



# FORUM ACUSTICUM EURONOISE 2025

## SUMMER SCHOOL

Fundamentals of Acoustics

# Course outline

Time table / Lecture modules	TOPIC
<b>DAY 1 - 21/06/2024</b>	
8:30-9:00 h	Registration
9:00-10:30 h	<b>Lecture:</b> Introduction (why do we study acoustics) Basic signal properties: Period, wavelength and frequency. Definitions, and fundamental equations for propagation of sound.
10:30-11:00 h	Coffee break
11:00-12:00 h	<b>Lecture:</b> More advanced signal properties: Pure and complex sounds, frequency decomposition and sound spectrum. Acoustic quantities: levels and their manipulation, Octave bands and physiology of hearing, binaural hearing.
12:00-13:00 h	Plenary session I. Deep learning, AI and acoustics (Prof. Hamid Krim. North Carolina State University) ( <b>All the topics</b> )
13:00-14:00 h	Lunch
14:00-15:30 h	<b>Tutorial:</b> Tools for acoustic analysis Recording of recorded sound Manipulation of sound
15:30-16:00 h	Coffee break
16:00-17:00 h	<b>Lecture:</b> Wave-surface interactions, 1D wave equation solution including reflection coefficient Sound absorption, types of sound absorption materials

# Course outline

DAY 2 - 22/06/2024	
9:00-10:30 h	Lecture: Sound insulation, governing principles. Tutorial: sound absorption and insulation
10:30-11:00	Coffee break
11:00-12:00 h	Lecture: Sound propagation in open and enclosed spaces.
12:00-13:00 h	Plenary session II. Carrier paths in Acoustics (All the topics)
13:00-14:00 h	Lunch
14:00-15:30 h	Tutorial: Sound propagation in open and enclosed spaces
	HBK demonstration
15:30-16:00	Coffee break
16:00-17:00 h	Test

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MENU

**TU/e** EINDHOVEN  
UNIVERSITY OF  
TECHNOLOGY

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# FACTS & FIGURES

Interested in key facts & figures of TU/e? Find the 2023 version [here](#)

Dutch [version](#)

# Building Acoustics TU/e

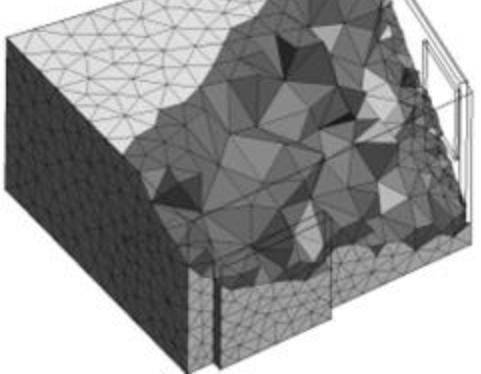
Research dealing with the propagation of sound in the built environment at multiple scales: city level, building level and building element level





