[1] Building Abstractions with functions

1.1 Getting Started

- Computer fundamentals:
    - representing information
    - specifying logic to process it
    - designing abstractions that manage the complexity of that topic
- Recommended textbook: Structure and Interpretation of Computer Programs (SICP)

1.1.1 Programming in Python

1.1.2 Installing Python 3

1.1.3 Interactive Sessions

- In an interactive Python session, you type some Python code after the “>>>”. The Python interpreter reads and executes what you type, carrying out your various commands.
- To start an interactive session, type “python3” at a terminal prompt.
- Interactive controls: each session keeps a history of what you have typed. To access that history, press [Ctrl-p] (previous) or [Ctrl-n] (next). You can exit a session with [Ctrl-d], which discards the history. Up and down arrows also cycle through history on some systems.

1.1.4 First Example

- Statements & Expressions: Python code consists of expressions and statements. Broadly, computer programs consist of instructions to either computer some value or carry out some actions. Statements typically describe actions. Expressions typically describe computations.
- Functions: functions encapsulates logic that manipulates data.
- Objects: an object seamlessly bundles together data and the logic that manipulates that data, in a way that manages the complexity of both.
- Interpreters: evaluating compound expressions requires a precise procedure that interprets code in a predictable way. A program that implements such a procedure, evaluating compound expressions, is called an interpreter. When compared with other computer programs, interpreters for programming languages are unique in their generality.

1.1.5 Errors

- Computers are rigid: even the smallest spelling and formatting changes can cause unexpected output and errors. Learning to interpret errors and diagnose the cause of unexpected errors is called debugging.
- Some guiding principles of debugging are:
    - Test incrementally
    - Isolate errors: trace the error to the smallest fragment of code you can before correcting it
    - Check your assumptions
    - Consult others

1.2 Elements of Programming

- Programs serve to communicate ideas among members for a programming community. Thus, programs must be written for people to read, and only incidentally for machines to execute.
- When we describe a language, we should pay particular attention to the means that the language provide for combining simple ideas to from more complex ideas. Each powerful language has three such mechanisms:
    - primitive expressions and statements: represent the simplest building blocks

means of combination: by which compound elements are built from simpler ones

means of abstraction: by which compound elements can be named and manipulated as units

1.2.1 Expressions

- primitive expressions like numbers in base 10: 42
- compound expressions like numbers combined with mathematical operators: -1 - -1

1.2.2 Call Expressions

- The most important kind of compound expression is a call expression, which applies a function to some arguments.
- A call expression can have subexpressions.
- The order of the arguments in a call function matters.
- Function notation has three principle advantages over the mathematical convention of infix notation

functions may take an arbitrary number of arguments

function notation extends in a straightforward way to nested expressions

mathematical notations has a great variety of forms, while this complexity can be unified via the notation of call expressions

1.2.3 Importing Library Functions

- Python defines a very large number of functions, including the operator functions mentioned in the preceding section, but does not make all of their names available by default. Instead, it organizes the functions and other quantities that it knows into modules., which together comprise the Python library. To use these elements, one imports them. Below are examples:

```
>>> from math import sqrt
>>> from operator import add, mul
```

1.2.4 Names and the Environment

- A critical aspect of a programming language it the means it provides for using names to refer to computational objects. If a value as been given a name, we say that the name binds to the value.
- In Python, we can establish new bindings using the assignment statement, like

```
>>> radius = 10
>>> radius
10
```

Here "=" is called the assignment operator. Assignment is our simplest means of abstraction.

- The possibility of binding names to values and later retrieving those values by names means that the interpreter must maintain some sort of memory that keeps track of the names, values and bindings. This memory is called an environment.
- Names can also be bound to functions, like

```
>>> f = max
>>> f
<built-in function max>
>>> f(1, 2, 3, 4)
4
```

- In Python, names are called variable names or variables, because they can be bound to different values in the course of executing a program. When a name is bound to a new value through assignment, it is no longer bound to any previous value. One can even bind built-in names to new values, like max.

- We can assign multiple values to multiple names in a single statement, like
    area, circumference = pi * radius * radius, 2 * pi * radius
    note that after assignment, even if we change the value of radius, area will not change
(because in nature, variables are bound to values, not expressions)

1.2.5 Evaluating Nested Expressions

- To evaluate a call expression, Python will do the following:
    evaluate the operator and operand subexpressions
    apply the function that is the value of the operator subexpression to the arguments that are
the values of the operand subexpressions
- Note that the evaluation procedure is recursive in nature. If we draw each expression that we evaluate, we can visualize the hierarchical structure of this process. This illustrations is called an expression tree. In computer science, trees conventionally grow from the top down. The objects at each point in a tree are called nodes, here they are expressions paired with values.

