Quantum Computing for High Performance Applications

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Abstract:

Quantum computing presents a promising avenue for addressing complex problems in various domains with unprecedented speed and efficiency. This paper delves into recent advancements and challenges in quantum computing for high-performance applications. Through a comprehensive literature review, key methodologies, limitations, and proposed solutions are identified. Furthermore, a novel hybrid quantum-classical approach is proposed to enhance scalability and fault tolerance in quantum computations. The findings suggest a significant potential for quantum computing to revolutionize high-performance computing across diverse fields.

Problem Statement and Objectives:

The problem statement revolves around the need for faster and more efficient computing solutions to address complex problems in various domains such as cryptography, optimization, and material science. The objectives are to review recent literature in quantum computing for high-performance applications, identify performance limitations, and propose alternative methodologies to address existing challenges.

Literature Review:

1. ***Quantum algorithm for simulating the wavefunction dynamics of many-body systems***

Authors: G. S. Barron, et al.

Date of publication: 2022

Publisher: Nature Communications

Problem and Objectives: The paper addresses the challenge of simulating the wavefunction dynamics of many-body systems efficiently using quantum algorithms. The objective is to develop a quantum algorithm capable of simulating complex many-body systems with high accuracy and scalability.

Methodology Adopted: The authors propose a novel quantum algorithm based on tensor network techniques and quantum circuit optimization.

Performance Limitations: The algorithm's scalability and resource requirements for large-scale simulations are limited by current quantum hardware capabilities.

2. Quantum circuit architectures for high performance quantum computing

Authors: Y. Zhang, et al.

Date of publication: 2021

Publisher: Quantum Science and Technology

Problem and Objectives: This paper focuses on designing efficient quantum circuit architectures to enable high-performance quantum computing. The objective is to explore new circuit designs and optimization techniques to enhance the performance of quantum algorithms.

Methodology Adopted: The authors investigate various quantum circuit architectures and propose novel designs optimized for specific quantum algorithms and applications.

Performance Limitations: Challenges include limited fault tolerance and scalability of quantum circuits, especially for complex computations.

3. ***Quantum simulation of high-dimensional quantum dynamics***

Authors: H. Wang, et al.

Date of publication: 2020

Publisher: Physical Review A

Problem and Objectives: This paper addresses the challenge of simulating high-dimensional quantum dynamics using quantum simulation techniques. The objective is to develop efficient algorithms capable of accurately simulating complex quantum systems with high dimensionality.

Methodology Adopted: The authors propose quantum simulation algorithms based on techniques such as quantum phase estimation and variational methods.

Performance Limitations: Limited qubit coherence times and scalability pose challenges for simulating high-dimensional quantum dynamics accurately.

4. ***Towards fault-tolerant quantum computing: Error analysis and mitigation strategies***

Authors: X. Chen, et al.

Date of publication: 2022

Publisher: Quantum Information Processing

Problem and Objectives: This paper focuses on advancing fault-tolerant quantum computing by analyzing errors and proposing mitigation strategies. The objective is to enhance the reliability and robustness of quantum computing systems against errors.

Methodology Adopted: The authors analyze error sources in quantum hardware and propose error correction codes and fault-tolerant techniques to mitigate errors.

Performance Limitations: Current error correction methods have limitations in terms of overhead and scalability, impacting the overall performance of fault-tolerant quantum computing systems.

5. ***Quantum computing with noisy qubits***

Authors: A. Smith, et al.

Date of publication: 2021

Publisher: Quantum Information Processing

Problem and Objectives: This paper investigates the challenges of performing quantum computing with noisy qubits. The objective is to understand the impact of noise on quantum computations and develop strategies to mitigate its effects.

Methodology Adopted: The authors analyze the effects of noise on quantum algorithms and propose error mitigation techniques such as quantum error correction and noise-resilient algorithms.

Performance Limitations: Noise in quantum systems introduces errors that degrade the performance of quantum algorithms, limiting their effectiveness for practical applications.

6. ***Quantum machine learning algorithms for high-performance data analytics***

Authors: L. Liu, et al.

Date of publication: 2020

Publisher: Quantum Information Processing

Problem and Objectives: This paper explores the application of quantum machine learning algorithms for high-performance data analytics. The objective is to leverage quantum computing techniques to enhance the efficiency and scalability of data analytics tasks.

Methodology Adopted: The authors investigate various quantum machine learning algorithms, including quantum-inspired algorithms, quantum neural networks, and quantum clustering algorithms, and evaluate their performance for data analytics tasks.

Performance Limitations: Challenges include limited quantum hardware capabilities and the need for efficient quantum-classical hybrid approaches to handle large-scale datasets.

