REAL-TIME IMPLEMENTATION OF THE ELASTO-PLASTIC BOW MODEL APPLIED TO FINITE-DIFFERENCE SCHEMES

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ABSTRACT

This is the template file for the proceedings of the 22nd International Conference on Digital Audio Effects (DAFx-19). This template has been derived from WASPAA'99 templates and aims at producing conference proceedings in electronic form. The format is essentially the one used for ICASSP conferences. Please use either this LaTeX or the accompanying Word formats when preparing your submission. The templates are available in electronic form on http://dafx2019.bcu.ac.uk/.

1. INTRODUCTION

This template can be found on the conference website.

1.1. Figures

All figures should be centred on the column (or page, if the figure spans both columns). Figure captions (in italic) should follow each figure and have the format given in Figure 1. Vectorial figures are preferred. For example when using `Matlab`, export using either Postscript or PDF format. Also, in order to provide a better readability, figure text font size should be at least identical to footnote font size. To do so using `Matlab`, use the `subplot` command before plotting. If bitmap figures are used, please make sure that the resolution is enough for print quality. Fig. **??** illustrates an example of a figure spanning two columns.

1.2. Tables

As for figures, all tables should be centered on the column (or page, if the table spans both columns). Table captions should be in italic, precede each table and have the format given in Table **??**.

1.3. Equations

Equations should be placed on separate lines and numbered:

$$X(e^{j\Omega}) = \sum_{n=0}^{N-1} x(n)e^{-j\Omega n} \quad (1)$$

where the sequence $x(n)$ in equation (1) is a windowed frame:

$$x(n) = s(n)w(n) \quad (2)$$

with a window function $w(n)$.

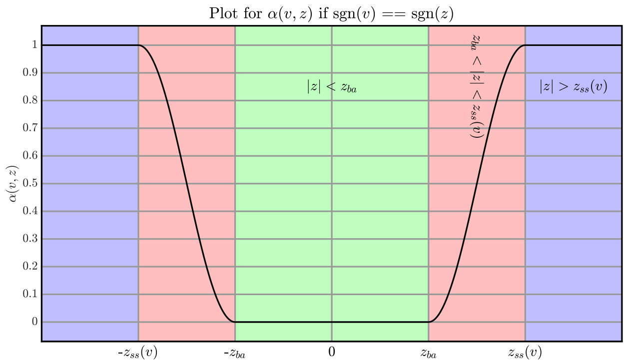

Figure 1: *A plot of the adhesion map $\alpha(v, z)$.*

1.4. Page Numbers

Page numbers will be added to the document in the post-processing stage, so *please leave the numbering as is*, that is, the first page will start at page DAFX-1 and the last page, at most, will have to be DAFX-8.

1.5. References

The references will be numbered in order of appearance [1], [2], [3] and [4]. Please avoid listing references that do not appear in the text (we did the opposite in this template).

1.5.1. Reference Format

The reference format is the standard IEEE one. We recommend to use BibTeX to create the reference list.

2. STIFF STRING

Using the subscripts t and x to denote a single derivative with respect to time and space respectively, the partial differential equation of the damped stiff string is defined as

$$u_{tt} = c^2 u_{xx} - \kappa^2 u_{xxxx} - 2\sigma_0 u_t + 2\sigma_1 u_{txx}, \quad (3)$$

where $c = \sqrt{T/\rho A}$ is the wave speed (in m/s) with tension T (in N), material density ρ (in kg·m^{-3}) and cross-sectional area A (in m^2), $\kappa = \sqrt{EI/\rho A}$ is the stiffness coefficient (in m^2/s) with Young's Modulus E (in Pa) and area moment of inertia I (in m^4) and frequency independent and frequency dependent damping coefficients $\sigma_0 \geq 0$ and $\sigma_1 \geq 0$.

