 23
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.431 Improvement: 0.08781676
## Iteration: 24
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.339 Improvement: 0.09237989
## Iteration: 25
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.239 Improvement: 0.09949855
## Iteration: 26
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.13 Improvement: 0.109785
## Iteration: 27
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1231.006 Improvement: 0.1240483
## Iteration: 28
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1230.862 Improvement: 0.1432658
## Iteration: 29
##   E-step: Computing responsibilities
```

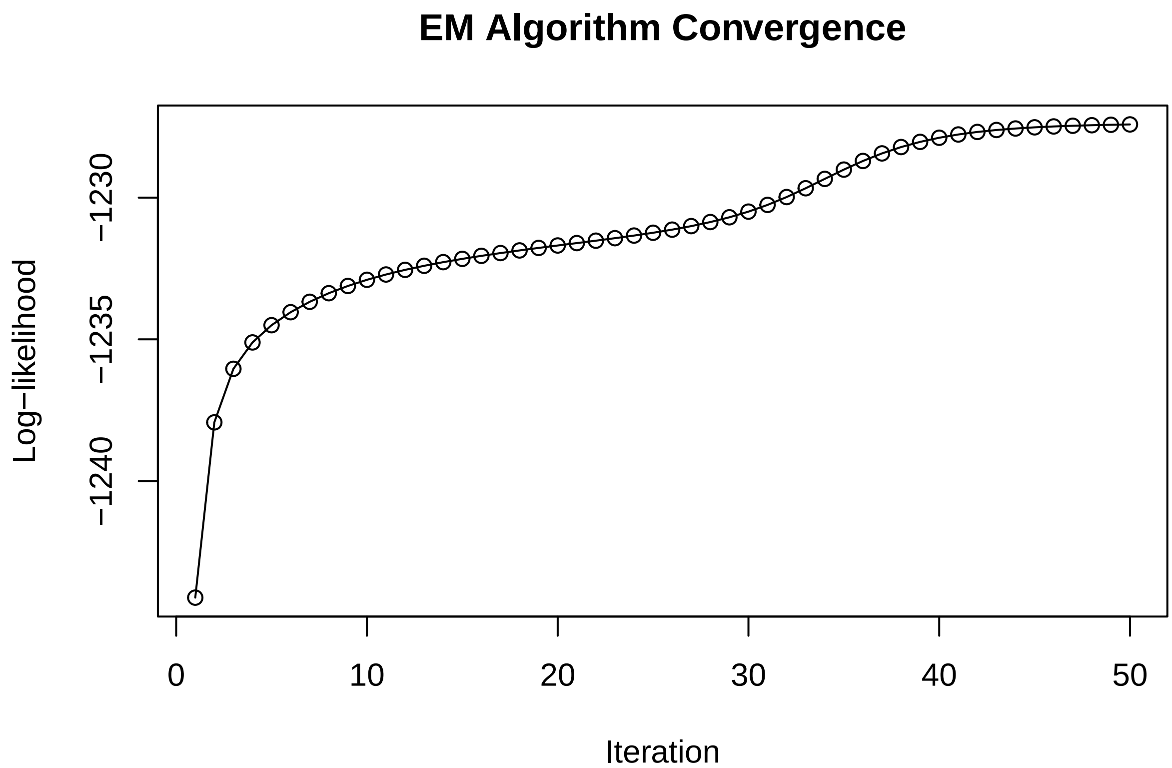
```
##   M-step: Updating parameters
##   Log-likelihood: -1230.694 Improvement: 0.1684187
## Iteration: 30
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1230.494 Improvement: 0.2000353
## Iteration: 31
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1230.257 Improvement: 0.2372651
## Iteration: 32
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1229.98 Improvement: 0.2765356
## Iteration: 33
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1229.669 Improvement: 0.3106394
## Iteration: 34
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1229.339 Improvement: 0.3301096
## Iteration: 35
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1229.011 Improvement: 0.3279218
## Iteration: 36
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1228.707 Improvement: 0.3043793
## Iteration: 37
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1228.44 Improvement: 0.2667686
## Iteration: 38
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1228.216 Improvement: 0.2240954
## Iteration: 39
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1228.033 Improvement: 0.1827812
## Iteration: 40
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.887 Improvement: 0.1460091
## Iteration: 41
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.773 Improvement: 0.1148538
## Iteration: 42
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.683 Improvement: 0.089327
```

