Two-Dimensional Packing Problems in Telecommunications

Silvano Martello
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IFORS Distinguished Lecture, CLAIO/SOBRAPO, September 2012, Rio de Janeiro

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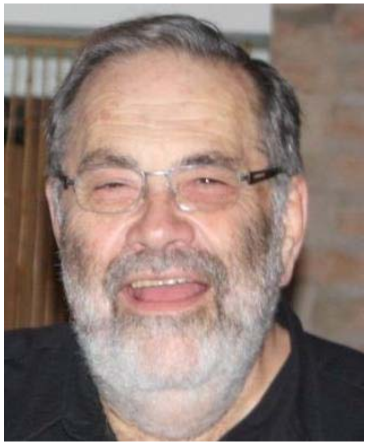

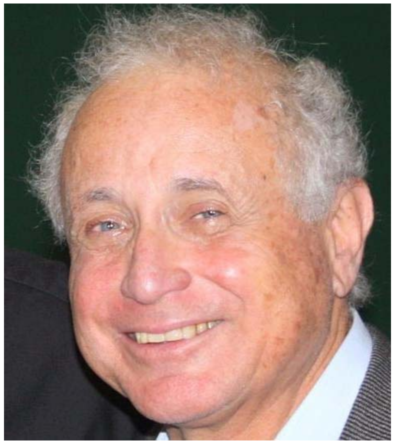


Happy
Birthday!


Happy
Anniversary!

Outline of this talk

- **Objective:** description of the development an interdisciplinary research applicable to real world problems.

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- The whole project has been described in:

  Lodi, Martello, etc ... Efficient two-dimensional packing algorithms for mobile WiMAX.
  Management Science, 2011.

Contents

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7. implementation and **experimental evaluation** on realistic scenarios.

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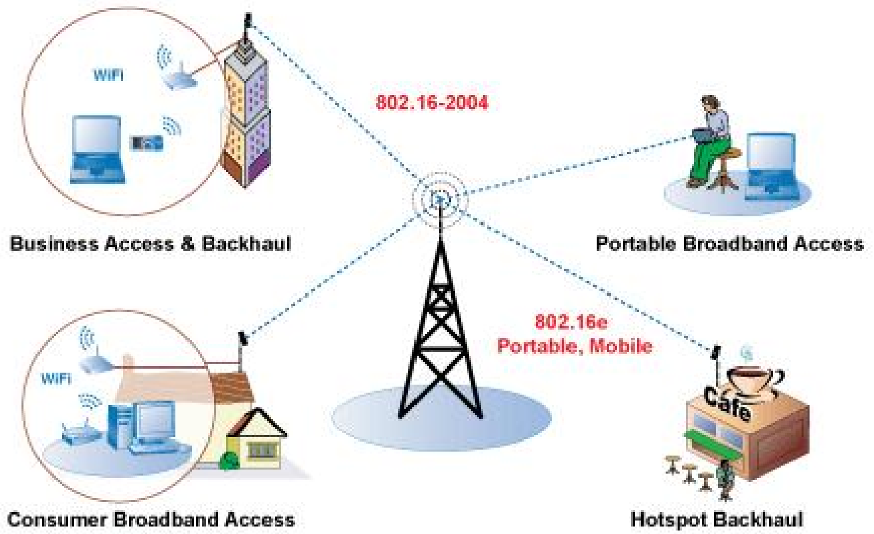

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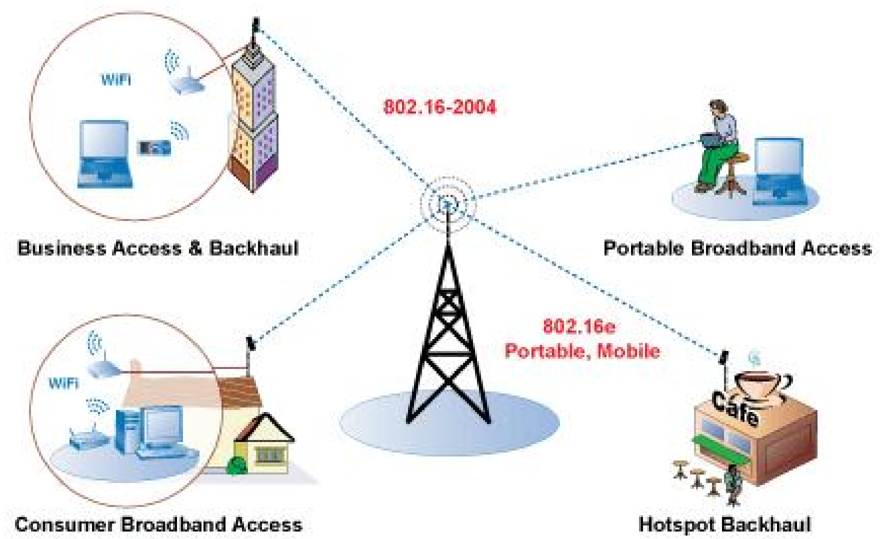

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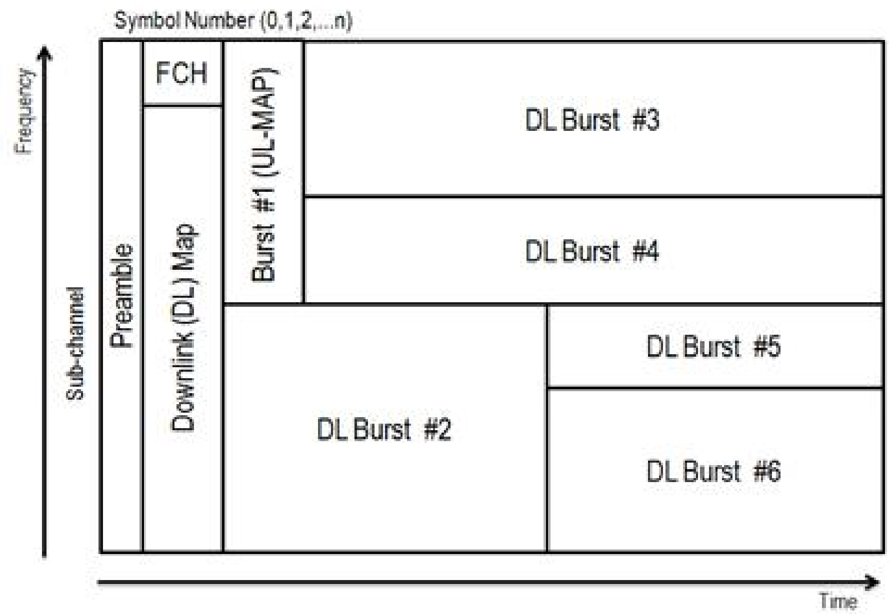

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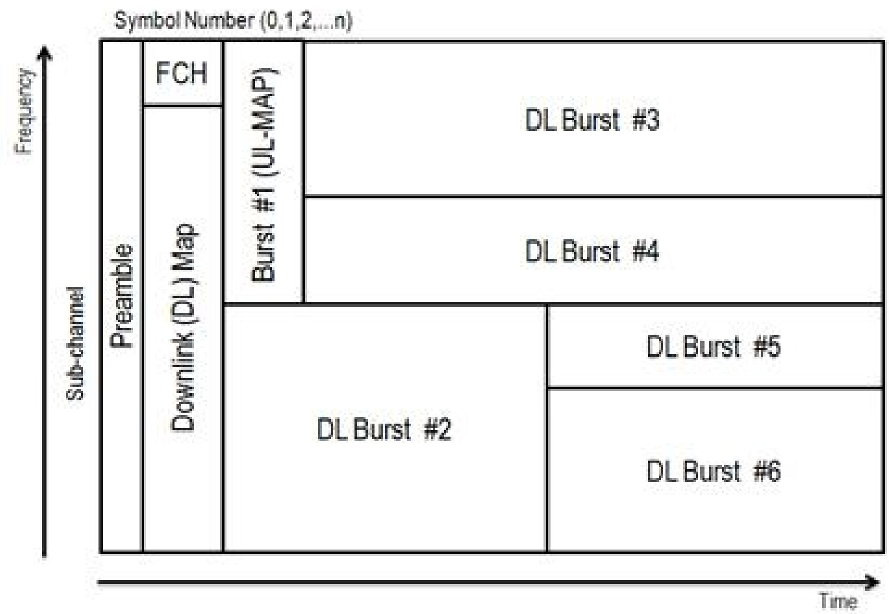

