Final Project

Course: Advanced Derivative Models

Instructor: Jeff Greco

Market Data and Model Assumptions

Market Quotes

Chicago Mercantile Exchange (CME) market settlement data as of close of business November 8, 2022 are supplied in the attached comma-separated values (CSV) file: cme.settle.20221108.s.csv.

Stein & Stein Stochastic Volatility Model Dynamics

Let F_t^T represent the time t CME Bitcoin futures price expiring at time T. Assume risk-neutral measure dynamics follow the Stein & Stein stochastic volatility model

$$
\begin{aligned}
\frac{dF_t^T}{F_t^T} &= \sigma_t dW_t \\
d\sigma_t &= \kappa\left(\theta - \sigma_t\right) dt + \alpha dZ_t \\
dW_t dZ_t &= \rho dt
\end{aligned}
$$

where W_t and Z_t are Brownian motions with respect to the risk-neutral measure. The model parameters satisfy $\kappa \geq 0$, $\theta \in \mathbb{R}$, $\alpha \geq 0$, $-1 < \rho < 1$, and $\sigma_0 \geq 0$.

Analytic Solutions

1. Prove that the volatility process σ_t is normally distributed. Find closed form expressions for its conditional mean $\mathbb{E}\left[\sigma_T \left| \mathcal{F}_t\right.\right]$ and conditional variance $\mathbb{E}\left[\left(\sigma_T - \mathbb{E}\left[\sigma_T \left| \mathcal{F}_t\right.\right]\right)^2 \left| \mathcal{F}_t\right.\right]$, where $T \geq t \geq 0$.

2. Find a closed form expression for the instantaneous T-forward variance $\xi_t^T = \mathbb{E}\left[\sigma_T^2 \left| \mathcal{F}_t\right.\right]$.

3. Using the above results, find an exact expression for the model break-even variance strike with expiration time T. What does it converge to in the limit $T \to +\infty$?

Calibration to Variance Strike Term Structure

4. Use the supplied CME Bitcoin options on futures quotes (11/8/2022 corresponds with $t = 0$) to accurately estimate the market implied (continuous) break-even variance strike for each available expiration time T.

5. Calibrate the parameters $(\kappa, \theta, \alpha, \sigma_0)$ by minimizing the mean square error (MSE) of the (continuous) break-even variance strikes between the Stein & Stein model and your estimates. Create a plot containing a series of your estimated break-even variance strikes and a series of your fitted Stein & Stein variance strikes. Describe the quality of fit.

Use the parameter values $(\kappa, \theta, \alpha, \sigma_0)$ that you calibrated for the remainder of this project.

Monte Carlo Simulation

6. Implement a Monte Carlo simulation routine that utilizes an Euler scheme for F_t^T and exactly captures the distribution of the volatility process σ_t. Use it to fix a value for ρ that minimizes the 12/30/2022 expiration option price MSE across all integer multiples of ± 0.05.

Use the parameter value ρ that you found and the expiration time T corresponding with 12/30/2022 for the remainder of this project. Your calibrated model parameter set $(\kappa, \theta, \alpha, \sigma_0, \rho)$ is now complete!

7. Plot the 12/30/2022 expiration market and model volatility smiles, using a logarithmic scale for the x-axis. Describe the quality of fit.

Pricing Exotic Options

8. Estimate the model continuous break-even volatility strike $\mathbb{E}\left[\sqrt{\frac{1}{T}\int_0^T \sigma_t^2 dt}\right]$.

9. A down-and-out put option with strike K and barrier B has payoff at time T equal to

$$P^{\text{down-and-out}}(T, K, B) = \begin{cases} \left(K - F_T^T\right)^+, & \min_{t\in[0,T]} F_t^T > B \\ 0, & \min_{t\in[0,T]} F_t^T \leq B. \end{cases}$$

Calculate and plot the price of the down-and-out put as a function of $(K, B) \in [0, 25000] \times [0, 18000]$.