1.2.6 The Non-Pure Print Function

- Pure functions: have some input (arguments) and return some output (the result of applying them). Pure functions have the property that applying them has no effects beyond returning a value. Moreover, a pure function must always return the same value when passed in same arguments.
- Non-pure function: can generate side effects, which make some changes to the state of the interpreter or computer. A common side effect is to generate additional output beyond the return value, using the print function. The value that print returns is always None (a special Python value that represents nothing)
- Below is a special case:

```
>>>print(print(1), print(2))
1
2
None, None
```

#Note that None evaluates to nothing, and will not be displayed by the interpreter as a value, like

```
>>> None
```

1.3 Defining New Functions

- 3 elements which make Python and other programming languages so powerful:
    Numbers and arithmetic operations are primitive built-in data values and functions
    Nested function application provides a means of combining operations
    Binding names to values provides a limited means of abstraction
- Function definition is a much more powerful abstraction technique by which a name can be bound to compound operation, which can then be referred to as a unit.
- How to define a function:

```
def <name> (<formal parameters>):
    return <return expression>
```

The return expression is not evaluated right away; it is stored as part of the newly defined function and evaluated only when the function is eventually applied.

- We can use the defined function as a building block in defining other function. Actually, user-defined functions are used in exactly the same way as built-in functions

1.3.1 Environments

- An environment in which an expression is evaluated consists of a sequence of frames, depicted as boxes. Each frame contains bindings, each of which associates a name with its corresponding value.
- global frame
- The name of a function can be repeated twice, once in the frame and again as part of the function itself. The name appearing in the function is called the intrinsic name. The name in a frame is a bound name. There is a difference between the two: different names may refer to the same function, but that function itself has only one intrinsic name. The name bound to a function in a frame is the one used during evaluation. The intrinsic name of a function does not play a role.
- Function signature: a description of the formal parameters of a function, like max(…). Note that the "…" here means max can take an arbitrary number of arguments.

1.3.2 Calling User-Defined Functions

- Applying a user-defined function introduces a second local frame, which is only accessible to that function. To apply a user-defined function to some arguments:

    Bind the arguments to the names of the function's formal parameters in a new local frame.

    Execute the body of the function in the environment that starts with the frame

Note that the first step: bound names, is very tricky! As we have mentioned before, binding is only between values and variable names. So here what we are really doing, is not changing names for the parameters, but evaluating their values and assigning these values to new names! [see HW1 Q5]

- Name evaluation: a name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.

1.3.3 Example: Calling a User-Defined Function

1.3.4 Local Names

- One detail of a function's implementation that should not affect the function's behavior is the implementer's choice of names for the function's formal parameters. The simplest function of it is that the parameter names of a function must remain local to the body of the function.

1.3.5 Choosing Names

- Important principles in choosing function and parameter names:

    Function names are lowercase, with words separated by underscores. Descriptive names are encouraged.

    Function names typically evoke operations applied to arguments by the interpreter, or the name of the quantity that results.

    Parameter names are lowercase, with words separated by underscores. Single-word names are preferred.

    Parameter names should evoke the role of the parameter in the function, not just the kind of argument that is allowed.

    Single letter parameter names are acceptable when their role is obvious, but avoid l, O or I to avoid confusion with numerals.

1.3.6 Functions as Abstractions

- To master the use of a functional abstraction, it is often useful to consider 3 core attributes:

    the domain of a function is the set of arguments it can take

    the range of a function is the set of values it can return the intent of a function is the relationship it computes between inputs and output (as well as any side effects it might generate)

1.3.7 Operators

- distinguish among / (normal division, which results to a floating point, or truediv in operator), // (rounds the result to an integer, or floordiv in operator), and % (mod)

1.4 Designing Functions

- The qualities of good functions:
  - each function should have exactly one job
  - don't repeat yourself (DRY principle)
  - functions should be defined generally

1.4.1 Documentation

- A function definition will often include documentation describing the function, called a docstring, which must be indented along with the function body. Docstrings are conventionally triple quoted. The first line describes the job of the function in one line. The following lines can describe arguments and clarify the behavior of the function.
- When you call [help] with the name of a function as an argument, you see its docstring (type [q] to exit), like

```
>>> help(pressure)
```

- Comments in Python can be attached to the end of a line following the "#" symbol

1.4.2 Default Argument Values

- In Python, we can provide default values for the arguments of a function. When calling that function, arguments with default values are optional. If they are not provided, then the default value is bound to the formal parameter name instead. Below is an example:

```
>>> def pressure(v, t, n = 6):
        ……
>>> pressure (1, 273.25) # pass in 2 parameters
2269
>>> pressure (1, 273.15, 6)
2269
```

1.5 Control

- Control statements are statements that control the flow of a program's execution based on the results of logical comparisons.

