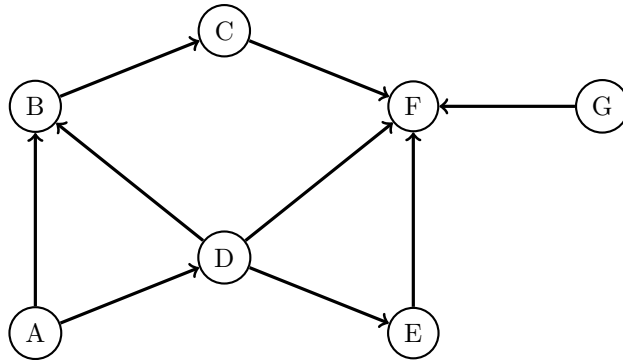


Graphs



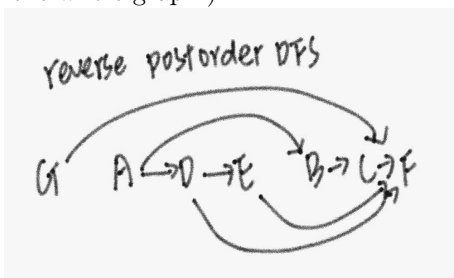
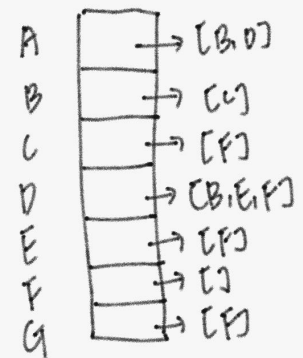
src \	A	B	C	D	E	F	G
A	0	1	0	1	0	0	0
B	0	0	1	0	0	0	0
C	0	0	0	0	0	1	0
D	0	1	0	0	1	1	0
E	0	0	0	0	0	1	0
F	0	0	0	0	0	0	0
G	0	0	0	0	0	1	0

- 1.1 Write the graph above as an adjacency matrix, then as an adjacency list. What would be different if the graph were undirected instead?

- 1.2 Give the DFS preorder, DFS postorder, and BFS order of the graph traversals starting from vertex A. Break ties alphabetically.

DFS preorder: A-B-C-F-D-E
DFS postorder: F-C-B-E-D-A
BFS order: A-B-D-C-E-F

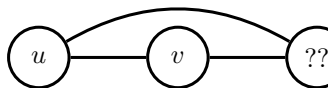
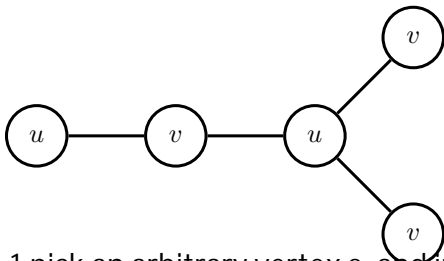
- 1.3 Give a valid topological sort of the graph. (Hint: Consider the reverse postorder of the whole graph.)



Graph Algorithm Design

we don't need this at all!

- 2.1 An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets U and V such that every edge connects an item in U to an item in V . For example below, the graph on the left is bipartite, whereas on the graph on the right is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?



the algorithm can be simplified with mark (keeping an extra array of size V). we traverse E times and mark a vertex to be u or v . When we want to mark a vertex to be u but it has already been marked v , or vice versa, return false. $\rightarrow O(V+E)$

1. pick an arbitrary vertex s , and insert it into U
2. traverse (BFS) all the other vertices E times
if their distance to s is even, insert them to U ; otherwise V
3. if U and V contain some same vertices, return false; otherwise true
 $O(E^2)$

the solution is traversal $s(O(V))$, find all vertices that have an edge pointing to s ($O(E)$), and run BFS on all those vertices ($O(V+E)$) $\rightarrow O(EV)$

- 2.2 Provide an algorithm that finds the shortest cycle (in terms of the number of edges used) in a directed graph in $O(EV)$ time and $O(E)$ space, assuming $E > V$.

keep track of a length = INT_MAX
for every vertex v :

do DFS & incrementing depth:

if the path cannot have a second $v \rightarrow$ continue; otherwise if length > depth \rightarrow length = depth
return length I think my solution is okay, although the space complexity is not $O(E)$

- 2.3 Consider the following implementation of DFS, which contains a crucial error:

```
create the fringe, which is an empty Stack
push the start vertex onto the fringe and mark it
while the fringe is not empty:
    pop a vertex off the fringe and visit it
    for each neighbor of the vertex:
        if neighbor not marked:
            push neighbor onto the fringe
            mark neighbor
```

we should only mark a vertex as true when visiting it

Give an example of a graph where this algorithm may not traverse in DFS order.

