Chapter 18

1. From Eq. 18-6, we see that the limiting value of the pressure ratio is the same as the absolute temperature ratio: (373.15 K)/(273.16 K) = 1.366.

2. We take p_3 to be 80 kPa for both thermometers. According to Fig. 18-6, the nitrogen thermometer gives 373.35 K for the boiling point of water. Use Eq. 18-5 to compute the pressure:

$$p_N = \frac{T}{273.16\,\text{K}}\,p_3 = \left(\frac{373.35\,\text{K}}{273.16\,\text{K}}\right)(80\,\text{kPa}) = 109.343\,\text{kPa}.$$

The hydrogen thermometer gives 373.16 K for the boiling point of water and

$$p_H = \left(\frac{373.16\,\text{K}}{273.16\,\text{K}}\right)(80\,\text{kPa}) = 109.287\,\text{kPa}.$$

(a) The difference is $p_N - p_H = 0.056$ kPa ≈ 0.06 kPa .

(b) The pressure in the nitrogen thermometer is higher than the pressure in the hydrogen thermometer.

3. Let T_L be the temperature and p_L be the pressure in the left-hand thermometer. Similarly, let T_R be the temperature and p_R be the pressure in the right-hand thermometer. According to the problem statement, the pressure is the same in the two thermometers when they are both at the triple point of water. We take this pressure to be p_3. Writing Eq. 18-5 for each thermometer,

$$T_L = (273.16\,\text{K})\left(\frac{p_L}{p_3}\right) \quad \text{and} \quad T_R = (273.16\,\text{K})\left(\frac{p_R}{p_3}\right),$$

we subtract the second equation from the first to obtain

$$T_L - T_R = (273.16\,\text{K})\left(\frac{p_L - p_R}{p_3}\right).$$

First, we take T_L = 373.125 K (the boiling point of water) and T_R = 273.16 K (the triple point of water). Then, $p_L - p_R$ = 120 torr. We solve

$$373.125\,\text{K} - 273.16\,\text{K} = (273.16\,\text{K})\left(\frac{120\,\text{torr}}{p_3}\right)$$

for p_3. The result is $p_3 = 328$ torr. Now, we let $T_L = 273.16$ K (the triple point of water) and T_R be the unknown temperature. The pressure difference is $p_L - p_R = 90.0$ torr. Solving the equation

$$273.16\,\text{K} - T_R = (273.16\,\text{K})\left(\frac{90.0\,\text{torr}}{328\,\text{torr}}\right)$$

for the unknown temperature, we obtain $T_R = 348$ K.

4. (a) Let the reading on the Celsius scale be x and the reading on the Fahrenheit scale be y. Then $y = \frac{9}{5}x + 32$. For $x = -71°\text{C}$, this gives $y = -96°\text{F}$.

(b) The relationship between y and x may be inverted to yield $x = \frac{5}{9}(y - 32)$. Thus, for $y = 134$ we find $x \approx 56.7$ on the Celsius scale.

5. (a) Let the reading on the Celsius scale be x and the reading on the Fahrenheit scale be y. Then $y = \frac{9}{5}x + 32$. If we require $y = 2x$, then we have

$$2x = \frac{9}{5}x + 32 \quad \Rightarrow \quad x = (5)(32) = 160°\text{C}$$

which yields $y = 2x = 320°\text{F}$.

(b) In this case, we require $y = \frac{1}{2}x$ and find

$$\frac{1}{2}x = \frac{9}{5}x + 32 \quad \Rightarrow \quad x = -\frac{(10)(32)}{13} \approx -24.6°\text{C}$$

which yields $y = x/2 = -12.3°\text{F}$.

6. We assume scales X and Y are linearly related in the sense that reading x is related to reading y by a linear relationship $y = mx + b$. We determine the constants m and b by solving the simultaneous equations:

$$-70.00 = m(-125.0) + b$$
$$-30.00 = m(375.0) + b$$

which yield the solutions $m = 40.00/500.0 = 8.000 \times 10^{-2}$ and $b = -60.00$. With these values, we find x for $y = 50.00$:

$$x = \frac{y - b}{m} = \frac{50.00 + 60.00}{0.08000} = 1375°\text{X}.$$

7. We assume scale X is a linear scale in the sense that if its reading is x then it is related to a reading y on the Kelvin scale by a linear relationship $y = mx + b$. We determine the constants m and b by solving the simultaneous equations:

$$373.15 = m(-53.5) + b$$
$$273.15 = m(-170) + b$$

which yield the solutions $m = 100/(170 - 53.5) = 0.858$ and $b = 419$. With these values, we find x for $y = 340$:

$$x = \frac{y - b}{m} = \frac{340 - 419}{0.858} = -92.1°\text{X}.$$

8. The increase in the surface area of the brass cube (which has six faces), which had side length L at $20°$, is

$$\Delta A = 6(L + \Delta L)^2 - 6L^2 \approx 12L\Delta L = 12\alpha_b L^2 \Delta T = 12 \ (19 \times 10^{-6} / \text{C}°) \ (30\,\text{cm})^2 (75°\text{C} - 20°\text{C})$$
$$= 11\,\text{cm}^2.$$

9. The new diameter is

$$D = D_0(1 + \alpha_{Al}\Delta T) = (2.725\,\text{cm})[1 + (23 \times 10^{-6} / \text{C}°)(100.0°\text{C} - 0.000°\text{C})] = 2.731\,\text{cm}.$$

10. The change in length for the aluminum pole is

$$\Delta\ell = \ell_0 \alpha_{Al}\Delta T = (33\,\text{m})(23 \times 10^{-6} / \text{C}°)(15\ °\text{C}) = 0.011\,\text{m}.$$

11. The volume at $30°\text{C}$ is given by

$$V' = V(1 + \beta\Delta T) = V(1 + 3\alpha\Delta T) = (50.00\,\text{cm}^3)[1 + 3(29.00 \times 10^{-6} / \text{C}°) \ (30.00°\text{C} - 60.00°\text{C})]$$
$$= 49.87\,\text{cm}^3$$

where we have used $\beta = 3\alpha$.

12. (a) The coefficient of linear expansion α for the alloy is

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{10.015\,\text{cm} - 10.000\,\text{cm}}{(10.01\,\text{cm})(100°\text{C} - 20.000°\text{C})} = 1.88 \times 10^{-5} / \text{C}°.$$

Thus, from $100°\text{C}$ to $0°\text{C}$ we have

$$\Delta L = L\alpha\Delta T = (10.015\,\text{cm})(1.88 \times 10^{-5} / \text{C}°)(0°\text{C} - 100°\text{C}) = -1.88 \times 10^{-2}\,\text{cm}.$$

The length at $0°\text{C}$ is therefore $L' = L + \Delta L = (10.015\,\text{cm} - 0.0188\,\text{cm}) = 9.996\,\text{cm}.$

(b) Let the temperature be T_x. Then from 20°C to T_x we have

$$\Delta L = 10.009\,\text{cm} - 10.000\,\text{cm} = \alpha L \Delta T = (1.88 \times 10^{-5}/\text{C}°)(10.000\,\text{cm})\Delta T,$$

giving $\Delta T = 48$ °C. Thus, $T_x = (20°\text{C} + 48$ °C $) = 68°\text{C}$.

13. **THINK** The aluminum sphere expands thermally when being heated, so its volume increases.

EXPRESS Since a volume is the product of three lengths, the change in volume due to a temperature change ΔT is given by $\Delta V = 3\alpha V \Delta T$, where V is the original volume and α is the coefficient of linear expansion (see Eq. 18-11).

ANALYZE With the volume of the sphere given by $V = (4\pi/3)R^3$, where $R = 10$ cm is the original radius of the sphere and $\alpha = 23 \times 10^{-6}/\text{C}°$, then

$$\Delta V = 3\alpha \left(\frac{4\pi}{3} R^3 \right) \Delta T = \left(23 \times 10^{-6}/\text{C}°\right)(4\pi)(10\,\text{cm})^3 (100°\text{C}) = 29\,\text{cm}^3.$$

The value for the coefficient of linear expansion is found in Table 18-2.

LEARN The change in volume can be expressed as $\Delta V / V = \beta \Delta T$, where $\beta = 3\alpha$ is the coefficient of volume expansion. For aluminum, we have $\beta = 3\alpha = 69 \times 10^{-6}/\text{C}°$.

14. (a) Since $A = \pi D^2/4$, we have the differential $dA = 2(\pi D/4)dD$. Dividing the latter relation by the former, we obtain $dA/A = 2\,dD/D$. In terms of Δ's, this reads

$$\frac{\Delta A}{A} = 2\frac{\Delta D}{D} \qquad \text{for} \quad \frac{\Delta D}{D} \ll 1.$$

We can think of the factor of 2 as being due to the fact that area is a two-dimensional quantity. Therefore, the area increases by $2(0.18\%) = 0.36\%$.

(b) Assuming that all dimensions are allowed to freely expand, then the thickness increases by 0.18%.

(c) The volume (a three-dimensional quantity) increases by $3(0.18\%) = 0.54\%$.

(d) The mass does not change.

(e) The coefficient of linear expansion is
$$\alpha = \frac{\Delta D}{D \Delta T} = \frac{0.18 \times 10^{-2}}{100°\text{C}} = 1.8 \times 10^{-5}/\text{C}°.$$

15. After the change in temperature the diameter of the steel rod is $D_s = D_{s0} + \alpha_s D_{s0} \, \Delta T$ and the diameter of the brass ring is $D_b = D_{b0} + \alpha_b D_{b0} \, \Delta T$, where D_{s0} and D_{b0} are the original diameters, α_s and α_b are the coefficients of linear expansion, and ΔT is the change in temperature. The rod just fits through the ring if $D_s = D_b$. This means

$$D_{s0} + \alpha_s D_{s0} \, \Delta T = D_{b0} + \alpha_b D_{b0} \, \Delta T.$$

Therefore,

$$\Delta T = \frac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}} = \frac{3.000\,\text{cm} - 2.992\,\text{cm}}{\left(19.00 \times 10^{-6}\,/\text{C}^\circ\right)\left(2.992\,\text{cm}\right) - \left(11.00 \times 10^{-6}\,/\text{C}^\circ\right)\left(3.000\,\text{cm}\right)}$$

$$= 335.0\,^\circ\text{C}.$$

The temperature is $T = (25.00^\circ\text{C} + 335.0\ ^\circ\text{C}) = 360.0^\circ\text{C}$.

16. (a) We use $\rho = m/V$ and

$$\Delta\rho = \Delta(m/V) = m\Delta(1/V) \simeq -m\Delta V/V^2 = -\rho(\Delta V/V) = -3\rho(\Delta L/L).$$

The percent change in density is

$$\frac{\Delta\rho}{\rho} = -3\frac{\Delta L}{L} = -3(0.23\%) = -0.69\%.$$

(b) Since $\alpha = \Delta L/(L\Delta T) = (0.23 \times 10^{-2}) / (100^\circ\text{C} - 0.0^\circ\text{C}) = 23 \times 10^{-6}\,/\text{C}^\circ$, the metal is aluminum (using Table 18-2).

17. **THINK** Since the aluminum cup and the glycerin have different coefficients of thermal expansion, their volumes would change by a different amount under the same ΔT.

EXPRESS If V_c is the original volume of the cup, α_a is the coefficient of linear expansion of aluminum, and ΔT is the temperature increase, then the change in the volume of the cup is $\Delta V_c = 3\alpha_a V_c \Delta T$ (See Eq. 18-11).

On the other hand, if β is the coefficient of volume expansion for glycerin, then the change in the volume of glycerin is $\Delta V_g = \beta V_c \Delta T$. Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\Delta V_g - \Delta V_c = (\beta - 3\alpha_a)V_c\Delta T = \left[(5.1\times10^{-4}\,/\text{C}^\circ) - 3(23\times10^{-6}\,/\text{C}^\circ)\right](100\,\text{cm}^3)(6.0\,^\circ\text{C})$$

$$= 0.26\,\text{cm}^3.$$

LEARN Glycerin spills over because $\beta > 3\alpha$, which gives $\Delta V_g - \Delta V_c > 0$. Note that since liquids in general have greater coefficients of thermal expansion than solids, heating a cup filled with liquid generally will cause the liquid to spill out.