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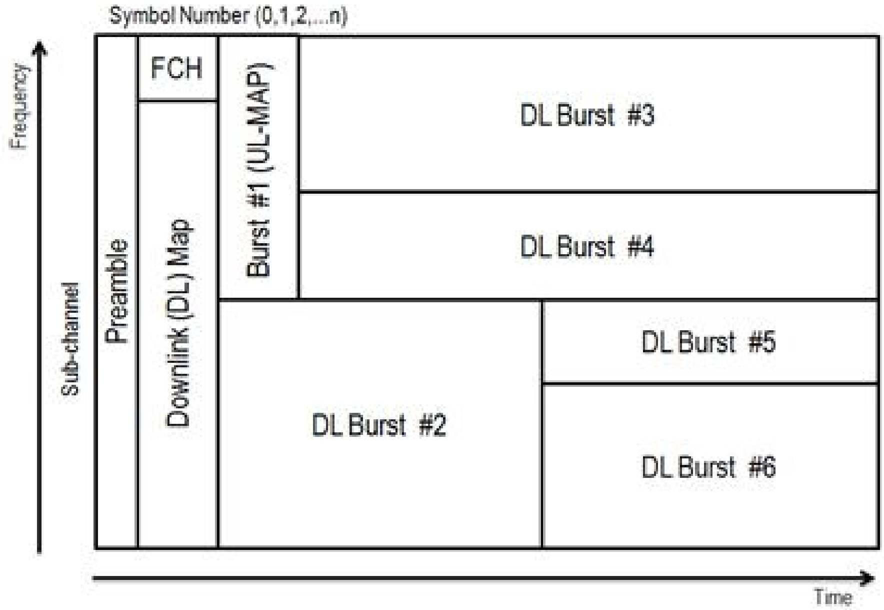

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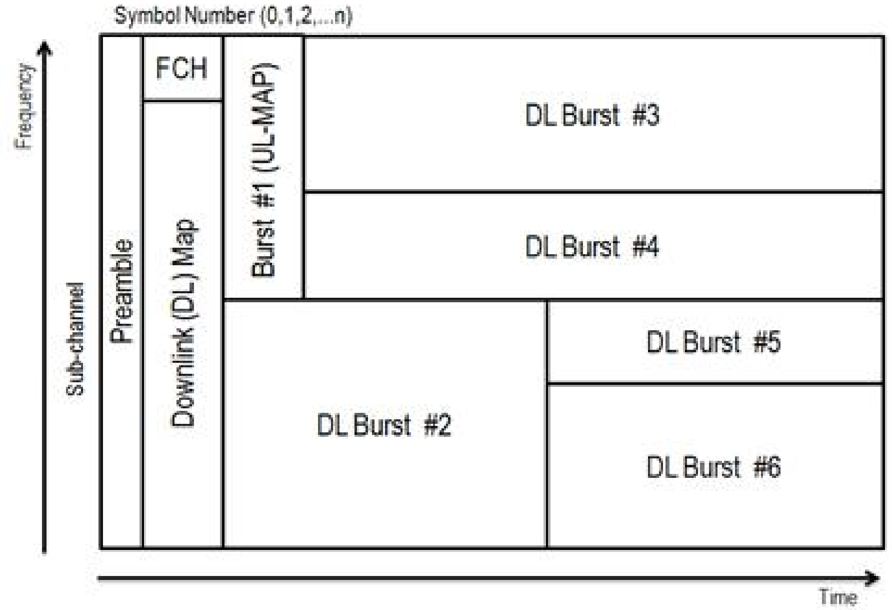

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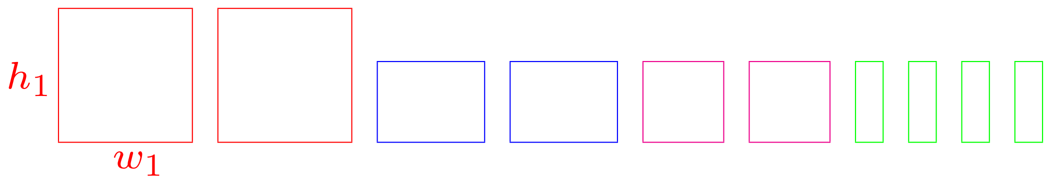
Standard **Two-Dimensional Bin Packing Problem (2BP)**

- given n **rectangles** (**items**), having width w_j and height h_j $(j = 1, \dots, n)$,



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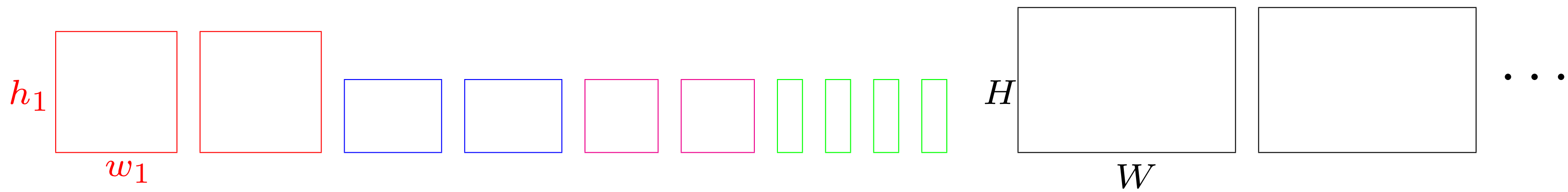

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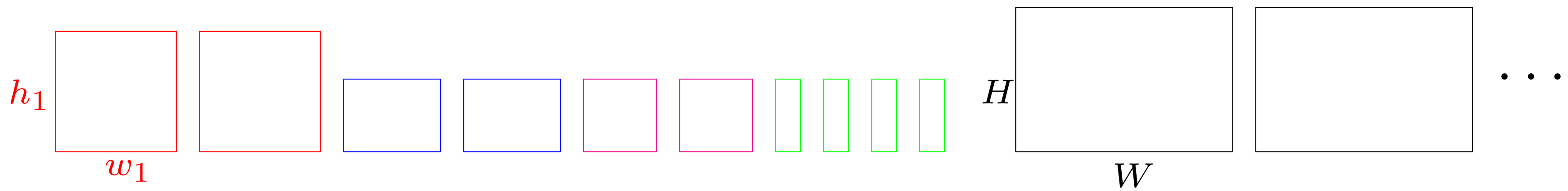

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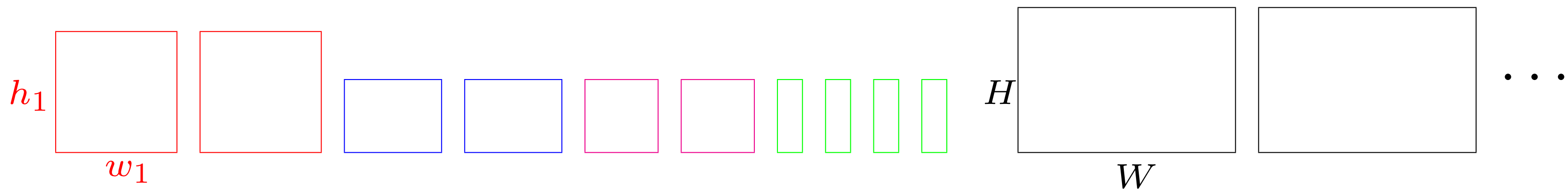

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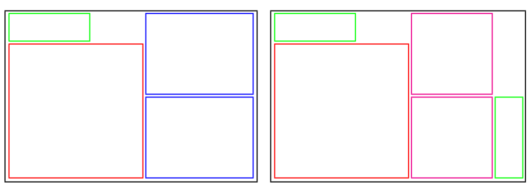

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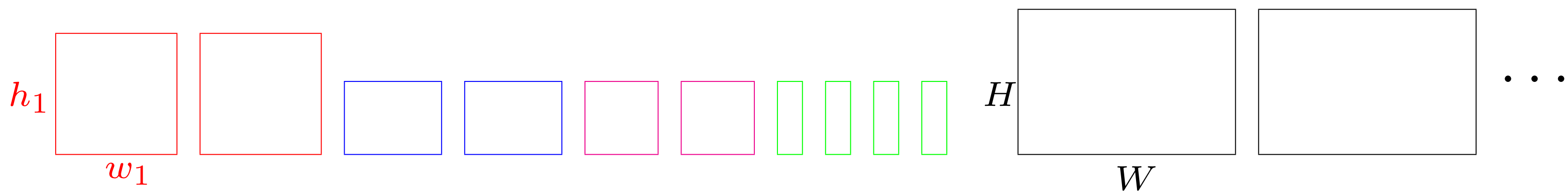