**Evaluating and experiencing  
room acoustics in VR**

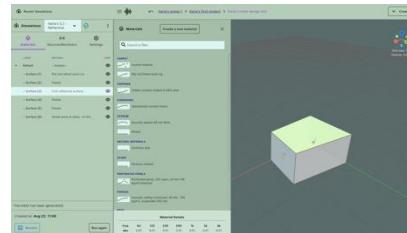
## Research



## Education



## Software



## Impact

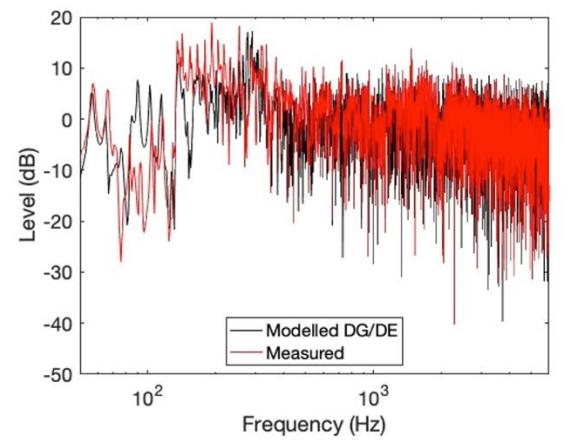




Measurement-based



Model-based



## My view on open science



MAARTEN HORNIKX ON WHY OPEN SCIENCE IS ESSENTIAL FOR SOCIETAL IMPACT

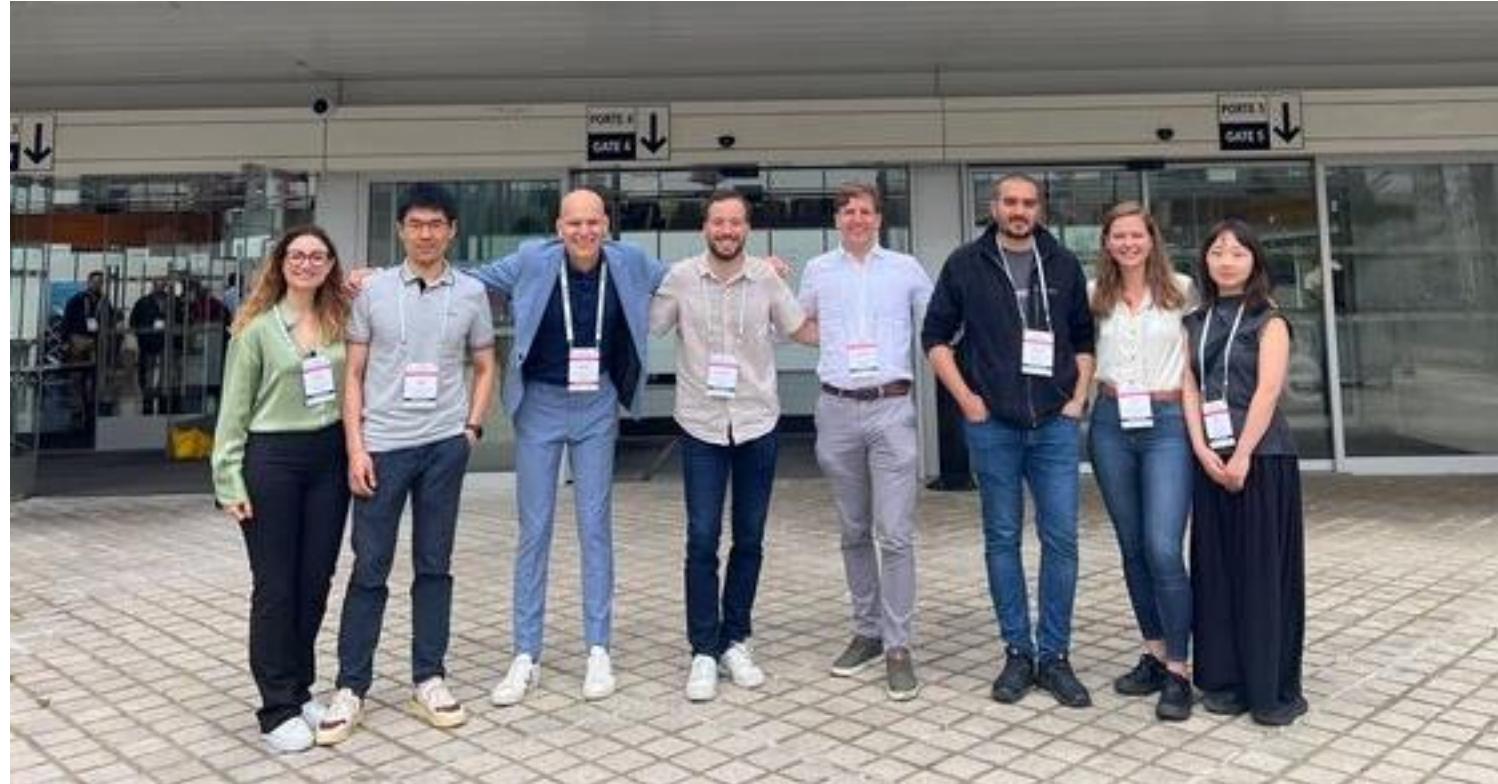
# 'Open science is not a bonus, but the essence of good research'

JUNE 2, 2025

Hornikx: "We need to make our non-traditional outputs, like software, much more visible – and show what kind of impact they have. That's when you can say: this is modern science, and here's the difference we're making."