1.5.1 Statements

- So far, we have seen 3 statements already: def, assignment, and return.
- Rather than being evaluated, statements are executed. Each statement describes some change to the interpreter state, and executing a statement applies that change.

1.5.2 Compound Statements

- In general, Python code is a sequence of statements. A simple statement is a single line that doesn't end in a colon. A compound statement is so-called because it's composed of other statements. Compound statements typically span multiple lines and start with a one-line header ending in colon, which identifies the type of statement. Together, a header and an indented suite of statements is called a clause. A compound statement consists of one or more clauses.

```
<header>:
      <statement>
      <statement>
      ……
<separating header>:
      <statement>
      <statement>
      …..
```

- We can catalog the statements we have learnt:
  Expressions, return statement, and assignment statements are simple statements.
  A def statement is a compound statement made up of clauses as the function body.
- We can understand multi-line programs: to execute a sequence of statements, execute the first statement. If that statement does not redirect control, then proceed to execute the rest of the sequence of statements, if any remain.

1.5.3 Defining Functions II: Local Assignment

- Whenever a user-defined function is called, the sequence of clauses in the suite of its definition is executed in a local environment - an environment starting with a local frame created by calling that function. A return statement redirects control: the process of function terminates whenever the first return statement is executed, and the value of the return expression is the returned value of the function being applied.

1.5.4 Conditional Statements

- A conditional statement in Python consists of a series of headers and suits: a required "if" clause, an optional sequence of "elif" clauses, and finally an optional "else" clause:

```
>>> if <expression>:
      <suite>
>>> elif <expression>:
      <suite>
>>> else:
      <suite>
```

- The computational process of executing a conditional clause follows:
  Evaluate th header's expression
  If it is a true value, execute the suite. Then, skip all subsequent clauses in the conditional statement.
- The expressions inside the header statements of conditional blocks are in boolean contexts: their truth value matter to control flow, but otherwise their values are not assigned or returned. Python includes several false values, including 0, None, "", [], and the boolean value False. All other numbers are true values.
- Python has two boolean values: True and False.
- Python has three boolean operators: and, or, and not, the first two of which has the so-called short-circuit evaluation behavior.

1. [not] returns the opposite truth value of the following expression (so not will always return either True of False).
2. [and] evaluates expressions in order and stops evaluating (short-circuits) once it reaches the first false value, and then returns it. If all values evaluate to a true value, the last value is

returned.

3. [or] short-circuits at the first true value and returns it. If all values evaluate to a false value, the last value is returned.

```
>>> not None
True
>>> -1 and 0 and 1
0
>>> False or 999 or 1/0
999
```

1.5.5 Iteration

- A while clause structure is like:

```
>>> while <expression>:
        <suite>
```

1.5.6 Testing

- Testing a function is the act of verifying that the function's behavior matches expectations. A test is a mechanism for systematically performing this verification. Tests typically take the form of another function that contains one more sample calls to this function being tested. The returned value is then verified against an expected result. Unlike most functions, which are meant to be general, tests involve selecting and validating calls with specific argument values. Tests also serve as documentation: they demonstrate how to call a function and what argument values are suitable.
- assert: when the expression being asserted evaluates to a true value, executing an assert statement has no effect. When it is a false value, assert causes an error that halts the execution.

```
>>> assert f(8) == 13
>>> def f_test():
        assert f(2) == 1
        assert f(3) == 1
        assert f(50) == 89
```

- Python provides a convenient method for placing simple tests directly in the docstring of a function. THE first line of a docstring should contain a one-line description of the function, followed by a blank line. A detailed description of arguments and behavior may follow. In addition, the docstring may include a sample interactive session that calls the function. Then, the interaction can be verified via the doctest module. Below is an example of usage:

```
>>> def f(n):
        """ Return the sum of the first n elements.

        >>> f(2)
        55
        >>> f(10)
        458
        """
        …..
>>> from doctest import testmod
```

```
>>> testmod()
    TestResults(failed = 0, attempted = 2)
```

- When writing Python in files, rather than directly into the interpreter, tests are typically written in the same file or a neighboring file with the suffix "_test.py". Then all doctests can be run by starting Python with the doctest command line option: [python3 -m doctest (-v) <python_file>]
- A test that applies a single function is called a unit test.
- autograder usage:

1. To prevent the ok autograder from interpreting print statements as output: print with 'DEBUG:' at the front of the outputted line
2. To open n interactive terminal to investigate a failing test for question sum_digits in assignment lab01: [python3 ok -q sum_digits -i]
3. To look at an environment diagram to investigate a failing test for question sum_digits in assignment lab01: [python3 ok -q sum_digits —trace]

1.6 Higher-Order Functions

- We need to construct functions that can accept other functions as arguments, or return functions as values. Functions that manipulate functions are called higher-order functions.

