Risk-averse Distributional Reinforcement Learning

A CVaR optimization approach

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Outline

1. Introduction
   - Motivation
   - Reinforcement Learning
   - Risk
   - Risk-averse Reinforcement Learning
2. CVaR Value Iteration
   - Previous results
   - Linear-time improvement
3. CVaR Q-learning
   - Var-based policy improvement
4. Deep CVaR Q-learning

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Ultimate goals of AI

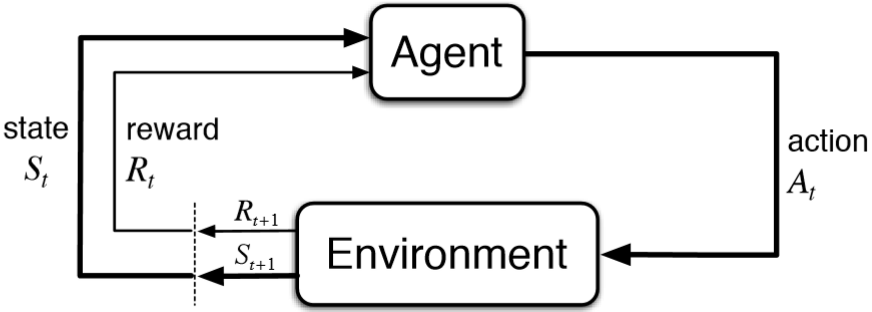
General AI

- Learning from experience
- Learning *tabula rasa*
- Beyond purpose-specific AI
- Beyond human-level performance

Safe AI

- Avoiding catastrophic events
- Robust to environment changes or adversaries

Reinforcement Learning



Recent successes

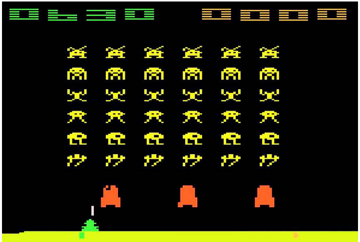


Figure: Atari games

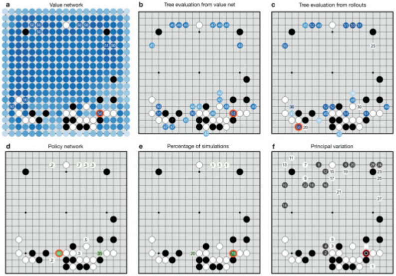


Figure: AlphaGo

Recent successes

Figure: LOXM - online trader

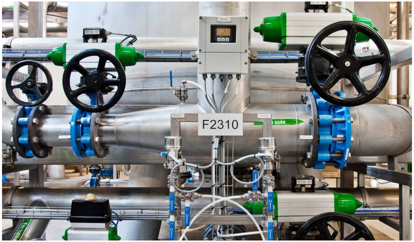
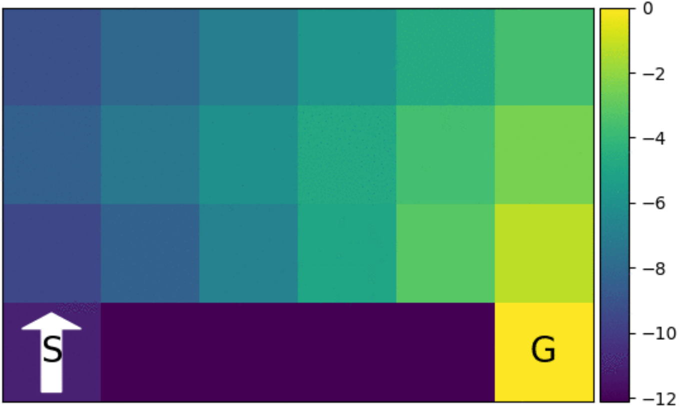


Figure: Google data centers: cooling

- Advertising recommendations (Microsoft)
- Dialog systems (Siri)

Markov Decision Process - example



Reinforcement Learning - Goal

Definition

$Z^\pi(x_t)$ Is a random variable representing the discounted reward along a trajectory generated by the MDP by following the policy π, starting at state x_t.

$$Z^\pi(x_t) = \sum_{t=0}^{\infty} \gamma^t R(x_t, \pi(x_t))$$

Reinforcement Learning goals

Our goal is to find a globally optimal policy π^*

$$\pi^* = \arg\max_{\pi} \mathbb{E}\, Z^\pi(x_0)$$

Potential problems

Problems

- RL is sample inefficient $\rightarrow$ expensive training
- Solutions must be **robust** to small model changes
- Solutions must avoid catastrophic events and be **safe**

Solution

Instead of maximizing the expected reward, focus on other criteria that take into account the **risk** of the potential reward.