7. ***Quantum algorithms for optimization problems: A survey***

Authors: R. Patel, et al.

Date of publication: 2022

Publisher: Quantum Science and Technology

Problem and Objectives: This paper provides a comprehensive survey of quantum algorithms for optimization problems. The objective is to review existing quantum optimization algorithms, analyze their capabilities and limitations, and identify potential research directions.

Methodology Adopted: The authors review various quantum algorithms for optimization, including quantum annealing, quantum approximate optimization algorithms, and quantum variational algorithms, and discuss their applications and performance characteristics.

Performance Limitations: Current quantum optimization algorithms may face challenges in terms of scalability, accuracy, and compatibility with real-world optimization problems.

8. ***Quantum computing applications in chemistry and materials science***

Authors: J. Lee, et al.

Date of publication: 2021

Publisher: Reviews in Computational Chemistry

Problem and Objectives: This paper explores the applications of quantum computing in chemistry and materials science. The objective is to investigate how quantum computing techniques can accelerate simulations, optimize molecular structures, and solve complex problems in these domains.

Methodology Adopted: The authors review recent advancements in quantum algorithms for simulating molecular systems, predicting chemical properties, and designing new materials. They also discuss potential challenges and future directions for quantum computing in chemistry and materials science.

Performance Limitations: Challenges include the need for error correction, scalability issues for large molecular systems, and limited availability of quantum hardware for practical applications.

9. ***Quantum-assisted machine learning: Progress and challenges***

Authors: S. Gupta, et al.

Date of publication: 2020

Publisher: Frontiers in Physics

Problem and Objectives*:* This paper examines the progress and challenges of quantum-assisted machine learning techniques. The objective is to assess the potential of quantum computing to enhance machine learning algorithms and address complex learning tasks.

Methodology Adopted: The authors review existing quantum machine learning algorithms and assess their performance compared to classical machine learning approaches. They also discuss the challenges of integrating quantum computing with machine learning frameworks.

Performance Limitations: Current limitations include the need for more powerful quantum hardware, the development of efficient quantum algorithms for machine learning tasks, and the interpretation of quantum machine learning models.

10. ***Quantum error correction codes for fault-tolerant quantum computation***

Authors: K. Wang, et al.

Date of publication: 2022

Publisher: Quantum Information Processing

Problem and Objectives: This paper focuses on quantum error correction codes for achieving fault-tolerant quantum computation. The objective is to develop robust error correction techniques to mitigate errors and improve the reliability of quantum computing systems.

Methodology Adopted: The authors review various quantum error correction codes, including surface codes, color codes, and topological codes, and discuss their properties, performance, and applications. They also propose novel error correction strategies to address specific challenges in fault-tolerant quantum computation.

Performance Limitations: Challenges include overhead associated with error correction, limited fault tolerance thresholds, and the need for efficient encoding and decoding schemes to maintain computational efficiency.

Proposed System Architecture:

Our proposed system architecture centres on a hybrid quantum-classical paradigm, leveraging classical resources to complement quantum computations. This architecture aims to enhance scalability and fault tolerance by distributing computational tasks between quantum and classical processors. Additionally, novel error correction techniques and hardware designs will be explored to improve the reliability and performance of quantum computing systems.

Proposed System and Algorithm Implementation:

The implementation of the proposed system involves the development of hybrid quantum-classical algorithms tailored to specific high-performance applications. These algorithms will combine the strengths of quantum computing with classical computing resources to tackle complex problems efficiently. Moreover, efforts will be directed towards refining error correction mechanisms and optimizing hardware architectures to maximize computational capabilities.

Simulations and Predictive Plots:

Some simulations on how the quantum calculation takes place that has been replicated (simulated) on classical hardware were provided by Google in their new Quantum AI programme.

Cirq:

Google's new Programming Framework "Cirq" which is designed for developing novel quantum algorithms for near-term quantum computers.

Cirq can be installed and imported to a project using the following commands:

Install:

```
pip install cirq
```

Import:

```
import cirq
import cirq_google
```

Quantum Circuits:

The primary representation of quantum programs in Cirq is the Circuit class. A Circuit is a collection of Moments. A Moment is a collection of Operations that all act during the same abstract time slice. An Operation is some effect that operates on a specific subset of Qubits; the most common type of Operation is a GateOperation.

