

1. Compute the distance from $P(3, -1, 2)$ to the line $\begin{cases} x + y - z = -1 \\ 2x - y + z = 4 \end{cases}$.
2. Given $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5, \vec{a} + \vec{b} + \vec{c} = \vec{0}$, compute $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.
3. Compute $L = \lim_{x \rightarrow 0} (|\vec{a} + x\vec{b}| - |\vec{a} - x\vec{b}|)/x$.
4. Compute the distance from point $P(1, 2, -1)$ to the line $L : (x - 1)/2 = (y + 1)/(-1) = (z - 2)/3$.
5. Find and classify the critical points of $f(x, y) = e^{-x}(x - y^3 + 3y)$.
6. Find points on $C : \begin{cases} x^2 + y^2 = 2z^2 \\ x + y + 3z = 5 \end{cases}$ with extremal distances to the xOy plane.
7. Given $A(1, 3, 4), B(3, 5, 6), C(2, 5, 8), D(4, 2, 10)$, compute V_{ABCD} .
8. A plane passes through the z -axis and forms an angle of $\pi/3$ with the plane $2x + y - \sqrt{5}z = 0$. Find the equation of the plane.
9. Prove that $L_1 : x/1 = y/2 = z/3$ and $L_2 : (x - 1)/1 = (y + 1)/1 = (z - 2)/1$ are skew lines, find their common perpendicular, and compute the distance between them.
10. Find the extreme points and extreme values of $z = z(x, y)$ from $x^2 - 6xy + 10y^2 - 2yz - z^2 + 18 = 0$.
11. Find the minimum and maximum values of u given: $\begin{cases} u = x^2 + y^2 + z^2 \\ z = x^2 + y^2 \\ x + y + z = 4 \end{cases}$.
12. Find the maximum value of xyz subject to $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$.
13. For $z = f(x, y) = \begin{cases} 0, & x^2 + y^2 = 0 \\ (x^2 + y^2) \sin(1/\sqrt{x^2 + y^2}), & x^2 + y^2 \neq 0 \end{cases}$
study: Continuity, Partial Derivatives, Differentiability, Continuity of Partial Derivatives.
14. Compute $\iint_D (x^2 - |x| \arctan y + x^3 e^y) dx dy$, where $D = \{(x, y) \mid x^2 + y^2 < 1\}$.
15. A thin plate occupies region D bounded by $y = x^2$ and $y = x$, with density $\mu(x, y) = x^2 y$. Find the centroid.
16. Compute $\iint_D (x^2 + y^2) d\sigma$, where $D = \{(x, y) \mid 0 \leq y \leq \sin x, 0 \leq x \leq \pi\}$.
17. Compute $\iint_D e^{\max\{b^2 x^2, a^2 y^2\}} d\sigma$, where $D = \{(x, y) \mid -a \leq x \leq a, -b \leq y \leq b\}$.
18. Compute $\lim_{r \rightarrow 0} [\iint_D e^{x^2 - y^2} \cos(x + y) d\sigma] / (\pi r^2)$, where $D = \{(x, y) \mid x^2 + y^2 \leq r^2\}$.
19. Compute $I = \int_0^1 \int_0^{1-x} \int_{x+y}^1 (\sin z) / z dz dy dx$.
20. Prove $\int_0^1 dx \int_x^1 dy \int_x^y f(x) f(y) f(z) dz = \frac{1}{6} (\int_0^1 f(t) dt)^3$.
21. Let Σ be the finite part of $z = (x^2 + y^2)/2$ cut by $z = 2$. Compute $\iint_{\Sigma} z dS$.
22. Let Σ be the outer surface of $x^2 + y^2 + z^2 = 9$. Compute $\iint_{\Sigma} z dx dy$.
23. Compute $I = \oint_D xy dx + z^2 dy + xz dz$, where D is the intersection of $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 = 2ax$ ($a > 0$), oriented counterclockwise when viewed from positive z -axis.

24. Let Σ be the lower side of $z = x^2 + y^2$ where $0 \leq z \leq a^2$. Compute:
 $\iint_{\Sigma} (y - x^2 + z^2) dy dz + (x - z^2 + y^2) dz dx + (z - y^2 + x^2) dx dy$.
25. Let Σ be the oriented surface $z = x^2 + y^2$ ($0 \leq z \leq 1$), with normal forming acute angle with positive z-axis. Compute $\iint_{\Sigma} (2x + z) dy dz + z dx dy$.
26. Given $f(0) = 0$, $f'(0) = 1$, and $[xy(x+y) - f(x)y]dx + [f'(x) + x^2y]dy = 0$ is exact. Find its general solution.
27. Given $f(0) = 1/2$ and $\int_L [e^x + f(x)]y dx - f(x)dy$ is path-independent. Compute from $(0, 0)$ to $(1, 1)$.
28. For all smooth oriented closed surfaces Σ in $x > 0$, $\iint_{\Sigma} x f(x) dy dz - xy f(x) dz dx - e^{2x} z dx dy = 0$. Given $\lim_{x \rightarrow 0^+} f(x) = 1$, find $f(x)$.
29. Compute $I = \iint_{\Sigma} x^2 dS$, where Σ is part of cylinder $x^2 + y^2 = a^2$ between $z = 0$ and $z = h$ ($h > 0$).
30. Let Σ be the part of $z = x^2 + y^2$ below $z = 1$. Compute $I = \iint_{\Sigma} |xyz| dS$.
31. Prove that $(yze^{xyz} + 2x)dx + (zxe^{xyz} + 3y^2)dy + (xye^{xyz} + 4z^3)dz$ is exact and find its potential function.
32. Compute $\lim_{t \rightarrow 0} [\iiint_{x^2+y^2+z^2 \leq t^2} f(\sqrt{x^2 + y^2 + z^2}) dv] / (\pi t^4)$.
33. Compute $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sum_{j=1}^n n / [(n+i)(n^2 + j^2)]$.
34. Given $r = \sqrt{x^2 + y^2 + z^2}$, Σ is outer surface of $x^2 + y^2 + z^2 = a^2$. Compute $\iint_{\Sigma} (x dy dz + y dz dx + z dx dy) / r^3$.
35. Compute $\iiint_{\Omega} z dx dy dz$, where Ω is bounded by cone $z = (h/R)\sqrt{x^2 + y^2}$ and plane $z = h$ ($R > 0$, $h > 0$).
36. Compute $S = \sum_{n=0}^{\infty} (-1)^n (n^2 - n + 1) / 2^n$.
37. Expand $f(x) = \cos x$ as a Sine Series on $[0, \pi]$.
38. If $\sum_{n=1}^{\infty} a_n$ converges, provide counterexamples showing the following may not converge:
 - (i) $\sum_{n=1}^{\infty} |a_n|$
 - (ii) $\sum_{n=1}^{\infty} (-1)^n a_n$
 - (iii) $\sum_{n=1}^{\infty} a_n a_{n+1}$
39. Compute $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{3^k} (1 + \frac{1}{k})^{k^2}$.
40. Given $f(x) = 2x^2 / (1 + x^2)$, compute $f^{(6)}(0)$.
41. Find the sum function of $\sum_{n=2}^{\infty} x^n / (n^2 - 1)$.
42. Expand $f(x) = \frac{1}{4} \ln((1+x)/(1-x)) + \frac{1}{2} \arctan x - x$ into a power series.
43. Study the power series $\sum_{n=1}^{\infty} x^{n+1} / [n(n+1)]$.
44. Prove convergence of $\sum_{n=1}^{\infty} \int_{n\pi}^{(n+1)\pi} (\sin x) / x dx$.