ReCEval: Evaluating Reasoning Chains via Correctness and Informativeness

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What does this paper state?

😵‍💫 Existing methods focus on only whether the reasoning leads to the correct answer.
😵‍💫 They don't really look at **how good the reasoning process itself** is.
😵‍💫 Cause shortcuts that give the *right* answer w/ *poor* logic.

Key Goal : Evaluate reasoning by analyzing if each step is **correct** and **informative** .

Author's Stance : The authors see reasoning chains as "informal proofs" and want to evaluate them based on if each step is logically sound AND helpful for the final answer.

Motivation: Why previous metrics are not enough? #1

Previous metrics focus on **surface-level similarity** or **likelihood** :

- **N-gram Overlap Metrics** (BLEU, ROUGE)
    - Compare surface-level token overlap with some reference
    - **Issue** : Don't capture multi-step logic, can't detect contradictions
- **Embedding-based Scores** (BERTScore, MoverScore)
    - Measure semantic similarity w.r.t. reference text
    - **Issue** : "logically correct reasoning" $\neq$ "fluent text"
- **Generative Probability Metrics** (BARTScore, CTC)
    - Use a pre-trained/fine-tuned LM to assess fluency/likelihood
    - **Issue** : May favor well-formed text but overlook stepwise correctness or new info

Motivation: Why previous metrics are not enough? #2

- **Existing Reasoning Metrics** (ROSCOE, etc.)
  - Partially address chain-of-thought evaluations
  - **Limitations** : Often rely on textual similarity or partial checks—may miss fine-grained step correctness or “helpfulness” (informativeness)

ReCEval specifically focuses on:

1. **Correctness** (both local and global consistency)
2. **Informativeness** (detects redundancy vs. genuine contribution)

The Proposed Framework: ReCEval

- **Reasoning Chain:** A sequence of multi-step rationales that lead to a final answer.
  - $R = \{s^{(1)}, s^{(2)}, \ldots, s^{(n)}\}$, Step $s^{(i)}$ eventually leads to the final (predicted) answer $\hat{a}$.
- **Step Decomposition:** Each step $s^{(i)}$ is broken down into smaller units.
- **Two Core Dimensions** :
  i. **Correctness** : Are intermediate conclusions valid and consistent with previous steps/context?
  ii. **Informativeness** : Does each step add meaningful new information that helps derive the final answer?

Step Decomposition: RCUs

We decompose each step $s^{(i)}$ into smaller units: **Reasoning Content Units (RCUs)** .

- **Premise RCUs** : Supporting facts or assumptions within that step.
  - $\{\mathrm{RCU}_{p_j}^{(i)}\}$ for $j = 1, \ldots, t$
- **Conclusion RCU** : The key inference or statement that the step arrives at. : $\mathrm{RCU}_c^{(i)}$

A step $s^{(i)}$ can be seen as a sequence of RCUs:

$$s^{(i)} = \left\{\mathrm{RCU}_{p_1}^{(i)}, \ldots, \mathrm{RCU}_{p_t}^{(i)},\ \mathrm{RCU}_c^{(i)}\right\}.$$

Why? : Fine-grained analysis: Evaluate each conclusion precisely against the premises.
How? : **SRL model*** decomposes a sentence into **'subject-verb-object' frames** .

Example Step

SRL model decomposes a sentence with [] brackets.

Then we compute the correctness / informativeness of each RCU😄

Correctness Evaluation, at two aspects

- **Intra-step** correctness:
  - Checks if the conclusion RCU is logically entailed by the premise RCUs.
  - Uses **NLI (Natural Language Inference)** or **PVI (pointwise V-information)** to score entailment.
- **Inter-step** correctness:
  - Ensures each conclusion does **not** contradict prior conclusions or the original input.
  - Uses an **NLI-based approach to detect contradictions** across steps.

Intra-step Correctness: How to Measure?

Entailment-based approach checks if the conclusion RCU is entailed by the premise RCUs. Formally:

$$\text{intra-correct}_{\text{entail}}^{(i)} = P_{\text{entail}}\Big(\mathrm{RCU}_p^{(i)};\ \mathrm{RCU}_c^{(i)} \Big),$$

where P_{entail} is the probability that $\mathrm{RCU}_c^{(i)}$ is entailed by the concatenated premises $\mathrm{RCU}_p^{(i)}$.

PVI-based approach uses pointwise V-information:

$$\text{intra-correct}_{\text{PVI}}^{(i)} = \mathrm{PVI}\Big(\mathrm{RCU}_p^{(i)} \to \mathrm{RCU}_c^{(i)}\Big).$$

Here, $\mathrm{PVI}(x \to y)$ measures how much information x provides to generate y under a model family V.

Inter-step Correctness

We also require that the conclusion in step i not contradict any previous information (the input X or previous conclusions). Specifically:

$$\text{inter-correct}^{(i)} = 1 - \max_{r \in X \cup \{\mathrm{RCU}_c^{(j)}\}_{j<i}} P_{\text{contr}}\big(r;\ \mathrm{RCU}_c^{(i)}\big),$$

where $P_{\text{contr}}(r; \mathrm{RCU}_c^{(i)})$ is the probability of contradiction between r and $\mathrm{RCU}_c^{(i)}$.