18. The change in length for the section of the steel ruler between its 20.05 cm mark and 20.11 cm mark is

$$\Delta L_s = L_s \alpha_s \Delta T = (20.11\text{cm})(11\times10^{-6}/\text{C}°)(270°\text{C} - 20°\text{C}) = 0.055\,\text{cm}.$$

Thus, the actual change in length for the rod is

$$\Delta L = (20.11 \text{ cm} - 20.05 \text{ cm}) + 0.055 \text{ cm} = 0.115 \text{ cm}.$$

The coefficient of thermal expansion for the material of which the rod is made is then

$$\alpha = \frac{\Delta L}{\Delta T} = \frac{0.115 \text{ cm}}{270°\text{C} - 20°\text{C}} = 23\times10^{-6}/\text{C}°.$$

19. The initial volume V_0 of the liquid is $h_0 A_0$ where A_0 is the initial cross-section area and $h_0 = 0.64$ m. Its final volume is $V = hA$ where $h - h_0$ is what we wish to compute. Now, the area expands according to how the glass expands, which we analyze as follows. Using $A = \pi r^2$, we obtain

$$dA = 2\pi r\, dr = 2\pi r \left(r\alpha\, dT\right) = 2\alpha(\pi r^2)dT = 2\alpha A\, dT .$$

Therefore, the height is

$$h = \frac{V}{A} = \frac{V_0\left(1 + \beta_{\text{liquid}}\Delta T\right)}{A_0\left(1 + 2\alpha_{\text{glass}}\Delta T\right)}.$$

Thus, with $V_0/A_0 = h_0$ we obtain

$$h - h_0 = h_0 \left(\frac{1 + \beta_{\text{liquid}}\Delta T}{1 + 2\alpha_{\text{glass}}\Delta T} - 1\right) = (0.64)\left(\frac{1 + \left(4\times10^{-5}\right)\left(10°\right)}{1 + 2\left(1\times10^{-5}\right)\left(10°\right)}\right) = 1.3\times10^{-4} \text{ m}.$$

20. We divide Eq. 18-9 by the time increment Δt and equate it to the (constant) speed $v = 100 \times 10^{-9}$ m/s.

$$v = \alpha L_0 \frac{\Delta T}{\Delta t}$$

where $L_0 = 0.0200$ m and $\alpha = 23 \times 10^{-6}/\text{C}°$. Thus, we obtain

$$\frac{\Delta T}{\Delta t} = 0.217 \frac{\text{C}°}{\text{s}} = 0.217 \frac{\text{K}}{\text{s}}.$$

21. **THINK** The bar expands thermally when heated. Since its two ends are held fixed, the bar buckles upward.

EXPRESS Consider half the bar. Its original length is $\ell_0 = L_0/2$ and its length after the temperature increase is $\ell = \ell_0 + \alpha \ell_0 \Delta T$. The old position of the half-bar, its new position, and the distance x that one end is displaced form a right triangle, with a hypotenuse of length ℓ, one side of length ℓ_0, and the other side of length x. The Pythagorean theorem yields

$$x^2 = \ell^2 - \ell_0^2 = \ell_0^2(1+\alpha \Delta T)^2 - \ell_0^2.$$

Since the change in length is small we may approximate $(1 + \alpha \Delta T)^2$ by $1 + 2\alpha \Delta T$, where the small term $(\alpha \Delta T)^2$ was neglected. Then,

$$x^2 = \ell_0^2 + 2\ell_0^2 \alpha \Delta T - \ell_0^2 = 2\ell_0^2 \alpha \Delta T$$

and $x \approx \ell_0 \sqrt{2\alpha \Delta T}$.

ANALYZE Substituting the values given, we obtain

$$x = \ell_0 \sqrt{2\alpha \Delta T} = \frac{3.77\,\mathrm{m}}{2}\sqrt{2\left(25\times 10^{-6}/\mathrm{C}°\right)\left(32°\mathrm{C}\right)} = 7.5\times 10^{-2}\,\mathrm{m}.$$

LEARN The length of the bar changes by $\Delta\ell = \alpha \ell_0 \Delta T \sim \alpha \Delta T$. However, to the leading order, the vertical distance the bar has risen is proportional to $(\alpha \Delta T)^{1/2}$.

22. (a) The water (of mass m) releases energy in two steps, first by lowering its temperature from 20°C to 0°C, and then by freezing into ice. Thus the total energy transferred from the water to the surroundings is

$$Q = c_w m\Delta T + L_F m = \left(4190\,\mathrm{J/kg\cdot K}\right)\left(125\,\mathrm{kg}\right)\left(20°\mathrm{C}\right) + \left(333\,\mathrm{kJ/kg}\right)\left(125\,\mathrm{kg}\right) = 5.2\times 10^7\,\mathrm{J}.$$

(b) Before all the water freezes, the lowest temperature possible is 0°C, below which the water must have already turned into ice.

23. **THINK** Electrical energy is supplied and converted into thermal energy to raise the water temperature.

EXPRESS The water has a mass $m = 0.100$ kg and a specific heat $c = 4190$ J/kg·K. When raised from an initial temperature $T_i = 23°$C to its boiling point $T_f = 100°$C, the heat input is given by $Q = cm(T_f - T_i)$. This must be the power output of the heater P multiplied by the time t: $Q = Pt$.

ANALYZE The time it takes to heat up the water is

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P} = \frac{(4190\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(100°\text{C} - 23°\text{C})}{200\,\text{J/s}} = 160\,\text{s}.$$

LEARN With a fixed power output, the time required is proportional to Q, which is proportional to $\Delta T = T_f - T_i$. In real life, it would take longer because of heat loss.

24. (a) The specific heat is given by $c = Q/m(T_f - T_i)$, where Q is the heat added, m is the mass of the sample, T_i is the initial temperature, and T_f is the final temperature. Thus, recalling that a change in Celsius degrees is equal to the corresponding change on the Kelvin scale,

$$c = \frac{314\,\text{J}}{(30.0\times10^{-3}\,\text{kg})(45.0°\text{C} - 25.0°\text{C})} = 523\,\text{J/kg}\cdot\text{K}.$$

(b) The molar specific heat is given by

$$c_m = \frac{Q}{N(T_f - T_i)} = \frac{314\,\text{J}}{(0.600\,\text{mol})(45.0°\text{C} - 25.0°\text{C})} = 26.2\,\text{J/mol}\cdot\text{K}.$$

(c) If N is the number of moles of the substance and M is the mass per mole, then $m = NM$, so

$$N = \frac{m}{M} = \frac{30.0\times10^{-3}\,\text{kg}}{50\times10^{-3}\,\text{kg/mol}} = 0.600\,\text{mol}.$$

25. We use $Q = cm\Delta T$. The textbook notes that a nutritionist's "Calorie" is equivalent to 1000 cal. The mass m of the water that must be consumed is

$$m = \frac{Q}{c\Delta T} = \frac{3500\times10^{3}\,\text{cal}}{(1\,\text{g/cal}\cdot\text{C}°)(37.0°\text{C} - 0.0°\text{C})} = 94.6\times10^{4}\,\text{g},$$

which is equivalent to 9.46 × 10⁴ g/(1000 g/liter) = 94.6 liters of water. This is certainly too much to drink in a single day!

26. The work the man has to do to climb to the top of Mt. Everest is given by

$$W = mgy = (73.0\text{ kg})(9.80\text{ m/s}^2)(8840\text{ m}) = 6.32 \times 10^{6}\text{ J}.$$

Thus, the amount of butter needed is

$$m = \frac{(6.32\times10^{6}\text{ J})\left(\frac{1.00\,\text{cal}}{4.186\,\text{J}}\right)}{6000\,\text{cal/g}} \approx 250\,\text{g} = 0.25\text{ kg}.$$

27. **THINK** Silver is solid at 15.0° C. To melt the sample, we must first raise its temperature to the melting point, and then supply heat of fusion.

EXPRESS The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0° C (= 288 K) to 1235 K. This requires heat

$$Q_1 = cm(T_f - T_i) = (236 \text{J/kg} \cdot \text{K})(0.130 \text{kg})(1235°C - 288°C) = 2.91 \times 10^4 \text{ J}.$$

Now the silver at its melting point must be melted. If L_F is the heat of fusion for silver this requires

$$Q_2 = mL_F = (0.130 \text{kg})(105 \times 10^3 \text{ J/kg}) = 1.36 \times 10^4 \text{ J}.$$

ANALYZE The total heat required is

$$Q = Q_1 + Q_2 = 2.91 \times 10^4 \text{ J} + 1.36 \times 10^4 \text{ J} = 4.27 \times 10^4 \text{ J}.$$

LEARN The heating process is associated with the specific heat of silver, while the melting process involves heat of fusion. Both the specific heat and the heat of fusion are chemical properties of the material itself.

28. The amount of water m that is frozen is

$$m = \frac{Q}{L_F} = \frac{50.2 \text{kJ}}{333 \text{kJ/kg}} = 0.151 \text{kg} = 151 \text{g}.$$

Therefore the amount of water that remains unfrozen is 260 g − 151 g = 109 g.

29. The power consumed by the system is

$$P = \left(\frac{1}{20\%}\right)\frac{cm\Delta T}{t} = \left(\frac{1}{20\%}\right)\frac{(4.18 \text{J/g} \cdot °C)(200 \times 10^3 \text{ cm}^3)(1 \text{g/cm}^3)(40°C - 20°C)}{(1.0 \text{h})(3600 \text{s/h})}$$
$$= 2.3 \times 10^4 \text{ W}.$$

The area needed is then $A = \dfrac{2.3 \times 10^4 \text{ W}}{700 \text{W/m}^2} = 33 \text{m}^2$.

30. While the sample is in its liquid phase, its temperature change (in absolute values) is $|\Delta T| = 30 °C$. Thus, with $m = 0.40$ kg, the absolute value of Eq. 18-14 leads to

$$|Q| = cm|\Delta T| = (3000 \text{ J/kg} \cdot °C)(0.40 \text{ kg})(30 °C) = 36000 \text{ J}.$$

The rate (which is constant) is

$$P = |Q|/t = (36000 \text{ J})/(40 \text{ min}) = 900 \text{ J/min},$$

which is equivalent to 15 W.

(a) During the next 30 minutes, a phase change occurs that is described by Eq. 18-16:

$$|Q| = Pt = (900 \text{ J/min})(30 \text{ min}) = 27000 \text{ J} = Lm.$$

Thus, with $m = 0.40$ kg, we find $L = 67500$ J/kg $\approx$ 68 kJ/kg.

(b) During the final 20 minutes, the sample is solid and undergoes a temperature change (in absolute values) of $|\Delta T| = 20$ C°. Now, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m \, |\Delta T|} = \frac{Pt}{m \, |\Delta T|} = \frac{(900)(20)}{(0.40)(20)} = 2250 \; \tfrac{\text{J}}{\text{kg·C°}} \approx 2.3 \; \tfrac{\text{kJ}}{\text{kg·C°}} .$$

31. Let the mass of the steam be m_s and that of the ice be m_i. Then

$$L_F m_c + c_w m_c (T_f - 0.0°C) = m_s L_s + m_s c_w (100°C - T_f),$$

where $T_f = 50°C$ is the final temperature. We solve for m_s:

$$
\begin{aligned}
m_s &= \frac{L_F m_c + c_w m_c (T_f - 0.0°C)}{L_s + c_w (100°C - T_f)} = \frac{(79.7 \,\text{cal}/\text{g})(150\,\text{g}) + (1\,\text{cal}/\text{g·°C})(150\,\text{g})(50°C - 0.0°C)}{539\,\text{cal}/\text{g} + (1\,\text{cal}/\text{g·C°})(100°C - 50°C)} \\
&= 33\,\text{g}.
\end{aligned}
$$

32. The heat needed is found by integrating the heat capacity:

$$
\begin{aligned}
Q &= \int_{T_i}^{T_f} cm \, dT = m \int_{T_i}^{T_f} c \, dT = (2.09) \int_{5.0°C}^{15.0°C} (0.20 + 0.14T + 0.023T^2) \, dT \\
&= (2.0)(0.20T + 0.070T^2 + 0.00767T^3) \Big|_{5.0}^{15.0} \text{(cal)} \\
&= 82 \, \text{cal}.
\end{aligned}
$$

33. We note from Eq. 18-12 that 1 Btu = 252 cal. The heat relates to the power, and to the temperature change, through
$$Q = Pt = cm\Delta T.$$

Therefore, the time t required is

$$
\begin{aligned}
t &= \frac{cm\Delta T}{P} = \frac{(1000\,\text{cal}/\text{kg·C°})(40\,\text{gal})(1000\,\text{kg}/264\,\text{gal})(100°F - 70°F)(5°C/9°F)}{(2.0 \times 10^5 \,\text{Btu}/\text{h})(252.0\,\text{cal}/\text{Btu})(1\,\text{h}/60\,\text{min})} \\
&= 3.0 \,\text{min}.
\end{aligned}
$$

The metric version proceeds similarly:

$$t = \frac{c\rho V \Delta T}{P} = \frac{(4190 \text{ J/kg·C°})(1000 \text{ kg/m}^3)(150 \text{ L})(1 \text{ m}^3/1000 \text{ L})(38°\text{C} - 21°\text{C})}{(59000 \text{ J/s})(60 \text{ s}/1\text{min})}$$

$$= 3.0 \text{ min.}$$

34. We note that the heat capacity of sample B is given by the reciprocal of the slope of the line in Figure 18-34(b) (compare with Eq. 18-14). Since the reciprocal of that slope is $16/4 = 4$ kJ/kg·C°, then $c_B = 4000$ J/kg·C° $= 4000$ J/kg·K (since a change in Celsius is equivalent to a change in Kelvins). Now, following the same procedure as shown in Sample Problem 18.03 —"Hot slug in water, coming to equilibrium," we find

$$c_A \, m_A \, (T_f - T_A) + c_B \, m_B \, (T_f - T_B) = 0$$

$$c_A \, (5.0 \text{ kg})(40°\text{C} - 100°\text{C}) + (4000 \text{ J/kg·C°})(1.5 \text{ kg})(40°\text{C} - 20°\text{C}) = 0$$

which leads to $c_A = 4.0 \times 10^2$ J/kg·K.