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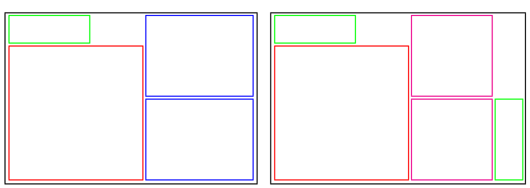

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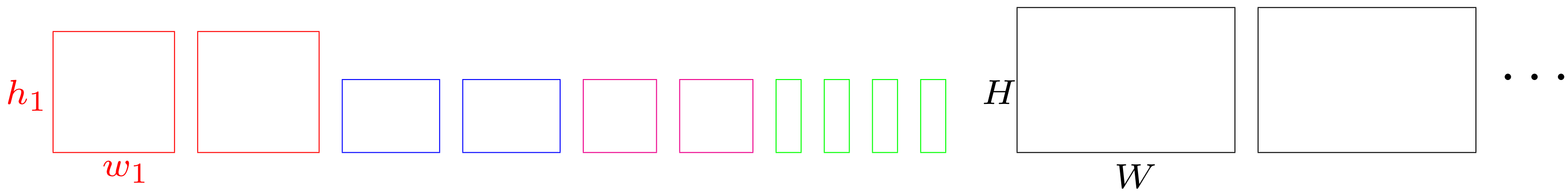

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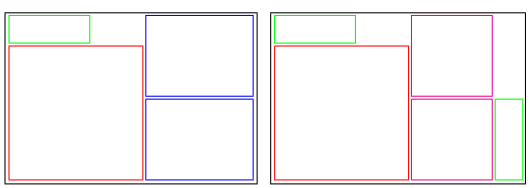

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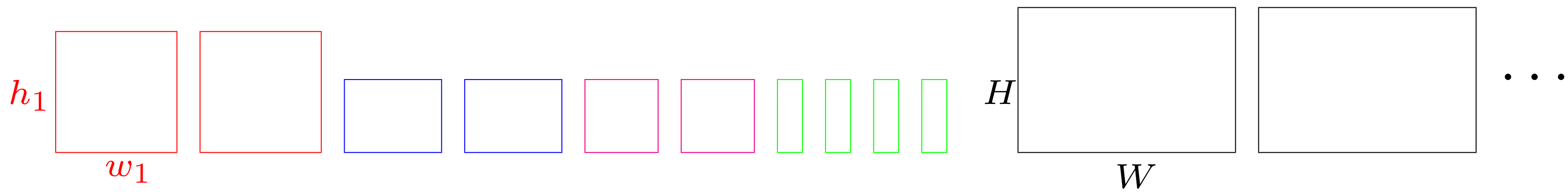

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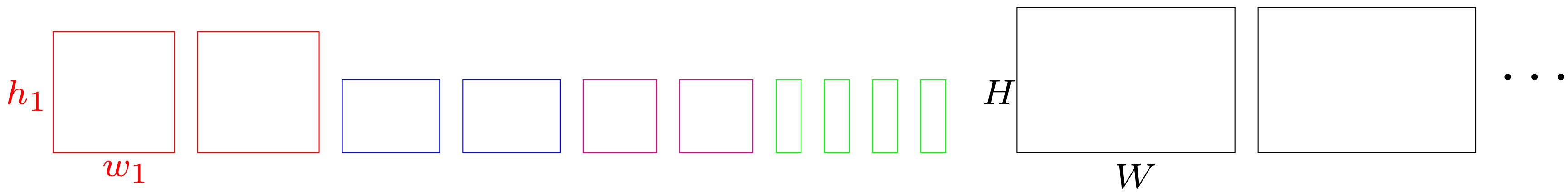

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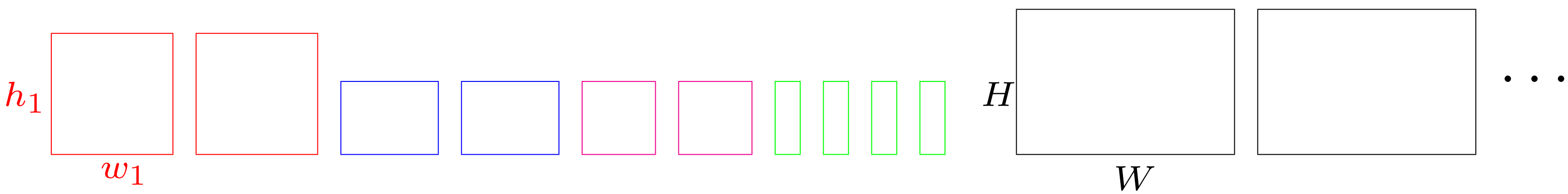

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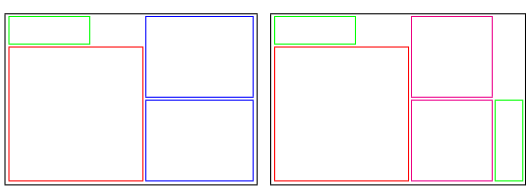

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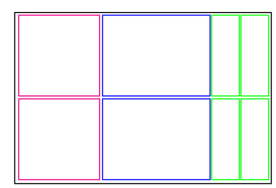

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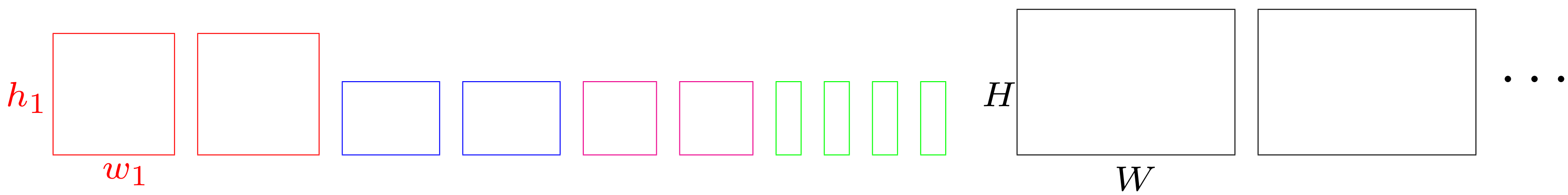

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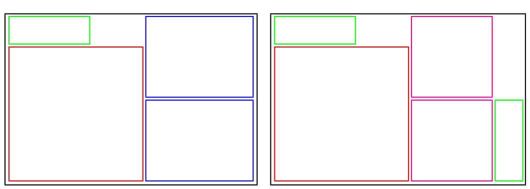

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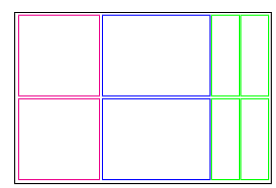

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  guillotine cutting may/may not be imposed (items must be obtained through a sequence of edge-to-edge cuts parallel to the edges of the bin); . . . **huge literature**

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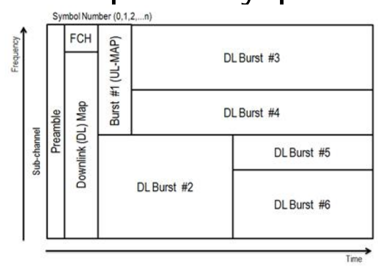

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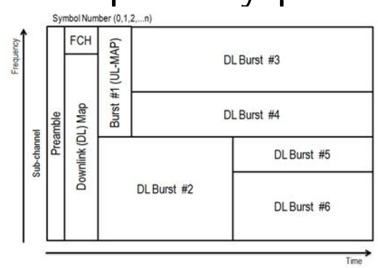

  - each packed rectangle needs information in the downlink zone (sizes, coordinates), i.e.,

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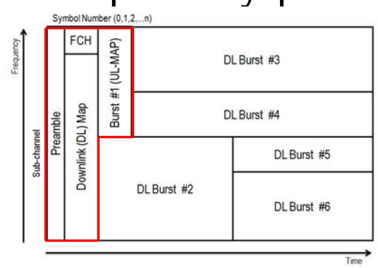

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  - part of the bin is used for **maps transmission**: size proportional to number of rectangles;

2. The models: our problems vs standard 2BPs

Main difference

- **Input to 2BP**: set of rectangles to be packed.
- **Input to the telecommunication problems**: set of data packets to be packed:
  - a **data packet** is an amount of information, in practice a **number**;
  - this number may be interpreted as an **area** a_j;
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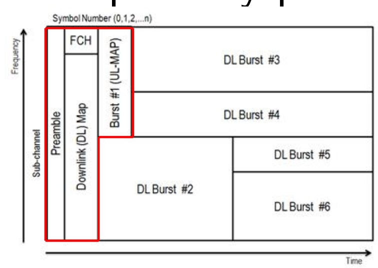