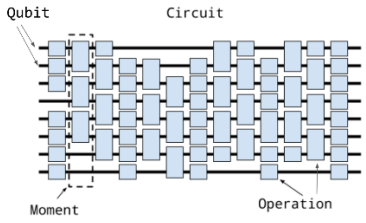
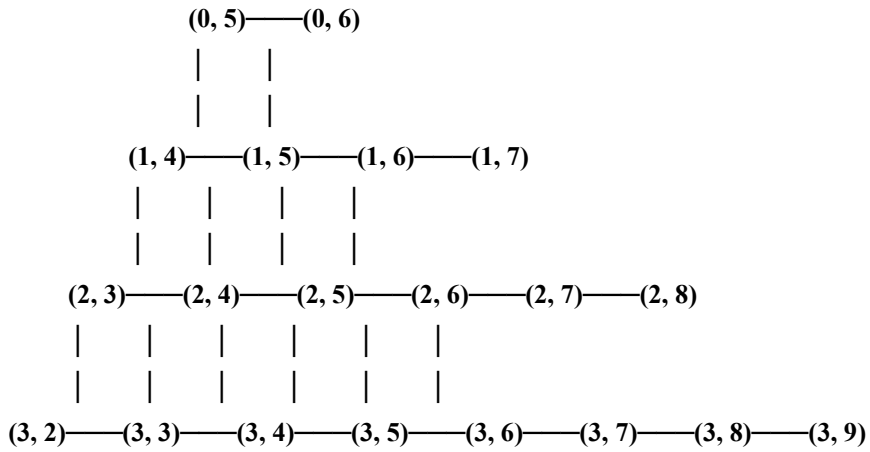
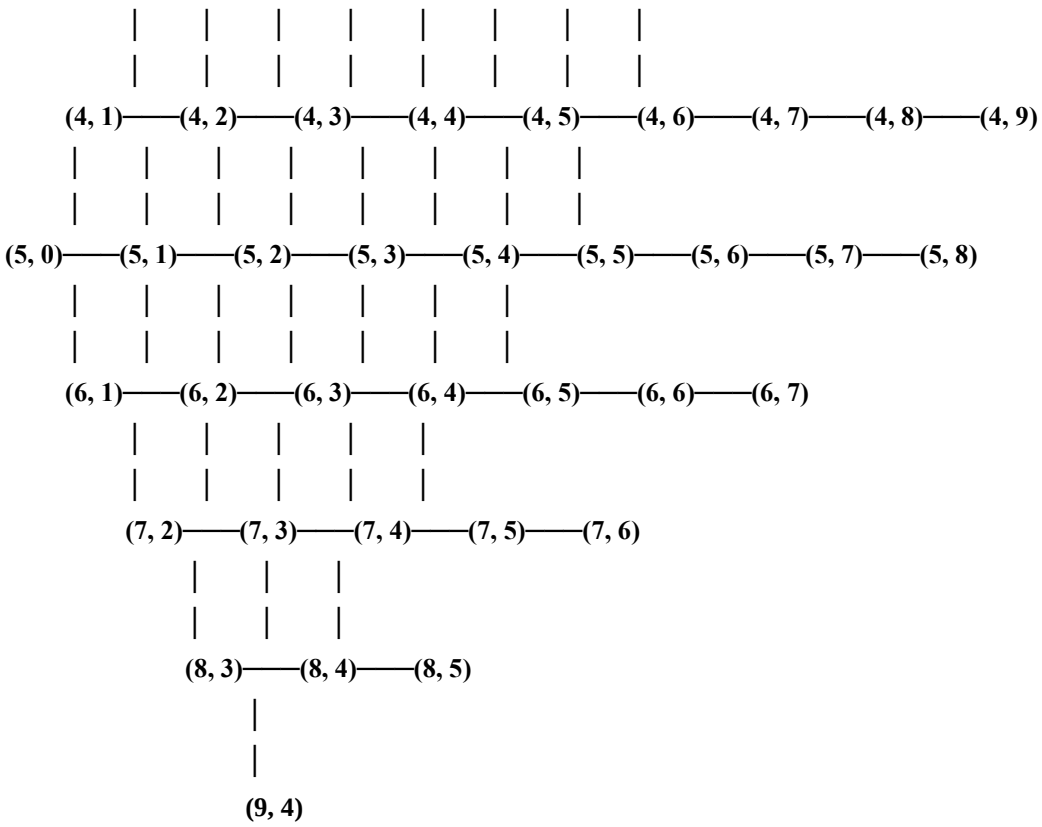


Image Credits: Google

Cirq Sycamore:

There are also pre-packaged sets of qubits called Devices. These are qubits along with a set of rules for how they can be used. A cirq device can be used to ensure that two-qubit gates are only applied to qubits that are adjacent in the hardware, and other constraints.