1.6.1 Functions as Arguments

- Below is an example:

```
>>> def summation(n, term):
    ……
>>> def identity(x):
    ……
>>> def sum_naturals(x, identity):
    ……
```

1.6.2 Functions as General Methods

- With higher-order functions, we begin to see a more powerful kind of abstraction: some functions express general methods of computation, independent of the particular functions they call.
- We learnt about two related big ideas in computer science:

1. Naming and functions allow us to abstract away a vast amount of complexity.
2. It is only by virtue of the fact that we have an extremely general evaluation procedure for the Python language that small components can be composed into complex processes.

- 1.6.3 Defining Functions III: Nested Definitions
- There are two negative consequences of passing functions as arguments:

1. The global frame becomes cluttered with names of small functions, which must all be unique.
2. We are constrained by particular function signatures

-> Nested function definitions address both of these problems, but require us to enrich our environment model.

- Below is an example:

```
>>> def sqrt(a):
    def sqrt_update(x):
        return average(x, a/x)
    def sqrt_close(x):
```

```
            return approx_eq(x * x, a)
        return improve(sqrt_update, sqrt_close)
```

Note that we now place function definitions inside the body of other definitions.

- Like local assignment, local def statements only affect the local frame. These functions are only in scope while sqrt is being evaluated. Locally defined functions also have access to the name bindings in the scope in which they are defined. This discipline of sharing names among nested definitions are called lexical scoping.
- We require two extensions to our environment model to enable lexical scoping:

1. Each user-defined function has a [parent environment where it was defined.
2. When a user-defined function is called, its local frame extends its parent environment.

- Functions values each have a new annotation that we will include in environment diagrams from now on: a parent. The parent of a function value is the first frame of the environment in which that function was defined. Functions without parent annotations were defined in the global environment. When a user-defined function is called, the frame created has the same parent as that function.

1.6.4 Functions as Returned Values

- Function composition: h(x) = f(g(x))

1.6.5 Example: Newton's Method

1.6.6 Currying

- Given a function f(x, y), we can define a function g such that g(x)(y) = f(x, y). Here, g is a higher-order function that takes in a single argument x and returns another function that takes in a single argument y. This transformation is called currying. Currying is useful when we require a function that takes in only a single argument (like the map function).
- Inverse Currying transformation:

1. given f, return g <-currying

```
>>> def curry(f):
    def g(x):
        def h(y):
            return f(x, y)
        return h
    return g
```

2. given g, return f <-uncurrying

```
>>> def uncurry(g):
    def f(x, y):
        return g(x)(y)
    return f
>>> uncurry(curry(f))
f
```

1.6.7 Lambda Expressions

- In Python, we can create function values on the fly using lambda expressions, which evaluate to unnamed functions. A lambda expression evaluates to a function that has a single return expression as its body. Assignment and control statements are not allowed. Below is an example:

```
>>> def compose(f, g):
    return lambda x: f(g(x))
```

We can understand the structure of a lambda expression by constructing a corresponding sentence:

lambda	x	:	f(g(x))
A function that	takes x	and returns	f(g(x))

- The result of a lambda expression is called a lambda function. It has no intrinsic name, but otherwise it behaves like any other function.

```
>>> s = lambda x: x* x
>>> s
<function <lambda> at 0xf3f490>
>>> s(12)
144
>>> (lambda x: x*x)(12)
144
```

- The main difference between lambda expressions and def statements is that, only the def statements give the function intrinsic names. All lambda expressions share the name [Greek letter(x)]

1.6.8 Abstractions and First-Class Functions

- Programming languages impose restrictions on the ways in which computational elements can be manipulated. Elements with the fewest restrictions are said to be have first-class status. Some of the "rights and privileges" of first-class elements are:

1. They may be bound to names.
2. They may be passed as arguments to functions.
3. They may be returned as the results of functions.
4. They may be included in data structures.