35. We denote the ice with subscript I and the coffee with c, respectively. Let the final temperature be T_f. The heat absorbed by the ice is

$$Q_I = \lambda_F m_I + m_I c_w \, (T_f - 0°\text{C}),$$

and the heat given away by the coffee is $|Q_c| = m_w c_w \, (T_I - T_f)$. Setting $Q_I = |Q_c|$, we solve for T_f:

$$T_f = \frac{m_w c_w T_I - \lambda_F m_I}{(m_I + m_c)c_w} = \frac{(130 \text{ g})(4190 \text{ J/kg·C°}) \, (80.0°\text{C}) - (333 \times 10^3 \text{ J/g})(12.0 \text{ g})}{(12.0 \text{ g} + 130 \text{ g})(4190 \text{ J/kg·C°})}$$

$$= 66.5°\text{C.}$$

Note that we work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. Therefore, the temperature of the coffee will cool by $|\Delta T| = 80.0°\text{C} - 66.5°\text{C} = 13.5\text{C°}$.

36. (a) Using Eq. 18-17, the heat transferred to the water is

$$Q_w = c_w m_w \Delta T + L_V m_s = (1 \text{cal/g·C°})(220 \text{ g})(100°\text{C} - 20.0°\text{C}) + (539 \text{ cal/g})(5.00 \text{ g})$$

$$= 20.3 \text{ kcal.}$$

(b) The heat transferred to the bowl is

$$Q_b = c_b m_b \Delta T = (0.0923 \text{cal/g·C°})(150 \text{ g})(100°\text{C} - 20.0°\text{C}) = 1.11 \text{ kcal.}$$

(c) If the original temperature of the cylinder be T_i, then $Q_w + Q_b = c_c m_c(T_i - T_f)$, which leads to

$$T_i = \frac{Q_w + Q_b}{c_c m_c} + T_f = \frac{20.3\,\text{kcal} + 1.11\,\text{kcal}}{(0.0923\,\text{cal/g}\cdot\text{C}°)(300\,\text{g})} + 100°\text{C} = 873°\text{C}.$$

37. We compute with Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. If the equilibrium temperature is T_f, then the energy absorbed as heat by the ice is

$$Q_I = L_F m_I + c_w m_I (T_f - 0°\text{C}),$$

while the energy transferred as heat from the water is $Q_w = c_w m_w(T_f - T_i)$. The system is insulated, so $Q_w + Q_I = 0$, and we solve for T_f:

$$T_f = \frac{c_w m_w T_i - L_F m_I}{(m_I + m_C)c_w}.$$

(a) Now $T_i = 90°\text{C}$ so

$$T_f = \frac{(4190\,\text{J}/\text{kg}\cdot\text{C}°)(0.500\,\text{kg})(90°\text{C}) - (333\times10^3\,\text{J}/\text{kg})(0.500\,\text{kg})}{(0.500\,\text{kg} + 0.500\,\text{kg})(4190\,\text{J}/\text{kg}\cdot\text{C}°)} = 5.3°\text{C}.$$

(b) Since no ice has remained at $T_f = 5.3°C$, we have $m_f = 0$.

(c) If we were to use the formula above with $T_i = 70°\text{C}$, we would get $T_f < 0$, which is impossible. In fact, not all the ice has melted in this case, and the equilibrium temperature is $T_f = 0°\text{C}$.

(d) The amount of ice that melts is given by

$$m_I' = \frac{c_w m_w (T_i - 0°\text{C})}{L_F} = \frac{(4190\,\text{J}/\text{kg}\cdot\text{C}°)(0.500\,\text{kg})(70\,\text{C}°)}{333\times10^3\,\text{J}/\text{kg}} = 0.440\,\text{kg}.$$

Therefore, the amount of (solid) ice remaining is $m_f = m_I - m_I' = 500$ g $- 440$ g $= 60.0$ g, and (as mentioned) we have $T_f = 0°\text{C}$ (because the system is an ice-water mixture in thermal equilibrium).

38. (a) Equation 18-14 (in absolute value) gives

$$|Q| = (4190 \text{ J/ kg}\cdot°\text{C})(0.530 \text{ kg})(40 \text{ °C}) = 88828 \text{ J}.$$

Since dQ/dt is assumed constant (we will call it P) then we have

$$P = \frac{88828 \text{ J}}{40 \text{ min}} = \frac{88828 \text{ J}}{2400 \text{ s}} = 37 \text{ W} .$$

(b) During that same time (used in part (a)) the ice warms by 20 C°. Using Table 18-3 and Eq. 18-14 again we have

$$m_{\text{ice}} = \frac{Q}{c_{\text{ice}} \Delta T} = \frac{88828}{(2220)(20°)} = 2.0 \text{ kg} .$$

(c) To find the ice produced (by freezing the water that has already reached 0°C, so we concerned with the 40 min $< t <$ 60 min time span), we use Table 18-4 and Eq. 18-16:

$$m_{\text{water becoming ice}} = \frac{Q_{20 \text{ min}}}{L_F} = \frac{44414}{333000} = 0.13 \text{ kg}.$$

39. To accomplish the phase change at 78°C,

$$Q = L_V m = (879 \text{ kJ/kg}) (0.510 \text{ kg}) = 448.29 \text{ kJ}$$

must be removed. To cool the liquid to −114°C,

$$Q = cm|\Delta T| = (2.43 \text{ kJ/ kg} \cdot \text{K}) (0.510 \text{ kg}) (192 \text{ K}) = 237.95 \text{ kJ}$$

must be removed. Finally, to accomplish the phase change at −114°C,

$$Q = L_F m = (109 \text{ kJ/kg}) (0.510 \text{ kg}) = 55.59 \text{ kJ}$$

must be removed. The grand total of heat removed is therefore (448.29 + 237.95 + 55.59) kJ = 742 kJ.

40. Let m_w = 14 kg, m_c = 3.6 kg, m_m = 1.8 kg, T_{i1} = 180°C, T_{i2} = 16.0°C, and T_f = 18.0°C. The specific heat c_m of the metal then satisfies

$$\left(m_w c_w + m_c c_m \right)\left(T_f - T_{i2} \right) + m_m c_m \left(T_f - T_{i1} \right) = 0$$

which we solve for c_m:

$$c_m = \frac{m_w c_w \left(T_{i2} - T_f \right)}{m_c \left(T_f - T_{i2} \right) + m_m \left(T_f - T_{i1} \right)} = \frac{\left(14 \text{kg}\right)\left(4.18 \text{kJ/kg} \cdot \text{K}\right)\left(16.0°\text{C} - 18.0°\text{C}\right)}{\left(3.6 \text{kg}\right)\left(18.0°\text{C} - 16.0°\text{C}\right) + \left(1.8 \text{kg}\right)\left(18.0°\text{C} - 180°\text{C}\right)}$$
$$= 0.41 \text{kJ/kg} \cdot \text{C}° = 0.41 \text{kJ/kg} \cdot \text{K}.$$

41. **THINK** Our system consists of both water and ice cubes. Initially the ice cubes are at −15°C (below freezing temperatures), so they must first absorb heat until 0°C is reached. The final equilibrium temperature reached is related to the amount of ice melted.

EXPRESS There are three possibilities:

• None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.

• The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.

• All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale.

First, suppose that no ice melts. The temperature of the water decreases from $T_{Wi} = 25°C$ to some final temperature T_f and the temperature of the ice increases from $T_{Ii} = -15°C$ to T_f. If m_W is the mass of the water and c_W is its specific heat then the water rejects heat

$$|Q| = c_W m_W (T_{Wi} - T_f).$$

If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{Ii}).$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (T_f - T_{Ii}).$$

The solution for the equilibrium temperature is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{c_W m_W + c_I m_I}$$

$$= \frac{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg})(25°C) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(-15°C)}{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})}$$

$$= 16.6°C.$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at $T_f = 0°C$, with mass m ($< m_I$) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_W m_W T_{Wi},$$

and the heat absorbed by the ice is
$$Q = c_I m_I (0 - T_{Ii}) + mL_F,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + mL_F.$$

This equation can be solved for the mass m of ice melted.

ANALYZE (a) Solving for m and substituting the values given, we find the amount of ice melted to be

$$m = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F}$$
$$= \frac{(4190 \, \text{J}/\text{kg} \cdot \text{K})(0.200 \, \text{kg})(25°C) + (2220 \, \text{J}/\text{kg} \cdot \text{K})(0.100 \, \text{kg})(-15°C)}{333 \times 10^3 \, \text{J}/\text{kg}}$$
$$= 5.3 \times 10^{-2} \, \text{kg} = 53 \, \text{g}.$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C. This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$|Q| = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{Ii}) + c_W m_I (T_f - 0) + m_I L_F.$$

The first term is the energy required to raise the temperature of the ice to 0°C, the second term is the energy required to raise the temperature of the melted ice from 0°C to T_f, and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{Ii}) + c_W m_I T_f + m_I L_F.$$

The solution for T_f is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii} - m_I L_F}{c_W (m_W + m_I)}.$$

Inserting the given values, we obtain $T_f = 2.5°C$.

LEARN In order to melt some ice, the energy released by the water must be sufficient to first raise the temperature of the ice to the melting point ($-c_I m_I T_{Ii}$ required, $T_{Ii} < 0$), with the remaining energy contributing to the heat of fusion. If the remaining energy is greater than $m_I L_F$, then all ice will be melted and the final temperature will be above 0°C.

42. If the ring diameter at 0.000°C is D_{r0}, then its diameter when the ring and sphere are in thermal equilibrium is

$$D_r = D_{r0}(1 + \alpha_c T_f),$$

where T_f is the final temperature and α_c is the coefficient of linear expansion for copper. Similarly, if the sphere diameter at T_i (= 100.0°C) is D_{s0}, then its diameter at the final temperature is

$$D_s = D_{s0}[1 + \alpha_a(T_f - T_i)],$$

where α_a is the coefficient of linear expansion for aluminum. At equilibrium the two diameters are equal, so

$$D_{r0}(1 + \alpha_c T_f) = D_{s0}[1 + \alpha_a(T_f - T_i)].$$

The solution for the final temperature is

$$
\begin{aligned}
T_f &= \frac{D_{r0} - D_{s0} + D_{s0}\alpha_a T_i}{D_{s0}\alpha_a - D_{r0}\alpha_c} \\
&= \frac{2.54000\,\text{cm} - 2.54508\,\text{cm} + (2.54508\,\text{cm})(23 \times 10^{-6}/\text{C}°)(100.0°\text{C})}{(2.54508\,\text{cm})(23 \times 10^{-6}/\text{C}°) - (2.54000\,\text{cm})(17 \times 10^{-6}/\text{C}°)} \\
&= 50.38°\text{C}.
\end{aligned}
$$

The expansion coefficients are from Table 18-2 of the text. Since the initial temperature of the ring is 0°C, the heat it absorbs is $Q = c_c m_r T_f$, where c_c is the specific heat of copper and m_r is the mass of the ring. The heat released by the sphere is

$$|Q| = c_a m_s (T_i - T_f)$$

where c_a is the specific heat of aluminum and m_s is the mass of the sphere. Since these two heats are equal,

$$c_c m_r T_f = c_a m_s (T_i - T_f),$$

we use specific heat capacities from the textbook to obtain

$$m_s = \frac{c_c m_r T_f}{c_a(T_i - T_f)} = \frac{(386\,\text{J/kg} \cdot \text{K})(0.0200\,\text{kg})(50.38°\text{C})}{(900\,\text{J/kg} \cdot \text{K})(100°\text{C} - 50.38°\text{C})} = 8.71 \times 10^{-3}\,\text{kg}.$$

43. (a) One part of path A represents a constant pressure process. The volume changes from 1.0 m³ to 4.0 m³ while the pressure remains at 40 Pa. The work done is

$$W_A = p\Delta V = (40\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 1.2 \times 10^2\,\text{J}.$$

(b) The other part of the path represents a constant volume process. No work is done during this process. The total work done over the entire path is 120 J. To find the work done over path B we need to know the pressure as a function of volume. Then, we can evaluate the integral $W = \int p\,dV$. According to the graph, the pressure is a linear function of the volume, so we may write $p = a + bV$, where a and b are constants. In order for the pressure to be 40 Pa when the volume is 1.0 m³ and 10 Pa when the volume is 4.00 m³ the values of the constants must be $a = 50$ Pa and $b = -10$ Pa/m³. Thus,

$$p = 50\ \text{Pa} - (10\ \text{Pa/m}^3)V$$

and

$$W_B = \int_1^4 p\,dV = \int_1^4 (50 - 10V)\,dV = \left(50V - 5V^2\right)\Big|_1^4 = 200\,\text{J} - 50\,\text{J} - 80\,\text{J} + 5.0\,\text{J} = 75\,\text{J}.$$

(c) One part of path C represents a constant pressure process in which the volume changes from 1.0 m³ to 4.0 m³ while p remains at 10 Pa. The work done is

$$W_C = p\Delta V = (10\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 30\,\text{J}.$$

The other part of the process is at constant volume and no work is done. The total work is 30 J. We note that the work is different for different paths.

