CS 61B Spring 2018 — More Asymptotic Analysis

Discussion 8: March 6, 2018

Here is a review of some formulas that you will find useful when doing asymptotic analysis.

- $\sum_{i=1}^{N} i = 1 + 2 + 3 + 4 + \cdots + N = \frac{N(N+1)}{2} = \mathbf{\frac{N^2+N}{2}}$
- $\sum_{i=0}^{N-1} 2^i = 1 + 2 + 4 + 8 + \cdots + 2^{N-1} = 2 \cdot 2^{N-1} - 1 = \mathbf{2^N - 1}$

The goal of this discussion is to help us analyze in trees, but I didn't get it...

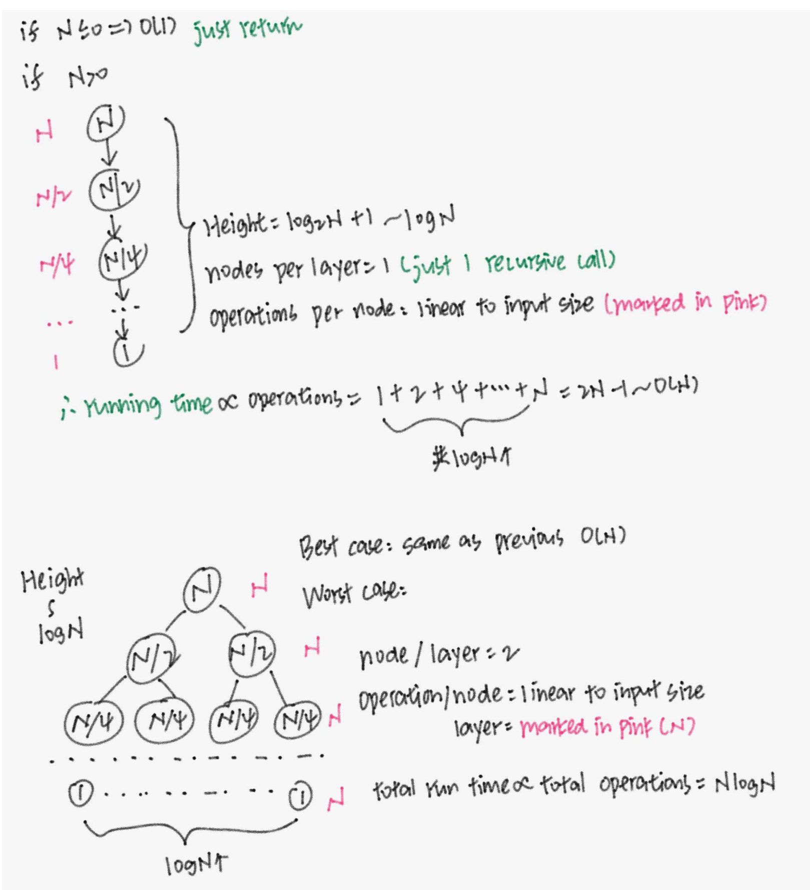
Intuition

For the following recursive functions, give the worst case and best case running time in the appropriate $O(\cdot)$, $\Omega(\cdot)$, or $\Theta(\cdot)$ notation.

1.1 Give the running time in terms of N.

Worst:Big Theta(N)

```java
public void andslam(int N) {
    if (N > 0) {
        for (int i = 0; i < N; i += 1) {
            System.out.println("datboi.jpg");
        }
        andslam(N / 2);
    }
}
```



next question

1.2 Give the running time for `andwelcome(arr, 0, N)` where N is the length of the input array `arr`.

```java
public static void andwelcome(int[] arr, int low, int high) {
    System.out.print("[ ");
    for (int i = low; i < high; i += 1) {
        System.out.print("loyal  ");
    }
    System.out.println("]");
    if (high - low > 0) {
        double coin = Math.random();
        if (coin > 0.5) {
            andwelcome(arr, low, low + (high - low) / 2);
        } else {
            andwelcome(arr, low, low + (high - low) / 2);
            andwelcome(arr, low + (high - low) / 2, high);
        }
    }
}
```