```
## Iteration: 43
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.614 Improvement: 0.06894645
## Iteration: 44
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.561 Improvement: 0.05301273
## Iteration: 45
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.52 Improvement: 0.04076181
## Iteration: 46
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.489 Improvement: 0.03145837
## Iteration: 47
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.465 Improvement: 0.02444955
## Iteration: 48
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.445 Improvement: 0.01918868
## Iteration: 49
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.43 Improvement: 0.01523782
## Iteration: 50
##   E-step: Computing responsibilities
##   M-step: Updating parameters
##   Log-likelihood: -1227.418 Improvement: 0.01225776
```

```r
# Plot results
plot(X, col = predict_cluster(model_2d, X), pch = 19)
points(model_2d$mu, col = 1:model_2d$K, pch = 8, cex = 2)
```



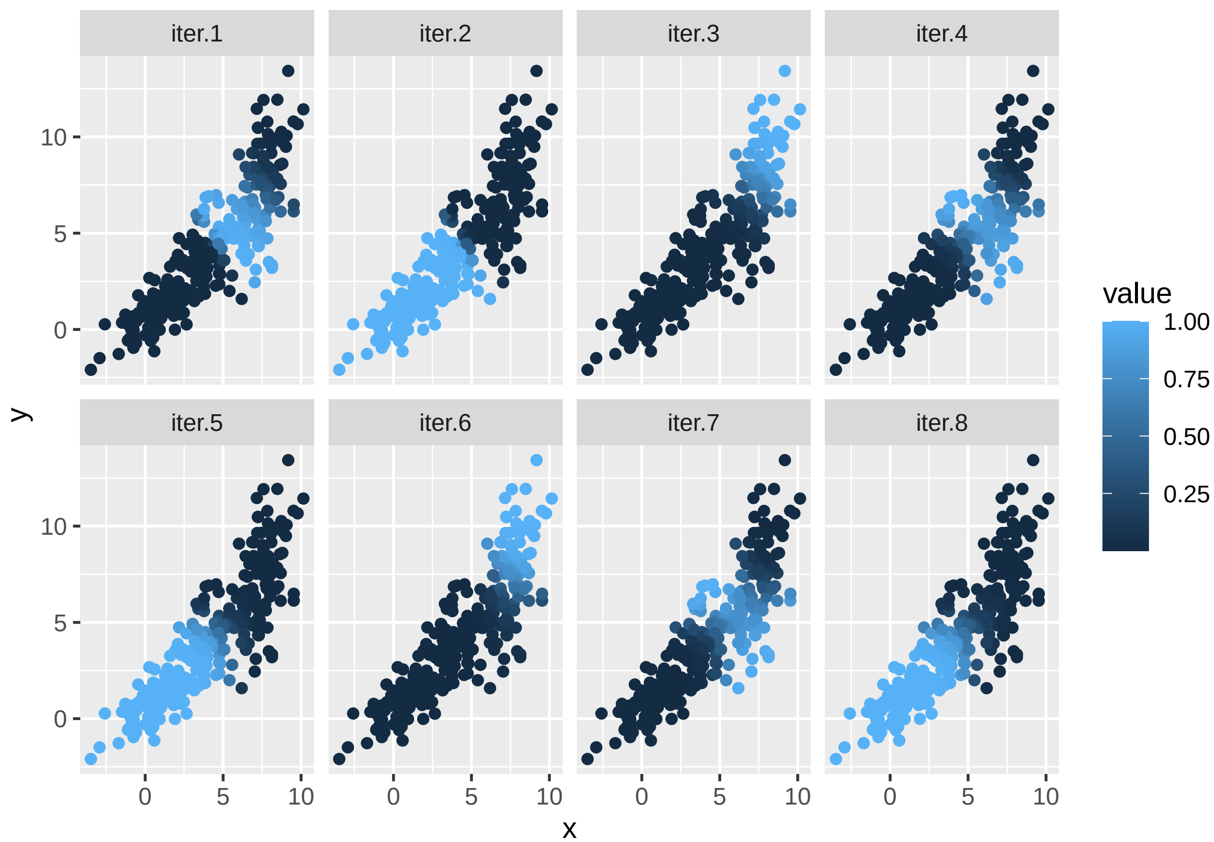