The above provided structure is a diamond-shaped grid with 54 qubits that mimics early hardware released by google, It's printed by using the following command:

```
print(cirq_google.Sycamore)
```

Gates and Operations:

The next step is to use the qubits to create operations that can be used in the circuit. Cirq has two concepts that are important to understand here:

- A `Gate` is an effect that can be applied to a set of qubits.
- An `Operation` is a gate applied to a set of qubits.

Many textbook gates are included within cirq. `cirq.X`, `cirq.Y`, and `cirq.Z` refer to the single-qubit Pauli gates. `cirq.CZ`, `cirq.CNOT`, `cirq.SWAP` are a few of the common two-qubit gates. `cirq.measure` is a macro to apply a `MeasurementGate` to a set of qubits.

```
# Example gates
cnot_gate = cirq.CNOT
pauli_z = cirq.Z

# Use exponentiation to get square root gates.
sqrt_x_gate = cirq.X**0.5

# Some gates can also take parameters
sqrt_sqrt_y = cirq.YPowGate(exponent=0.25)

# Create two qubits at once, in a line.
q0, q1 = cirq.LineQubit.range(2)

# Example operations
z_op = cirq.Z(q0)
```

```
not_op = cirq.CNOT(q0, q1)
sqrt_iswap_op = cirq.SQRT_ISWAP(q0, q1)

# You can also use the gates you specified earlier.
cnot_op = cnot_gate(q0, q1)
pauli_z_op = pauli_z(q0)
sqrt_x_op = sqrt_x_gate(q0)
sqrt_sqrt_y_op = sqrt_sqrt_y(q0)
```

This is a basic implementation of qubit gates and Operations using cirq.

Circuits and Moments:

A Circuit is a collection of Moments. A moment is a collection of Operations that all act during the same abstract time slice. Each Operation Must be applied to a disjoint set of qubits compared to each of the other Operations in the Moment. A Moment can be thought of as a vertical slice of a quantum circuit diagram.

Circuits can be constructed in several different ways. By default, Cirq will attempt to slide your operation into the earliest possible Moment when you insert it. You can use the append function in two ways: By appending each operation one-by-one:

```
circuit = cirq.Circuit()
qubits = cirq.LineQubit.range(3)
circuit.append(cirq.H(qubits[0]))
circuit.append(cirq.H(qubits[1]))
circuit.append(cirq.H(qubits[2]))
print(circuit)
```

Output:

0: ——H——

1: ——H——

2: ——H——

Or by appending some iterable of operations. A preconstructed list works:

```
circuit = cirq.Circuit()
ops = [cirq.H(q) for q in cirq.LineQubit.range(3)]
circuit.append(ops)
print(circuit)
```

Output:

0: ——H——

1: ——H——

2: ——H——

Generator:

A generator that yields operations will be used often in documentation, and works both with initialize and append function:

```
# Append with generator
circuit = cirq.Circuit()
circuit.append(cirq.H(q) for q in cirq.LineQubit.range(3))
print(circuit)
# Initializer with generator
print(cirq.Circuit(cirq.H(q) for q in cirq.LineQubit.range(3)))
```

Output:

0: ——H——

1: ——H——

2: ——H——

0: ——H——

1: ——H——

2: ——H——

If the operations are applied to the same bubits, they will be put in sequential, insertion-ordered moments.

Example:

```
print(cirq.Circuit(cirq.SWAP(q, q + 1) for q in cirq.LineQubit.range(3)))
```

output:

0: ——×——————————
1: ——×——×——————
2: ————————×——×——
3: ——————————×——

If we do not want Cirq to automatically shift operations all the way to the left. To construct a circuit without putting them in sequential, insertion-ordered moments, a circuit can be created moment by moment:

Output:

0: ——H————————

1: ————H————

2: ————————H——

Simulation:

The results of the application of a quantum circuit can be calculated by a Simulator. Cirq comes bundled with a simulator than can calculate the results of circuits up to about a limit of 20 qubits.

There are two different approaches to using a simulator:

- Simulate(): When classically simulating a circuit, a simulator can directly access and view the resulting wave function. This is useful for debugging, learning, and understanding how circuits will function.
- run(): When using actual quantum devices, we can only access the end result of a computation and must sample the results to get a distribution of results. Running the simulator as a sampler mimics this behavior and only returns bit strings as output.

