

1 Derivative

1.1

$$f(x) = \frac{1}{\ln(1+x)} - \frac{1}{x}, \quad x \in [0, 1], \quad \lambda = \max_{x \in [0, 1]} f(x), \quad \mu = \min_{x \in [0, 1]} f(x) \quad (1)$$

1.2

Assume $f(x)$ has continuous second-order derivatives on $[a, b]$. Prove that:

$$\lim_{n \rightarrow \infty} n^2 \left[\int_a^b f(x) dx - \frac{b-a}{n} \sum_{k=1}^n f \left(a + \frac{2k-1}{2n}(b-a) \right) \right] = \frac{(b-a)^2}{24} [f'(b) - f'(a)] \quad (2)$$

2 Limitation

2.1

$$I = \lim_{x \rightarrow 0} \frac{\ln(\cos x)}{x^2} \quad (3)$$

2.2

$$I = \lim_{x \rightarrow 0} \frac{\ln(\cos x + x \sin 2x)}{e^{x^2} - \sqrt[3]{1-x^2}} \quad (4)$$

3 Integral

3.1

$$I = \int \sec x dx \quad (5)$$

3.2

$$I = \int \frac{dx}{x^2 + a^2} \quad (6)$$

3.3

$$I = \int \frac{dx}{\sqrt{x^2 + a^2}} \quad (7)$$

4 Trigonometric and Multiple Integrals

4.1

Let $D = \{(x, y) \mid x^2 + y^2 \leq \pi\}$, compute:

$$I = \iint_D \left[\sin(x^2) \cos(y^2) + x \sqrt{x^2 + y^2} \right] dx dy \quad (8)$$

4.2

Compute the surface integral:

$$I = \iint_S (x^2 - x) dy dz + (y^2 - y) dz dx + (z^2 - z) dx dy \quad (9)$$

where S is the **upper side** of the upper hemisphere $x^2 + y^2 + z^2 = R^2$ ($z \geq 0$).