```
plot_convergence(model_2d)
```




```
to.plot = data.frame(x = data[,1],
                     y = data[,2],
                     iter = model_2d$Alpha[,1:8])

to.plot = melt(to.plot, id.vars = c("x", "y"))

# model_2d$Alpha
ggplot(to.plot)+aes(x,y, color = value)+
  geom_point()+facet_wrap(~variable, nrow = 2)
```



c) 3 dimensions

Solution:

```
library(rgl)
```

```
## Warning: package 'rgl' was built under R version 4.4.2
```

```
Mu1 = c(1,1,1)
Mu2 = c(5,5,5)
Sigma1 = matrix(c(2, 1, 1, 1, 2, 1,1,1,2), 3,3)
Sigma2 = matrix(c(2, 2, 1, 2, 2, 2,1,2,2), 3,3)
Sigma2
```

```
##      [,1] [,2] [,3]
## [1,]    2    2    1
## [2,]    2    2    2
## [3,]    1    2    2
```

```
pi = 1/3
n = 300

set.seed(100)
```

```
data = matrix(0, n, 3)
z = rep(0,n)
for (i in 1:n){
  z[i] = rbinom(1,1,pi)
  if (z[i] ==1){
    data[i,] = rmvnorm(1, Mu1,Sigma1)
  }else{
    data[i,] = rmvnorm(1, Mu2,Sigma2)
  }
}
```

```
## Warning in rmvnorm(1, Mu2, Sigma2): sigma is numerically not positive
## semidefinite
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```
data
```

```
##              [,1]        [,2]        [,3]
##   [1,]  2.94507293  2.92424882  3.55491860
##   [2,]  5.29027484  4.92989181  4.59151289
##   [3,]  1.62681502  0.82451894  1.68627562
##   [4,]  1.30776813  1.00985972  1.95133417
##   [5,]  5.79018576  7.28721495  8.06638037
##   [6,]  0.40565003  3.62976104  0.88137424
##   [7,]  2.64955233  1.68400919  2.90237554
##   [8,]  3.94294047  4.14317709  4.61233557
##   [9,]  3.54881455  3.94094953  4.66547739
##  [10,]  0.54902458  0.57576070 -0.00306013
##  [11,]  6.01417795  6.65608939  6.77822170
##  [12,]  2.72746077  1.65562952  3.16046224
##  [13,]  4.79699252  5.01227482  5.22370455
##  [14,]  5.64335112  6.00461466  6.05057051
##  [15,]  5.58865050  5.04612124  4.48911640
##  [16,]  3.09946715 -1.06992404  2.18246559
##  [17,]  2.83557477  3.21035402  4.14683037
##  [18,]  0.50142977  0.69458152  0.38454199
##  [19,]  3.23412743  0.24060108  2.59185560
##  [20,]  5.16496312  5.02119917  4.87078165
##  [21,] -0.16538474  2.52530297  1.80323637
##  [22,]  2.38023760  2.47058047  3.35480527
##  [23,]  6.80096467  6.91032377  6.42010991
##  [24,]  3.50709967  2.11351696  1.62588390
##  [25,]  2.66713653  1.38886925  2.53255741
##  [26,]  6.85442668  5.84486149  4.57012864
##  [27,]  1.06240789  0.21826981 -0.62779474
```