## Building Acoustics Research group

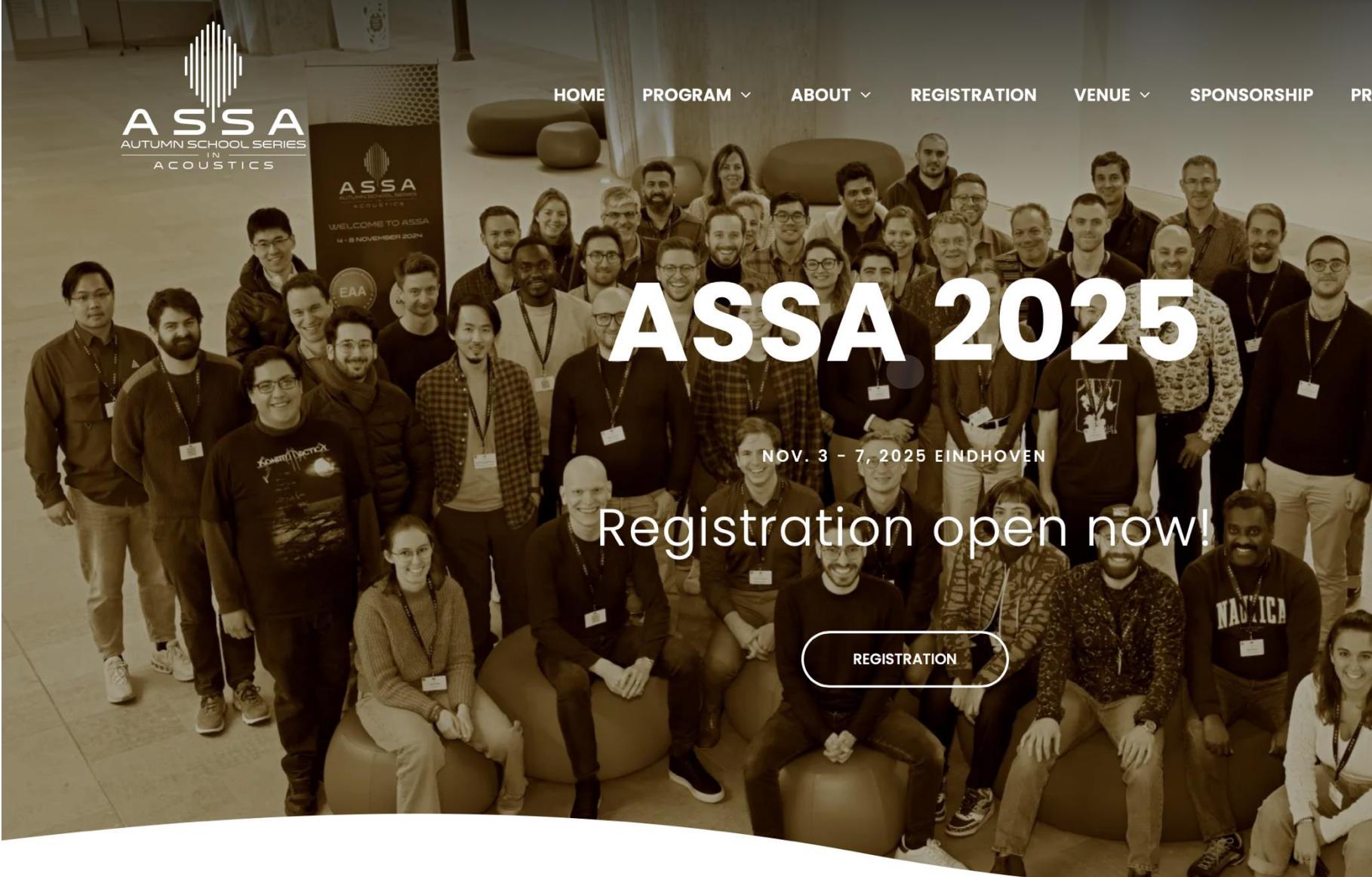


Website



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ASSA 2025

NOV. 3 - 7, 2025 EINDHOVEN

Registration open now!

