Animal Barn Problem

Woo Sik Choi

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You are a farmer who has $n \geq 0$ rabbits and $m \geq 0$ chickens as pets in a barn such that $m + n > 0$. Every day, you go into the barn and **randomly** choose 2 animals from the barn to "give away." You have an endless supply of chickens in another shed. If the two animals you took out of the barn are both chickens, you put 1 chicken back into the barn. If the two animals you took out of the barn are both rabbits, you take 1 chicken from the shed and put it into the barn. If the two animals you took out of the barn are one of each, you put 1 rabbit back in the barn.

It should be obvious that the barn is getting smaller and smaller each day. You take out 2 animals per day and replace it with 1 animal. Eventually there will be 1 animal left in the barn. Given the fact that you have n rabbits, m chickens and the above algorithm, what animal is the last one remaining? Provide a loop invariant to prove your answer is correct. Prove your loop invariant is correct.