PHYSICAL MODEL OF THE TROMBONE USING DYNAMIC GRIDS FOR FINITE DIFFERENCE SCHEMES

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ABSTRACT

The trombone...

1. INTRODUCTION

Introintro

Main challenge is to include time-varying length

2. CONTINUOUS

The behaviour of the air in an acoustic tube can be approximated using a 1-dimensional model.

Consider a tube of time-varying length $L = L(t)$ (in m) defined over spatial domain $x \in [0, L]$ and time $t \geq 0$. A system of first-order PDEs can then be written as

$$\frac{S}{\rho_0 c^2}\partial_t p = -\partial_x (Sv) \tag{1a}$$

$$\rho_0 \partial_t v = -\partial_x p \tag{1b}$$

with pressure $p = p(x,t)$ (in N/m^2), particle velocity $v = v(x,t)$ (in m/s) and cross-sectional area $S(x)$ (in m^2). Furthermore, ρ_0 is the density of air (in kg/m^3) and c is the speed of sound in air (in m/s).

2.1. Coupling to a Lip Reed

To excite the system, a lip reed can be modelled as a simple oscillating mass according to

$$M_\mathrm{r}\frac{d^2 y}{dt^2} = -Ky - M_\mathrm{r}\sigma_\mathrm{r}\frac{dy}{dt} + S_\mathrm{r}\Delta p, \tag{2}$$

with displacement from the equilibrium $y = y(t)$, lip mass M_r (in kg), externally supplied (angular) frequency of oscillation $w_0 = w_0(t) = \sqrt{K/M_\mathrm{r}}$ (in rad/s), stiffness $K = K(t)$ (in N/m), effective surface area S_r (in m^2) and

$$\Delta p = P_\mathrm{m} - p(0,t) \tag{3}$$

is the difference between the pressure in the mouth P_m and the pressure in the mouth piece $p(0,t)$ (all in Pa). See Figure 2 for a schematic of the lip reed model. This pressure difference causes a volume flow velocity following the Bernoulli equation

$$U_\mathrm{B} = w_\mathrm{r}[y + H_0]_+ \mathrm{sgn}(\Delta p)\sqrt{\frac{2|\Delta p|}{\rho_0}}, \tag{4}$$

(in m/s) with effective lip-reed width w_r (m), static equilibrium separation H_0 (in m) and $[\cdot]_+ = 0.5(\cdot + |\cdot|)$ describes the "positive part of". Notice that when $y + H_0 \leq 0$, the lips are closed and the volume velocity U_B is 0. Another volume flow is generated by the lip reed itself according to

$$U_\mathrm{r} = S_\mathrm{r}\frac{dy}{dt} \tag{5}$$

(in m/s). Assuming that the volume flow velocity is conserved the total air volume entering the system is defined as

$$S(0)v(0,t) = U_\mathrm{B}(t) + U_\mathrm{r}(t). \tag{6}$$

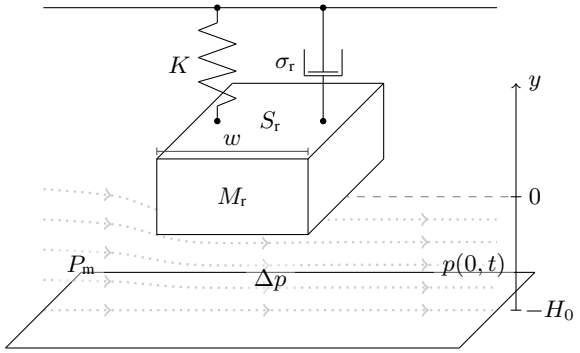

Figure 1: Lipsystem with the equilibrium at 0 and the distance from the lower lip H_0.

2.2. Radiation

The radiation model used is the one for the unflanged cylindrical pipe proposed by Levine and Schwinger in [1] and discretised by Silva *et al.* in [2]. As it is not important for the contribution of this work it will not be detailed here in full. The reader is instead referred to [3] for a comprehensive explanation.

3. DISCRETISATION

$l = [0, \dots, N]$ where

As done in [3], it is useful to place either p or v on an interleaved grid; both in space and time. Following Harrison, we place v on this interleaved grid. Accordingly, system (1) is discretised into the following finite difference scheme (FDS)

$$\frac{\bar{S}_l}{\rho_0 c^2}\delta_{t+} p_l^n = -\delta_{x-}(S_{l+1/2} v_{l+1/2}^{n+1/2}), \tag{7a}$$

$$\rho_0 \delta_{t-} v_{l+1/2}^{n+1/2} = -\delta_{x+} p_l^n, \tag{7b}$$

Boundary conditions:

At $l = 0$

after which the update schemes become

$$p_l^{n+1} = p_l^n - \frac{\rho_0 c \lambda}{\bar{S}_l}(S_{l+1/2} v_{l+1/2}^{n+1/2} - S_{l-1/2} v_{l-1/2}^{n+1/2}), \tag{8a}$$

$$v_{l+1/2}^{n+1/2} = v_{l+1/2}^{n-1/2} - \frac{\lambda}{\rho_0 c}(p_{l+1}^n - p_l^n), \tag{8b}$$

where $\lambda = ck/h$ is referred to as the Courant number and

$$\lambda \leq 1 \tag{9}$$

in order for the scheme to be stable.