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# Saturday 21 June

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## Sound and vibration signals: harmonic motions

$$s(t) = \hat{s} \cos(\omega t + \phi)$$

$s(t)$  Displacement (m)

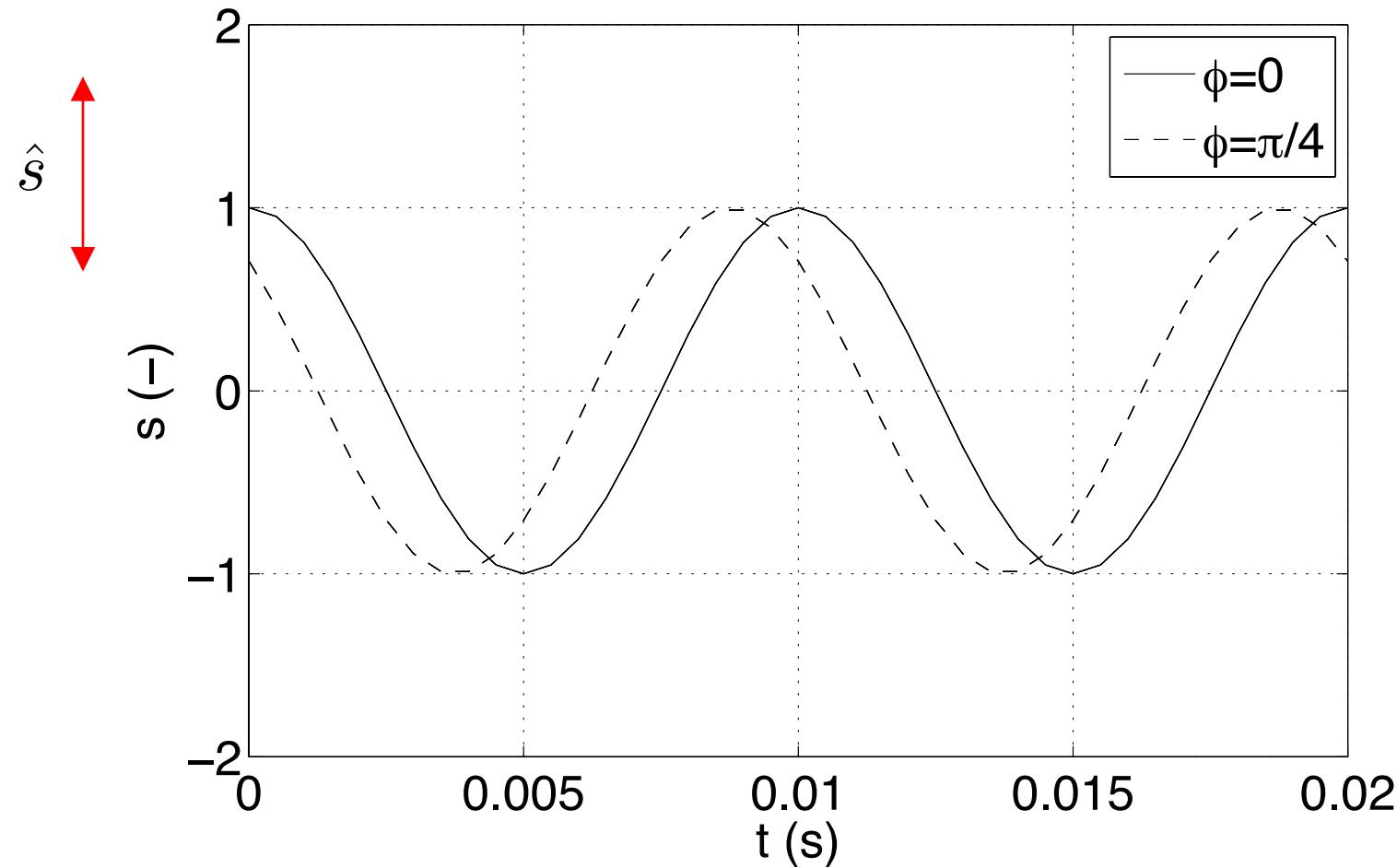
$\hat{s}$  Displacement amplitude

$\omega$  Angular frequency

$\phi$  Phase shift

## Sound and vibration signals: harmonic motions

$$s(t) = \hat{s} \cos(\omega t + \phi)$$