44. During process $A \to B$, the system is expanding, doing work on its environment, so $W > 0$, and since $\Delta E_{\text{int}} > 0$ is given then $Q = W + \Delta E_{\text{int}}$ must also be positive.

(a) $Q > 0$.

(b) $W > 0$.

During process $B \to C$, the system is neither expanding nor contracting. Thus,

(c) $W = 0$.

(d) The sign of ΔE_{int} must be the same (by the first law of thermodynamics) as that of Q, which is given as positive. Thus, $\Delta E_{\text{int}} > 0$.

During process $C \to A$, the system is contracting. The environment is doing work on the system, which implies $W < 0$. Also, $\Delta E_{\text{int}} < 0$ because $\sum \Delta E_{\text{int}} = 0$ (for the whole cycle)

and the other values of ΔE_{int} (for the other processes) were positive. Therefore, $Q = W + \Delta E_{int}$ must also be negative.

(e) $Q < 0$.

(f) $W < 0$.

(g) $\Delta E_{int} < 0$.

(h) The area of a triangle is $\frac{1}{2}$ (base)(height). Applying this to the figure, we find

$$|W_{net}| = \tfrac{1}{2}(2.0\,\mathrm{m}^3)(20\,\mathrm{Pa}) = 20\,\mathrm{J}.$$

Since process $C \rightarrow A$ involves larger negative work (it occurs at higher average pressure) than the positive work done during process $A \rightarrow B$, then the net work done during the cycle must be negative. The answer is therefore $W_{net} = -20$ J.

45. **THINK** Over a complete cycle, the internal energy is the same at the beginning and end, so the heat Q absorbed equals the work done: $Q = W$.

EXPRESS Over the portion of the cycle from A to B the pressure p is a linear function of the volume V and we may write $p = a + bV$. The work done over this portion of the cycle is

$$W_{AB} = \int_{V_A}^{V_B} p\,dV = \int_{V_A}^{V_B} (a + bV)\,dV = a(V_B - V_A) + \frac{1}{2}b\left(V_B^2 - V_A^2\right).$$

The *BC* portion of the cycle is at constant pressure and the work done by the gas is

$$W_{BC} = p_B \Delta V_{BC} = p_B (V_C - V_B).$$

The *CA* portion of the cycle is at constant volume, so no work is done. The total work done by the gas is

$$W = W_{AB} + W_{BC} + W_{CA}.$$

ANALYZE The pressure function can be written as

$$p = \frac{10}{3}\ \mathrm{Pa}\ + \left(\frac{20}{3}\ \mathrm{Pa/m}^3\right)V,$$

where the coefficients a and b were chosen so that $p = 10$ Pa when $V = 1.0$ m^3 and $p = 30$ Pa when $V = 4.0$ m^3. Therefore, the work done going from A to B is

$$W_{AB} = a(V_B - V_A) + \frac{1}{2}b\left(V_B^2 - V_A^2\right)$$

$$= \left(\frac{10}{3} \text{Pa}\right)(4.0 \text{ m}^3 - 1.0 \text{ m}^3) + \frac{1}{2}\left(\frac{20}{3} \text{ Pa/m}^3\right)\left[(4.0 \text{ m}^3)^2 - (1.0 \text{ m}^3)^2\right]$$

$$= 10 \text{ J} + 50 \text{ J} = 60 \text{ J}$$

Similarly, with $p_B = p_C = 30 \text{ Pa}$, $V_C = 1.0 \text{ m}^3$ and $V_B = 4.0 \text{ m}^3$, we have

$$W_{BC} = p_B(V_C - V_B) = (30 \text{ Pa})(1.0 \text{ m}^3 - 4.0 \text{ m}^3) = -90 \text{ J}.$$

Adding up all contributions, we find the total work done by the gas to be

$$W = W_{AB} + W_{BC} + W_{CA} = 60 \text{ J} - 90 \text{ J} + 0 = -30 \text{ J}.$$

Thus, the total heat absorbed is $Q = W = -30$ J. This means the gas loses 30 J of energy in the form of heat.

LEARN Notice that in calculating the work done by the gas, we always start with Eq. 18-25: $W = \int p\,dV$. For isobaric process where $p = $ constant, $W = p\Delta V$, and for isochoric process where $V = $ constant, $W = 0$.

46. (a) Since work is done *on* the system (perhaps to compress it) we write $W = -200$ J.

(b) Since heat leaves the system, we have $Q = -70.0 \text{ cal} = -293$ J.

(c) The change in internal energy is $\Delta E_{\text{int}} = Q - W = -293 \text{ J} - (-200 \text{ J}) = -93$ J.

47. **THINK** Since the change in internal energy ΔE_{int} only depends on the initial and final states, it is the same for path *iaf* and path *ibf*.

EXPRESS According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, where Q is the heat absorbed and W is the work done by the system. Along *iaf*, we have

$$\Delta E_{\text{int}} = Q - W = 50 \text{ cal} - 20 \text{ cal} = 30 \text{ cal}.$$

ANALYZE (a) The work done along path *ibf* is given by

$$W = Q - \Delta E_{\text{int}} = 36 \text{ cal} - 30 \text{ cal} = 6.0 \text{ cal}.$$

(b) Since the curved path is traversed from f to i the change in internal energy is $\Delta E_{\text{int}} = -30 \text{ cal}$, and

$$Q = \Delta E_{\text{int}} + W = -30 \text{ cal} - 13 \text{ cal} = -43 \text{ cal}.$$

(c) Let $\Delta E_{int} = E_{int, f} - E_{int, i}$. We then have

$$E_{int, f} = \Delta E_{int} + E_{int, i} = 30 \text{ cal} + 10 \text{ cal} = 40 \text{ cal}.$$

(d) The work W_{bf} for the path bf is zero, so

$$Q_{bf} = E_{int, f} - E_{int, b} = 40 \text{ cal} - 22 \text{ cal} = 18 \text{ cal}.$$

(e) For the path ibf, $Q = 36$ cal so $Q_{ib} = Q - Q_{bf} = 36 \text{ cal} - 18 \text{ cal} = 18 \text{ cal}$.

LEARN Work W and heat Q in general are path-dependent quantities, i.e., they depend on how the finial state is reached. However, the combination $\Delta E_{int} = Q - W$ is path independent; it is a *state function*.

48. Since the process is a complete cycle (beginning and ending in the same thermodynamic state) the change in the internal energy is zero, and the heat absorbed by the gas is equal to the work done by the gas: $Q = W$. In terms of the contributions of the individual parts of the cycle $Q_{AB} + Q_{BC} + Q_{CA} = W$ and

$$Q_{CA} = W - Q_{AB} - Q_{BC} = +15.0 \text{ J} - 20.0 \text{ J} - 0 = -5.0 \text{ J}.$$

This means 5.0 J of energy leaves the gas in the form of heat.

49. We note that there is no work done in the process going from d to a, so $Q_{da} = \Delta E_{int \ da}$ = 80 J. Also, since the total change in internal energy around the cycle is zero, then

$$\Delta E_{int \ ac} + \Delta E_{int \ cd} + \Delta E_{int \ da} = 0$$

$$-200 \text{ J} + \Delta E_{int \ cd} + 80 \text{ J} = 0$$

which yields $\Delta E_{int \ cd} = 120$ J. Thus, applying the first law of thermodynamics to the c to d process gives the work done as

$$W_{cd} = Q_{cd} - \Delta E_{int \ cd} = 180 \text{ J} - 120 \text{ J} = 60 \text{ J}.$$

50. (a) We note that process a to b is an expansion, so $W > 0$ for it. Thus, $W_{ab} = +5.0$ J. We are told that the change in internal energy during that process is +3.0 J, so application of the first law of thermodynamics for that process immediately yields $Q_{ab} = +8.0$ J.

(b) The net work (+1.2 J) is the same as the net heat ($Q_{ab} + Q_{bc} + Q_{ca}$), and we are told that $Q_{ca} = +2.5$ J. Thus we readily find $Q_{bc} = (1.2 - 8.0 - 2.5) \text{ J} = -9.3$ J.

51. We use Eqs. 18-38 through 18-40. Note that the surface area of the sphere is given by $A = 4\pi r^2$, where $r = 0.500$ m is the radius.

(a) The temperature of the sphere is $T = (273.15 + 27.00) \text{ K} = 300.15$ K. Thus

$$P_r = \sigma \varepsilon A T^4 = \left(5.67 \times 10^{-8} \text{ W}/\text{m}^2 \cdot \text{K}^4\right)(0.850)(4\pi)(0.500 \text{m})^2 (300.15 \text{K})^4 = 1.23 \times 10^3 \text{ W}.$$

(b) Now, $T_{\text{env}} = 273.15 + 77.00 = 350.15$ K so

$$P_a = \sigma \varepsilon A T_{\text{env}}^4 = (5.67 \times 10^{-8} \text{ W}/\text{m}^2 \cdot \text{K}^4)(0.850)(4\pi)(0.500 \text{m})^2 (350.15 \text{K})^4 = 2.28 \times 10^3 \text{ W}.$$

(c) From Eq. 18-40, we have

$$P_n = P_a - P_r = 2.28 \times 10^3 \text{ W} - 1.23 \times 10^3 \text{ W} = 1.05 \times 10^3 \text{ W}.$$

52. We refer to the polyurethane foam with subscript p and silver with subscript s. We use Eq. 18-32 to find $L = kR$.

(a) From Table 18-6 we find $k_p = 0.024$ W/m·K, so

$$L_p = k_p R_p$$
$$= \left(0.024 \text{ W/m} \cdot \text{K}\right)\left(30 \text{ft}^2 \cdot \text{F}° \cdot \text{h/Btu}\right)(1 \text{m}/3.281 \text{ft})^2 (5 \text{C}°/9\text{F}°)(3600 \text{s/h})(1 \text{Btu}/1055 \text{J})$$
$$= 0.13 \text{m}.$$

(b) For silver $k_s = 428$ W/m·K, so

$$L_s = k_s R_s = \left(\frac{k_s R_s}{k_p R_p}\right) L_p = \left[\frac{428(30)}{0.024(30)}\right](0.13 \text{m}) = 2.3 \times 10^3 \text{m}.$$

53. **THINK** Energy is transferred as heat from the hot reservoir at temperature T_H to the cold reservoir at temperature T_C. The conduction rate is the amount of energy transferred per unit time.

EXPRESS The rate of heat flow is given by

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L},$$

where k is the thermal conductivity of copper (401 W/m·K), A is the cross-sectional area (in a plane perpendicular to the flow), L is the distance along the direction of flow between the points where the temperature is T_H and T_C. The thermal conductivity is found in Table 18-6 of the text. Recall that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale.