Motivation

Figure: Simulation vs Real world

Figure: Critical Applications

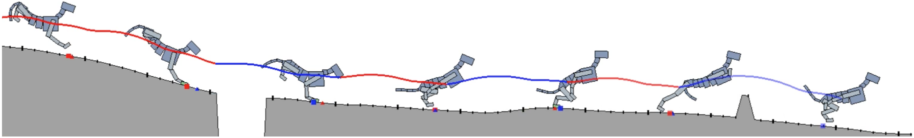


Figure: AI safety

Risk

Definition

Risk is the potential of gaining or losing something of value.

Risk-averse: disinclined or reluctant to take risks

Risk-neutral: indifferent to or balanced with respect to risk.

Risk-seeking: inclined or eager to take risks

Example

Choose between paying:

1. $100 in 100% cases
2. $1,000 in 10% cases and $0 in 90% cases

Risk-averse Reinforcement Learning - example

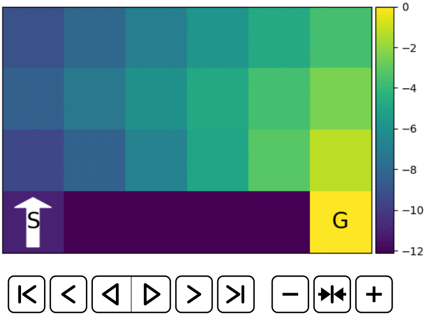


Figure: Greedy agent

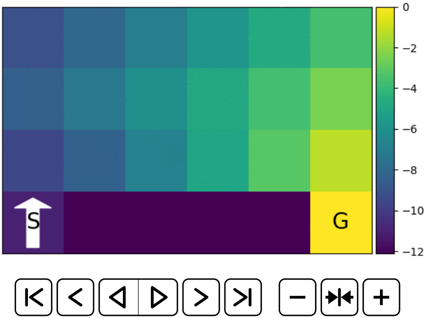
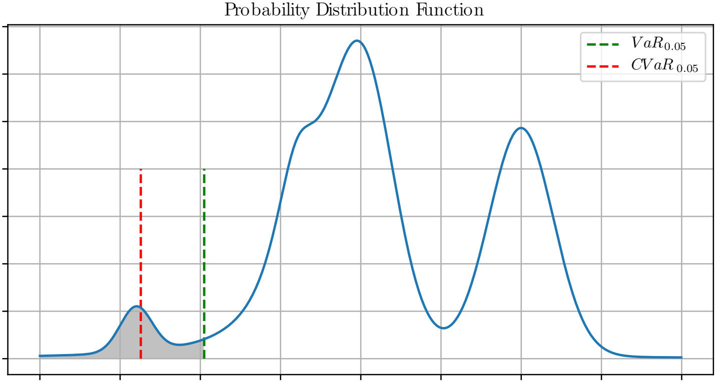


Figure: Risk-averse agent

Value-at-Risk, Conditional Value-at-Risk



Risk-averse Reinforcement Learning - goals

Definition

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$$Z^\pi(x_t) = \sum_{t=0}^{\infty} \gamma^t R(x_t, \pi(x_t))$$

Reinforcement Learning with CVaR

For a given α, our goal is to find a globally optimal policy π^*

$$\pi^* = \arg\max_\pi \mathrm{CVaR}_\alpha(Z^\pi(x_0))$$

Original Contributions

1. **Faster CVaR Value Iteration**
   - Polynomial $\rightarrow$ linear time.
   - Formally proved for increasing, unbounded distributions.
   - Experimentally verified for general distributions.
2. **CVaR Q-learning**
   - Sampling version of CVaR Value Iteration.
   - Based on the distributional approach.
   - Experimentally verified.
3. **Distributional Policy improvement**
   - Proved monotonic improvement for distributional RL.
   - Used as a heuristic for extracting π^* from CVaR Q-learning.
4. **Deep CVaR Q-learning**
   - TD update $\rightarrow$ loss function.
   - Experimentally verified in a deep learning context.