```
## [28,]  4.12125169  0.84307314  2.08908878
## [29,]  6.25291144  7.35012456  7.70972914
## [30,]  0.16943085  1.48145382 -0.81941553
## [31,]  5.42181720  5.29696612  5.07890944
## [32,]  0.99915318  1.27918049  1.36118826
## [33,]  4.20379721  3.49689068  3.26174904
## [34,]  5.94091859  5.33281694  4.62025758
## [35,]  2.36885424  2.19198570  2.16312357
## [36,]  0.86537718 -1.09492015 -0.06840800
## [37,]  5.23974845  4.81733048  4.45224504
## [38,]  0.38916256 -0.04342378  1.49600319
## [39,]  4.87757557  6.05487058  6.90108460
## [40,]  5.58754216  5.69724699  5.58811435
## [41,]  3.88011560  4.42677175  5.15334093
## [42,] -0.82610968 -0.95791897 -0.28162294
## [43,]  3.85556611  4.57085416  5.42083365
## [44,]  6.93282378  6.29760290  5.25511723
## [45,]  7.85345971  7.41755461  6.22287742
## [46,]  5.38017540  5.15063659  4.87381908
## [47,] 10.03643539  9.54628482  7.62924030
## [48,] -0.15402437  0.22327614 -0.21465624
## [49,]  4.30279031  4.56210720  4.95886084
## [50,]  5.11624835  5.15946618  5.15263406
## [51,]  2.93145041  2.58151945  2.99065120
## [52,]  6.66561356  6.03935616  5.08688713
## [53,]  5.65439934  6.55007940  6.95925256
## [54,]  4.45598430  3.76602923  3.46336742
## [55,]  4.97129126  4.14444870  3.58612890
## [56,]  6.13794141  6.25248417  5.97392309
## [57,]  0.37054945  0.70197927 -0.03387040
## [58,]  4.89915574  5.36934409  5.72361035
## [59,]  2.39709725  1.09546349  1.72682141
## [60,] -1.24622945 -3.25448767  0.09867690
## [61,]  1.51777238  2.93903224  1.48130065
## [62,] -1.08606691  1.93428575  0.36836416
## [63,]  8.80771593  8.99883706  7.93488583
## [64,]  5.02283891  5.13832907  5.21040336
## [65,]  4.33955197  5.09407384  5.81906976
## [66,]  6.18591585  5.87032699  5.28157788
## [67,]  3.00212776  2.65298008  0.93662312
## [68,]  4.15132841  4.83956197  5.57815051
## [69,]  1.38031666  1.32606326  1.00346318
## [70,]  1.84436261  2.42351864  3.81132740
## [71,]  0.43857481  0.36094389  1.00187332
## [72,]  7.29449202  5.90905134  4.23829642
## [73,]  4.21099330  4.28039862  4.57565756
## [74,]  4.01784463  4.23720679  4.69597872
## [75,]  7.85434919  7.60931704  6.54532637
## [76,]  6.16949039  6.48360318  6.33207326
## [77,]  1.98398894 -0.32898701  1.23650957
## [78,]  2.17893692  2.05894055  2.72756679
## [79,]  6.04595071  6.86793909  7.10365734
## [80,]  0.50886234  0.92273260 -2.30587262
## [81,]  0.44999182  2.25939149  0.80768975
```