- First-class functions, in nature, are functions that can be manipulated as values. They are opposite to higher-order functions, which take functions as argument values, or return a function as a return value

1.6.9 Function Decorators

- Python provides special syntax to apply higher-order functions as part of executing a def statement, called a decorator. Below is the most common examples — trace:

```
>>> def trace(fn):
    def wrapped(x):
        print('->', fn, '(', x, ')')
        return fn(x)
    return wrapped
>>> @trace
    def triple(x):
        return 3 * x
>>> triple(12)
-> <function triple at 0x102a39848> (12)
36
```

In code, the decorator is equivalent to:

```
>>> def triple(x):
    return 3*x
>>> triple = trace(triple)
```

- The decorator symbol "@" may also be followed by a call expression. The expression following @ is evaluated first (just as the name trace was evaluated first), the def statement second, and finally the result of evaluating the decorator expression is applied to the newly defined function, and the result if bound to the name in the def statement.

1.7 Recursive Functions

- A function is called recursive if the body of the function calls the function itself.

1.7.1 The Anatomy of Recursive Functions

- A common pattern can be found in the body of many recursive functions. the body begins with a base case, a conditional statement that defines the behavior of the function for the inputs that are simplest to process. Some recursive functions will have multiple base cases. They are then followed by one or more recursive calls, which always have a certain character: they simplify the original problem incrementally.
- Treating a recursive call as a functional abstraction has been called a recursive leap of faith. We define a function in terms of itself, but simply trust that the simpler cases will work correctly when verifying the correctness of the function. Verifying the correctness of a recursive function is a form of proof by induction.

1.7.2 Mutual Recursion

- When a recursive procedure is divided among two functions that call each other, the functions are said to be mutually cursive.

1.7.3 Printing in Recursive Functions

1.7.4 Tree Recursions

- Another common pattern of computation is called tree recursion, in which a function calls itself more than once.
- A function with multiple recursive calls is said to be tree recursive, because each of call branches into multiple smaller calls, each of which branches into yet smaller calls, just as the branch of a tree become smaller but more numerous as they extend from the trunk.

1.7.5 Example: Partitions

[2] Building Abstractions with Data
2.1 Introduction
2.1.1 Native data Types
- Every value in Python has a class that determines what type of value it is. The built-in [type] function allows us to inspect the class of any value.

```
>>> type(2)
<class 'int'>
```

- Native data types have the following properties:

1. There are expressions that evaluate to values of native types, called literals.
2. There are built-in functions and operators to manipulate values of native types.

- Python includes three numeric types: int (integers), float (real numbers) and complex (complex numbers).
- The difference between int and float is very important: int objects represent integers exactly; float objects can represent a wide range of fractional numbers, but not all numbers can be represented exactly.
- There are also non-numeric types of data, such as the bool class for values True and False.

2.2 Data Abstraction
- The general technique of isolating the parts of a program that deal with how data are represented from the parts that deal with how data are manipulated is a powerful design methodology called data abstraction. Data abstraction makes programs much easier to design, maintain, and modify.

2.2.1 Example: Rational Numbers
2.2.2 Pairs
- List:

```
>>> pair = [10, 20]
>>> x, y = pair
>>> x
10
>>> pair[0]
10
>>> from operator import getitem
>>> getitem(pair, 0)
10
```

2.2.3 Abstraction Barriers
- Function can be divided into several layers of abstraction. In each layer, the functions enforce an abstraction barrier. These functions are called by a higher level and implemented using a lower level of abstraction. An abstraction barrier violation occurs whenever a part of the program that can use higher level function instead uses a function in a lower level.

2.2.4 The Properties of data

2.3 Sequences
- A sequence is an ordered collection of values. There are many different kinds of sequences, but they all share common behaviors:

1. A sequence has a finite length. An empty sequence has length 0.
2. A sequence has an element corresponding to any non-negative integer index less than its length, starting from 0 for the first element.

2.3.1 Lists

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1, 0]
30
```

2.3.2 Sequence Iteration

- In many cases, we would like to iterate over the elements of a sequence and perform some computation for each element in turn. This pattern is so common that Python has an additional control statement to process sequential data: the for loop.
- A for statement consists of a single clause with the form:

```
for <name> in <expression>:
    <suite>
```

And a for statement is executed by the following procedure:

1. Evaluate the header <expression>, which must yield an iterable value.
2. For each element value in that iterable value, in order:
    1. Bind <name> to that value in the current frame
    2. Execute the <suite>

- A common pattern in programs is to have a sequence of elements that are themselves sequences, but all of a fixed length. A for statement may include multiple names in its header to "unpack" each element sequence into its respective elements. Such pattern of binding multiple names to multiple values in a fixed-length sequence is called sequence unpacking.

