期末复习第二单元

数学社

24计量测试与仪器学院智能感知工程1班吴奕铭

2025年12月27日

试卷年份	小题分值	大题分值	总分值
2018	4	6	10
2019	4	6	10
2020	4	6	10

1. 导数定义

定义

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

2. 链式法则

公式

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

3. 参数方程

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

变换

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$$

$$y = u(x)^{v(x)} \quad (u > 0)$$

对数求导法

$$\ln y = v(x) \ln u(x)$$

$$y'/y = v' \ln u + v \cdot u'/u$$

补充：三角函数的求导公式

$$
\begin{aligned}
(\sin x)' &= \cos x, & (\cos x)' &= -\sin x, \\
(\tan x)' &= \sec^2 x, & (\cot x)' &= -\csc^2 x, \\
(\sec x)' &= \sec x \tan x, & (\csc x)' &= -\csc x \cot x.
\end{aligned}
$$

$$
(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, (\arccos x)' = -\frac{1}{\sqrt{1-x^2}},
$$

$$
(\arctan x)' = \frac{1}{1+x^2}, \quad (\operatorname{arccot} x)' = -\frac{1}{1+x^2}.
$$

补充：反函数求导法则

$$y = \arcsin x \Rightarrow x = \sin y$$

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \cos y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\cos y}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1-x^2}} \quad (-1 < x < 1)$$

$$y = f\left(x^3\right), \quad \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} =$$

$$y = f\left(x^3\right), \quad \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} =$$

答案

$$6xf'\left(x^3\right) + 9x^4 f''\left(x^3\right)$$

$$\begin{cases} x = \ln(1+t^2) \\ y = t - \arctan t \end{cases}, \quad \frac{\mathrm{d}y}{\mathrm{d}x} =$$

U2-2023-2-02

$$\begin{cases} x = \ln(1+t^2) \\ y = t - \arctan t \end{cases}, \quad \frac{\mathrm{d}y}{\mathrm{d}x} =$$

答案

$$= \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{(t - \arctan t)'}{(\ln(1+t^2))'} = \frac{1 - \frac{1}{1+t^2}}{\frac{2t}{1+t^2}} = \frac{t}{2}$$

$$\lim_{\Delta x \to 0} \frac{f(x_0 - 2\Delta x) - f(x_0)}{\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{f(x_0-2\Delta x)-f(x_0)}{\Delta x}$$

答案

$$-2f'(x_0)$$

$$e^y = 1 - xy, \quad y = y(x), \quad \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}$$

U2-2024-2-02

$$e^y = 1 - xy, \quad y = y(x), \quad \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}$$

答案

$$y' = -\frac{y}{x + e^y}, \quad y'(0) = 0$$

$$y = \frac{1}{2}\arctan\frac{2x}{1-x^2}, \qquad \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$y = \frac{1}{2}\arctan\frac{2x}{1-x^2}$$

答案

$$= \frac{1}{2}\cdot\frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2}\cdot\frac{2(1-x^2)-2x(-2x)}{(1-x^2)^2} = \frac{1}{1+x^2}$$

$$y = \sqrt[x]{x}, \quad x > 0, \quad y' =$$

$$y = \sqrt[x]{x}, \quad x > 0, \quad y' =$$

答案

$$y' = \sqrt[x]{x}\frac{1 - \ln x}{x^2}$$

谢谢大家

第三单元，夏鑫老师有请