```
##  [82,]  6.02330825  6.21764941  6.02981993
##  [83,]  3.88693622  3.60138312  3.75479900
##  [84,]  3.95868512  4.40621191  5.04010463
##  [85,]  6.80638630  6.39966019  5.55363765
##  [86,]  1.02688635 -1.10717021 -1.64259104
##  [87,]  4.76177688  5.46214211  6.01745973
##  [88,]  6.39176598  6.26254235  5.73705801
##  [89,]  4.27831341  4.80814602  5.39819379
##  [90,]  4.16548271  4.02691945  4.19376661
##  [91,]  4.60692849  3.57247997  2.98607194
##  [92,]  1.93070996 -0.07237935  2.04109897
##  [93,]  3.46460182  3.94329180  4.75363952
##  [94,]  6.74757601  7.18961430  6.94442170
##  [95,]  4.18235069  5.05386736  5.90847725
##  [96,]  5.85323432  5.61395864  5.18198630
##  [97,]  4.01317823  4.11032351  4.48670207
##  [98,]  5.24003566  4.47483335  3.87445950
##  [99,]  3.20966927  2.72875805  4.71148988
## [100,]  6.10235156  6.02273040  5.62211576
## [101,]  4.03512799  3.41706077  3.29581380
## [102,]  6.25415149  6.63843520  6.50848073
## [103,]  2.20111525  3.19133455  4.74922040
## [104,]  9.80071670  9.42329570  7.65758204
## [105,]  5.16462145  5.16164700  5.10793813
## [106,]  5.44933296  5.58905080  5.54388955
## [107,]  1.32190793  1.08099061 -0.47117610
## [108,] -1.23048135  1.11379401  1.12417557
## [109,]  5.15472291  6.29303104  7.02550931
## [110,]  3.72324739  3.18705255  3.21986819
## [111,]  1.07872701  2.23457141  2.45699670
## [112,]  4.75140862  5.56318622  6.19820257
## [113,]  0.97418868 -0.84060181  2.04431080
## [114,]  1.92911013  1.64605626  2.41566895
## [115,]  5.23699270  4.88928011  4.57631799
## [116,]  3.14361213  4.06416799  5.27844346
## [117,]  7.19419691  5.66287960  3.92351134
## [118,]  6.69231690  7.26361213  7.12445156
## [119,]  5.62478700  6.09216734  6.21676076
## [120,] -2.04455422 -0.18130455 -0.96380123
## [121,]  5.50530970  6.71712491  7.39000443
## [122,]  6.00164926  5.73493267  5.23755060
## [123,]  5.18587415  4.56356893  4.07824167
## [124,]  5.65558175  5.41629872  5.04635645
## [125,]  3.73845521  4.45255221  5.33847081
## [126,]  1.43777979 -2.07232790 -0.36806760
## [127,]  4.44499511  4.47058449  4.66233586
## [128,]  2.33595243  0.04180287  0.80264957
## [129,]  2.39319596  2.72880977  3.77725785
## [130,]  6.09531835  5.58713951  4.89468145
## [131,]  4.98716842  5.10451447  5.18905767
## [132,]  3.94688992  3.82266196  4.06795253
## [133,]  6.10636896  6.79198812  6.91517507
## [134,]  0.43076862  1.42702033  0.24325096
## [135,]  7.72555497  7.22343377  6.02346713
```

```
## [136,] -0.76586516  0.41606464  3.00655698
## [137,]  7.70012779  7.19852240  6.00689022
## [138,]  1.50835446  2.58675148  1.70601663
## [139,]  6.99326244  6.30474642  5.20672355
## [140,]  0.06926732  0.77604639  0.47314127
## [141,]  2.95133055  1.31359323  4.13558527
## [142,]  3.04311756  2.99570895  3.57736581
## [143,]  4.61791088  4.58052358  4.67479287
## [144,]  4.74795799  3.36828982  2.50074913
## [145,]  5.93785712  3.99032616  2.35969077
## [146,]  4.63986853  4.88772181  5.17081466
## [147,]  0.25523507  0.22431411  0.05006581
## [148,]  5.52021611  4.04559565  2.87052390
## [149,]  4.93522998  4.72388139  4.59919520
## [150,]  0.85939767  3.52039042  3.38193553
## [151,]  3.98701998  3.93239548  4.21284863
## [152,]  6.36447529  7.53034168  7.90203670
## [153,]  5.23557317  4.67709710  4.21996712
## [154,]  5.04370121  5.29883001  5.46016822
## [155,]  4.65240511  4.45891829  4.43525502
## [156,]  5.48912146  5.67618418  5.65102018
## [157,] -0.40130794  1.51743886  0.51535209
## [158,] -0.19950532  1.38384294  0.92960832
## [159,]  8.28426016  6.79209673  4.73746701
## [160,]  2.39943781  3.65152420  3.33398345
## [161,]  1.72315971 -0.10856454  1.18183989
## [162,]  4.81487089  3.79245049  3.14903078
## [163,]  6.31689028  6.47133282  6.16398381
## [164,]  6.08456844  6.06073391  5.70397813
## [165,]  2.23669870  2.41182408  3.39927265
## [166,]  7.55822195  7.44353443  6.56192081
## [167,]  1.72490219  2.51590723  4.08656797
## [168,]  5.04648312  4.81618297  4.64357551
## [169,]  1.61446062  0.38020077  0.58405289
## [170,]  4.77050208  4.10039151  3.71263147
## [171,]  5.29065028  5.64344154  5.79428266
## [172,]  9.55254846  8.88814184  7.00340559
## [173,]  5.00624693  5.74224632  6.24528477
## [174,]  4.87355447  4.04353716  3.51371463
## [175,]  4.38828750  5.90718825  7.14135949
## [176,]  2.66432271  4.01852442  5.68077139
## [177,]  5.62667450  5.25734950  4.80725295
## [178,]  6.72833905  5.86263974  4.72619289
## [179,] -1.99032114 -0.52817794  0.01276754
## [180,]  5.56804603  5.94201656  6.02032640
## [181,]  0.21201277 -0.46875927 -0.21244616
## [182,]  6.23795905  5.99285494  5.43613404
## [183,]  0.48872638 -1.17948827  0.20969475
## [184,]  3.03336379  3.56214774  4.54221505
## [185,]  5.29901853  6.14481744  6.63130471
## [186,]  0.36750251  1.70910648  4.08358811
## [187,]  6.05292840  6.27829623  6.10245884
## [188,]  6.75351610  7.86917062  8.08430915
## [189,]  6.75933045  8.09239172  8.45487697
```