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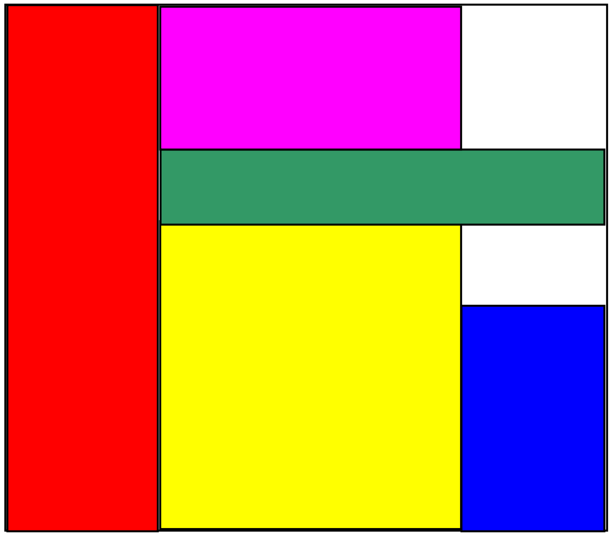
- 8 x 7

- 14
- 6
- 8
- 6
- 16

- 14 = 2 x 7
- 6 = 2 x 3
- 8 = 4 x 2
- 6 = 6 x 1
- 16 = 4 x 4



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- Of course, if the optimal solution to the optimization version has value n, i.e., a unique rectangular sub-area is created for each original area, then the recognition version has answer "YES".
- This version makes sense by itself as a very naïve approximation of the application at hand. In other words, the best configuration is obtained by minimizing the number of sub-areas.

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  by further splitting only when necessary.

  C. Post-optimize the solution. (Not needed for the worst-case guarantee.)

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Instance with $W = 15, H = 10$

area	a_j	$\widetilde{w}_j$	$\widetilde{h}_j$
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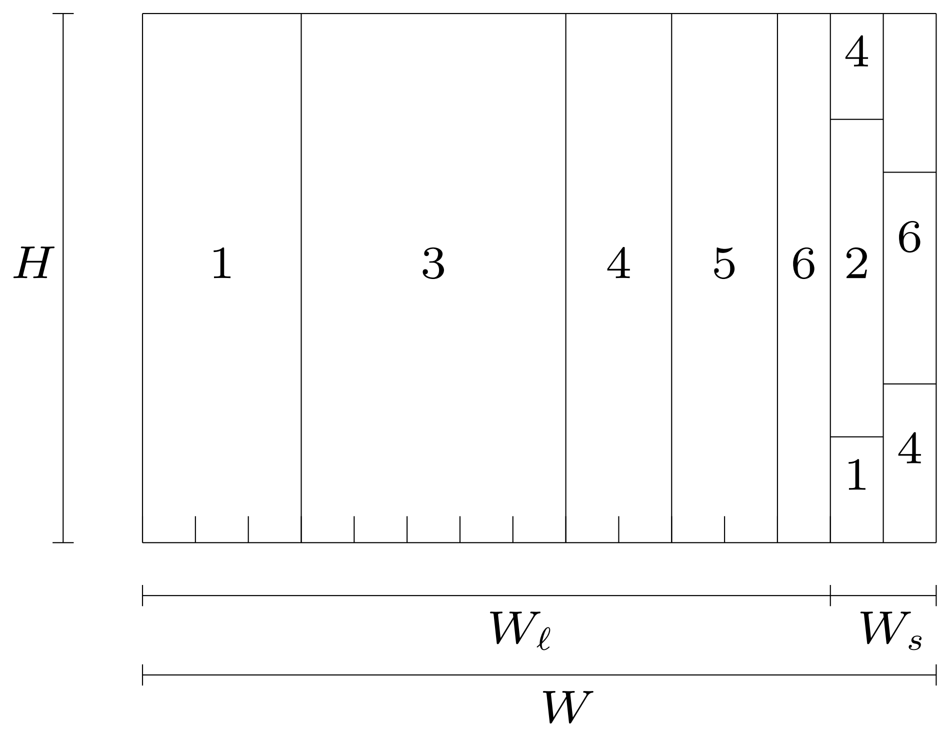

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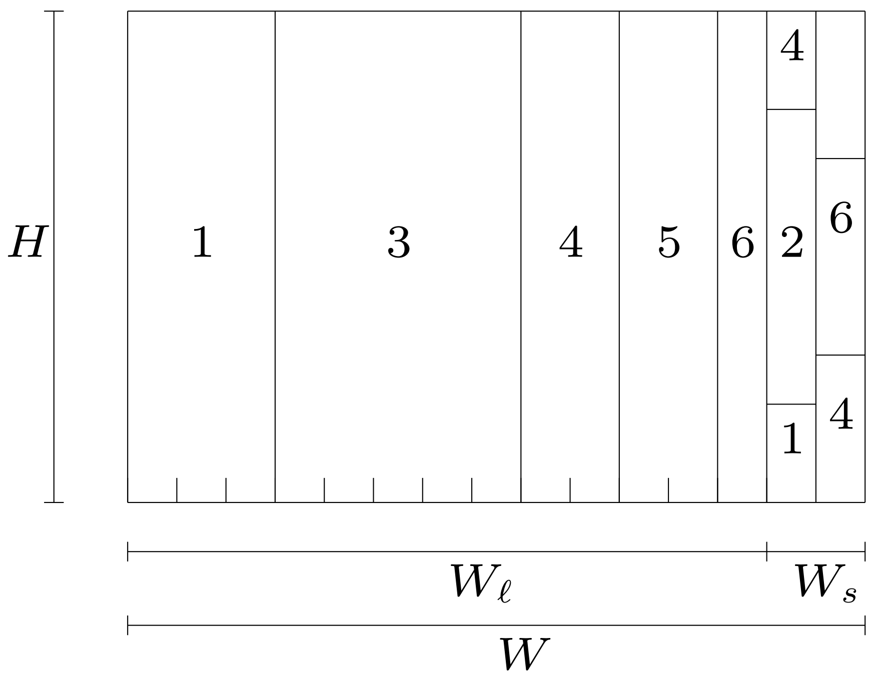

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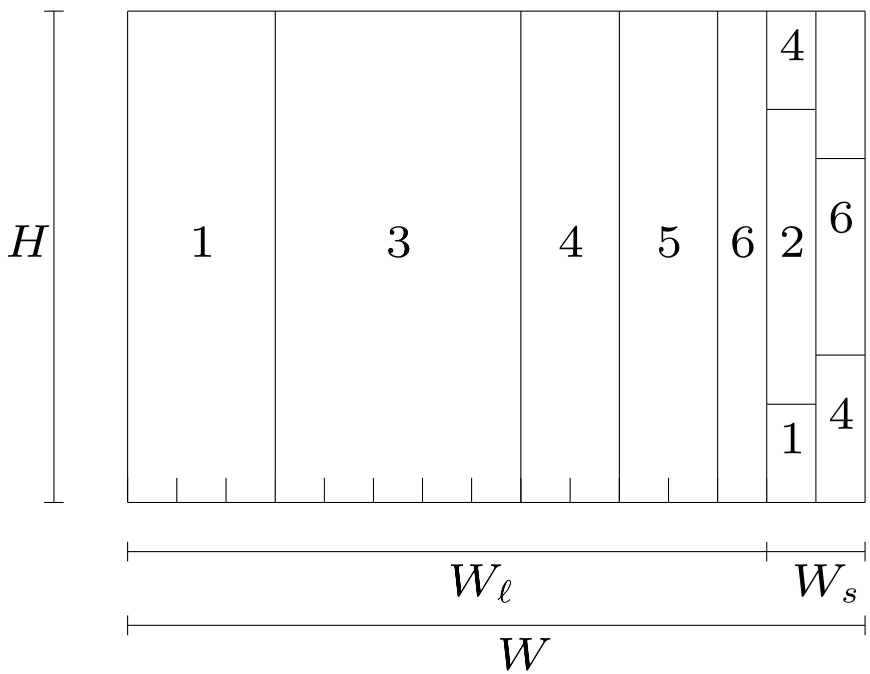

- It can be shown that the bound is tight.

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- Conjecture: Every instance possesses a feasible solution with at most two rectangles per area.