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Value Iteration

Definition

Value function $V(x)$ represents the expected return when starting in state x and following the optimal policy π^* thereafter.

Value Iteration

Initialize $V_0(x)$ for each state (arbitrary value, e.g. 0).
Update each state:

$$V_{k+1}(x) = \max_a \left[R(x, a) + \gamma \sum_{x'} p(x'|x, a) V_k(x') \right]$$

Repeat.

The algorithm converges to the optimal policy π^*: $\lim_{k\to\infty} V_k(x) = V(x)$

CVaR Value Iteration

Theorem (CVaR Value Iteration)

The following Bellman operator is a contraction:

$$\mathbf{T}C(x,y) = \max_a \left[R(x,a) + \gamma \min_\xi \sum_{x'} p(x'|x,a)\xi(x')C\left(x', y\xi(x')\right) \right]$$

The operator $\mathbf{T}$ describes the following relationship:

$$\mathbf{T}CVaR_y(Z(x)) = \max_a \left[R(x,a) + \gamma CVaR_y(Z(x')) \right]$$
$$x' \sim p(\cdot|x,a)$$

CVaR VI computational complexity

Problem

- CVaR Value Iteration requires computing a Linear Program for each state and atom.
- LP computation is slow

Solution

CVaR Value Iteration with quantile representation.

αCVaR$_\alpha$ describes a quantile function

Lemma

Any discrete distribution has a piece-wise linear $\alpha CVaR_\alpha$ function. Similarly, any a piece-wise linear $\alpha CVaR_\alpha$ function can be seen as representing a certain discrete distribution.

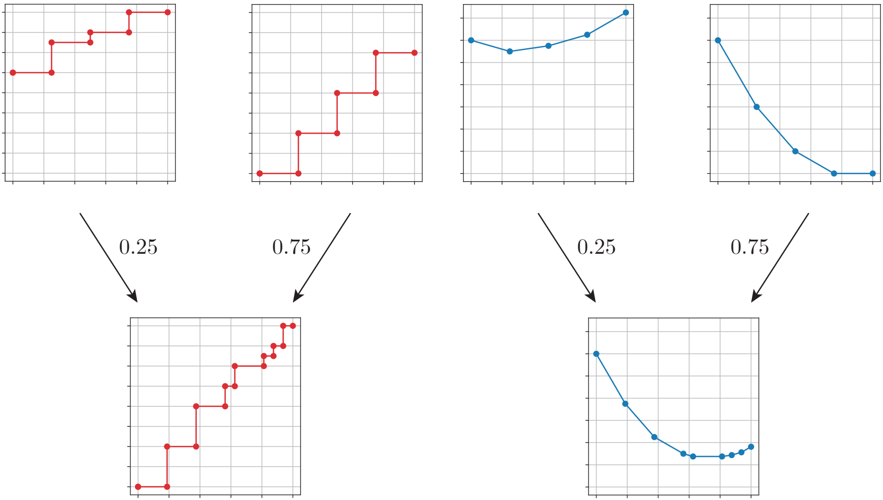
αCVaR$_\alpha \Rightarrow$ VaR

$$\frac{\partial}{\partial\alpha}\alpha\mathrm{CVaR}_\alpha(Z) = \frac{\partial}{\partial\alpha}\int_0^\alpha VaR_\beta(Z)d\beta = VaR_\alpha(Z)$$