> **Note** : Authors use a similar NLI-based approach to detect contradictions.

Informativeness Evaluation

> **Key Question** : Does this step reduce uncertainty about the final answer?

- Uses **V-information / PVI** : Higher difference ⇒ more informative.

Measure whether adding step $s^{(i)}$ actually helps move closer to the final answer $\hat{a}$. Using conditional PVI:

$$\text{info-gain}_{\text{PVI}}^{(i)} = \text{PVI}\left(s^{(i)} \;\to\; \hat{a} \;\middle|\; s^{(<i)}\right).$$

Intuitively, it compares the likelihood of generating $\hat{a}$ **with** step $s^{(i)}$ included v.s. **without** it.

Putting It All Together

- **ReCEval** runs through each step, computes:
  - $\text{score}_{\text{intra}} = \min_{1 \le i \le n} \left\{\text{intra-correct}^{(i)}\right\},$
  - $\text{score}_{\text{inter}} = \min_{1 \le i \le n} \left\{\text{inter-correct}^{(i)}\right\}$
  - $\text{score}_{\text{info}} = \min_{1 \le i \le n} \left\{\text{info-gain}^{(i)}\right\}.$
- Chain-level scores: **minimum** of all **minimized** steps' scores

Outcome : A single final correctness score + a single final informativeness score.

Experiments #1 Can ReCEval Be Used for Evaluation?

Goal : Meta-evaluate ReCEval against existing metrics and compare its correlation with human judgments.

Dataset Name	Domain	Input / Output
Entailment Bank	Science QA	Perbutated egations, hallucinations
GSM-8K	Math Word Problems	Human-annotated error categories
DROP	Discrete Reasoning	Paragraphs to answers

- **Baselines** : ROUGE, BERTScore, ROSCOE
- **Metrics** : Somer’s D (correlation with human judgments)

Results: Higher Sensitivity to Errors

Detected logic/hallucination mistakes better than typical similarity metrics (ROUGE, BERTScore, etc.) or older reasoning metrics (ROSCOE).

Results: Better Correlation with Human Judgments

Strong performance across multiple error types (factuality, redundancy, logic).

Experiments #2 Can ReCEval Improve Downstream Tasks?

> **Goal** : Evaluate if selecting chains with higher ReCEval scores leads to improved final accuracy in tasks like math word problem solving.

Setup :

- **Task** : GSM-8K (math word problems)
- **Model** : LM (e.g., FLAN T5-XXL) with multiple candidate chains-of-thought (CoT).
- **Selection** : Compare different chain selection strategies:
  - Baseline: **Greedy** / **Random**
  - Ours: **ReCEval** -guided (rank by correctness + informativeness, pick top)

🤔 If ReCEval can help select better chains, we should see higher final answer accuracy.

Results: Downstream Gains 👍

Chain sampling based on ReCEval scores leads to improved final accuracy in math word problems.

Key Findings

1. **Higher Sensitivity to Errors**
   - RECEVAL detects subtle logic/hallucination mistakes better than typical similarity metrics (ROUGE, BERTScore, etc.) or older reasoning metrics (ROSCOE).
2. **Better Correlation with Human Judgments**
   - Strong performance across multiple error types (factuality, redundancy, logic).
3. **Downstream Gains**
   - Selecting chains with higher ReCEval scores leads to improved final accuracy in tasks like math word problem solving.

Takeaways and Future Directions

- **Formalizes "Reasoning Chain Quality"** :
  - Beyond just the final answer → focuses on **stepwise correctness** and **informativeness** .
- **Generalizable to Various Reasoning Tasks** :
  - Science QA, math word problems, discrete reading comprehension, etc.
- **Opens Doors to** :
  - More advanced chain-of-thought evaluation.
  - Automatic selection of the best rationales to boost QA accuracy.
  - Potential synergy with larger LMs (GPT-3.5, etc.) for "self-check" of reasoning.

In a sentence : ReCEval is a practical, reference-free way to measure how logical and genuinely useful each step of a model's reasoning is.

Appendix

Further Related Paper

If you are interested in evaluating mathematical reasoning, you can check out the followings:

- GAIR-Lab are publishing a series of papers on reasoning chains (also evaluating them):
    - Evaluating Mathematical Reasoning Beyond Accuracy (AAAI 2025, *Oral*)
    - O1 Replication Journey: A Strategic Progress Report -- Part 1

Meta-Evaluation Metrics: Somer's D

To measure how well these metrics correlate with *ground truth* error annotations (e.g., "hallucination" or "redundancy"), the authors use **Somer's D** based on **Kendall's τ** :

$$D_{S|E} = \frac{\tau(E,S)}{\tau(E,E)},$$

where

- $E \in \{0,1\}$ indicates whether a given error type is present,
- S is the score assigned by a particular metric.