## Harmonic motion: time averaged signal

$$\langle s^2 \rangle = \frac{1}{t_0} \int_0^{t_0} [s(t)]^2 dt = \tilde{s}^2$$

$$\langle s^2 \rangle = \tilde{s}^2$$

Effective value

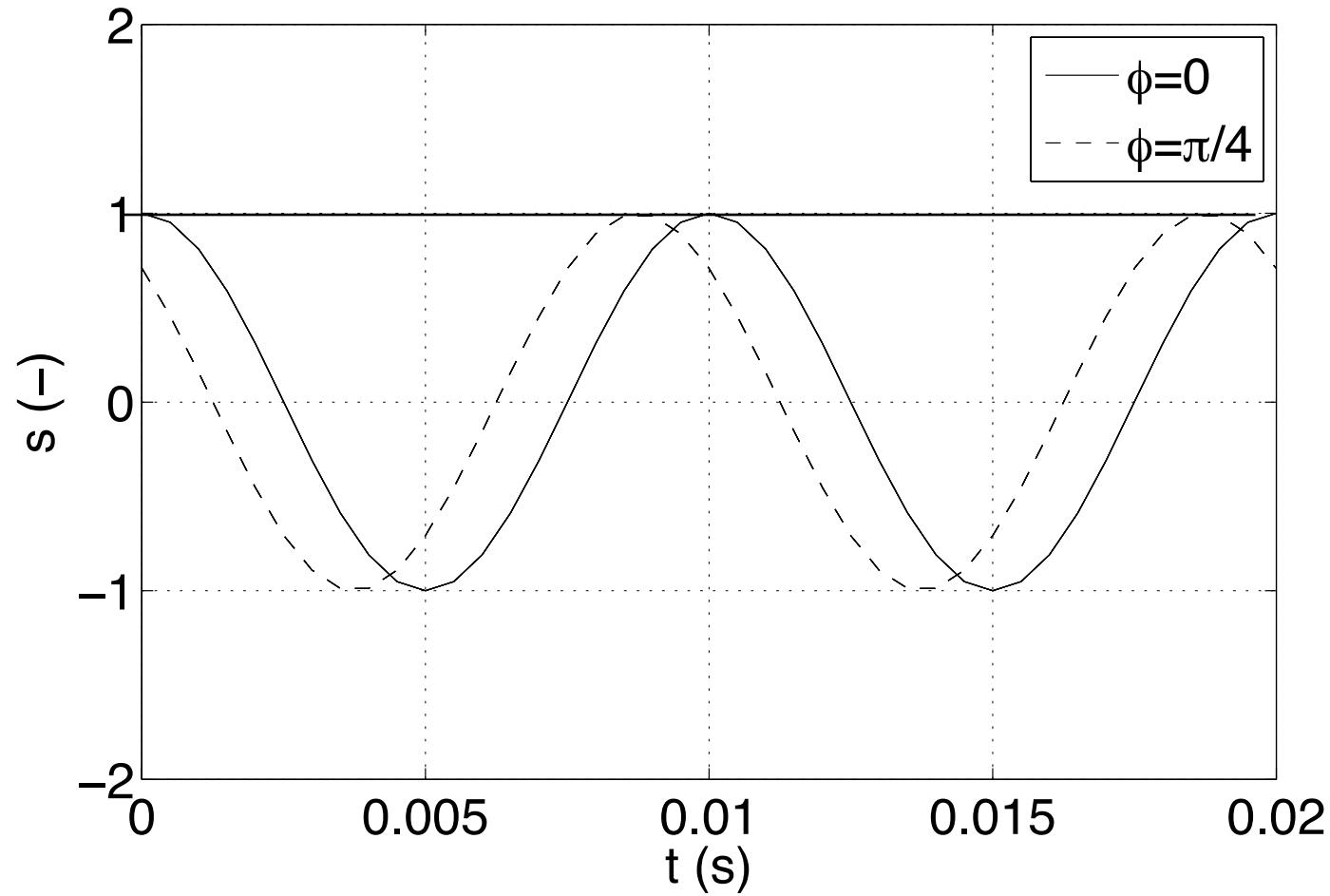
$$\tilde{s} = \hat{s} / \sqrt{2}$$

For harmonic signals

## Harmonic motion: time averaged signal

$$\tilde{s} = \hat{s}/\sqrt{2}$$

For harmonic signals



## Complex numbers

Easy way to compute the amplitude (effective value) and phase of a harmonic signal