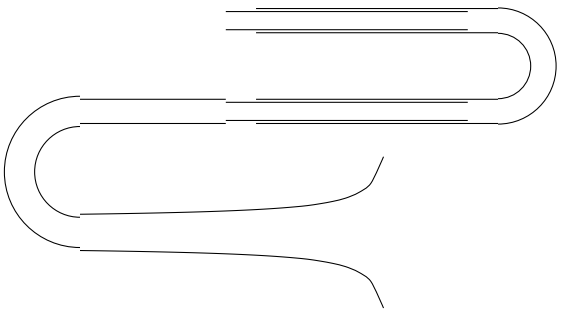

Figure 2: Lipsystem with the equilibrium at 0 and the distance from the lower lip H_0.

4. DYNAMIC GRID

We can split the FDS shown in (7) into two systems connected at their inner boundaries. The system is split at the end of the slide

For simplicity, the points are added at the far end of the slide so that there is an (approximately) equal slide-length on both sides of where the points are added.

$$\frac{\bar{S}_l}{\rho_0 c^2}\delta_{t+} p_l^n = -\delta_{x-}(S_{l+1/2} v_{l+1/2}^{n+1/2}), \tag{10a}$$

$$\rho_0 \delta_{t-} v_{l+1/2}^{n+1/2} = -\delta_{x+} p_l^n, \tag{10b}$$

$$\frac{\bar{S}_m}{\rho_0 c^2}\delta_{t+} q_m^n = -\delta_{x-}(S_{m+1/2} w_{m+1/2}^{m+1/2}), \tag{11a}$$

$$\rho_0 \delta_{t-} w_{m+1/2}^{n+1/2} = -\delta_{x+} q_l^n, \tag{11b}$$

Name	Symbol (unit)	Value
Tube		
Length	L (m)	$2.685 \leq L \leq 3.718^\star$
Air density	ρ_0 (kg/m^3)	1.1769**
Wave speed	c (m/s)	347.23**
Geometry	S (m^2)	See Table **??**
Lip reed		
Mass	M_r (kg)	$5.37 \cdot 10^{-5}$*
Frequency	ω_0 (rad/s)	?? $\leq \omega_0 \leq$??
Mouth pressure	P_m (Pa)	$0 \leq P_\mathrm{m} \leq$ 6000??
Damping	σ_r (s^{-1})	5*
Eff. surface area	S_r (m^2)	$1.46 \cdot 10^{-5}$*
Width	w_r (m)	0.01*
Equilibrium	H_0 (m)	$2.9 \cdot 10^{-4}$*

Table 1: List of parameter values used for the simulation. Taken from *[4], *[3] or **[5] with temperature $T = 26.85^\circ C$.

where q is the pressure of the right side of the tube Though the paper shows changes in the wavespeed c rather than the length L, the effect of a change in either of these parameters has an identical effect on the system.

As long a the geometry is unchanged for the grid points.

To stick to what is physically logical, L is changed

One can change the

As the geometry varies it matters a lot where points are added and subtracted.

5. IMPLEMENTATION

5.1. Parameters

5.2. Order of Calculation

6. CONCLUSION AND FUTURE WORK

Investigate the possibility of adding / removing grid points at points where the cross-sectional area is varying.

7. REFERENCES

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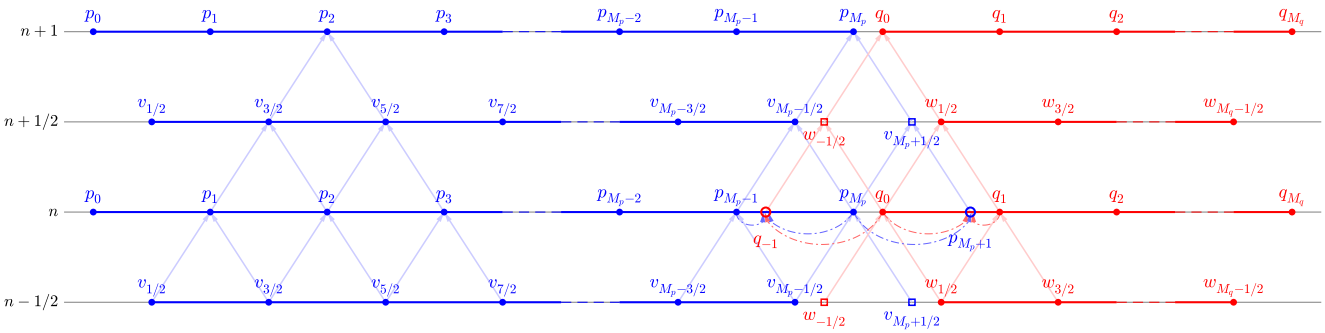

Figure 3: Schematic showing data flow of how different grid points at time index $n+1$ are calculated. To prevent cluttering, arrows going straight up (indicating that the state of a grid point at time step n is needed to calculate the state of that grid point at $n+1$) are suppressed. As an example of the usual case, the points required to calculate p_2^{n+1} are shown. Furthermore, the points needed to calculate $p_{M_p}^{n+1}$ and q_0^{n+1} are shown. The most important difference with the usual case is that the virtual grid points q_{-1}^n and $p_{M_p+1}^n$ are calculated from known values of p^n and q^n as opposed to values of $v^{n-1/2}$ and $w^{n-1/2}$.

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