```
## [190,]  6.00393232  5.88269843  5.48442140
## [191,]  6.36691141  6.98751850  6.98432435
## [192,]  5.51098249  3.63850658  2.19334810
## [193,] -2.11584253  1.23494043  0.23695933
## [194,]  5.28811207  6.08555444  6.54228541
## [195,]  1.90130911  1.55675191  2.23595964
## [196,]  5.31887693  4.81031369  4.36128527
## [197,]  0.58338043  2.05481192  2.95556022
## [198,]  4.47352946  3.46083722  2.93122559
## [199,] -0.41089944  1.79487965  2.68479223
## [200,]  0.31742499 -0.60384229  0.76741978
## [201,]  0.72241780  2.44640675  3.16672607
## [202,]  5.04077797  6.46134325  7.42325230
## [203,]  1.28157586 -0.15898745 -1.45810759
## [204,] -0.99399730  1.50485057  0.88982025
## [205,]  5.88703936  5.79813029  5.45872057
## [206,]  2.59839538  3.38706817  4.66213593
## [207,]  1.02787280  2.16866213 -0.56037001
## [208,]  0.96275909 -0.33533385 -0.53205166
## [209,]  5.23636809  5.19822148  5.09786121
## [210,]  4.65851616  4.11622430  3.85131370
## [211,]  2.77019355  1.98410308  1.62873186
## [212,]  2.17750638  0.51537898  0.88448998
## [213,]  3.28041734  4.31634111  5.56683761
## [214,]  0.28265699  0.31204469  0.20418643
## [215,]  1.79785570  2.19490571  3.47236075
## [216,]  3.02947894  2.10488297  2.08894652
## [217,]  2.49912791  2.12210901  2.64834307
## [218,]  4.62530155  3.09759406  4.55715568
## [219,]  6.07802885  6.07843647  5.74036673
## [220,]  3.77694848  4.91856679  6.08574367
## [221,]  3.76199386  3.75745042  4.14289277
## [222,]  3.63076707  4.74852219  4.46862626
## [223,]  2.92746926  2.82379057  3.40313553
## [224,]  2.20581218  2.16925598  1.09920384
## [225,]  5.86700146  6.13020707  6.03868663
## [226,]  1.74741629  2.40246870  2.23473127
## [227,]  5.58577882  6.67653416  7.24109360
## [228,] -0.39344837  1.78980752 -1.14837580
## [229,] -0.90466711 -0.58809104 -0.58717735
## [230,]  4.98206339  5.35923839  5.62366307
## [231,]  4.22735232  3.58877275  3.39312004
## [232,]  0.48642743  0.64416994  2.45513938
## [233,]  6.52095939  5.51619937  4.34942537
## [234,]  0.41339136  1.89954164  1.63287096
## [235,]  5.76734813  5.12955256  4.45109571
## [236,]  4.42168912  4.35715510  4.49438395
## [237,]  1.51049096  1.26696757  2.50857709
## [238,]  2.87124318  2.58384914  2.94566326
## [239,]  5.15624813  4.89567764  4.66784970
## [240,]  5.43579295  5.56257340  5.51278494
## [241,]  0.55648987 -0.04000271 -0.02785067
## [242,]  3.01813777  3.41618778  3.06470608
## [243,]  0.05013209  1.62692413 -0.15642026
```