$$s(t) = \hat{s} e^{j(\omega t + \phi)}$$

$$e^{jz} = \cos(z) + j \sin(z)$$

Physical signal is the real value of the complex signal, thus

$$\hat{s} \cos(\omega t + \phi)$$

## Complex numbers

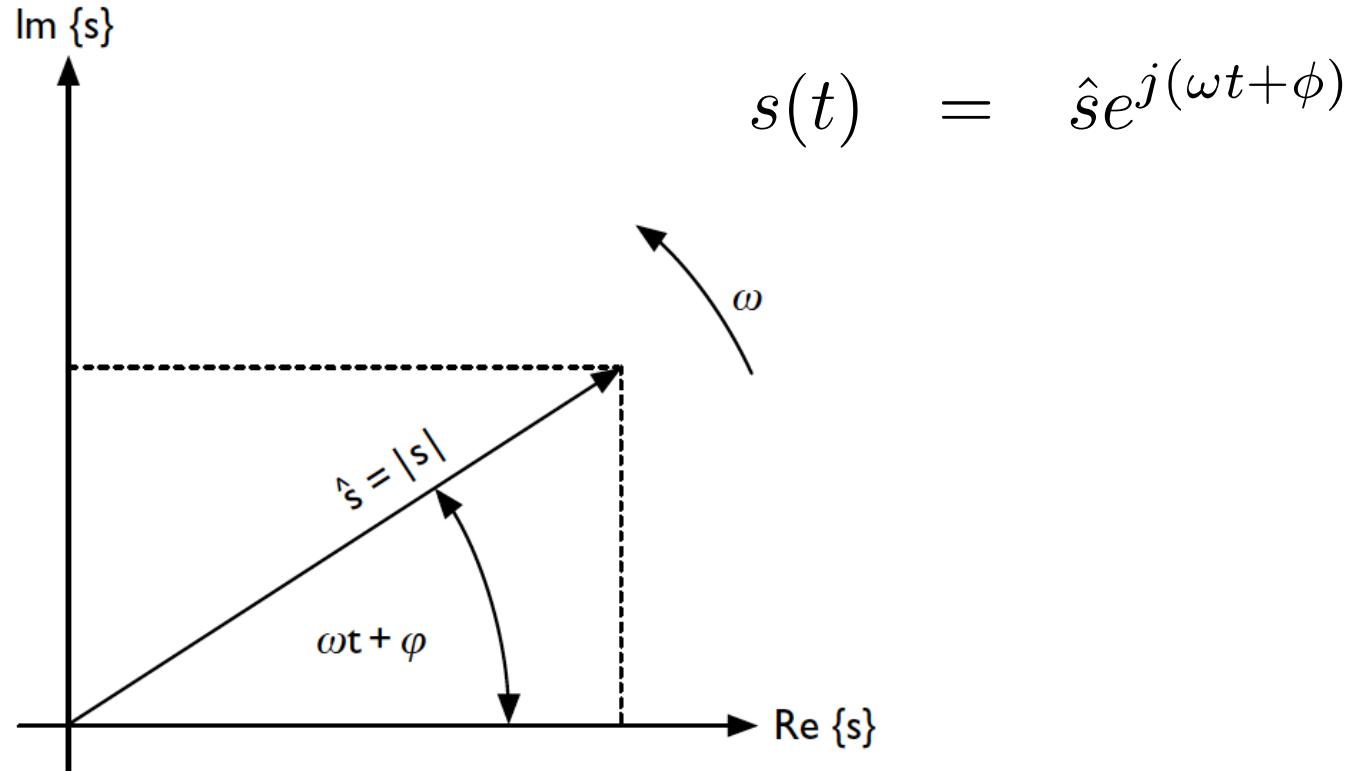


Figure 2.4 Phasor representation of a harmonic oscillation.