ANALYZE Substituting the values given, we find the rate to be

$$P_{cond} = \frac{(401\,\text{W/m} \cdot \text{K})(90.0 \times 10^{-4}\,\text{m}^2)(125°\text{C} - 10.0°\text{C})}{0.250\,\text{m}} = 1.66 \times 10^3\,\text{J/s}.$$

LEARN The thermal resistance (R-value) of the copper slab is

$$R = \frac{L}{k} = \frac{0.250\,\text{m}}{401\,\text{W/m} \cdot \text{K}} = 6.23 \times 10^{-4}\,\text{m}^2 \cdot \text{K/W}.$$

The low value of R is an indication that the copper slab is a good conductor.

54. (a) We estimate the surface area of the average human body to be about 2 m^2 and the skin temperature to be about 300 K (somewhat less than the internal temperature of 310 K). Then from Eq. 18-37

$$P_r = \sigma \varepsilon A T^4 \approx (5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4)(0.9)(2.0\,\text{m}^2)(300\,\text{K})^4 = 8 \times 10^2\,\text{W}.$$

(b) The energy lost is given by $\Delta E = P_r \Delta t = (8 \times 10^2\,\text{W})(30\,\text{s}) = 2 \times 10^4\,\text{J}.$

55. (a) Recalling that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale, we find that the rate of heat conduction is

$$P_{cond} = \frac{kA(T_H - T_C)}{L} = \frac{(401\,\text{W/m} \cdot \text{K})(4.8 \times 10^{-4}\,\text{m}^2)(100°\text{C})}{1.2\,\text{m}} = 16\,\text{J/s}.$$

(b) Using Table 18-4, the rate at which ice melts is

$$\left| \frac{dm}{dt} \right| = \frac{P_{cond}}{L_F} = \frac{16\,\text{J/s}}{333\,\text{J/g}} = 0.048\,\text{g/s}.$$

56. The surface area of the ball is $A = 4\pi R^2 = 4\pi(0.020\,\text{m})^2 = 5.03 \times 10^{-3}\,\text{m}^2$. Using Eq. 18-37 with $T_i = 35 + 273 = 308\,\text{K}$ and $T_f = 47 + 273 = 320\,\text{K}$, the power required to maintain the temperature is

$$P_r = \sigma \varepsilon A(T_f^4 - T_i^4) \approx (5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4)(0.80)(5.03 \times 10^{-3}\,\text{m}^2)\left[(320\,\text{K})^4 - (308\,\text{K})^4\right]$$
$$= 0.34\,\text{W}.$$

Thus, the heat each bee must produce during the 20-minute interval is

$$\frac{Q}{N} = \frac{P_r t}{N} = \frac{(0.34\,\text{W})(20\,\text{min})(60\,\text{s/min})}{500} = 0.81\,\text{J}.$$

57. (a) We use

$$P_{cond} = kA\frac{T_H - T_C}{L}$$

with the conductivity of glass given in Table 18-6 as 1.0 W/m·K. We choose to use the Celsius scale for the temperature: a temperature difference of

$$T_H - T_C = 72°F - (-20°F) = 92 \ °F$$

is equivalent to $\frac{5}{9}(92) = 51.1C°$. This, in turn, is equal to 51.1 K since a change in Kelvin temperature is entirely equivalent to a Celsius change. Thus,

$$\frac{P_{cond}}{A} = k\frac{T_H - T_C}{L} = (1.0\,W/m\cdot K)\left(\frac{51.1°C}{3.0\times10^{-3}m}\right) = 1.7\times10^4 \ W/m^2.$$

(b) The energy now passes in succession through 3 layers, one of air and two of glass. The heat transfer rate P is the same in each layer and is given by

$$P_{cond} = \frac{A(T_H - T_C)}{\sum L/k}$$

where the sum in the denominator is over the layers. If L_g is the thickness of a glass layer, L_a is the thickness of the air layer, k_g is the thermal conductivity of glass, and k_a is the thermal conductivity of air, then the denominator is

$$\sum\frac{L}{k} = \frac{2L_g}{k_g} + \frac{L_a}{k_a} = \frac{2L_g k_a + L_a k_g}{k_a k_g}.$$

Therefore, the heat conducted per unit area occurs at the following rate:

$$\frac{P_{cond}}{A} = \frac{(T_H - T_C)k_a k_g}{2L_g k_a + L_a k_g} = \frac{(51.1°C)(0.026\,W/m\cdot K)(1.0\,W/m\cdot K)}{2(3.0\times10^{-3}m)(0.026\,W/m\cdot K) + (0.075m)(1.0\,W/m\cdot K)}$$
$$= 18\,W/m^2.$$

58. (a) The surface area of the cylinder is given by

$$A_1 = 2\pi r_1^2 + 2\pi r_1 h_1 = 2\pi(2.5\times10^{-2}m)^2 + 2\pi(2.5\times10^{-2}m)(5.0\times10^{-2}m) = 1.18\times10^{-2}m^2,$$

its temperature is $T_1 = 273 + 30 = 303$ K, and the temperature of the environment is $T_{env} = 273 + 50 = 323$ K. From Eq. 18-39 we have

$$P_1 = \sigma \varepsilon A_1 \left(T_{\text{env}}^4 - T^4 \right) = (0.85)(1.18 \times 10^{-2} \text{ m}^2)\left((323 \text{ K})^4 - (303 \text{ K})^4 \right) = 1.4 \text{ W}.$$

(b) Let the new height of the cylinder be h_2. Since the volume V of the cylinder is fixed, we must have $V = \pi r_1^2 h_1 = \pi r_2^2 h_2$. We solve for h_2:

$$h_2 = \left(\frac{r_1}{r_2} \right)^2 h_1 = \left(\frac{2.5 \text{cm}}{0.50 \text{cm}} \right)^2 (5.0 \text{cm}) = 125 \text{cm} = 1.25 \text{m}.$$

The corresponding new surface area A_2 of the cylinder is

$$A_2 = 2\pi r_2^2 + 2\pi r_2 h_2 = 2\pi (0.50 \times 10^{-2} \text{ m})^2 + 2\pi (0.50 \times 10^{-2} \text{ m})(1.25 \text{ m}) = 3.94 \times 10^{-2} \text{ m}^2.$$

Consequently,

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} = \frac{3.94 \times 10^{-2} \text{ m}^2}{1.18 \times 10^{-2} \text{ m}^2} = 3.3.$$

59. We use $P_{\text{cond}} = kA\Delta T/L \propto A/L$. Comparing cases (a) and (b) in Fig. 18-45, we have

$$P_{\text{cond } b} = \left(\frac{A_b L_a}{A_a L_b} \right) P_{\text{cond } a} = 4 P_{\text{cond } a}.$$

Consequently, it would take 2.0 min/4 = 0.50 min for the same amount of heat to be conducted through the rods welded as shown in Fig. 18-45(b).

60. (a) As in Sample Problem 18.06 — "Thermal conduction through a layered wall," we take the rate of conductive heat transfer through each layer to be the same. Thus, the rate of heat transfer across the entire wall P_w is equal to the rate across layer 2 (P_2). Using Eq. 18-37 and canceling out the common factor of area A, we obtain

$$\frac{T_H - T_c}{(L_1/k_1 + L_2/k_2 + L_3/k_3)} = \frac{\Delta T_2}{(L_2/k_2)} \quad \Rightarrow \quad \frac{45 \text{ C}°}{(1 + 7/9 + 35/80)} = \frac{\Delta T_2}{(7/9)}$$

which leads to $\Delta T_2 = 15.8\ °C$.

(b) We expect (and this is supported by the result in the next part) that greater conductivity should mean a larger rate of conductive heat transfer.

(c) Repeating the calculation above with the new value for k_2, we have

$$\frac{45 \text{ C}°}{(1 + 7/11 + 35/80)} = \frac{\Delta T_2}{(7/11)}$$

which leads to $\Delta T_2 = 13.8\ °C$. This is less than our part (a) result, which implies that the temperature gradients across layers 1 and 3 (the ones where the parameters did not change) are greater than in part (a); those larger temperature gradients lead to larger conductive heat currents (which is basically a statement of "Ohm's law as applied to heat conduction").

61. **THINK** As heat continues to leave the water via conduction, more ice is formed and the ice slab gets thicker.

EXPRESS Let h be the thickness of the ice slab and A be its area. Then, the rate of heat flow through the slab is

$$P_{cond} = \frac{kA(T_H - T_C)}{h},$$

where k is the thermal conductivity of ice, T_H is the temperature of the water ($0°C$), and T_C is the temperature of the air above the ice ($-10°C$). The heat leaving the water freezes it, the heat required to freeze mass m of water being $Q = L_F m$, where L_F is the heat of fusion for water. Differentiate with respect to time and recognize that $dQ/dt = P_{cond}$ to obtain

$$P_{cond} = L_F \frac{dm}{dt}.$$

Now, the mass of the ice is given by $m = \rho A h$, where ρ is the density of ice and h is the thickness of the ice slab, so $dm/dt = \rho A(dh/dt)$ and

$$P_{cond} = L_F \rho A \frac{dh}{dt}.$$

We equate the two expressions for P_{cond} and solve for dh/dt:

$$\frac{dh}{dt} = \frac{k(T_H - T_C)}{L_F \rho h}.$$

ANALYZE Since 1 cal = 4.186 J and 1 cm = 1×10^{-2} m, the thermal conductivity of ice has the SI value

$$k = (0.0040\ \text{cal/s·cm·K})\ (4.186\ \text{J/cal})/(1 \times 10^{-2}\ \text{m/cm}) = 1.674\ \text{W/m·K}.$$

The density of ice is $\rho = 0.92\ \text{g/cm}^3 = 0.92 \times 10^3\ \text{kg/m}^3$. Thus, we obtain

$$\frac{dh}{dt} = \frac{(1.674\ \text{W/m·K})(0°C\ +\ 10°C)}{(333 \times 10^3\ \text{J/kg})(0.92 \times 10^3\ \text{kg/m}^3)(0.050\text{m})} = 1.1 \times 10^{-6}\ \text{m/s} = 0.40\ \text{cm/h}.$$

LEARN The rate of ice formation is proportional to the conduction rate – the faster the energy leaves the water, the faster the water freezes.

62. (a) Using Eq. 18-32, the rate of energy flow through the surface is

$$P_{cond} = \frac{kA(T_s - T_w)}{L} = (0.026 \text{ W/m} \cdot \text{K})(4.00 \times 10^{-6} \text{ m}^2)\frac{300°\text{C} - 100°\text{C}}{1.0 \times 10^{-4} \text{ m}} = 0.208 \text{W} \approx 0.21 \text{ W.}$$

(Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.)

(b) With $P_{cond}t = L_V m = L_V(\rho V) = L_V(\rho Ah)$, the drop will last a duration of

$$t = \frac{L_V \rho Ah}{P_{cond}} = \frac{(2.256 \times 10^6 \text{ J/kg})(1000 \text{ kg/m}^3)(4.00 \times 10^{-6} \text{ m}^2)(1.50 \times 10^{-3} \text{ m})}{0.208 \text{W}} = 65 \text{ s.}$$

63. We divide both sides of Eq. 18-32 by area A, which gives us the (uniform) rate of heat conduction per unit area:

$$\frac{P_{cond}}{A} = k_1\frac{T_H - T_1}{L_1} = k_4\frac{T - T_C}{L_4}$$

where $T_H = 30°\text{C}$, $T_1 = 25°\text{C}$ and $T_C = -10°\text{C}$. We solve for the unknown T.

$$T = T_C + \frac{k_1 L_4}{k_4 L_1}(T_H - T_1) = -4.2°\text{C.}$$

64. (a) For each individual penguin, the surface area that radiates is the sum of the top surface area and the sides:

$$A_r = a + 2\pi rh = a + 2\pi\sqrt{\frac{a}{\pi}}h = a + 2h\sqrt{\pi a},$$

where we have used $r = \sqrt{a/\pi}$ (from $a = \pi r^2$) for the radius of the cylinder. For the huddled cylinder, the radius is $r' = \sqrt{Na/\pi}$ (since $Na = \pi r'^2$), and the total surface area is

$$A_h = Na + 2\pi r'h = Na + 2\pi\sqrt{\frac{Na}{\pi}}h = Na + 2h\sqrt{N\pi a}.$$

Since the power radiated is proportional to the surface area, we have

$$\frac{P_h}{NP_r} = \frac{A_h}{NA_r} = \frac{Na + 2h\sqrt{N\pi a}}{N(a + 2h\sqrt{\pi a})} = \frac{1 + 2h\sqrt{\pi/Na}}{1 + 2h\sqrt{\pi/a}}.$$

With $N = 1000$, $a = 0.34$ m^2, and $h = 1.1$ m, the ratio is

$$\frac{P_h}{NP_r} = \frac{1 + 2h\sqrt{\pi/Na}}{1 + 2h\sqrt{\pi/a}} = \frac{1 + 2(1.1 \text{ m})\sqrt{\pi/(1000 \cdot 0.34 \text{ m}^2)}}{1 + 2(1.1 \text{ m})\sqrt{\pi/(0.34 \text{ m}^2)}} = 0.16.$$

(b) The total radiation loss is reduced by $1.00 - 0.16 = 0.84$, or 84%.