3. ELASTO-PLASTIC BOW MODEL

As opposed to less complex bow models, such as the hyperbolic [source] and exponential [source] models, the elasto-plastic bow model assumes that the friction between the bow and the string is caused by a large quantity of bristles, each of which contributes to the total amount of friction. The bristles are assumed to be stiff springs with damping and can 'break' after a given 'break-away displacement'. An extra term can be added to (3) to include the bowing interaction

$$u_{tt} = \ldots - f(v, z), \tag{4}$$

where bowing function f describes the input of the scheme and is dependent on the relative velocity between the string and the bow at bowing point x_B

$$v = u_t(x_\text{B}) - v_\text{B} \tag{5}$$

and the aforementioned average bristle displacement z. The excitation (or bowing) function is defined as

$$f(v, z) = s_0 z + s_1 \dot{z} + s_2 v \tag{6}$$

where s_0 is the bristle stiffness (in N/m), s_1 is the damping coefficient and s_2

Moreover, the rate of change of z is

$$\dot{z}(v, z) = v\left[1 - \alpha(v, z)\frac{z}{z_\text{ss}(v)}\right] \tag{7}$$

where z_ss is the steady-state function

$$z_\text{ss} = \frac{\text{sgn}(v)}{s_0}\left[f_\text{c} + (f_\text{s} - f_\text{c})e^{-(v/v_\text{s})^2}\right]. \tag{8}$$

with stribeck velocity v_s (in m/s) coulomb force $f_\text{c} = f_\text{N}\mu_\text{c}$ and stiction force $f_\text{s} = f_\text{N}\mu_\text{s}$ (both in N). Here f_N is the normal force (in N) and μ_c and μ_s are the dynamic and static friction coefficient respectively. Furthermore, the adhesion map between the bow and the string is defined as

$$\alpha(v, z)\begin{cases} \left.\begin{matrix} 0 & |z| < z_\text{ba} \\ \alpha_\text{m}(v, z) & z_\text{ba} < |z| < z_\text{ss}(v) \\ 1 & |z| > z_\text{ss}(v) \end{matrix}\right\} & \text{if } \text{sgn}(v) = \text{sgn}(z) \\ 0 & \text{if } \text{sgn}(v) \neq \text{sgn}(z), \end{cases} \tag{9}$$

where the transition between the elastic and plastic behaviour is defined as

$$\alpha_\text{m} = \frac{1}{2}\left[1 + \sin\left(\pi\frac{z - \frac{1}{2}(z_\text{ss}(v) + z_\text{ba})}{z_\text{ss}(v) - z_\text{ba}}\right)\right], \tag{10}$$

with break-away displacement z_ba, i.e. where the bristles start to break, as mentioned at the beginning of this section.

4. DISCRETISATION

Equation (3) can be discretised at times $t = nk$, with sample $n \in \mathbb{N}$ and time-step $k = 1/f_\text{s}$ with sample-rate f_s and locations $x = lh$, with grid points $l \in [0, N]$, where $N+1$ is the total number of grid points and grid spacing h which needs to abide the following condition

$$h \geq h_\text{min} = \sqrt{\frac{c^2k^2 + 4\sigma_1 k + \sqrt{(c^2k^2 + 4\sigma_1 k)^2 + 16\kappa^2k^2}}{2}}. \tag{11}$$

The closer h is to h_min, the more accurate the scheme will be. Approximations for the derivatives in the equations found in 3 are described in the following way:

$$u_{xx} \approx \delta_{xx}u_l^n = \frac{1}{h^2}\left(u_{l+1}^n - 2u_l^n + u_{l-1}^n\right), \tag{12a}$$

$$u_t \approx \delta_{t\cdot}u_l^n = \frac{1}{2k}\left(u_l^{n+1} - u_l^{n-1}\right), \tag{12b}$$

$$u_{tt} \approx \delta_{tt}u_l^n = \frac{1}{k^2}\left(u_l^{n+1} - 2u_l^n + u_l^{n-1}\right), \tag{12c}$$

$$\begin{aligned} u_{txx} \approx \delta_{t-}\delta_{xx}u_l^n = \frac{1}{hk^2}\big(&u_{l+1}^n - 2u_l^2 + u_{l-1}^n \\ &- u_{l+1}^{n-1} + 2u_l^{n-1} - u_{l-1}^{n-1}\big), \end{aligned} \tag{12d}$$

Using the discretised variable u_l^n is $u(x,t)$ at the nth time step and the lth point on the string, and the approximations shown in (12) we can discretise (3)

$$\begin{aligned} \delta_{tt}u_l^n = {}& c^2\delta_{xx}u_l^n - \kappa^2\delta_{xxxx}u_l^n - 2\sigma_0\delta_t u_l^n \\ &+ 2\sigma_1\delta_t\delta_{xx}u_l^n \end{aligned} \tag{13}$$

adding onto this

At the bowing point $l = x_\text{B}$, the FDS for the bowed string looks like:

$$\begin{aligned} \delta_{tt}u_l^n = {}& c^2\delta_{xx}u_l^n - \kappa^2\delta_{xxxx}u_l^n - 2\sigma_0\delta_{t\cdot}u_l^n \\ &+ 2\sigma_1\delta_{t-}\delta_{xx}u_l^n - f(v, z), \end{aligned} \tag{14}$$

where we need to iteratively solve for two unknown variables: the relative velocity between the bow and the string v and the mean bristle displacement z of the bow. The excitation (or bowing) function is defined as

$$f(v, z) = s_0 z + s_1 \dot{z} + s_2 v \tag{15}$$

where

$$\dot{z}(v, z) = v\left[1 - \alpha(v, z)\frac{z}{z_\text{ss}(v)}\right] \tag{16}$$

with adhesion map

$$\alpha(v, z)\begin{cases} \left.\begin{matrix} 0 & |z| < z_\text{ba} \\ \alpha_\text{m}(v, z) & z_\text{ba} < |z| < z_\text{ss}(v) \\ 1 & |z| > z_\text{ss}(v) \end{matrix}\right\} & \text{if } \text{sgn}(v) = \text{sgn}(z) \\ 0 & \text{if } \text{sgn}(v) \neq \text{sgn}(z), \end{cases} \tag{17}$$

the transition between the elastic and plastic behaviour

$$\alpha_\text{m} = \frac{1}{2}\left[1 + \sin\left(\pi\frac{z - \frac{1}{2}(z_\text{ss}(v) + z_\text{ba})}{z_\text{ss}(v) - z_\text{ba}}\right)\right], \tag{18}$$

and the steady-state function

$$z_\text{ss} = \frac{\text{sgn}(v)}{s_0}\left[f_\text{c} + (f_\text{s} - f_\text{c})e^{-(v/v_\text{s})^2}\right]. \tag{19}$$

We can solve (14) this we can use the following identities

$$\delta_{tt}u_l^n = \frac{2}{k}\left(\delta_{t\cdot}u_l^n - \delta_{t-}u_l^n\right) \quad \text{and} \quad \delta_{t\cdot}u_l^n = v + v_\text{B}, \tag{20}$$

resulting in

$$\frac{2}{k}v + \frac{2}{k}v_\text{B} - \delta_{t-}u_l^n = c^2\delta_{xx}u_l^n - \kappa^2\delta_{xxxx}u_l^n - 2\sigma_0 v - 2\sigma_0 v_\text{B} + 2\sigma_1\delta_{t-}\delta_{xx}u_l^n - f \tag{21}$$

This can be rewritten to

$$s_0 z + s_1 \dot{z} + s_2 v + \frac{2}{k} v + 2\sigma_0 v + b = 0 \quad \text{where} \tag{22}$$

$$b = \frac{2}{k} v_{\mathrm{B}} - \frac{2}{k}\delta_{t-} u_l^n - c^2 \delta_{xx} u_l^n + \kappa^2 \delta_{xxxx} u_l^n + 2\sigma_0 v_{\mathrm{B}} - 2\sigma_1 \delta_{t-}\delta_{xx} u_l^n, \tag{23}$$

where b can be pre-computed.

Newton's method (or Newton-Raphson) is defined as

$$x^{i+1} = x^i - \frac{g(x^i)}{g'(x^i)} \tag{24}$$

where $g(x)$ is an arbitrary function dependent on to-be-calculated variable x. In this case, g is (16) $x = \dot{z}$

$$\dot{z}^{i+1} = \dot{z}^i - \frac{\dot{z}(v,z)}{\dot{z}'(v,z)} \tag{25}$$

where

$$\dot{z}'(v,z) = \frac{d\dot{z}}{dv} + \frac{d\dot{z}}{dz}, \tag{26}$$

i.e. the derivative of $\dot{z}$ with respect to the relative velocity v plus the derivative of $\dot{z}$ with respect to the average displacement z. In the iteration, we use the newly calculated value for $\dot{z}$ and the value of z at the previous time step to calculate an estimate of z using

$$z^n = k\dot{z}^{i+1} - z^{n-1}. \tag{27}$$

Inserting this into (22) we can caluclate v using

$$v = \frac{-s_0 z - s_1 \dot{z} - b}{s_2 + \frac{2}{k} + 2\sigma_0}. \tag{28}$$

5. CONCLUSIONS

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6. ACKNOWLEDGMENTS

Many thanks to the great number of anonymous reviewers!

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8. APPENDIX: MARGIN CHECK

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