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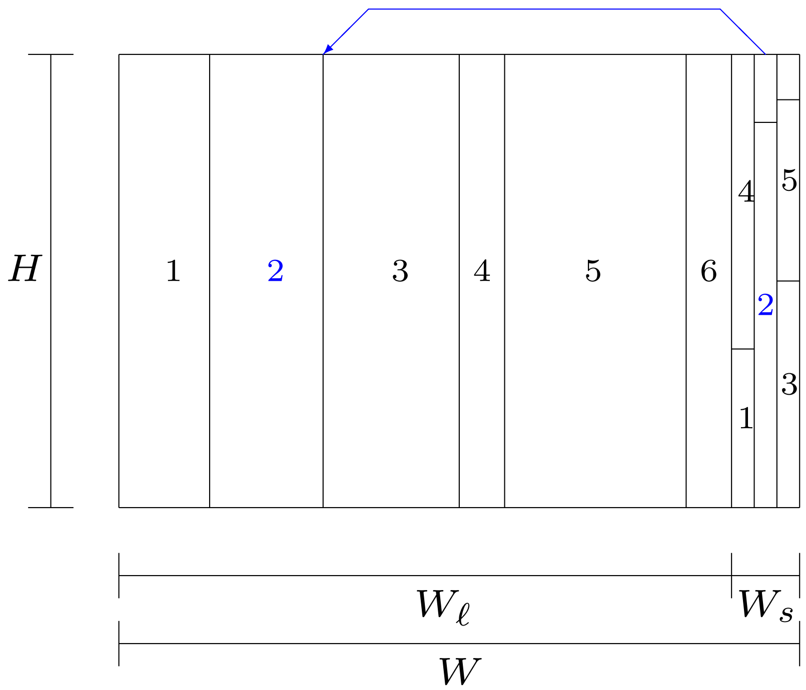

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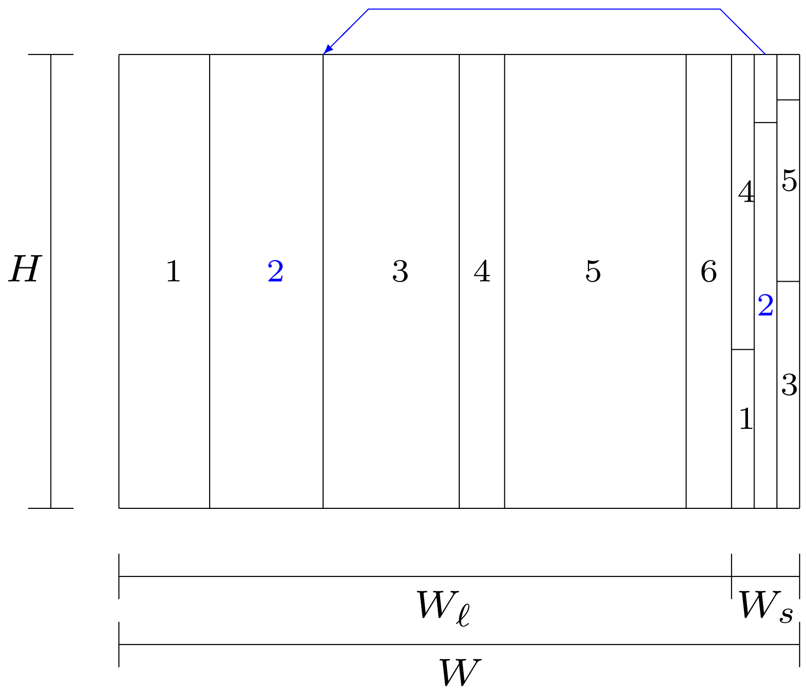

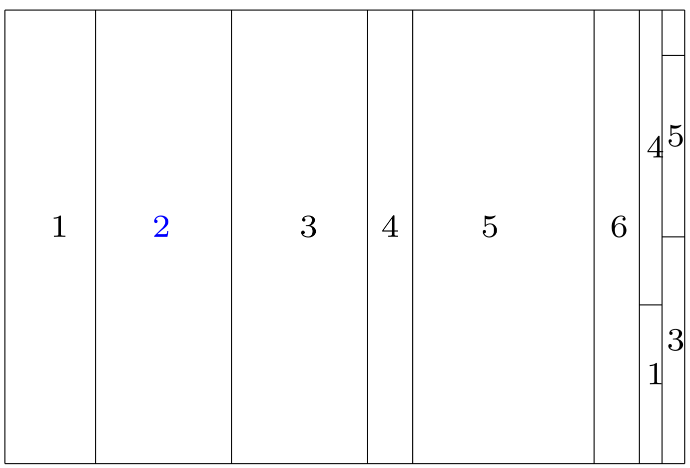

- Move the 1-dimensional bin that packs strip j close to the rectangle associated with area j.

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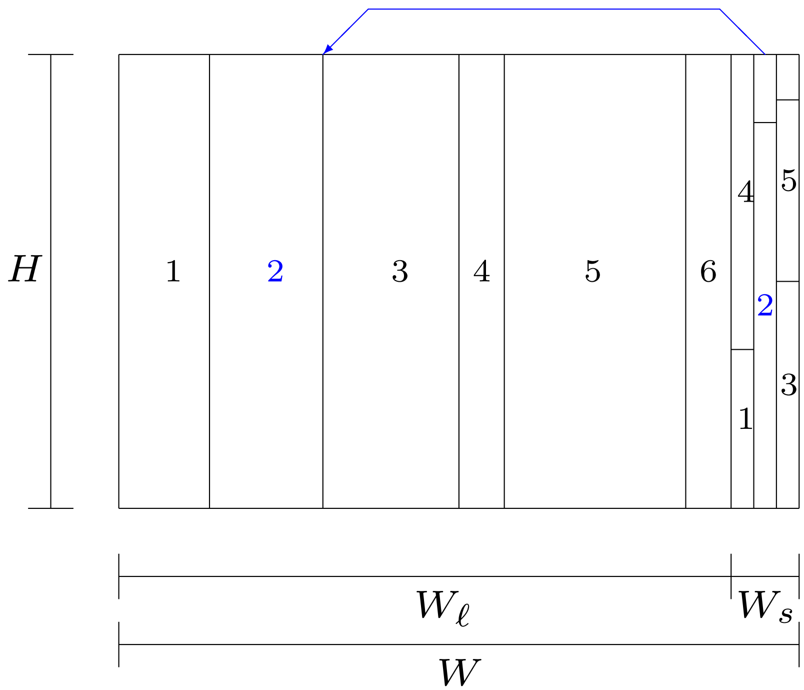

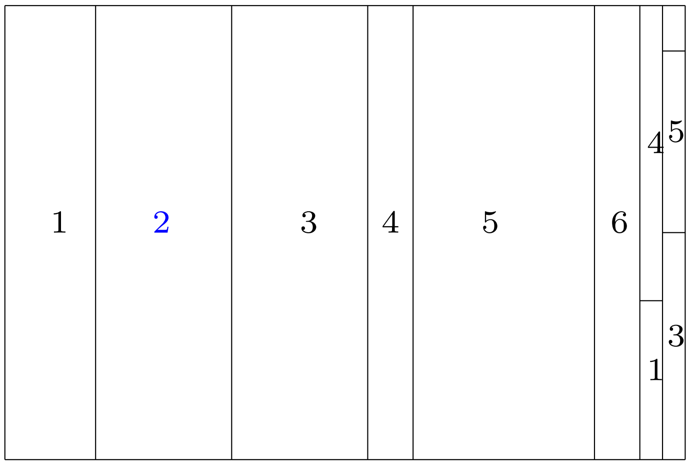

- Move the 1-dimensional bin that packs strip j close to the rectangle associated with area j.

- $\Rightarrow$ New solution in which area j is packed with a unique rectangle $(\widetilde{w}_j + 1) \times H$.