65. We assume (although this should be viewed as a "controversial" assumption) that the top surface of the ice is at $T_C = -5.0°C$. Less controversial are the assumptions that the bottom of the body of water is at $T_H = 4.0°C$ and the interface between the ice and the water is at $T_X = 0.0°C$. The primary mechanism for the heat transfer through the total distance $L = 1.4$ m is assumed to be conduction, and we use Eq. 18-34:

$$\frac{k_{\text{water}} A (T_H - T_X)}{L - L_{\text{ice}}} = \frac{k_{\text{ice}} A (T_X - T_C)}{L_{\text{ice}}} \implies \frac{(0.12) A (4.0° - 0.0°)}{1.4 - L_{\text{ice}}} = \frac{(0.40) A (0.0° + 5.0°)}{L_{\text{ice}}}.$$

We cancel the area A and solve for thickness of the ice layer: $L_{\text{ice}} = 1.1$ m.

66. The condition that the energy lost by the beverage can be due to evaporation equals the energy gained via radiation exchange implies

$$L_V \frac{dm}{dt} = P_{\text{rad}} = \sigma \varepsilon A (T_{\text{env}}^4 - T^4).$$

The total area of the top and side surfaces of the can is

$$A = \pi r^2 + 2\pi rh = \pi (0.022 \text{ m})^2 + 2\pi (0.022 \text{ m})(0.10 \text{ m}) = 1.53 \times 10^{-2} \text{ m}^2.$$

With $T_{\text{env}} = 32°C = 305$ K, $T = 15°C = 288$ K, and $\varepsilon = 1$, the rate of water mass loss is

$$\frac{dm}{dt} = \frac{\sigma \varepsilon A}{L_V} (T_{\text{env}}^4 - T^4) = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.0)(1.53 \times 10^{-2} \text{ m}^2)}{2.256 \times 10^6 \text{ J/kg}} \left[(305 \text{ K})^4 - (288 \text{ K})^4 \right]$$

$$= 6.82 \times 10^{-7} \text{ kg/s} \approx 0.68 \text{ mg/s}.$$

67. We denote the total mass M and the melted mass m. The problem tells us that work/M = p/ρ, and that all the work is assumed to contribute to the phase change $Q = Lm$ where L = 150×10^3 J/kg. Thus,

$$\frac{p}{\rho} M = Lm \implies m = \frac{5.5 \times 10^6}{1200} \frac{M}{150 \times 10^3}$$

which yields $m = 0.0306M$. Dividing this by 0.30 M (the mass of the fats, which we are told is equal to 30% of the total mass), leads to a percentage $0.0306/0.30 = 10\%$.

68. The heat needed is

$$Q = (10\%)mL_F = \left(\frac{1}{10}\right)(200,000 \text{ metric tons}) \ (1000 \text{ kg} / \text{metric ton}) \ (333 \text{ kJ/kg}) = 6.7 \times 10^{12} \text{ J}.$$

69. (a) Regarding part (a), it is important to recognize that the problem is asking for the total work done during the two-step "path": $a \rightarrow b$ followed by $b \rightarrow c$. During the latter part of this "path" there is no volume change and consequently no work done. Thus, the answer to part (b) is also the answer to part (a). Since ΔU for process $c \rightarrow a$ is -160 J, then $U_c - U_a = 160$ J. Therefore, using the First Law of Thermodynamics, we have

$$160 = U_c - U_b + U_b - U_a$$
$$= Q_{b \rightarrow c} - W_{b \rightarrow c} + Q_{a \rightarrow b} - W_{a \rightarrow b}$$
$$= 40 - 0 + 200 - W_{a \rightarrow b}.$$

Therefore, $W_{a \rightarrow b \rightarrow c} = W_{a \rightarrow b} = 80$ J.

(b) $W_{a \rightarrow b} = 80$ J.

70. We use $Q = cm\Delta T$ and $m = \rho V$. The volume of water needed is

$$V = \frac{m}{\rho} = \frac{Q}{\rho C \Delta T} = \frac{\left(1.00 \times 10^6 \text{ kcal/day}\right)(5 \text{ days})}{\left(1.00 \times 10^3 \text{ kg/m}^3\right)(1.00 \text{ kcal/kg})\left(50.0°C - 22.0°C\right)} = 35.7 \text{ m}^3.$$

71. The graph shows that the absolute value of the temperature change is $|\Delta T| = 25 °C$. Since a watt is a joule per second, we reason that the energy removed is

$$|Q| = (2.81 \text{ J/s})(20 \text{ min})(60 \text{ s/min}) = 3372 \text{ J}.$$

Thus, with $m = 0.30$ kg, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m \ |\Delta T|} = 4.5 \times 10^2 \text{ J/kg} \cdot \text{K}.$$

72. We use $P_{\text{cond}} = kA(T_H - T_C)/L$. The temperature T_H at a depth of 35.0 km is

$$T_H = \frac{P_{\text{cond}}L}{kA} + T_C = \frac{\left(54.0 \times 10^{-3} \text{ W/m}^2\right)\left(35.0 \times 10^3 \text{ m}\right)}{2.50 \text{ W/m} \cdot \text{K}} + 10.0°C = 766°C.$$

73. Its initial volume is $5^3 = 125$ cm^3, and using Table 18-2, Eq. 18-10, and Eq. 18-11, we find

$$\Delta V = (125\,\text{m}^3)\,(3 \times 23 \times 10^{-6}/\text{C}°)\,(50.0\,\text{C}°) = 0.432\,\text{cm}^3.$$

74. As is shown Sample Problem 18.03 — "Hot slug in water, coming to equilibrium," we can express the final temperature in the following way:

$$T_f = \frac{m_A c_A T_A + m_B c_B T_B}{m_A c_A + m_B c_B} = \frac{c_A T_A + c_B T_B}{c_A + c_B}$$

where the last equality is made possible by the fact that $m_A = m_B$. Thus, in a graph of T_f versus T_A, the "slope" must be $c_A/(c_A + c_B)$, and the "y intercept" is $c_B/(c_A + c_B)T_B$. From the observation that the "slope" is equal to 2/5 we can determine, then, not only the ratio of the heat capacities but also the coefficient of T_B in the "y intercept"; that is,

$$c_B/(c_A + c_B)T_B = (1 - \text{"slope"})T_B.$$

(a) We observe that the "y intercept" is 150 K, so

$$T_B = 150/(1 - \text{"slope"}) = 150/(3/5)$$

which yields $T_B = 2.5 \times 10^2$ K.

(b) As noted already, $c_A/(c_A + c_B) = \frac{2}{5}$, so $5\,c_A = 2c_A + 2c_B$, which leads to $c_B/c_A = \frac{3}{2} = 1.5$.

75. We note that there is no work done in process $c \to b$, since there is no change of volume. We also note that the *magnitude* of work done in process $b \to c$ is given, but not its sign (which we identify as negative as a result of the discussion in Section 18-8). The total (or *net*) heat transfer is $Q_{\text{net}} = [(-40) + (-130) + (+400)]$ J $= 230$ J. By the First Law of Thermodynamics (or, equivalently, conservation of energy), we have $Q_{\text{net}} = W_{\text{net}}$, or

$$230\,\text{J} = W_{a \to c} + W_{c \to b} + W_{b \to a} = W_{a \to c} + 0 + (-80\,\text{J}).$$

Therefore, $W_{a \to c} = 3.1 \times 10^2$ J.

76. From the law of cosines, with $\phi = 59.95°$, we have

$$L_{\text{Invar}}^2 = L_{\text{alum}}^2 + L_{\text{steel}}^2 - 2L_{\text{alum}}L_{\text{steel}} \cos \phi$$

Plugging in $L = L_0\,(1 + \alpha \Delta T)$, dividing by L_0 (which is the same for all sides) and ignoring terms of order $(\Delta T)^2$ or higher, we obtain

$$1 + 2\alpha_{\text{Invar}}\Delta T = 2 + 2\,(\alpha_{\text{alum}} + \alpha_{\text{steel}})\,\Delta T - 2\,(1 + (\alpha_{\text{alum}} + \alpha_{\text{steel}})\,\Delta T)\cos\phi \;.$$

This is rearranged to yield

$$\Delta T = \frac{\cos \phi - \frac{1}{2}}{(\alpha_{alum} + \alpha_{steel})(1 - \cos \phi) - \alpha_{Invar}} = \approx 46°C,$$

so that the final temperature is $T = 20.0° + \Delta T = 66°$ C. Essentially the same argument, but arguably more elegant, can be made in terms of the differential of the above cosine law expression.

77. **THINK** The heat absorbed by the ice not only raises its temperature but could also change its phase – to water.

EXPRESS Let m_I be the mass of the ice cube and c_I be its specific heat. The energy required to bring the ice cube to the melting temperature $(0 \ C°)$ is

$$Q_1 = c_I m_I (0 \ C° - T_{Ii}) = (2220 \ \text{J/kg} \cdot \text{K})(0.700 \ \text{kg})(150 \ \text{K}) = 2.331 \times 10^5 \ \text{J}.$$

Since the total amount of energy transferred to the ice is $Q = 6.993 \times 10^5 \ \text{J}$, and $Q_1 < Q$, some or all the ice will melt. The energy required to melt all the ice is

$$Q_2 = m_I L_F = (0.700 \ \text{kg})(3.33 \times 10^5 \ \text{J/kg}) = 2.331 \times 10^5 \ \text{J}.$$

However, since

$$Q_1 + Q_2 = 4.662 \times 10^5 \ \text{J} < Q = 6.993 \times 10^5 \ \text{J},$$

this means that all the ice will melt and the extra energy

$$\Delta Q = Q - (Q_1 + Q_2) = 6.993 \times 10^5 \ \text{J} - 4.662 \times 10^5 \ \text{J} = 2.331 \times 10^5 \ \text{J}$$

would be used to raise the temperature of the water.

ANALYZE The final temperature of the water is given by $\Delta Q = m_I c_{water} T_f$. Substituting the values given, we have

$$T_f = \frac{\Delta Q}{m_I c_{water}} = \frac{2.331 \times 10^5 \ \text{J}}{(0.700 \ \text{kg})(4186.8 \ \text{J/kg} \cdot \text{K})} = 79.5°C$$

LEARN The key concepts in this problem are outlined in the Sample Problem 18.04 – "Heat to change temperature and state." An important difference with part (b) of the sample problem is that, in our case, the final state of the H_2O is *all liquid* at $T_f > 0$. As discussed in part (a) of that sample problem, there are three steps to the total process.

78. (a) Using Eq. 18-32, we find the rate of energy conducted upward to be

$$P_{cond} = \frac{Q}{t} = kA\frac{T_H - T_C}{L} = (0.400 \ \text{W/m} \cdot °\text{C})A\frac{5.0 \ °\text{C}}{0.12 \ \text{m}} = (16.7A) \ \text{W}.$$

Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.

(b) The heat of fusion in this process is $Q = L_F m$, where $L_F = 3.33 \times 10^5$ J/kg. Differentiating the expression with respect to t and equating the result with P_{cond}, we have

$$P_{cond} = \frac{dQ}{dt} = L_F \frac{dm}{dt}.$$

Thus, the rate of mass converted from liquid to ice is

$$\frac{dm}{dt} = \frac{P_{cond}}{L_F} = \frac{16.7A \text{ W}}{3.33 \times 10^5 \text{ J/kg}} = (5.02 \times 10^{-5} A) \text{ kg/s}.$$

(c) Since $m = \rho V = \rho A h$, differentiating both sides of the expression gives

$$\frac{dm}{dt} = \frac{d}{dt}(\rho A h) = \rho A \frac{dh}{dt}.$$

Thus, the rate of change of the icicle length is

$$\frac{dh}{dt} = \frac{1}{\rho A} \frac{dm}{dt} = \frac{5.02 \times 10^{-5} \text{kg/m}^2 \cdot \text{s}}{1000 \text{ kg/m}^3} = 5.02 \times 10^{-8} \text{ m/s}$$

79. **THINK** The work done by the expanding gas is given by Eq. 18-24: $W = \int p \, dV$.