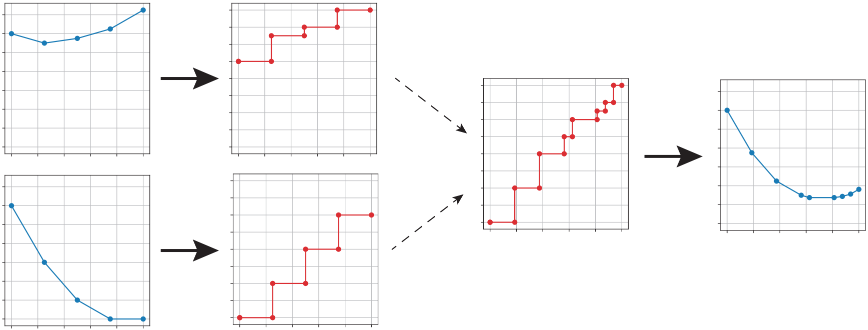
αCVaR$_\alpha \Leftarrow$ VaR

$$\alpha\mathrm{CVaR}_\alpha(Z) = \int_0^\alpha VaR_\beta(Z)\mathrm{d}\beta$$

Next state CVaR computation



Next state CVaR computation



Linear-time Computation

Theorem

Solution to minimization problem

$$\min_{\xi \in \mathcal{U}_{CVaR}(\alpha, p(\cdot|x,a))} \sum_{x'} p(x'|x,a)\xi(x') CVaR_{\xi(x')\alpha}\left(Z^{\pi}(x')\right)$$

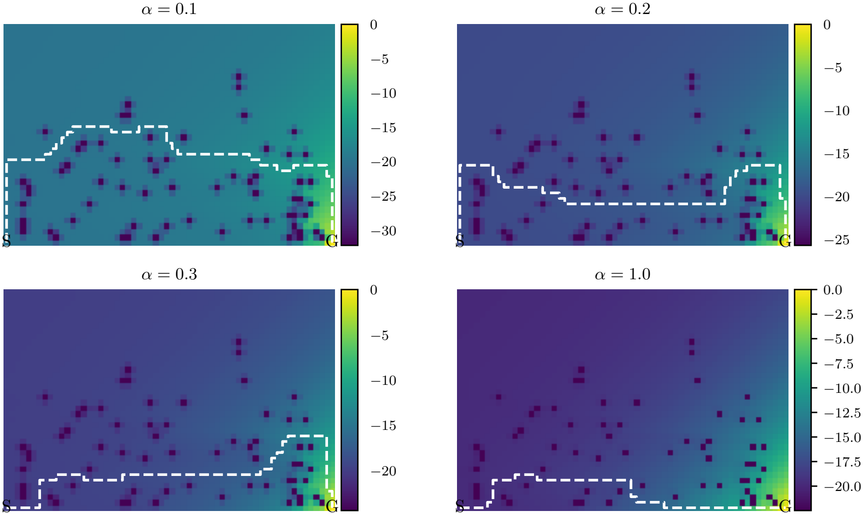
can be computed by setting

$$\xi(x') = \frac{F_{Z(x')}(F^{-1}_{Z(x,a)}(\alpha))}{\alpha}$$

The computational complexity is $O(n \cdot m)$ where n is the number of transition states and m is the number of atoms.

- Proved for increasing unbounded distributions

CVaR Value Iteration - Experiments



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Q-learning

Problem

- In practice, we often don't have access to the transition probabilities $p(x'|x, a)$
- We need to learn through direct interaction with the environment.

Solution

Q-learning: Sampling version of Value Iteration.

Action Value function

Definition

Value function $V(x)$ represents the expected return when starting in state x and following the optimal policy π^* thereafter.

Definition

Action-Value function $Q(x, a)$ represents the expected return when starting in state x, performing action a and following the optimal policy π^* thereafter.

Q-learning

Value Iteration

$$Q(x, a) \leftarrow \mathcal{T} Q(x, a)$$

$$Q(x, a) \leftarrow R(x, a) + \gamma \sum_{x'} p(x'|x, a) \max_{a'} Q(x', a')$$

Q-learning

$$Q(x, a) \leftarrow (1 - \beta) Q(x, a) + \beta \mathcal{T} Q(x, a)$$

$$Q(x, a) \leftarrow (1 - \beta) Q(x, a) + \beta \left[R(x, a) + \gamma \max_{a'} Q(x', a') \right]$$

CVaR estimation

Requirements:

- Store a single value
- Expected value of updates is CVaR

Recursive CVaR Estimation

$$V_{t+1} = V_t + \beta_t \left[1 - \frac{1}{\alpha} \mathbb{1}_{(V_t \geq r)} \right]$$
$$C_{t+1} = (1 - \beta_t) C_t + \beta_t \left[V_t + \frac{1}{\alpha} (r - V_t)^- \right]$$