```
>>> pairs = [[1, 2], [2, 2], [2, 3], [3, 4]]
>>> same = False
>>> for x, y in pairs:
    if x == y:
        same = True
>>> same
True
```

- A range is another built-in type of sequence in Python, which represents a range of integers. range takes in two integer arguments: the first number and one beyond the last number in the desired range. If only one argument is given, it is interpreted as one beyond the last value for a range that starts at 0.
- A common convention is to use a single underscore character for the name in the for header if the name is unused in the suite. This underscore is just another name in the environment as far as the interpreter is concerned, but has a conventional meaning among programmers that indicates the name will not appear in any future expressions.

2.3.3 Sequence Processing

- Many sequence processing operations can be expressed by evaluating a fixed expressions for each element in a sequence and collecting the resulting values in a result sequence. In Python, a list comprehension is an expression that performs such a computation.

```
>>> odds = [1, 3, 5, 7, 9]
>>> [x+1 for x in odds]
[2, 4, 6, 8, 10]
```

Notice the above for keyword is contained within square brackets, therefore, it is not part of a for statement, but instead part of a list comprehension. The subexpression x+1 is evaluated with x bound to each element of adds in turn, and the resulting value are collected into a list.

- List comprehension can also help to select a subset of values that satisfy some condition.

```
>>> [x for x in odds if 25 % x == 0]
[1, 5]
```

The general form of a list comprehension is

[<map expression> for <name> in <sequence expression> if <filter expression>]

To evaluate a list comprehension, Python evaluates the <sequence expression>, which must return an iterable value. Then, for each element in order, the element value is bound to <name>, the filter expression is evaluated, and if it yields a true value, the map expression is evaluated. The values of the map expression are collected into a list.

- A third common pattern in sequence processing is to aggregate all values in a sequence into a single value. The built-in functions sum, min, and max are all examples.
- The common patterns we have observed in sequence processing can be expressed using higher-order functions.

2.3.4 Sequence Abstraction

- A value can be tested for membership in a sequence. Python has two operators [in] and [not in] that evaluates to True or False depending on whether an element appears in a sequence.

```
>>> digits
[1, 8, 2, 8]
>>> 2 in digits
True
```

- Sequence contain smaller sequences within them. A slice of a sequence is any contiguous span of the original sequence, designated by a pair of integers. As with the range constructor, the first integer indicates the starting index of the slice and the second indicates one beyond the ending index. In Python, sequence slicing is expressed similarly to element selection, using square brackets. A colon separates the starting and ending indices. Any bound that is omitted is assumed to be an extreme value: 0 for the starting index, and the length of the sequence for the ending index.

```
>>> digits[0:2]
[1, 8]
>>> digits[1:]
[8, 2, 8]
```

2.3.5 Strings

- The native data type for text in Python is called a string, and corresponds to the constructor str.
- String literals can express arbitrary text, surrounded by either single or double quotation marks (If you use double quotes, then you can use single quotes inside of it. Otherwise you cannot.).

- The elements of a string are themselves strings that have only a single character. A character is any single letter of the alphabet, punctuation mark, or other symbol. Unlike many other programming languages, Python does not have a separate character type.
- Like lists, strings can also be combined via addition and multiplication.

```
>>> 'Berkeley' + ', CA'
'Berkeley, CA'
>>> 'Shabu' * 2
'Shabu Shabu'
```

- The membership operator [in] applies to strings, but has an entirely different behavior than when it is applied to sequences. It matches substrings rather than elements.

```
>>> 'here' in 'Where's she'
True
```

- Strings are not limited to a single line. Triple quotes delimit string literals that span multiple lines. '\n' is a single element that represents a new line, and is considered a single character for the purpose of length and element selection.
- A string can be created from any object in Python by calling the str constructor function with an object value as its argument

2.3.6 Trees

- Our ability to use lists as the elements of other lists provides a new means of combination in our programming language. The ability is called a closure property of a data type. In general, a method for combining data values has closure property if the result of combination can itself be combined using the same method. Closure is the key to power in any means of combination because it permits hierarchical structures.
- Nesting lists within lists can introduce complexity. The tree is a fundamental data abstraction that imposes regularity on how hierarchical values are structured and manipulated. A tree has a root label and a sequence of branches. Each branch of a tree is a tree. A tree with no branches is called a leaf. Any tree contained with a tree is called a sub-tree of that tree. The root of each sub-tree is called a node in that tree. A tree is well-formed only if it has a root label and all branches are also trees.

2.3.7 Linked Lists

- A common representation of a sequence constructed from nested pairs is called a linked list.
- Linked lists have recursive structure: the rest of a linked list or 'empty'.

2.4 Mutable Data

- We need strategies to help us structure large systems to be modular, meaning that they divide naturally into coherent parts that can be separately developed and maintained. One powerful technique for creating modular programs is to incorporate data that may change state over time. In this way, a single data object can represent something that evolves independently of the rest of the program. Adding state to data is a central ingredient of a paradigm called object-oriented programming.

2.4.1 The Object Metaphor

- Objects combine data values with behavior. Objects are both information and processes, bundled together to represent the properties, interactions, and behaviors of complex things.