EXPRESS Let V_i and V_f be the initial and final volumes, respectively. With $p = aV^2$, the work done by the gas is

$$W = \int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} aV^2 dV = \frac{1}{3} a \left(V_f^3 - V_i^3 \right).$$

ANALYZE With $a = 10 \text{ N/m}^8$, $V_i = 1.0 \text{ m}^3$ and $V_f = 2.0 \text{ m}^3$, we obtain

$$W = \frac{1}{3} a \left(V_f^3 - V_i^3 \right) = \frac{1}{3} \left(10 \text{ N/m}^8 \right) \left[(2.0 \text{ m}^3)^3 - (1.0 \text{ m}^3)^3 \right] = 23 \text{ J}.$$

LEARN In this problem, the initial and final pressures are

$$p_i = aV_i^2 = (10 \text{ N/m}^8)(1.0 \text{ m}^3)^2 = 10 \text{ N/m}^2 = 10 \text{ Pa}$$
$$p_f = aV_f^2 = (10 \text{ N/m}^8)(2.0 \text{ m}^3)^2 = 40 \text{ N/m}^2 = 40 \text{ Pa}$$

In this case, since $p \sim V^2$, the work done would be proportional to V^3 after volume integration.

80. We use $Q = -\lambda_F m_{ice} = W + \Delta E_{int}$. In this case $\Delta E_{int} = 0$. Since $\Delta T = 0$ for the ideal gas, then the work done on the gas is

$$W' = -W = \lambda_F m_i = (333\,\text{J/g})(100\,\text{g}) = 33.3\,\text{kJ}.$$

81. **THINK** The work done is the "area under the curve:" $W = \int p\, dV$.

EXPRESS According to the first law of thermodynamics, $\Delta E_{int} = Q - W$, where Q is the heat absorbed and W is the work done by the system. For process 1,

$$W_1 = p_i(V_b - V_i) = p_i(5.0V_i - V_i) = 4.0 p_i V_i$$

so that

$$\Delta E_{int} = Q - W_1 = 10 p_i V_i - 4.0 p_i V_i = 6.0 p_i V_i \ .$$

Path 2 involves more work than path 1 (note the triangle in the figure of area $\frac{1}{2}(4V_i)(p_i/2)$ $= p_i V_i$). Thus, $W_2 = W_1 + p_i V_i = 5.0 p_i V_i$. Note that $\Delta E_{int} = 6.0 p_i V_i$ is the same for all three paths.

ANALYZE (a) The energy transferred to the gas as heat in process 2 is

$$Q_2 = \Delta E_{int} + W_2 = 6.0 p_i V_i + 5.0 p_i V_i = 11 p_i V_i.$$

(b) Path 3 starts at a and ends at b (same as paths 1 and 2), so $\Delta E_{int} = 6.0 p_i V_i$.

LEARN Work W and heat Q in general are path-dependent quantities, i.e., they depend on how the finial state is reached. However, the combination $\Delta E_{int} = Q - W$ is path independent; it is a *state function*.

82. (a) We denote T_H = 100°C, T_C = 0°C, the temperature of the copper–aluminum junction by T_1. and that of the aluminum-brass junction by T_2. Then,

$$P_{cond} = \frac{k_c A}{L}(T_H - T_1) = \frac{k_a A}{L}(T_1 - T_2) = \frac{k_b A}{L}(T_2 - T_c).$$

We solve for T_1 and T_2 to obtain

$$T_1 = T_H + \frac{T_C - T_H}{1 + k_c(k_a + k_b)/k_a k_b} = 100°C + \frac{0.00°C - 100°C}{1 + 401(235 + 109)/[(235)(109)]} = 84.3°C$$

(b) and

$$T_2 = T_c + \frac{T_H - T_C}{1 + k_b(k_c + k_a)/k_c k_a} = 0.00°C + \frac{100°C - 0.00°C}{1 + 109(235 + 401)/[(235)(401)]}$$

$$= 57.6°C.$$

83. **THINK** The Pyrex disk expands as a result of heating, so we expect $\Delta V > 0$.

EXPRESS The initial volume of the disk (thought of as a short cylinder) is $V_0 = \pi r^2 L$ where $L = 0.50$ cm is its thickness and $r = 8.0$ cm is its radius. After heating, the volume becomes

$$V = \pi(r + \Delta r)^2(L + \Delta L) = \pi r^2 L + \pi r^2 \Delta L + 2\pi r L \Delta r + ...$$

where we ignore higher-order terms. Thus, the change in volume of the disk is

$$\Delta V = V - V_0 \approx \pi r^2 \Delta L + 2\pi r L \Delta r$$

ANALYZE With $\Delta L = L\alpha\Delta T$ and $\Delta r = r\alpha\Delta T$, the above expression becomes

$$\Delta V = \pi r^2 L\alpha\Delta T + 2\pi r^2 L\alpha\Delta T = 3\pi r^2 L\alpha\Delta T.$$

Substituting the values given ($\alpha = 3.2 \times 10^{-6}/C°$ from Table 18-2), we obtain

$$\Delta V = 3\pi r^2 L\alpha\Delta T = 3\pi(0.080 \text{ m})^2(0.0050 \text{ m})(3.2\times10^{-6}/°C)(60°C - 10°C)$$

$$= 4.83\times10^{-8} \text{ m}^3$$

LEARN All dimensions of the disk expand when heated. So we must take into consideration the change in radius as well as the thickness.

84. (a) The rate of heat flow is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(0.040 \text{ W/m}\cdot\text{K})(1.8\text{ m}^2)(33°C - 1.0°C)}{1.0\times10^{-2}\text{ m}} = 2.3\times10^2 \text{ J/s.}$$

(b) The new rate of heat flow is

$$P'_{\text{cond}} = \frac{k'P_{\text{cond}}}{k} = \frac{(0.60 \text{ W/m}\cdot\text{K})(230\text{J/s})}{0.040 \text{ W/m}\cdot\text{K}} = 3.5\times10^3 \text{ J/s,}$$

which is about 15 times as fast as the original heat flow.

85. **THINK** Since the system remains thermally insulated, the total energy remains unchanged. The energy released by the aluminum lump raises the water temperature.

EXPRESS Let T_f be the final temperature of the aluminum lump-water system. The energy transferred from the aluminum is $Q_{Al} = m_{Al} c_{Al} (T_{i,Al} - T_f)$. Similarly, the energy transferred as heat into water is $Q_{water} = m_{water} c_{water} (T_f - T_{i,\,water})$. Equating Q_{Al} with Q_{water} allows us to solve for T_f.

ANALYZE With

$$m_{Al} c_{Al} (T_{i,Al} - T_f) = m_{water} c_{water} (T_f - T_{i,\,water}),$$

we find the final equilibrium temperature to be

$$
\begin{aligned}
T_f &= \frac{m_{Al} c_{Al} T_{i,Al} + m_{water} c_{water} T_{i,water}}{m_{Al} c_{Al} + m_{water} c_{water}} \\
&= \frac{(2.50\,\text{kg})(900\,\text{J}/\text{kg}\cdot\text{K})(92^\circ\text{C}) + (8.00\,\text{kg})(4186.8\,\text{J/kg}\cdot\text{K})(5.0^\circ\text{C})}{(2.50\,\text{kg})(900\,\text{J}/\text{kg}\cdot\text{K}) + (8.00\,\text{kg})(4186.8\,\text{J/kg}\cdot\text{K})} \\
&= 10.5^\circ\text{C}.
\end{aligned}
$$

LEARN No phase change is involved in this problem, so the thermal energy transferred from the aluminum can only change the water temperature.

86. If the window is L_1 high and L_2 wide at the lower temperature and $L_1 + \Delta L_1$ high and $L_2 + \Delta L_2$ wide at the higher temperature, then its area changes from $A_1 = L_1 L_2$ to

$$A_2 = (L_1 + \Delta L_1)(L_2 + \Delta L_2) \approx L_1 L_2 + L_1\,\Delta L_2 + L_2\,\Delta L_1$$

where the term $\Delta L_1\,\Delta L_2$ has been omitted because it is much smaller than the other terms, if the changes in the lengths are small. Consequently, the change in area is

$$\Delta A = A_2 - A_1 = L_1\,\Delta L_2 + L_2\,\Delta L_1.$$

If ΔT is the change in temperature then $\Delta L_1 = \alpha L_1\,\Delta T$ and $\Delta L_2 = \alpha L_2\,\Delta T$, where α is the coefficient of linear expansion. Thus

$$\Delta A = \alpha (L_1 L_2 + L_1 L_2)\,\Delta T = 2\alpha L_1 L_2 \Delta T = 2\left(9 \times 10^{-6}/\text{C}^\circ\right)(30\,\text{cm})\,(20\,\text{cm})\,(30^\circ\text{C}) = 0.32\,\text{cm}^2.$$

87. For a cylinder of height h, the surface area is $A_c = 2\pi r h$, and the area of a sphere is $A_o = 4\pi R^2$. The net radiative heat transfer is given by Eq. 18-40.

(a) We estimate the surface area A of the body as that of a cylinder of height 1.8 m and radius $r = 0.15$ m plus that of a sphere of radius $R = 0.10$ m. Thus, we have $A \approx A_c + A_o = 1.8\,\text{m}^2$. The emissivity $\varepsilon = 0.80$ is given in the problem, and the Stefan-Boltzmann constant is found in Section 18-11: $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K^4. The "environment"

temperature is T_{env} = 303 K, and the skin temperature is $T = \frac{5}{9}(102 - 32) + 273 = 312$ K. Therefore,

$$P_{net} = \sigma \varepsilon A \left(T_{env}^4 - T^4 \right) = -86 \, \text{W}.$$

The corresponding sign convention is discussed in the textbook immediately after Eq. 18-40. We conclude that heat is being lost by the body at a rate of roughly 90 W.

(b) Half the body surface area is roughly A = 1.8/2 = 0.9 m². Now, with T_{env} = 248 K, we find

$$| P_{net} | = | \sigma \varepsilon A \left(T_{env}^4 - T^4 \right) | \approx 2.3 \times 10^2 \, \text{W}.$$

(c) Finally, with T_{env} = 193 K (and still with A = 0.9 m²) we obtain $|P_{net}| = 3.3 \times 10^2$ W.

88. We take absolute values of Eq. 18-9 and Eq. 12-25:

$$| \Delta L | = L \alpha \, | \Delta T | \quad \text{and} \quad \left| \frac{F}{A} \right| = E \left| \frac{\Delta L}{L} \right|.$$

The ultimate strength for steel is $(F/A)_{rupture} = S_u = 400 \times 10^6$ N/m² from Table 12-1. Combining the above equations (eliminating the ratio $\Delta L/L$), we find the rod will rupture if the temperature change exceeds

$$| \Delta T | = \frac{S_u}{E \alpha} = \frac{400 \times 10^6 \, \text{N/m}^2}{\left(200 \times 10^9 \, \text{N/m}^2 \right) \left(11 \times 10^{-6} / \text{C}^\circ \right)} = 182 ^\circ \text{C}.$$

Since we are dealing with a temperature decrease, then, the temperature at which the rod will rupture is T = 25.0°C − 182°C = −157°C.

89. (a) Let the number of weight lift repetitions be N. Then $Nmgh = Q$, or (using Eq. 18-12 and the discussion preceding it)

$$N = \frac{Q}{mgh} = \frac{(3500 \, \text{Cal})(4186 \, \text{J/Cal})}{(80.0 \, \text{kg}) \left(9.80 \, \text{m/s}^2 \right) (1.00 \, \text{m})} \approx 1.87 \times 10^4.$$

(b) The time required is

$$t = (18700)(2.00 \, \text{s}) \left(\frac{1.00 \, \text{h}}{3600 \, \text{s}} \right) = 10.4 \, \text{h}.$$

90. For isotropic materials, the coefficient of linear expansion α is related to that for volume expansion by $\alpha = \frac{1}{3}\beta$ (Eq. 18-11). The radius of Earth may be found in the Appendix. With these assumptions, the radius of the Earth should have increased by approximately

$$\Delta R_E = R_E \alpha \Delta T = \left(6.4 \times 10^3 \text{ km}\right)\left(\frac{1}{3}\right)\left(3.0 \times 10^{-5} / \text{K}\right) \ (3000 \text{ K} - 300 \text{ K}) = 1.7 \times 10^2 \text{ km}.$$

91. We assume the ice is at 0°C to begin with, so that the only heat needed for melting is that described by Eq. 18-16 (which requires information from Table 18-4). Thus,

$$Q = Lm = (333 \text{ J/g})(1.00 \text{ g}) = 333 \text{ J}.$$

92. One method is to simply compute the change in length in each edge (x_0 = 0.200 m and y_0 = 0.300 m) from Eq. 18-9 (Δx = 3.6 × 10^{-5} m and Δy = 5.4 × 10^{-5} m) and then compute the area change:

$$A - A_0 = \left(x_0 + \Delta x\right)\left(y_0 + \Delta y\right) - x_0 y_0 = 2.16 \times 10^{-5} \text{ m}^2.$$

Another (though related) method uses $\Delta A = 2\alpha A_0 \Delta T$ (valid for $\Delta A / A \ll 1$) which can be derived by taking the differential of $A = xy$ and replacing d 's with Δ's.