CVaR Q-learning

Pseudocode

1. Sample a transition x, a, x', r
2. Create a target distribution $\mathbf{d}$
3. Sample from the target distribution
4. Update current estimates of VaR and CVaR towards the sample

Improvement

- ~~Sample from target distribution~~
- Update proportionally to the target distribution

CVaR Q-learning

CVaR Q-learning: General case

input: x, a, x', r

for each i **do**

  $C(x', y_i) = \max_{a'} C(x', a', y_i)$

end for

$\mathbf{d} = \text{extractDistribution}\left(C(x', \bullet), \mathbf{y}\right)$

for each i **do**

  $V(x, a, y_i) = V(x, a, y_i) + \beta\, \mathbb{E}_j \left[1 - \frac{1}{y_i} \mathbb{1}_{(V(x,a,y_i) \geq r + \gamma d_j)}\right]$

  $C(x, a, y_i) =$

  $(1 - \beta) C(x, a, y_i) + \beta\, \mathbb{E}_j \left[V(x, a, y_i) + \frac{1}{y_i} \left(r + \gamma d_j - V(x, a, y_i)\right)^{-}\right]$

end for

Optimal policy extraction

Standard RL: Optimal policy

$$\pi^*(x) = \arg\max_a Q(x, a)$$

CVaR VI: Optimal policy

$$\begin{aligned}
\pi^*(x_0) &= \arg\max_a C(x, a, \alpha) \\
\pi^*(x_1) &= \arg\max_a C(x_1, a, \alpha\xi^*(x_0)) \\
&\vdots \\
\pi^*(x_t) &= \arg\max_a C(x_t, a, y_{t-1}\xi^*(x_{t-1}))
\end{aligned}$$

VaR-based Policy Improvement

Theorem

Let π be a fixed policy, $\alpha \in (0,1]$. By following policy π' from the following algorithm, we will improve $CVaR_\alpha(Z)$ in expectation:

$$CVaR_\alpha(Z^\pi) \leq CVaR_\alpha(Z^{\pi'})$$

VaR-based Policy Improvement

$a = \arg\max_a \mathrm{CVaR}_\alpha(Z(x_0, a))$
$s = \mathrm{VaR}_\alpha(Z(x_0, a))$
Take action a, observe x, r
while x is not terminal **do**
$\quad s = \frac{s - r}{\gamma}$
$\quad a = \arg\max_a \mathbb{E}\left[(Z(x,a) - s)^-\right]$
$\quad$Take action a, observe x, r
end while

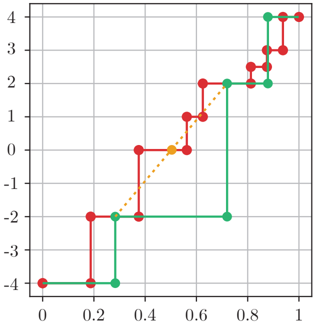
VaR-based heuristic

Problem

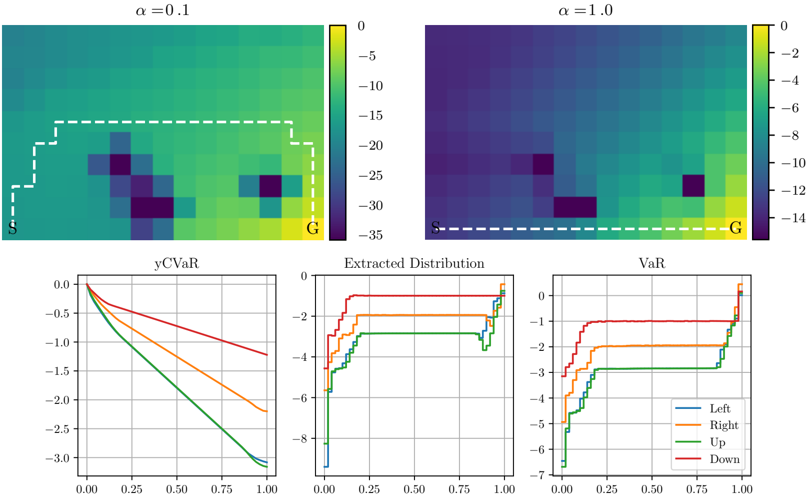
When using quantile discretization, we don't have access to the exact *VaR*.