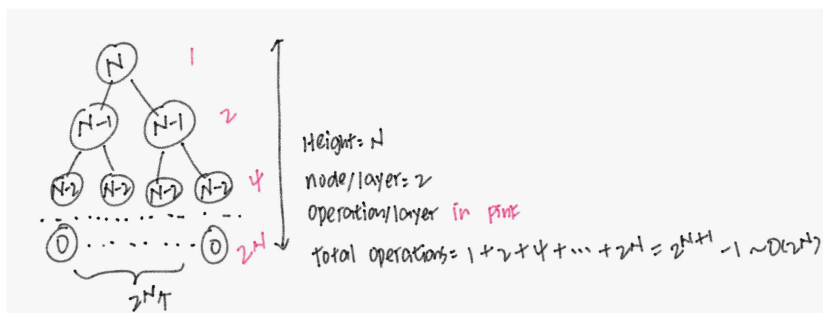
Big Theta(N or logN)

two lines here!

1.3 Give the running time in terms of N.

```java
public int tothe(int N) {
    if (N <= 1) {
        return N;
    }
    return tothe(N - 1) + tothe(N - 1);
}
```

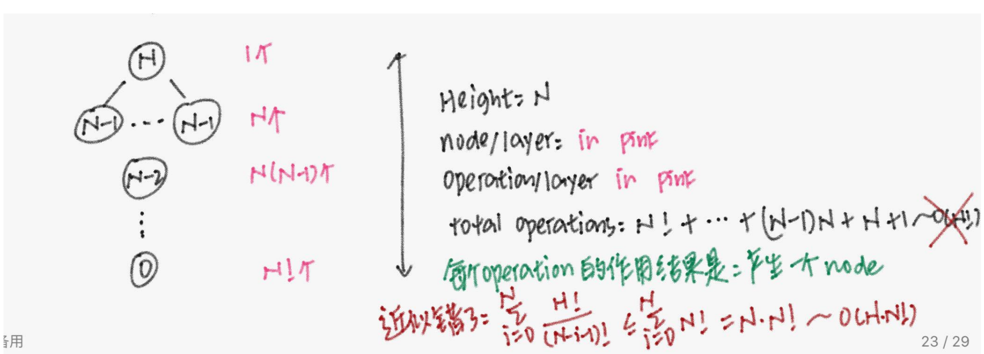
Big Theta(2^N)



1.4 Give the running time in terms of N.

```java
public static void spacejam(int N) {
    if (N <= 1) {
        return;
    }
    for (int i = 0; i < N; i += 1) {
        spacejam(N - 1);
    }
}
```

Worst: Big Theta(N*N!)



Hey you watchu gon do

2.1 For each example below, there are two algorithms solving the same problem. Given the asymptotic runtimes for each, is one of the algorithms **guaranteed** to be faster? If so, which? And if neither is always faster, explain why.

(a) Algorithm 1: $\Theta(N)$, Algorithm 2: $\Theta(N^2)$

For large input, Algorithm 2 is faster

(b) Algorithm 1: $\Omega(N)$, Algorithm 2: $\Omega(N^2)$

No guarantee

(c) Algorithm 1: $O(N)$, Algorithm 2: $O(N^2)$

No guarantee

(d) Algorithm 1: $\Theta(N^2)$, Algorithm 2: $O(\log N)$

For large input, Algorithm 2 is faster

(e) Algorithm 1: $O(N \log N)$, Algorithm 2: $\Omega(N \log N)$

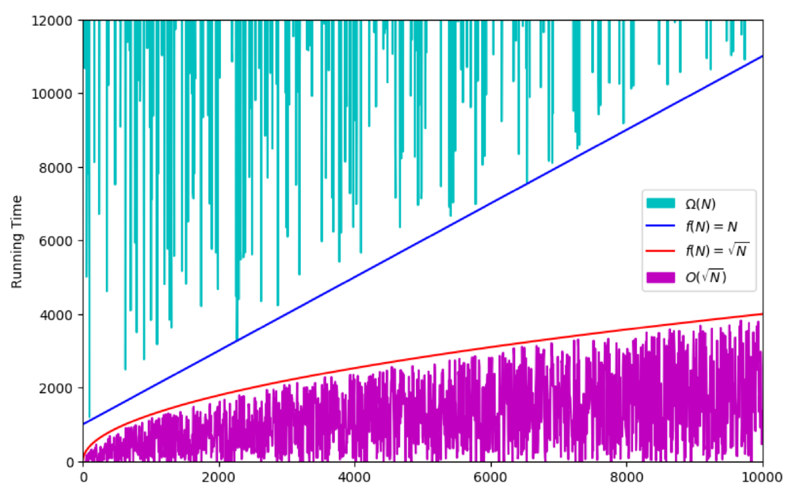
For large input, Algorithm 1 is faster No, they can both be Big Theta(N log N)