A higher Somer's $D \rightarrow$ better alignment with human judgments of error presence/absence.

Step Decomposition: How to Split Steps into RCUs

1. **Run an SRL model on the step**
    - For a step $s^{(i)}$ (which may be one or more sentences), feed it into an off-the-shelf SRL system (e.g., AllenNLP).
    - The SRL model outputs multiple "frames" in the form of $[\mathrm{ARG0}, \mathrm{VERB}, \mathrm{ARG1}, ...]$. These are the potential **Reasoning Content Units (RCUs)** .
2. **Remove overlapping frames**
    - If the SRL model returns multiple frames that overlap heavily in text, filter them to keep only the largest or most complete frames.
    - For instance, if one frame is almost entirely contained within a larger one, you would keep the larger one and discard the subset.

Step Decomposition: How to Split Steps into RCUs (cont.)

3. **Identify premise vs. conclusion**
   - Use simple heuristics (or rules) to decide which frames are "premise RCUs" and which frame is the "conclusion RCU."
   - For example:
     - After **"because"** , **"since"** , or **"due to"** → **premise** .
     - After **"so"** , **"thus"** , or **"hence"** , **conclusion** .
     - If there is no explicit cue word, you can still treat the final statement or main claim as the conclusion, and the rest as premises.
4. **Result**
   - Each step $s^{(i)}$ is split into:

$$s^{(i)} = \{\underbrace{\mathrm{RCU}_{p_1}^{(i)}, \mathrm{RCU}_{p_2}^{(i)}, \ldots,}_{\text{premises}} \underbrace{\mathrm{RCU}_{c}^{(i)}}_{\text{conclusion}}\}.$$

Example Step Decomposition

> "Allen is 12 years old and Bob is 15, so Bob is older."

1. **SRL Frames** might produce something like:
   - Frames: `(Allen) [is] (12 years old)`, `(Bob) [is] (15)`, `(Bob) [is] (older)`
2. **Heuristic labeling** :
   - **"so"** near the end ⇒ The phrase "Bob is older" is the conclusion.
   - The rest ("Allen is 12 years old and Bob is 15") forms the premises.

Hence:

- **Premise RCUs** : $\{\text{"Allen is 12 years old"},\ \text{"Bob is 15"}\}$
- **Conclusion RCU** : $\{\text{"Bob is older"}\}$

ROSCOE: A Fine-Grained Scorer for Reasoning Chains

Evaluate multi-step reasoning on **four** dimensions, each in $[0, 1]$.

1. **Semantic Alignment (SA)**
    - Measures alignment of hypothesis steps $\{h_i\}$ to source s (and reference r).

$$\text{Faithfulness-Step}(h \to s) \;=\; \frac{1}{N}\sum_{i=1}^{N}\text{r-align}(h_i \to s)$$

2. **Semantic Similarity (SS)**
    - Embedding-based measure of how similar the overall chain is to source or reference.

$$\text{Info-Chain}(h \to s) \;=\; \frac{1 + \cos(\mathbf{h}, \mathbf{s})}{2}$$

higher: more semantically aligned at the chain level.

ROSCOE: Logical Inference & Language Coherence (cont.)

3. **Logical Inference (LI)**
   - Uses an NLI model (contradiction probability p_{contr}) to detect logical conflicts:

$$\text{Self-Consistency}(h_i \leftrightarrow h_j) = 1 - \max_{i<j} p_{\text{contr}}\big(h_i, h_j\big).$$

   - Punishes chains with **contradictory steps** .

4. **Language Coherence (LC)**
   - Checks grammar $\left(p_{\text{gram}}\right)$ and perplexity (PPL).
   - Example:

$$\text{Perplexity-Chain}(h) = \frac{1}{\text{PPL}(h)},$$

   inverting PPL to keep scores in $[0, 1]$.

What's Good?

- **Fine-Grained** : Multiple angles (alignment, similarity, logical inference, coherence).
- Can be **Reference-free** or **Reference-based** .
- Intuitive stepwise measures (e.g., alignment vectors per step).

What's Not So Good?

- SA/SS rely on **embedding similarity** → can be confused by rephrasings or partial logic.
- LI depends on **NLI models** → might miss complex multi-step contradictions.
- LC focuses on **language fluency** → a chain can be grammatically fine but logically wrong.

Does ReCEval "Fix" ROSCOE?

✅ Yes, Partly

- **Step-level correctness** (intra-step entailment) addresses a gap in ROSCOE's alignment-based approach.
- **Information gain** is more direct than "repetition detection" or "semantic coverage," since it asks *how* each step reduces uncertainty about the final answer.

❌ No, Not Completely

- **ROSCOE** offers a *suite* of metrics
  - some can detect grammar or coherence issues (LC) that ReCEval does not focus on.
- Large language models used in ReCEval for NLI/PVI can themselves be imperfect.
  - If the chain depends on deeper background knowledge, an NLI model could fail—just as embedding-based approaches sometimes do.