Solution

Use linear interpolation as a heuristic.



CVaR Q-learning - Experiments



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Approximate Q-learning

Problem

Q-learning is intractable for large state spaces.

Solution

Use approximate Q-learning.

- Formulate CVaR Q-learning update as a minimizing argument
- Use methods of convex optimization to find the optimal point

TD update → loss function

Standard RL

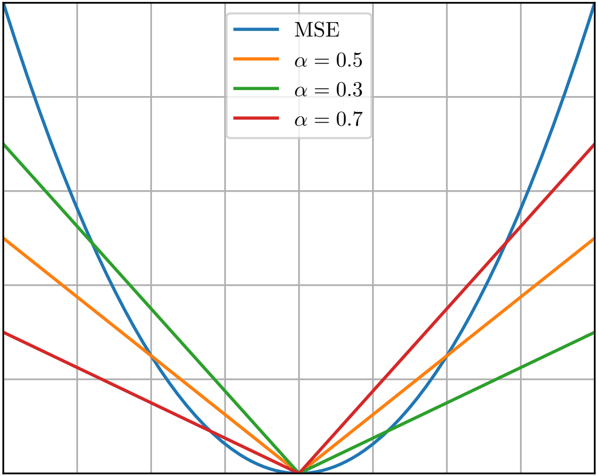
$$Q(x,a) = (1-\beta)Q(x,a) + \beta\mathcal{T}Q(x,a)$$
$$\downarrow$$
$$\min_\theta \mathbb{E}\left[\left(Q_\theta(x,a) - \mathcal{T}Q(x,a)\right)^2\right]$$

CVaR RL

$$V(x,a,y_i) = V(x,a,y_i) + \beta \mathop{\mathbb{E}}_j\left[1 - \frac{1}{y_i}\mathbb{1}_{(V(x,a,y_i)\geq \mathcal{T}d_j)}\right]$$
$$C(x,a,y_i) = (1-\beta)C(x,a,y_i) + \beta \mathop{\mathbb{E}}_j\left[V(x,a,y_i) + \frac{1}{y_i}\left(r + \gamma d_j - V(x,a,y_i)\right)^-\right]$$
$$\downarrow$$

???

Quantile loss



TD update $\rightarrow$ loss function

VaR loss

$$\mathcal{L}_{\text{VaR}} = \sum_{i=1}^{N} \mathbb{E}_{j} \left[(r + \gamma d_j - V_i(x,a))(y_j - \mathbb{1}_{(V_i(x,a) \geq r + \gamma d_j)}) \right]$$

CVaR loss

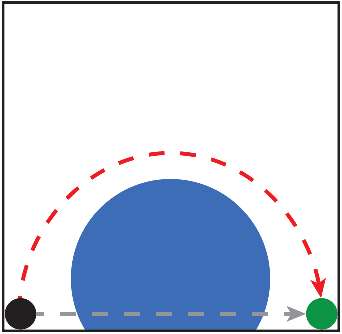
$$\mathcal{L}_{\text{CVaR}} = \sum_{i=1}^{N} \mathbb{E}_{j} \left[\left(V_i(x,a) + \frac{1}{y_i} (r + \gamma d_j - V_i(x,a))^{-} - C_i(x,a) \right)^2 \right]$$

$$\mathcal{L} = \mathbb{E}\left[\mathcal{L}_{\text{VaR}} + \mathcal{L}_{\text{CVaR}}\right]$$

Deep CVaR Q-learning

- Model: Convolutional Neural Network
  1. Input: $84 \times 84 \times 4$
  2. Convolution: $8 \times 8 \times 32$ (stride 4)
  3. Convolution: $4 \times 4 \times 64$ (stride 2)
  4. Convolution: $3 \times 3 \times 32$ (stride 1)
  5. Fully connected: 256 hidden units
  6. Output: $|\mathcal{A}| \times 100$
- Replay Memory
- Target network C'
- Optimizer: Adam (Stochastic Gradient Descent)

Deep CVaR Q-learning - Experiments



1. Video: $\alpha = 1$
2. Video: $\alpha = 0.3$

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