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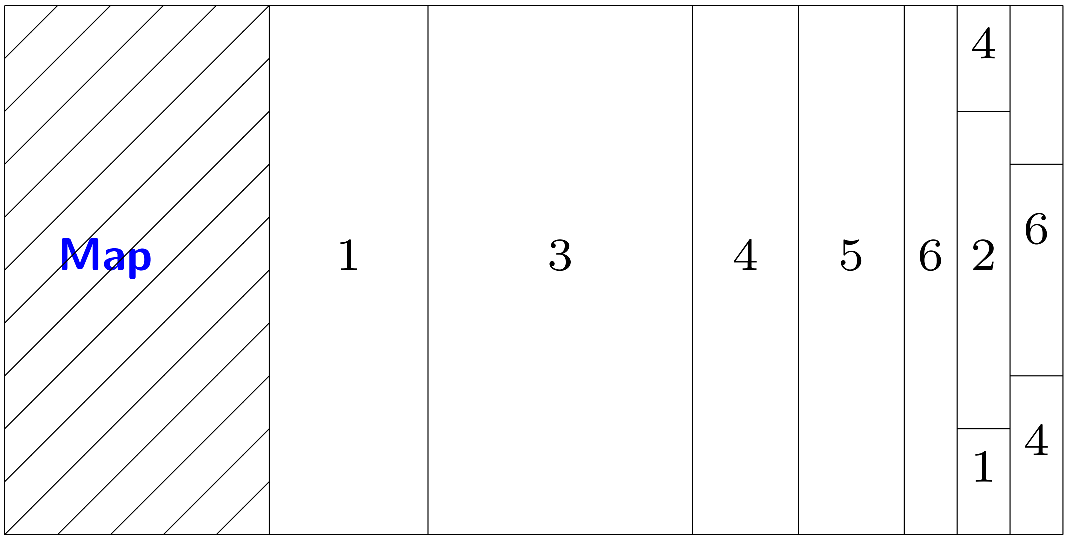

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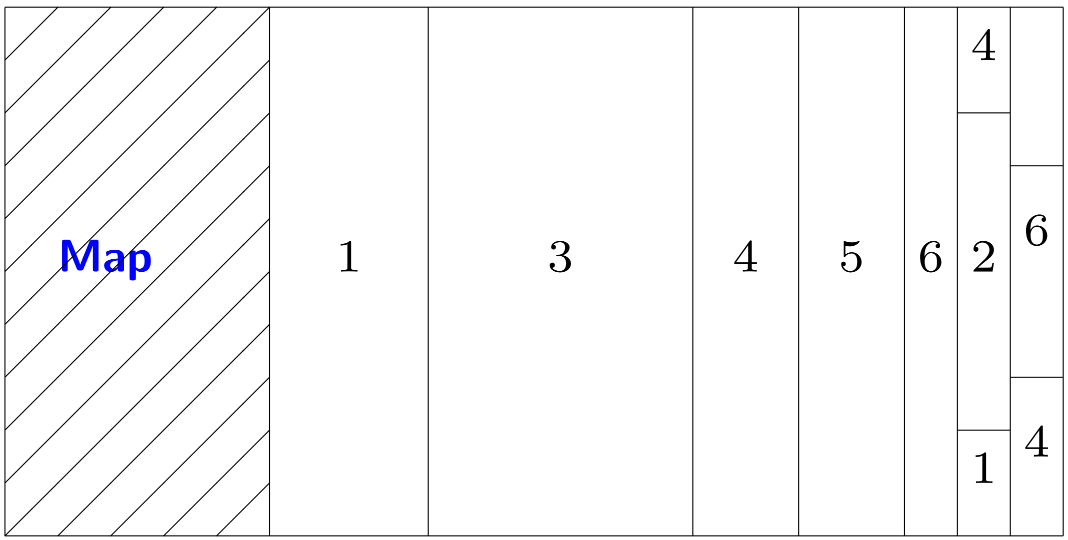

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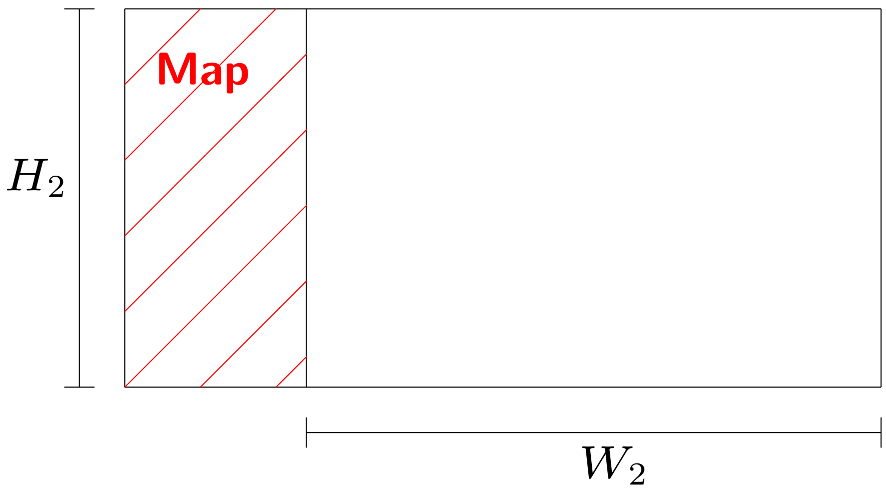

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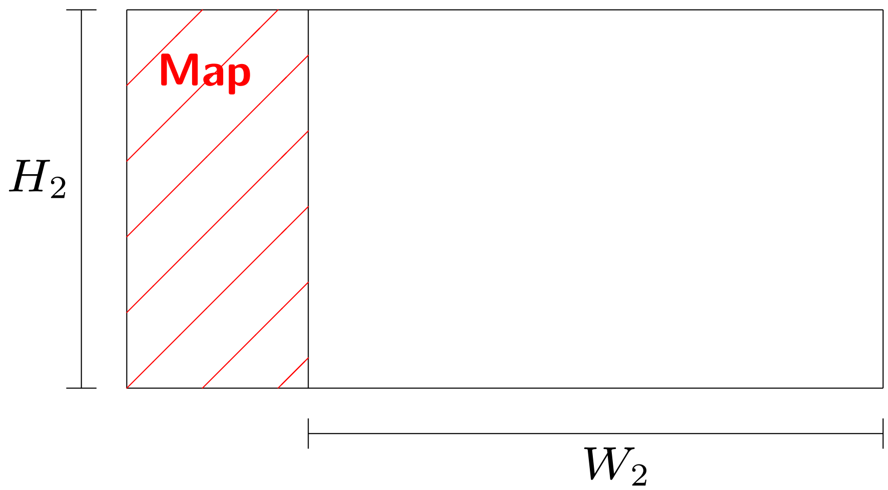
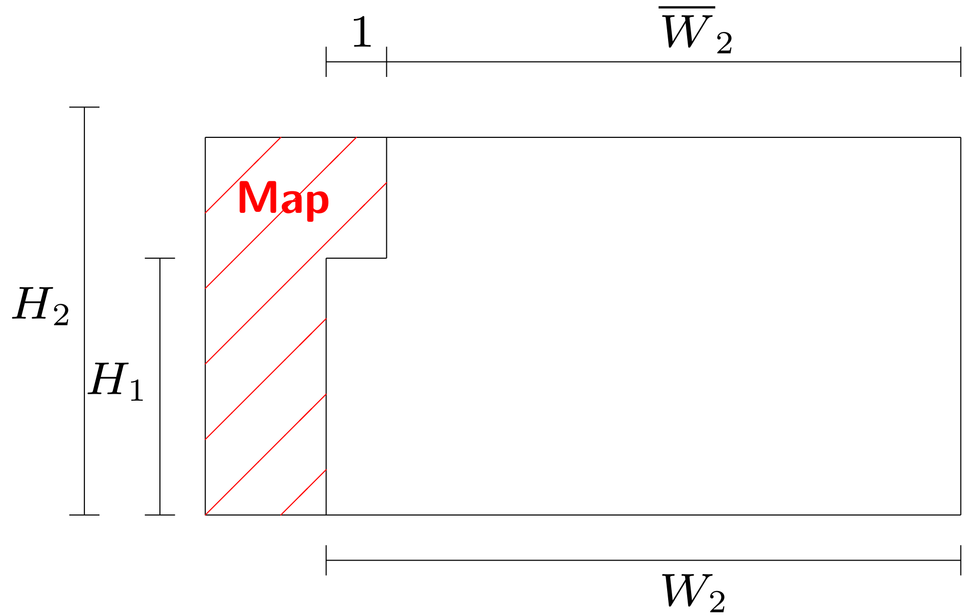