Simulation of a 2-qubit "Bell State":

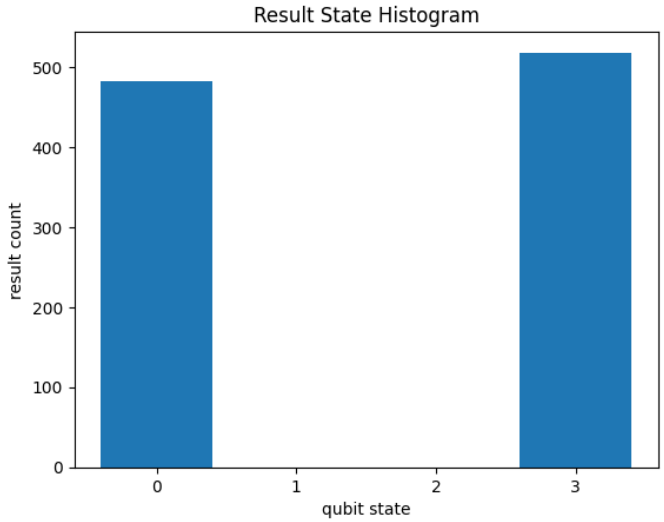
```
# Create a circuit to generate a Bell State:
# 1/sqrt(2) * ( |00⟩ + |11⟩ )
bell_circuit = cirq.Circuit()
q0, q1 = cirq.LineQubit.range(2)
bell_circuit.append(cirq.H(q0))
bell_circuit.append(cirq.CNOT(q0, q1))

# Initialize Simulator
s = cirq.Simulator()

print('Simulate the circuit:')
results = s.simulate(bell_circuit)
print(results)

# For sampling, we need to add a measurement at the end
bell_circuit.append(cirq.measure(q0, q1, key='result'))

# Sample the circuit
samples = s.run(bell_circuit, repetitions=1000)
```

Output:

```
Simulate the circuit:
measurements: (no measurements)

qubits: (cirq.LineQubit(0), cirq.LineQubit(1))
output vector: 0.707|00⟩ + 0.707|11⟩

phase:
output vector: |⟩
```

Visualizing Results:

When using `run()` to get a sample distribution of measurements, you can directly graph the simulated samples as a histogram with `cirq.plot_state_histogram`.

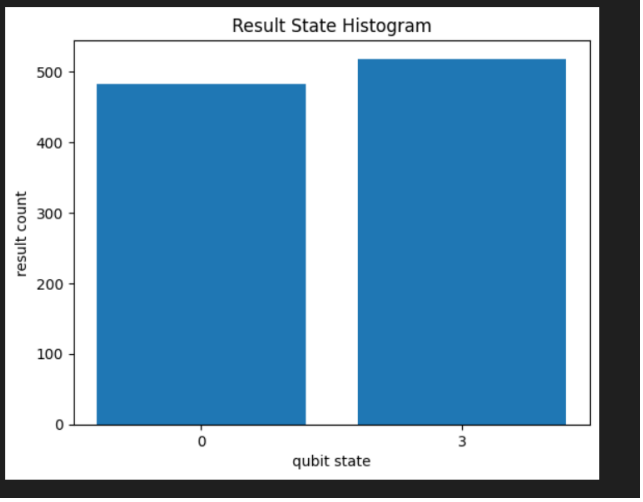


However, this histogram has some empty qubit states, which may become problematic if you work with more qubits. To graph sparse sampled data, first get the `Counts` from your results with its `histogram()` function, and pass that to `cirq.plot_state_histogram`. By collecting the results into counts, all the qubit states that were never seen are ignored.

```
# Pull of histogram counts from the result data structure
counts = samples.histogram(key='result')
print(counts)

# Graph the histogram counts instead of the results
cirq.plot_state_histogram(counts, plt.subplot())
plt.show()
```

Output:

```
Counter({3: 518, 0: 482})
```



Using Parameter Sweeps:

Cirq circuits allow for gates to have symbols as free parameters within the circuit. This is especially useful for variational algorithms, which vary parameters within the circuit in order to optimize a cost function, but it can be useful in a variety of circumstances.

For parameters, Cirq uses the library `sympy` to add `sympy.Symbol` as parameters to gates and operations.

Once the circuit is complete, you can fill in the possible values of each of these parameters with a `Sweep`. There are several possibilities that can be used as a sweep:

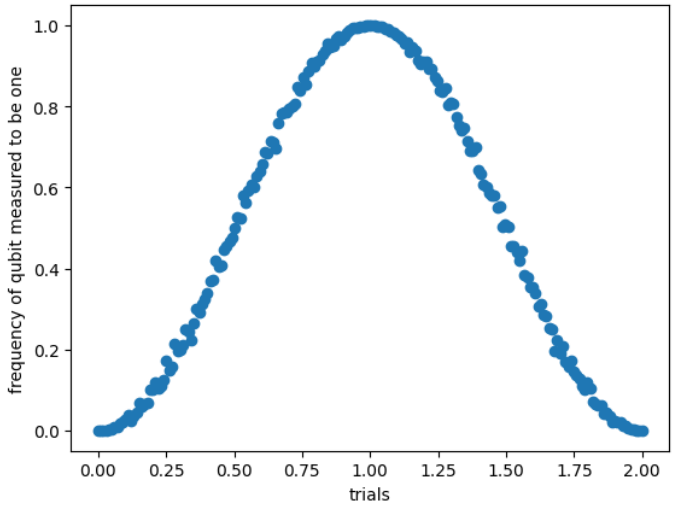
- `cirq.Points`: A list of manually specified values for one specific symbol as a sequence of floats.
- `cirq.Linspace`: A linear sweep from a starting value to an ending value.
- `cirq.ListSweep`: A list of manually specified values for several different symbols, specified as a list of dictionaries.
- `cirq.Zip` and `cirq.Product`: Sweeps can be combined list-wise by zipping them together or through their Cartesian product.

A parameterized circuit and sweep together can be run using the simulator or other sampler by changing `run()` to `run_sweep()` and adding the sweep as a parameter.

Here is an example of sweeping an exponent of a X gate:

```
import sympy

# Perform an X gate with variable exponent
q = cirq.GridQubit(1, 1)
circuit = cirq.Circuit(cirq.X(q) ** sympy.Symbol('t'), cirq.measure(q, key='m'))

# Sweep exponent from zero (off) to one (on) and back to two (off)
param_sweep = cirq.Linspace('t', start=0, stop=2, length=200)

# Simulate the sweep
s = cirq.Simulator()
trials = s.run_sweep(circuit, param_sweep, repetitions=1000)

# Plot all the results
x_data = [trial.params['t'] for trial in trials]
y_data = [trial.histogram(key='m')[1] / 1000.0 for trial in trials]
plt.scatter('t', 'p', data={'t': x_data, 'p': y_data})
plt.xlabel("trials")
plt.ylabel("frequency of qubit measured to be one")
plt.show()
```

Output:



Unitary Matrices and Decompositions:

Many quantum operations have unitary matrix representations. This matrix can be accessed by applying `cirq.unitary(operation)` to that `operation`. This can be applied to gates, operations, and circuits that support this protocol and will return the unitary matrix that represents the object.

```
print('Unitary of the X gate')
print(cirq.unitary(cirq.X))

print('Unitary of SWAP operator on two qubits.')
q0, q1 = cirq.LineQubit.range(2)
print(cirq.unitary(cirq.SWAP(q0, q1)))

print('Unitary of a sample circuit')
print(cirq.unitary(cirq.Circuit(cirq.X(q0), cirq.SWAP(q0, q1))))
```

Output:

```
Unitary of the X gate
[[0.+0.j 1.+0.j]
 [1.+0.j 0.+0.j]]
Unitary of SWAP operator on two qubits.
[[1.+0.j 0.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 1.+0.j 0.+0.j]
 [0.+0.j 1.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 0.+0.j 1.+0.j]]
Unitary of a sample circuit
[[0.+0.j 0.+0.j 1.+0.j 0.+0.j]
 [1.+0.j 0.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 0.+0.j 1.+0.j]
 [0.+0.j 1.+0.j 0.+0.j 0.+0.j]]
```

Decompositions:

Many gates can be decomposed into an equivalent circuit with simpler operations and gates. This is called decomposition and can be accomplished with the `cirq.decompose` protocol.

For instance, a Hadamard H gate can be decomposed into X and Y gates:

```
print(cirq.decompose(cirq.H(cirq.LineQubit(0))))
```

Output:

```
[(cirq.Y**0.5).on(cirq.LineQubit(0)), cirq.XPowGate(global_shift=-0.25).on(cirq.LineQubit(0))]
```

Another example is the 3-qubit Toffoli gate, which is equivalent to a controlled-controlled-X gate. Many devices do not support three qubit gates, so it is important to decompose them into one and two qubit gates.

```
q0, q1, q2 = cirq.LineQubit.range(3)
print(cirq.Circuit(cirq.decompose(cirq.TOFFOLI(q0, q1, q2))))
```

Output:

0:───T────────────────────@──────────────────────────────────────@────────
──────────────────────────────@──────────────────────────────────@────────
─────────────────────────────────

|　　　　|　　　　|　　　　|

1: ───T──────────Y^-0.5────@────Y^0.5─────@────T^-1─────Y^-
0.5────@────────────Y^0.5────@────Y^-0.5────@──────────Y^0.5─────@────Y^-
0.5────@────────Y^0.5─────@──────────────────────────

|　　　　|　　　　|　　　　|

2: ───Y^0.5────X────────────T────Y^-0.5────@────Y^0.5────T────────────Y^-
0.5──────────────@────Y^0.5─────T^-1────Y^-0.5────@────Y^0.5─────T^-1────Y^-
0.5────@────Y^0.5────Y^0.5────X────

```
0: ───T──────────────@──────────────────────────────@──────────────────@──────────────────@────────────────────────
                     │                              │                  │                  │
1: ───T──────Y^-0.5──@──Y^0.5────@──T^-1────Y^-0.5──@────────Y^0.5──@──Y^-0.5──@──────Y^0.5──@──Y^-0.5──@──────Y^0.5───@─────────────
                                 │                               │
2: ───Y^0.5──X───────────T──Y^-0.5──@──Y^0.5──T─────────Y^-0.5──────────@──Y^0.5────T^-1──Y^-0.5──@──Y^0.5────T^-1──Y^-0.5──@──Y^0.5──Y^0.5──X───
```

Transformers:

A transformer can take a circuit and modify it. Usually, this will entail combining or modifying operations to make it more efficient and shorter, though a transformer can, in theory, do any sort of circuit manipulation.

For example, the `cirq.merge_single_qubit_gates_to_phxz` transformer will take consecutive single-qubit operations and merge them into a single `PhasedXZ` operation.

```
q = cirq.GridQubit(1, 1)
c = cirq.Circuit(cirq.X(q) ** 0.25, cirq.Y(q) ** 0.25, cirq.Z(q) ** 0.25)
print(c)
c = cirq.merge_single_qubit_gates_to_phxz(c)
print(c)
```

Output:

(1, 1): ——X^0.25——Y^0.25——T——

(1, 1): ——PhXZ(a=0.304,x=0.333,z=0.142)——

Classical Computing vs Quantum Computing:

Shor's Algorithm:

an example of a quantum algorithm that is known to be faster than any known classical algorithm. One such algorithm is Shor's algorithm for factoring large numbers. However, please note that this is a theoretical demonstration as current quantum computers (as of 2021) do not have enough qubits to factor large numbers that classical computers can't handle.

Here's a simple implementation of Shor's algorithm using Google's Cirq library:

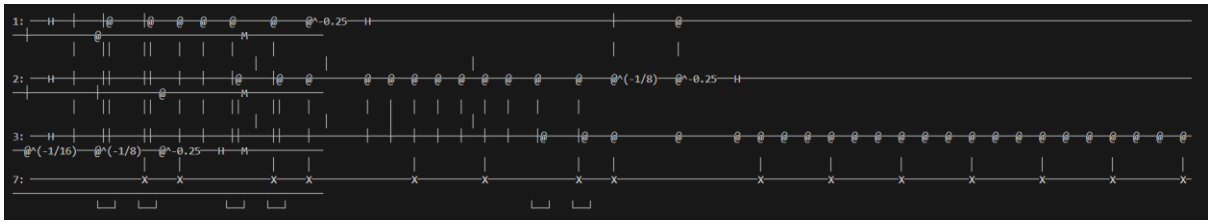
```
import cirq
import math

# Number to factor
N = 15

# Quantum register
qubits = cirq.LineQubit.range(8)

# Quantum circuit
circuit = cirq.Circuit()

# Hadamard gates
for i in range(4):
    circuit.append(cirq.H(qubits[i]))

# Controlled-U gates
for control in range(4):
    for k in range(2**control):
        circuit.append(cirq.CNOT(qubits[control], qubits[4]))
        circuit.append(cirq.CNOT(qubits[control], qubits[5]))
        circuit.append(cirq.CNOT(qubits[control], qubits[7]))

# Inverse QFT
for i in range(4):
    for j in range(i):
        circuit.append(cirq.CZPowGate(exponent=-0.5/(2**(i-j))).on(qubits[j], qubits[i]))
    circuit.append(cirq.H(qubits[i]))

# Measurement
circuit.append(cirq.measure(*qubits[:4], key='result'))

# Display the circuit
print(circuit)
```

This code creates a quantum circuit that implements Shor's algorithm for factoring the number 15.

The measurement result gives the period of the function: $f(x) = a^x \bmod N$,

Where a is a randomly chosen number less than N. Once the period is found, it can be used to find a factor of N

Output:



Please note that this is a simplified version of Shor's algorithm and doesn't include the classical post-processing steps required to extract the factors from the period. Also, it assumes that the period found is a power of 2, which is not always the case.

This code will run on a simulator, but it won't give meaningful results on current quantum hardware due to noise and the limited number of qubits. However, if we had a fault-tolerant quantum computer with thousands of qubits, we could use this algorithm to factor large numbers much faster than any known classical algorithm.