93. The problem asks for 0.5% of E, where $E = Pt$ with t = 120 s and P given by Eq. 18-38. Therefore, with $A = 4\pi r^2 = 5.0 \times 10^{-3} \text{ m}^2$, we obtain

$$\left(0.005\right) Pt = \left(0.005\right)\sigma \varepsilon A T^4 t = 8.6 \text{ J}.$$

94. Let the initial water temperature be T_{wi} and the initial thermometer temperature be T_{ti}. Then, the heat absorbed by the thermometer is equal (in magnitude) to the heat lost by the water:

$$c_t m_t \left(T_f - T_{ti}\right) = c_w m_w \left(T_{wi} - T_f\right).$$

We solve for the initial temperature of the water:

$$T_{wi} = \frac{c_t m_t \left(T_f - T_{ti}\right)}{c_w m_w} + T_f = \frac{\left(0.0550 \text{ kg}\right)\left(0.837 \text{ kJ/kg} \cdot \text{K}\right)\left(44.4 - 15.0\right) \text{K}}{\left(4.18 \text{ kJ} / \text{kg} \cdot \text{C}°\right)\left(0.300 \text{ kg}\right)} + 44.4°\text{C} = 45.5°\text{C}.$$

95. The net work may be computed as a sum of works (for the individual processes involved) or as the "area" (with appropriate ± sign) inside the figure (representing the cycle). In this solution, we take the former approach (sum over the processes) and will need the following fact related to processes represented in pV diagrams:

$$\text{for a straight line: Work} = \frac{p_i + p_f}{2}\Delta V$$

which is easily verified using the definition Eq. 18-25. The cycle represented by the "triangle" *BC* consists of three processes:

• "tilted" straight line from (1.0 m³, 40 Pa) to (4.0 m³, 10 Pa), with

$$\text{Work} = \frac{40\,\text{Pa} + 10\,\text{Pa}}{2}\left(4.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 75\,\text{J}$$

• horizontal line from (4.0 m³, 10 Pa) to (1.0 m³, 10 Pa), with

$$\text{Work} = \left(10\,\text{Pa}\right)\left(1.0\,\text{m}^3 - 4.0\,\text{m}^3\right) = -30\,\text{J}$$

• vertical line from (1.0 m³, 10 Pa) to (1.0 m³, 40 Pa), with

$$\text{Work} = \frac{10\,\text{Pa} + 40\,\text{Pa}}{2}\left(1.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 0$$

(a) and (b) Thus, the total work during the *BC* cycle is (75 − 30) J = 45 J. During the *BA* cycle, the "tilted" part is the same as before, and the main difference is that the horizontal portion is at higher pressure, with Work = (40 Pa)(−3.0 m³) = −120 J. Therefore, the total work during the *BA* cycle is (75 − 120) J = − 45 J.

96. (a) The total length change of the composite bar is

$$\Delta L = \Delta L_1 + \Delta L_2 = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T = (\alpha_1 L_1 + \alpha_2 L_2)\Delta T.$$

Writing $\Delta L = \alpha L \Delta T$ and equating the two expressions leads to $\alpha = \dfrac{\alpha_1 L_1 + \alpha_2 L_2}{L}$.

(b) The coefficients of thermal expansions are $\alpha_1 = 11 \times 10^{-6}/\text{C}°$ for steel and $\alpha_2 = 19 \times 10^{-6}/\text{C}°$ for brass. We solve the system of equations

$$\alpha = 13 \times 10^{-6}/\text{C}° = \frac{(11 \times 10^{-6}/\text{C}°)L_1 + (19 \times 10^{-6}/\text{C}°)L_2}{L_1 + L_2}$$

$$L = L_1 + L_2 = 52.4 \text{ cm}$$

and obtain $L_1 = 39.3$ cm, and

(c) $L_2 = 13.1$ cm.

97. The heat required to raise the water of mass *m* from an initial temperature T_i to final temperature T_f is $Q = cm(T_f - T_i)$, where *c* is the specific heat of water. On the other hand, each shake supplies an energy $\Delta U_1 = mgh$, where *h* is the vertical distance the water has moved during each shake. Thus, with 27 shakes/min, the time required to raise the water temperature to T_f is

$$\Delta t = \frac{Q}{R(\Delta U_1)} = \frac{cm(T_f - T_i)}{Rmgh} = \frac{c(T_f - T_i)}{Rgh} = \frac{(4186.8\,\text{J/kg} \cdot \text{C}^\circ)(100^\circ\text{C} - 19^\circ\text{C})}{(27\,\text{shakes/min})(9.8\,\text{m/s}^2)(0.32\,\text{m})}$$

$$= 4.0 \times 10^3 \ \text{min}.$$

98. Since the combination "p_1V_1" appears frequently in this derivation we denote it as "x. Thus for process 1, the heat transferred is $Q_1 = 5x = \Delta E_{\text{int 1}} + W_1$, and for path 2 (which consists of two steps, one at constant volume followed by an expansion accompanied by a linear pressure decrease) it is $Q_2 = 5.5x = \Delta E_{\text{int 2}} + W_2$. If we subtract these two expressions and make use of the fact that internal energy is state function (and thus has the same value for path 1 as for path 2) then we have

$$5.5x - 5x = W_2 - W_1 = \text{"area" inside the triangle} = \frac{1}{2}(2\,V_1)(p_2 - p_1).$$

Thus, dividing both sides by x (= p_1V_1), we find $0.5 = (p_2/p_1) - 1$, which leads immediately to the result: $p_2/p_1 = 1.5$.

99. The cube has six faces, each of which has an area of $(6.0 \times 10^{-6}\ \text{m})^2$. Using Kelvin temperatures and Eq. 18-40, we obtain

$$P_{\text{net}} = \sigma\varepsilon A\,(T_{\text{env}}^4 - T^4)$$

$$= \left(5.67 \times 10^{-8}\,\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\right)(0.75)\left(2.16 \times 10^{-10}\ \text{m}^2\right)\left((123.15\ \text{K})^4 - (173.15\ \text{K})^4\right)$$

$$= -6.1 \times 10^{-9}\ \text{W}.$$

100. We denote the density of the liquid as ρ, the rate of liquid flowing in the calorimeter as μ, the specific heat of the liquid as c, the rate of heat flow as P, and the temperature change as ΔT. Consider a time duration dt, during this time interval, the amount of liquid being heated is $dm = \mu\rho dt$. The energy required for the heating is

$$dQ = Pdt = c(dm)\,\Delta T = c\mu\Delta T dt.$$

Thus,

$$c = \frac{P}{\rho\mu\Delta T} = \frac{250\,\text{W}}{\left(8.0 \times 10^{-6}\ \text{m}^3/\text{s}\right)\left(0.85 \times 10^3\ \text{kg/m}^3\right)(15^\circ\text{C})}$$

$$= 2.5 \times 10^3\ \text{J/kg} \cdot \text{C}^\circ = 2.5 \times 10^3\ \text{J/kg} \cdot \text{K}.$$

101. Consider the object of mass m_1 falling through a distance h. The loss of its mechanical energy is $\Delta E = m_1gh$. This amount of energy is then used to heat up the temperature of water of mass m_2: $\Delta E = m_1gh = Q = m_2c\Delta T$. Thus, the maximum possible rise in water temperature is

$$\Delta T = \frac{m_1 gh}{m_2 c} = \frac{(6.00\,\text{kg})(9.8\,\text{m/s}^2)(50.0\,\text{m})}{(0.600\,\text{kg})(4190\,\text{J/kg}\cdot\text{C}°)} = 1.17\,°\text{C}.$$

102. When the temperature changes from T to $T + \Delta T$ the diameter of the mirror changes from D to $D + \Delta D$, where $\Delta D = \alpha D\,\Delta T$. Here $\alpha = 3.2 \times 10^{-6}/\text{C}°$ is the coefficient of linear expansion for Pyrex glass. The range of values for the diameters can be found by setting ΔT equal to the temperature range. Thus

$$\Delta L = \alpha D \Delta T = (3.2 \times 10^{-6}/\text{C}°)\left(170\ \text{in.}\cdot\frac{0.0254\ \text{m}}{1\ \text{in.}}\right)(32\,°\text{C} - (-16\,°\text{C}))$$

$$= 6.63 \times 10^{-4}\,\text{m} \approx 660\ \mu\text{m}.$$

103. The change in area for the plate is

$$\Delta A = (a + \Delta a)(b + \Delta b) - ab \approx a\Delta b + b\Delta a = 2ab\alpha\Delta T = 2\alpha A \Delta T$$

$$= 2(32 \times 10^{-6}/\text{C}°)(1.4\ \text{m}^2)(89\,°\text{C}) = 7.97 \times 10^{-3}\,\text{m}^2 \approx 8.0 \times 10^{-3}\,\text{m}^2.$$

104. The relative volume change is

$$\frac{\Delta V}{V} = \beta\Delta T = (6.6 \times 10^{-4}/\text{C}°)(12\,°\text{C}) = 7.92 \times 10^{-3}.$$

Since the expansion the glass tube can be ignored, the cross-sectional area of the liquid remains unchanged, and we have $\dfrac{\Delta h}{h} = \dfrac{\Delta V}{V} = 7.92 \times 10^{-3}$.

105. (a) We note that if the pendulum shortens, its frequency of oscillation will increase, thereby causing it to record more units of time ("ticks") than have actually passed during an interval. Thus, as the pendulum contracts (this problem involves cooling the brass wire), the pendulum will "run fast."

(b) The period of the pendulum is $\tau = 2\pi\sqrt{L/g}$ (so not to be confused with temperature T). Differentiating τ with respect to L gives

$$\frac{d\tau}{dL} = \frac{d}{dL}\left(2\pi\sqrt{\frac{L}{g}}\right) = \pi\frac{1}{\sqrt{Lg}} = \frac{1}{2L}\left(2\pi\sqrt{\frac{L}{g}}\right) = \frac{\tau}{2L}.$$

Thus,

$$\Delta\tau = \frac{\tau\Delta L}{2L} = \frac{1}{2}\tau\alpha\Delta T.$$

Substituting the values given, the change in period is

$$\Delta\tau = \frac{1}{2}\tau\alpha\Delta T = \frac{1}{2}\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)(19\times10^{-6}/\text{C}°)(23\,\text{C}°) = 0.787\,\text{s/h}.$$

106. Recalling that 1 W = 1 J/s, the heat Q which is added to the room in 6.9 h is

$$Q = 4(100\,\text{W})(0.73)(6.9\,\text{h})\left(\frac{3600\,\text{s}}{1.00\,\text{h}}\right) = 7.25\times10^{6}\,\text{J}.$$

107. With 1 Calorie = 1000 cal, we find the athlete's rate of dissipating energy to be

$$P = 4000\,\text{Cal/day} = \frac{\left(4000\times10^{3}\,\text{cal}\right)\left(4.1868\,\text{J/cal}\right)}{\left(1\,\text{day}\right)\left(86400\,\text{s/day}\right)} = 193.83\,\text{W},$$

which is about 1.9 times as much as the power of a 100 W light bulb.

108. The initial speed of the car is $v_i = 83\,\text{km/h} = (83\,\text{km/h})\left(\dfrac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 23.056\,\text{m/s}.$
The deceleration a of the car is given by $v_f^2 - v_i^2 = -v_i^2 = 2ad$, or

$$a = -\frac{(23.056\,\text{m/s})^2}{2(93\,\text{m})} = -2.86\,\text{m/s}^2.$$

The time Δt it takes for the car to stop is then

$$\Delta t = \frac{v_f - v_i}{a} = \frac{-23.056\,\text{m/s}}{-2.86\,\text{m/s}^2} = 8.07\,\text{s}.$$

The change in kinetic energy of the car is

$$\Delta K = -\frac{1}{2}mv_i^2 = -\frac{1}{2}(1700\,\text{kg})(23.056\,\text{m/s})^2 = -4.52\times10^{5}\,\text{J}.$$

Thus, the average rate at which mechanical energy is transferred to thermal energy is

$$P = \frac{\Delta E_{\text{th}}}{\Delta t} = \frac{-\Delta K}{\Delta t} = \frac{4.52\times10^{5}\,\text{J}}{8.07\,\text{s}} = 5.6\times10^{4}\,\text{W}.$$