```
>>> from datetime import date
```

The name [date] is bound to a class. As we have seen, a class represents a kind of value. Individual dates are called instances of that class. Instances are constructed by calling the class on arguments that characterize the instance.

- Objects have attributes, which are named values that are part of the object. in Python, like many other programming languages, we use dot notation to designate an attribute of an object.

<expression>.<name>

The <expression> evaluates to an object, and <name> is the name of an attribute for that object. These attributes are not available in the general environment.

- Objects also have methods, which are function-valued attributes. By implementation, methods are functions that compute their values from both their arguments and their object. The expression for attribute also applies to methods.
- All values in Python are objects.

2.4.2 Sequence Objects

- Instances of primitive built-in values such as numbers are immutable. The values themselves cannot change over the course of program execution. Lists on the other hand are mutable.
- Mutable objects are used to represent values that change over time. An object may have changing properties due to muting operations. Most changes are performed by invoking methods on list objects.
- Here are many list modification operations:

```
>>> chinese = ['coin', 'string', 'myriad']
>>> suits = chinese
>>> suits.pop()
'myriad' #remove and return the final element
>>> suits.remove('string') #remove the first element that equals the argument
>>> suits.append('cup') #add an element to the end
>>> suits.extend(['sword', 'club']) #add all elements of a sequence to the end
>>> suits[2] = 'spade' #replace an element
>>> suits[0:2] = ['heart', 'diamond'] #replace a slice
>>> suits
['heart', 'diamond', 'spade', 'club']
```

Note all of these mutation operations change the value of the list; they do not create new list objects. Since the object bound to the name chinese has also changed, because it is the same list object that was bound to suits! This behavior is different from what we have learned from primitive data types. With mutable data, methods called on one name can affect another name at the same time.

- Lists can be copied using the [list] constructor function. Changes to one list do not affect another, unless they share structure.

```
>>> new = list(suits)
```

Note that we can use slicing to shallow copy lists, but actually the mechanism is: for lists or something that we must store in memory (not other prmitive data types like ints), we copied the references to them instead of the items themselves. Therefore, I can say that both elements [is] the same thing, but modifying one element means changing that reference without change the item, and the other will not be affected.

```
>>> suits.insert(2, 'Joker') #insert an element at index 2, shifting the rest
```

- Because two lists may have the same contents but in fact be different lists, we require a means to test whether two objects are the same. Python includes two comparison operators, called [is] and [is not], that test whether two expressions in fact evaluate to the identical object. Two objects are identical if they are equal in their current value, and any change to on will always be reflected in the other. Identity is a stronger condition than equality.

```
>>> suits is ['heart, 'diamond', 'spade', 'club'] #check for identity
False
>>> suits == ['heart, 'diamond', 'spade', 'club'] #check for equality
True
```

- A list comprehension always creates a new list.
- It is very dangerous to use a mutable data type as the default value for a function argument (list lists). It is because in the function definition of Python, all default values for function arguments are only computed once, instead of computing each time the function is called. Therefore, every time the function is called, its default value will be changed (since it is mutable).
-
- A tuple, an instance of the built-in tuple type, is an immutable sequence. Tuples are created using a tuple literal that separates element expressions by commas. Parentheses are optional but used commonly in practice. Any objects can be placed into tuples.

```
>>> 1, 2 + 3
(1, 5)
```

There are also empty and one-element tuples which has special syntax.

```
>>> ()
()
>>> tuple()
()
>>> (10, )
(10, )
```

Here are some methods of tuples which are shared between it and lists: in & not in; mul and add; len; []; slice

```
>>> code = (1, 1, 2, 2, 3, 4, 3, 4)
>>> code.count(2)
2
>>> code.index(3)
4
```

But note that the methods for manipulating the contents of a list are not available for tuples, because they are immutable. It is possible to change the value of a mutable element contained within a tuple.

```
>>> nested = (10, 20, [30, 40])
>>> nested[2].pop()
```

Note that you can change the elements inside of a mutable element, but you cannot change the whole element itself, for it is part of the immutable tuple.