# Complex numbers

Absolute value of  $s$

$$\begin{aligned} |s(t)| &= \sqrt{\operatorname{Re}\{s(t)\}^2 + \operatorname{Im}\{s(t)\})^2} \\ &= \sqrt{\operatorname{Re}\{\cos(\omega t + \phi)\}^2 + \operatorname{Im}\{\sin(\omega t + \phi)\})^2} \end{aligned}$$

Argument of  $s$

$$\begin{aligned} \angle s &= \operatorname{atan} \left( \frac{\operatorname{Im}\{s(t)\}}{\operatorname{Re}\{s(t)\}} \right) \quad \text{for } \operatorname{Im}\{s(t)\} > 0 \\ &= \pi + \operatorname{atan} \left( \frac{\operatorname{Im}\{s(t)\}}{\operatorname{Re}\{s(t)\}} \right) \quad \text{for } \operatorname{Im}\{s(t)\} < 0, \operatorname{Re}\{s(t)\} > 0 \\ &= -\pi + \operatorname{atan} \left( \frac{\operatorname{Im}\{s(t)\}}{\operatorname{Re}\{s(t)\}} \right) \quad \text{for } \operatorname{Im}\{s(t)\} < 0, \operatorname{Re}\{s(t)\} < 0 \end{aligned}$$

## Beats

Example of calculating with complex numbers`

$$s_1(t) = \hat{s}e^{j(\omega + \Delta\omega)t}$$

$$s_2(t) = \hat{s}e^{j(\omega - \Delta\omega)t}$$

$$s_1 + s_2 = \hat{s} (e^{-j\Delta\omega t} + e^{j\Delta\omega t}) e^{j\omega t} = 2\hat{s} \cos(\Delta\omega t) e^{j\omega t}$$