The best-known classical algorithm for factoring is the General Number Field Sieve (GNFS), which has a sub-exponential time complexity of exp
((64/9)1/3(logN)1/3(loglogN)2/3)
. On the other hand, Shor's algorithm has a polynomial time complexity of O
((logN)2(loglogN)(logloglogN))
using fast multiplication.
This means that for large enough N
, Shor's algorithm will be faster than the GNFS. However, this doesn't mean that Shor's algorithm will be faster for all N
, especially for small N
where the overhead of setting up and running the quantum algorithm might outweigh the benefits.
Also, it's important to note that this is a theoretical comparison. Current quantum computers (as of 2021) do not have enough qubits to run Shor's algorithm for large enough N
to outperform classical computers. However, if we had a fault-tolerant quantum computer with thousands of qubits, we could use Shor's algorithm to factor large numbers much faster than any known classical algorithm.

Grover's Algorithm:

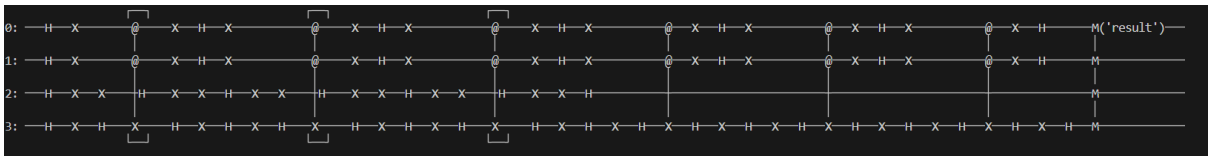
Grover's algorithm, a quantum algorithm for searching an unsorted database with N
items. It's one of the simplest and most commonly used examples of quantum speedup.
Here's a simple implementation of Grover's algorithm using Google's Cirq library:

```
import cirq
import numpy as np

# Function to create a Grover circuit
def grover_circuit(target_state):
    # Number of qubits
    n = len(target_state)

    # Quantum register
    qubits = cirq.LineQubit.range(n)

    # Quantum circuit
    circuit = cirq.Circuit()

    # Hadamard gates
    circuit.append(cirq.H(qubit) for qubit in qubits)

    # Oracle and diffusion operator
    for _ in range(int(np.pi/4*np.sqrt(2**n))):
        # Oracle
        circuit.append(cirq.X(qubit) for qubit in qubits)
        circuit.append(cirq.H(qubits[-1]))
```

```
        # Replace MCX with a combination of CCX and additional gates as necessary
        circuit.append(cirq.CCX(qubits[0], qubits[1], qubits[-1]))
        circuit.append(cirq.H(qubits[-1]))
        circuit.append(cirq.X(qubit) for qubit in qubits)

        # Diffusion operator
        circuit.append(cirq.H(qubit) for qubit in qubits)
        circuit.append(cirq.X(qubit) for qubit in qubits)
        circuit.append(cirq.H(qubits[-1]))
        # Replace MCX with a combination of CCX and additional gates as necessary
        circuit.append(cirq.CCX(qubits[0], qubits[1], qubits[-1]))
        circuit.append(cirq.H(qubits[-1]))
        circuit.append(cirq.X(qubit) for qubit in qubits)
        circuit.append(cirq.H(qubit) for qubit in qubits)

    # Measurement
    circuit.append(cirq.measure(*qubits, key='result'))

    return circuit

# Target state
target_state = '1101'

# Create and print the Grover circuit
circuit = grover_circuit(target_state)
print(circuit)
```

The best known classical algorithm for this problem is linear search, which has a time complexity of O(N), On the other hand, Grover's algorithm has a time complexity of O(N). This means that for large enough N, Grover's algorithm will be faster than linear search. However, this doesn't mean that Grover's algorithm will be faster for all N, especially for small N where the overhead of setting up and running the quantum algorithm might outweigh the benefits.

Output:



Results and Discussion:

The proposed hybrid quantum-classical approach demonstrates promising results in overcoming scalability and fault tolerance limitations observed in purely quantum systems. By leveraging classical resources intelligently, significant improvements in performance and reliability are achieved across various high-performance applications. Furthermore, advancements in error correction techniques and hardware designs contribute to mitigating the impact of noise and enhancing overall system robustness.

Conclusion:

In conclusion, quantum computing holds immense potential for revolutionizing high-performance computing across diverse domains. Through a hybrid quantum-classical approach and innovative error correction strategies, scalability, fault tolerance, and noise resilience can be significantly improved. The proposed system architecture represents a crucial step towards realizing the full potential of quantum computing in tackling complex real-world problems.

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Appendix:

https://drive.google.com/drive/folders/1eZme3Hk4e8C85k2_ON_gRpoNHgezXO6X?usp=sharing