```
>>> lst = [1, 2, [0]]
>>> lst[2] = [1]
```

Error
Tuples are often used implicitly in multiple assignment. An assignment of two values to two names creates a two-element tuple and then unpacks it.
2.4.3 Dictionaries
- Dictionaries are Python's built-in data type for storing and manipulating correspondence relationships. A dictionary contains key-value pairs, where both the keys and values are objects. The purpose of a dictionary is to provide an abstraction for storing and retrieving values that are indexed not by consecutive integers, but by descriptive keys.
- Look up values:

>>> numerals = ['i': 1.0, 'v': 5, 'x':10]
>>> numerals['x']
10

- A dictionary can have at most one value for each key. Adding new key-value pairs and changing the existing value for a key can both be achieved with assignment statements:

>>> numerals['i'] = 1
>>> numerals['l'] = 50

Note that the newly added key-values pair is not added to the end of the dictionary, because dictionaries are unordered collections of key-value pairs. When we print a dictionary, the keys are values are rendered in some order, but as users of the language we cannot predict what the order will be. The order may change when running a program multiple times.
- The methods keys, values and items all return iterable values.
- A list of key-value pairs can be converted into a dictionary by calling the [dict] constructor.
- Dictionaries have some restrictions:

1. A key of a dictionary cannot be or contain mutable value.
2. There can be at most one value for a given key.

According to the first restriction, tuples are often used as keys in dictionaries instead of lists. On the other hand, a list not allowed be used as keys for dictionaries (unhashable list). Tuples can contain anything, including lists; but those containing a list as an element cannot be used as keys.
- A useful method implemented by dictionaries is [get], which returns either the value for a key, if the key is present or a default value. The arguments to get are the key and the default value.

>>> numerals.get('a', 0)
0
>>> numerals.get('v', 5)
5

- We can also remove one key-value pair using key

>>> numerals.pop('v')
5

- Dictionaries have a comprehension syntax analogous to those of lists. A key expression and a vaue expression are separated by a colon. Evaluating a dictionary comprehension creates a new dictionary object.

>>> [x: x * x fr x in range(3, 6)]
[3 : 9; 4 : 16; 5 : 25]
2.4.4 Local State
- Lists and dictionaries have local state: they are changing values that have some particular

contents at any point in the execution of a program. The word "state" implies an evolving process in which that state may change.

- Functions can also have local state.

nonlocal balance

The nonlocal statement declares that whenever we change the binding of the name balance, the binding is changed in the first frame in which balance is already bound. If balance has not previously been bound to a value, then the nonlocal statement will give an error.

- Ever since we first encountered nested def statements, we have observed that a locally defined function can look up names outside of its local frames. No nonlocal statement is required to access a non-local name. By contrast, only after a nonlocal statement can a function change the binding of names in these frames.

2.4.5 The Benefits of Non-Local Assignment

- Non-local assignment is an important step on our path to viewing a program as a collection of independent and autonomous objects, which interact with each other but each manage their own internal state. In particular, non-local assignment has given us the ability to maintain some state that is local to a function, but evolves over successive calls to that function.

2.4.6 The Cost of Non-Local Assignment

- When two names are both bound to a function, it does matter whether they are bound to the same function, or different instances of that function. By introducing non-pure functions that change the non-local environment, we have changed the nature of expressions. An expression that contains only pure function calls is referentially transparent: its value does not change if we substitute one of its subexpression with the value of that subexpression. Re-binding operations violate the conditions of referential transparency because they do more than return a value.

2.4.7 Implementing Lists and Dictionaries

- None is None, but [] is not [] (they are equal, but not identical).
- We design our mutable linked list function as a dispatch function, and its arguments are first a message, followed by additional arguments to parameterize that method. This message is a string naming what the function should do. Dispatch functions are effectively many functions in one: the message determines the behavior of the function, and the additional arguments are used in that behavior.
- The approach, which encapsulates the logic for all operations on a data value within one function that responds to different messages, is a discipline called message passing. A program that uses message passing defines dispatch functions, each of which may have local state, and organizes computation by passing "messages" as the first argument to those functions. The messages are strings that correspond to particular behaviors.

2.4.8 Dispatch Dictionaries

2.4.9 Propagating Constraints

- Declarative programming: in which a programmer declares the structure of a program to be solved, but abstracts away the details of exactly how the solution to the problem is computed.
- Computer programs are traditionally organized as one-directional computations, which perform operations on pre-specified arguments to produce desired outputs. On the other hand, we often want to model systems in terms of relations among quantities. Such a model is not one-directional.
- We can use a message passing system to coordinate constraints and connectors. Constraints are

dictionaries that do not hold local states themselves. Their responses to messages are non-pure functions that change the connectors that they constrain. Connectors are dictionaries that hold a current value and respond to messages that manipulate that value. Constraints will not change the value of connectors directly, but will do so by sending messages, so that the connector can notify other constraints in response to the change. In this way, a connector represents a number, but also encapsulates connector behavior.