⟵ Bin structure for **Problem P1**

Bin structure for **Problem P2** ⟶



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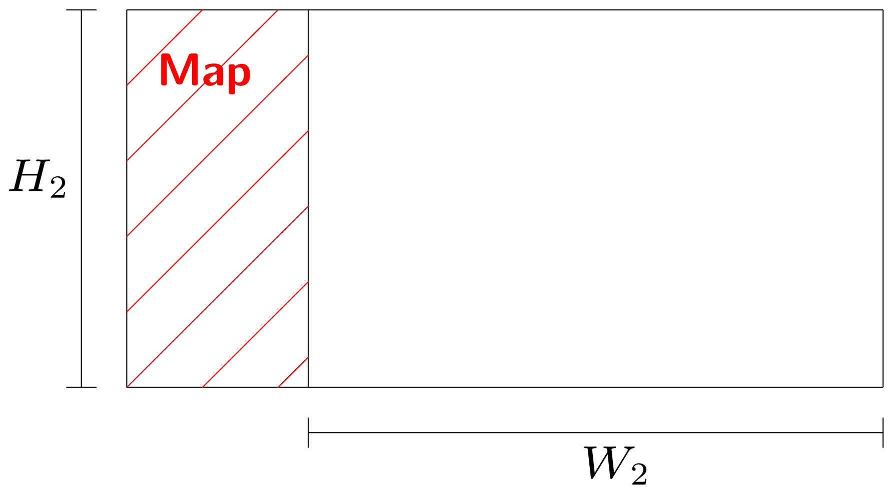

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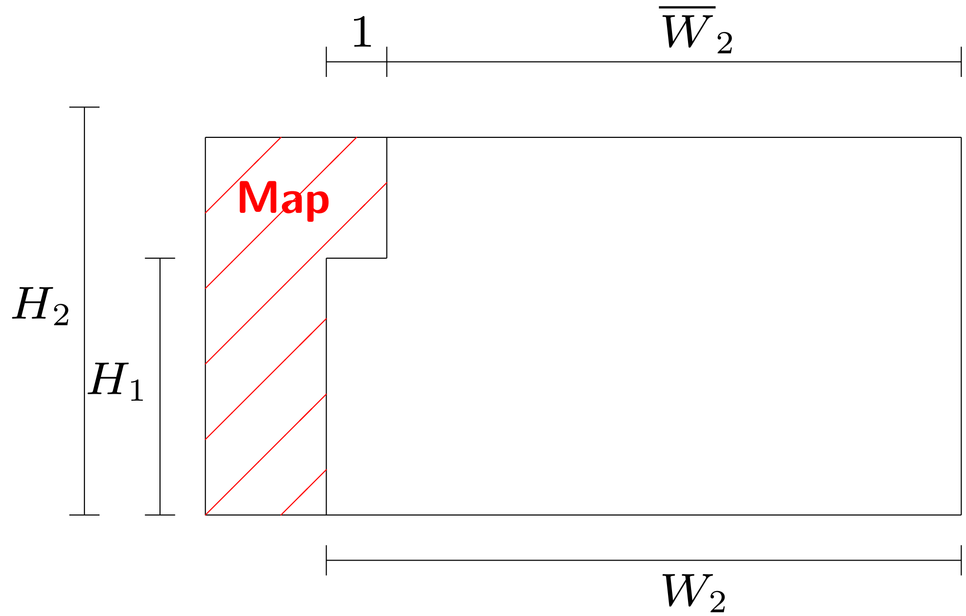

- A third real-world problem (**P3**) will be discussed later.

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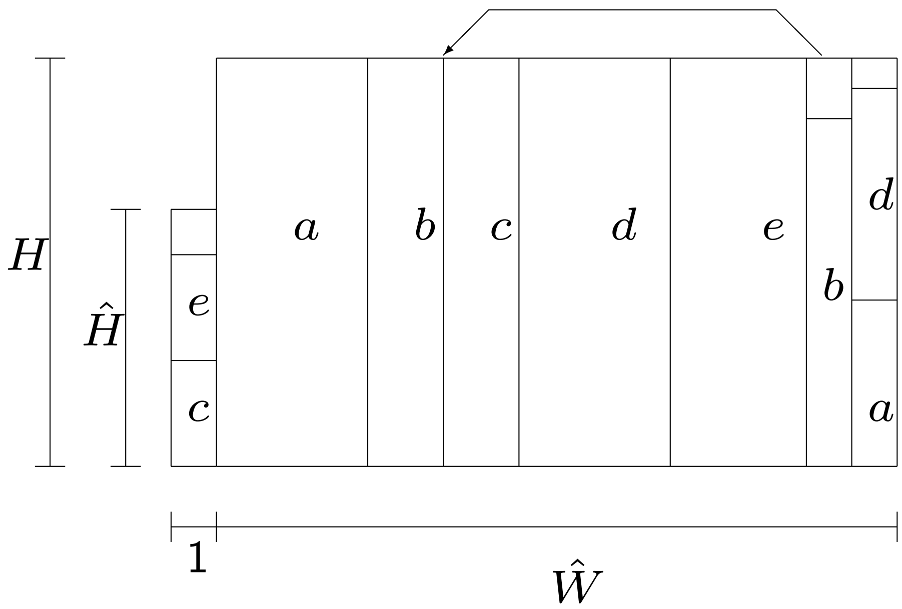

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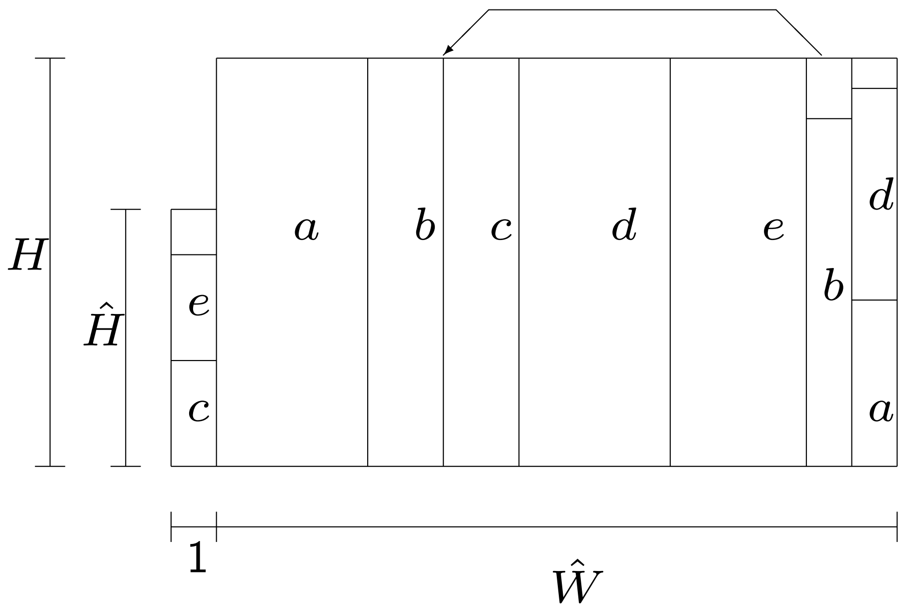

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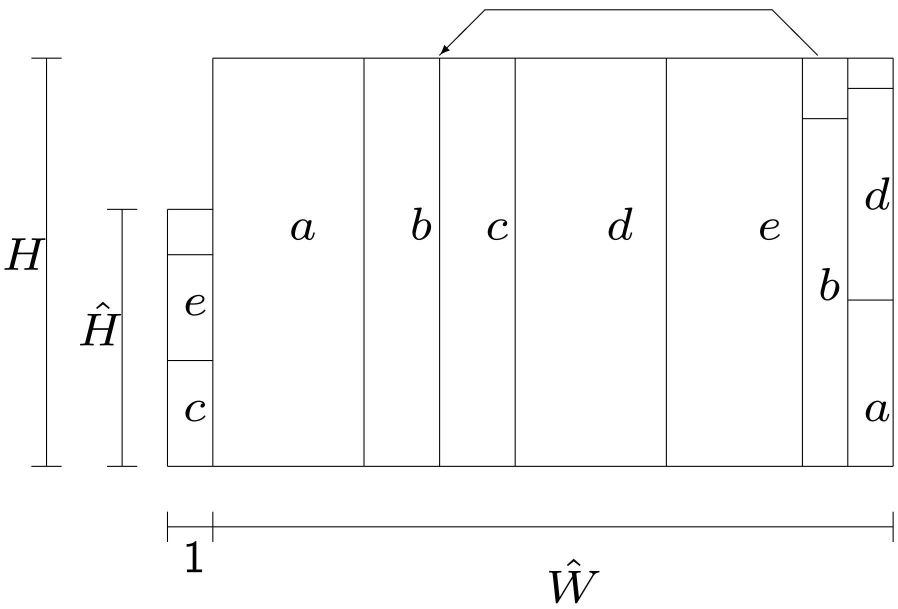