```
## [244,]  3.54951867  2.34289330  0.95564811
## [245,]  1.67834913  1.80174763  2.92894750
## [246,]  6.27578846  6.34790855  5.99697496
## [247,] -0.05217252 -0.20127869  0.19063295
## [248,]  3.33795191  3.51442056  1.92045787
## [249,]  7.60296355  8.19812772  7.78952964
## [250,]  1.18749077  0.47522157  0.99227095
## [251,]  0.72881439  2.70895898  2.75190873
## [252,]  2.45403071  2.12393492  2.69651900
## [253,]  4.70512306  5.31366974  5.82376825
## [254,]  1.90906637  2.49622296 -0.14806946
## [255,]  4.94263636  4.12732690  3.58591403
## [256,]  4.38262585  2.85234502  2.00483304
## [257,]  3.90267660  4.35736915  5.01375740
## [258,]  4.82195980  4.74584053  4.74949159
## [259,]  4.57504987  3.83843969  3.46639606
## [260,]  6.99958928  5.51004654  3.86042092
## [261,]  3.77283937  3.40948755  3.54533291
## [262,]  3.05666750  0.27099533  2.76980703
## [263,]  3.34228082  3.08656004  3.43139026
## [264,]  3.41495517  4.39470488  5.56443211
## [265,] -0.12339278  0.36307248  0.09891415
## [266,]  5.14377417  5.37049353  5.48093004
## [267,]  5.99500626  4.78686238  3.64561373
## [268,]  4.57504035  4.52053001  4.61650580
## [269,]  3.31132793  4.06796548  5.11713076
## [270,]  3.23649743  3.77579839  4.69932646
## [271,]  5.60518480  5.16911585  4.67996831
## [272,]  3.85860693  3.70942633  3.96530433
## [273,]  5.07775485  5.33294103  5.48363055
## [274,] -0.24831942 -1.08300643 -0.62491904
## [275,]  1.81933578  3.03145913  2.29088519
## [276,] -0.55566848  1.25945341 -1.14224521
## [277,] -0.39122659 -2.12151813  0.68684364
## [278,]  5.26913750  4.68918323  4.20678171
## [279,]  0.33845545 -0.01992513  0.65077041
## [280,]  5.73787011  6.05731129  6.04490545
## [281,]  1.27624930  0.24518497  1.45848546
## [282,]  0.57875634 -1.33951194 -0.62904342
## [283,]  1.93474139  3.61730153  2.57184503
## [284,]  4.37160195  4.97791080  5.59115256
## [285,]  5.46286486  5.77508252  5.84403329
## [286,]  2.08719401  2.89073837  4.35629418
## [287,] -0.08724756  1.17450487 -0.05766964
## [288,]  6.20843813  6.61495245  6.51459887
## [289,]  2.76252190  2.84972995  3.61182034
## [290,]  5.35816739  4.47727371  3.76044256
## [291,]  6.06568284  5.80242783  5.28732336
## [292,]  0.68404762  0.98556471  1.00902484
## [293,]  0.38627195  1.01734112 -0.09959872
## [294,]  3.97535081  4.59482637  5.34146945
## [295,]  5.11143146  5.90715346  6.41815688
## [296,] -0.79238075 -1.30796368 -0.43519518
## [297,]  0.41297860 -0.09753303  0.49828114
```

```
## [298,]  6.75337042  7.03115070  6.67143536
## [299,]  6.14041078  6.63896456  6.62311401
## [300,]  5.36099776  5.70563767  5.82880661
```

```
to.plot = data.frame(x = data[,1],
                     y = data[,2],
                     z = data[,2],
                     class =  as.factor(z))

mycolors <- c('royalblue1', 'darkorange')
color <- mycolors[ as.numeric(to.plot$class) ]
plot3d(to.plot, col = color)
```

Point 3

Once you know the algorithm works, apply it to segment three images where you think there are different classes and show the result.