Would your answers above change if we did not assume that N was very large (for example, if there was a maximum value for N, or if N was constant)?

Sure

Asymptotic Notation

3.1 Draw the running time graph of an algorithm that is $O(\sqrt{N})$ in the best case and $\Omega(N)$ in the worst case. Assume that the algorithm is also trivially $\Omega(1)$ in the best case and $O(\infty)$ in the worst case.



Extra learned

abou

3.2 Are the statements in the right column true or false? If false, correct the asymptotic notation ($\Omega(\cdot)$, $\Theta(\cdot)$, $O(\cdot)$). Be sure to give the tightest bound. $\Omega(\cdot)$ is the opposite of $O(\cdot)$, i.e. $f(n) \in \Omega(g(n)) \iff g(n) \in O(f(n))$.

$f(n) = 20501$	$g(n) = 1$	$f(n) \in O(g(n))$	True
$f(n) = n^2 + n$	$g(n) = 0.000001n^3$	$f(n) \in \Omega(g(n))$	False -> Big O
$f(n) = 2^{2n} + 1000$	$g(n) = 4^n + n^{100}$	$f(n) \in O(g(n))$	True
$f(n) = \log(n^{100})$	$g(n) = n \log n$	$f(n) \in \Theta(g(n))$	False -> Big O
$f(n) = n \log n + 3^n + n$	$g(n) = n^2 + n + \log n$	$f(n) \in \Omega(g(n))$	True
$f(n) = n \log n + n^2$	$g(n) = \log n + n^2$	$f(n) \in \Theta(g(n))$	True
$f(n) = n \log n$	$g(n) = (\log n)^2$	$f(n) \in O(g(n))$	False -> Big Omega

Fall 2015 *Extra*

4.1 If you have time, try to answer this challenge question. For each answer true or false. If true, explain why and if false provide a counterexample.

(a) If $f(n) \in O(n^2)$ and $g(n) \in O(n)$ are positive-valued functions (that is for all n, $f(n), g(n) > 0$), then $\frac{f(n)}{g(n)} \in O(n)$.

False; like f(n) is Big Theta (n^2), g(n) is Big Theta(1), then f(n)/g(n) is actually Big Theta(n^2)

(b) If $f(n) \in \Theta(n^2)$ and $g(n) \in \Theta(n)$ are positive-valued functions, then $\frac{f(n)}{g(n)} \in \Theta(n)$.

True

Theorem 1. *If $f(n) \in \Theta(n^2)$ and $g(n) \in \Theta(n)$ are positive-valued functions, then $\frac{f(n)}{g(n)} \in \Theta(n)$.*

Proof. Given that $f \in \Theta(n^2)$ is positive, by definition there exists $k_0, k_0' > 0$ such that for all $n > N$, the following holds.

$$k_0 n^2 \leq f(n) \leq k_0' n^2$$

Similarly, $g \in \Theta(n)$ implies there exists $k_1, k_1' > 0$ such that

$$k_1 n \leq g(n) \leq k_1' n$$

Now consider $\frac{f(n)}{g(n)}$.

$$\frac{f(n)}{g(n)} \leq \frac{k_0' n^2}{k_1 n} = \frac{k_0' n}{k_1} \in O(n) \qquad \frac{f(n)}{g(n)} \geq \frac{k_0 n^2}{k_1' n} = \frac{k_0 n}{k_1'} \in \Omega(n)$$

As $\frac{f(n)}{g(n)}$ is in $O(n)$ and $\Omega(n)$ then it is in $\Theta(n)$. $\square$