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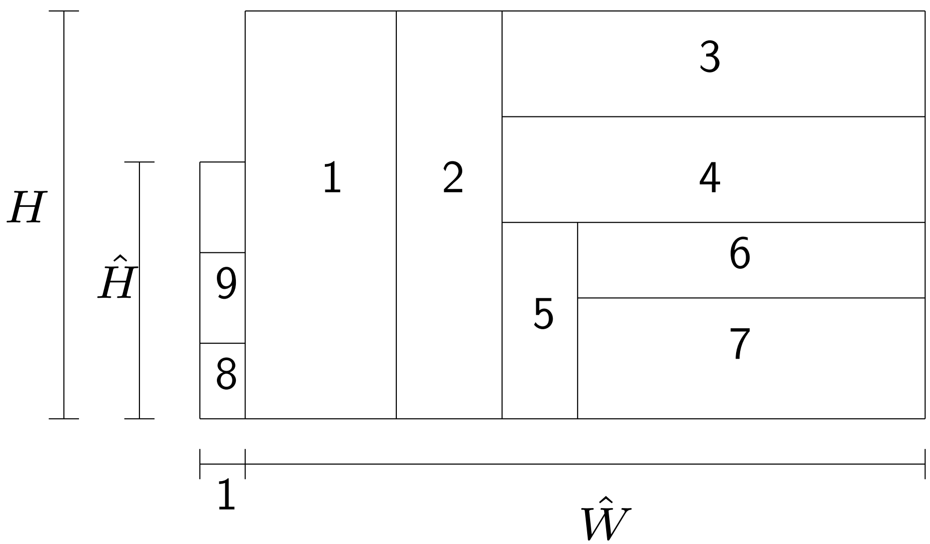

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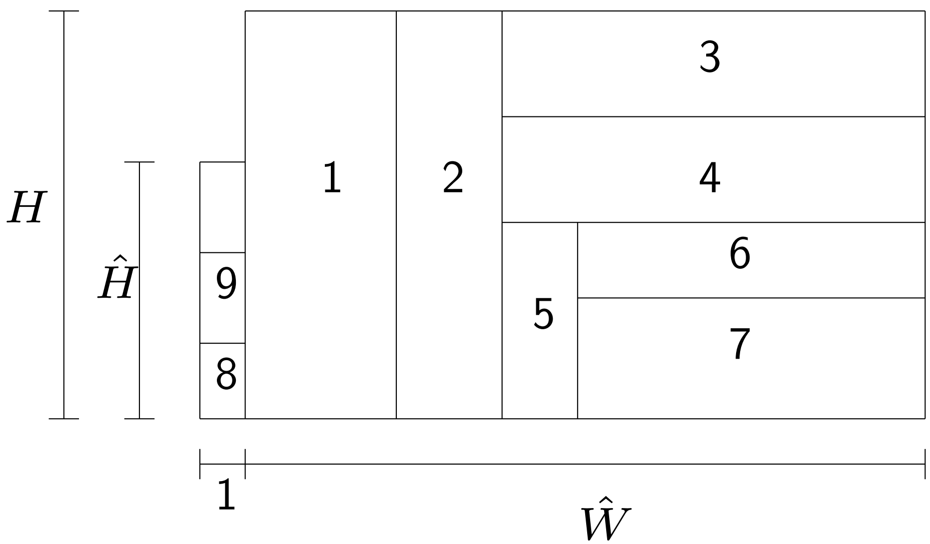

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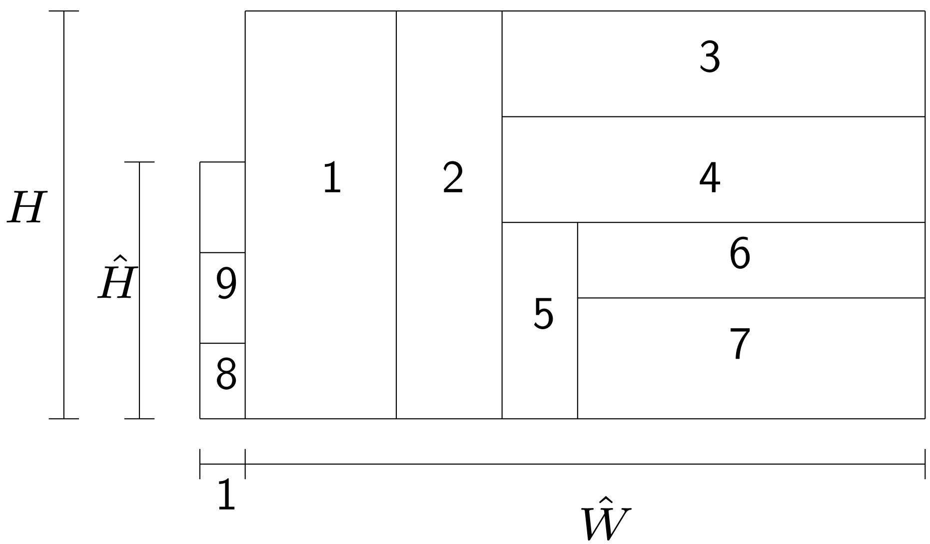

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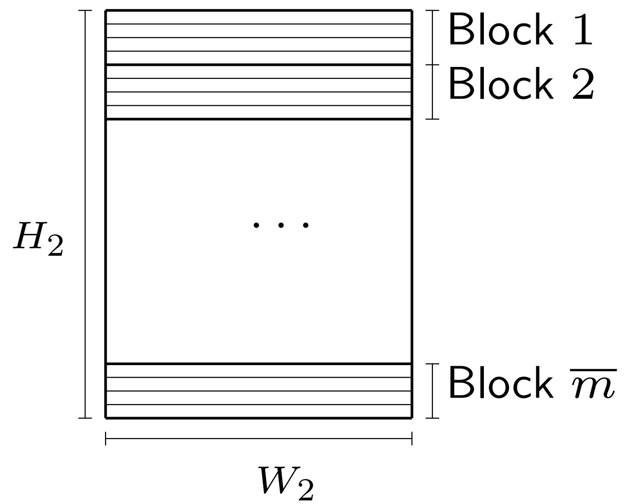

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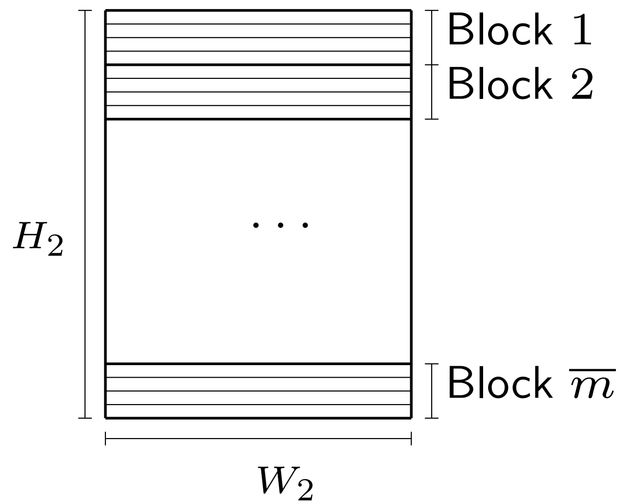

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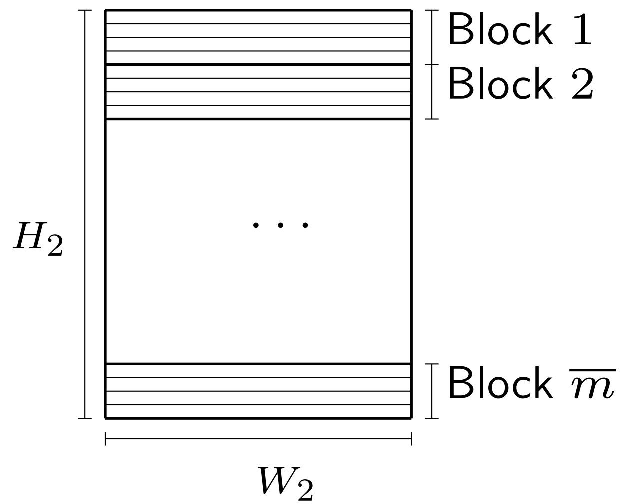

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B2	23,040	[1,15]	23,040	21,840	23,014	0.9977	0.078	0.54
C1	23,210	[1,15]	10,158	8,340	13,719	0.9241	0.085	0.55
C2	23,317	[1,26]	2,512	1,788	4,544	0.8378	0.196	0.96

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C2	23,317	[1,26]	2,512	1,788	4,544	0.8378	0.196	0.96

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- More than 90,000 instances representing different scenarios of transmission
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Thank you for your attention