$$Re\{s_1 + s_2\} = 2\hat{s} \cos(\Delta\omega t) \cos(\omega t)$$

## Beats

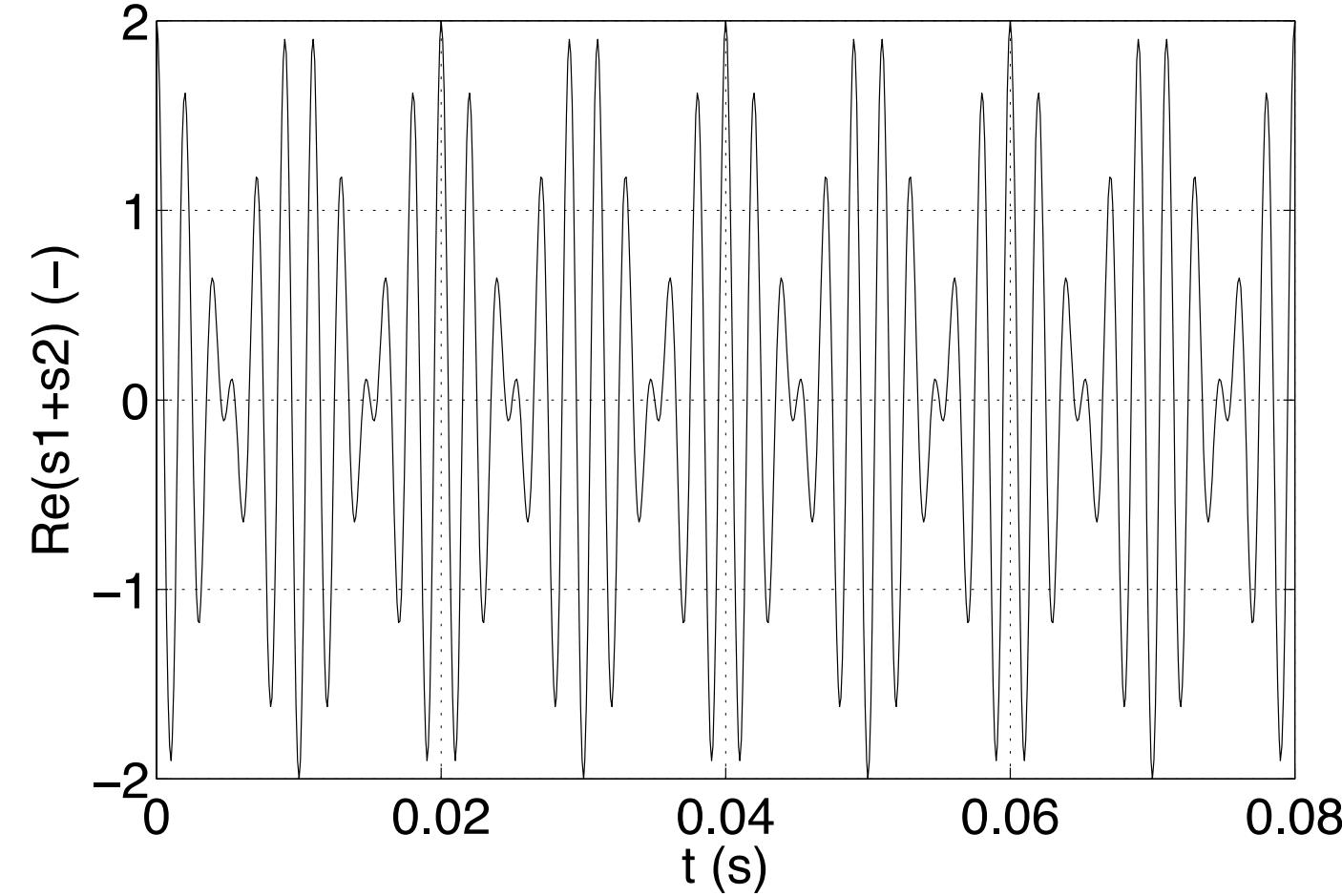
S1



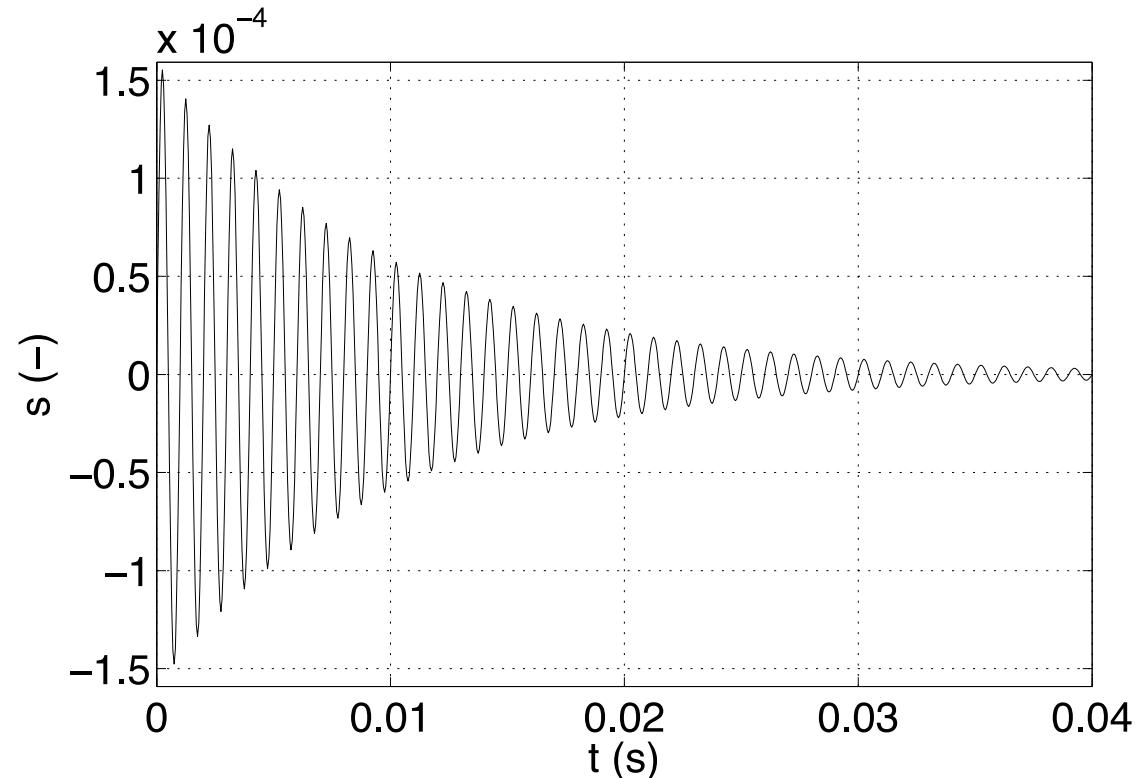
S2



S1+S2

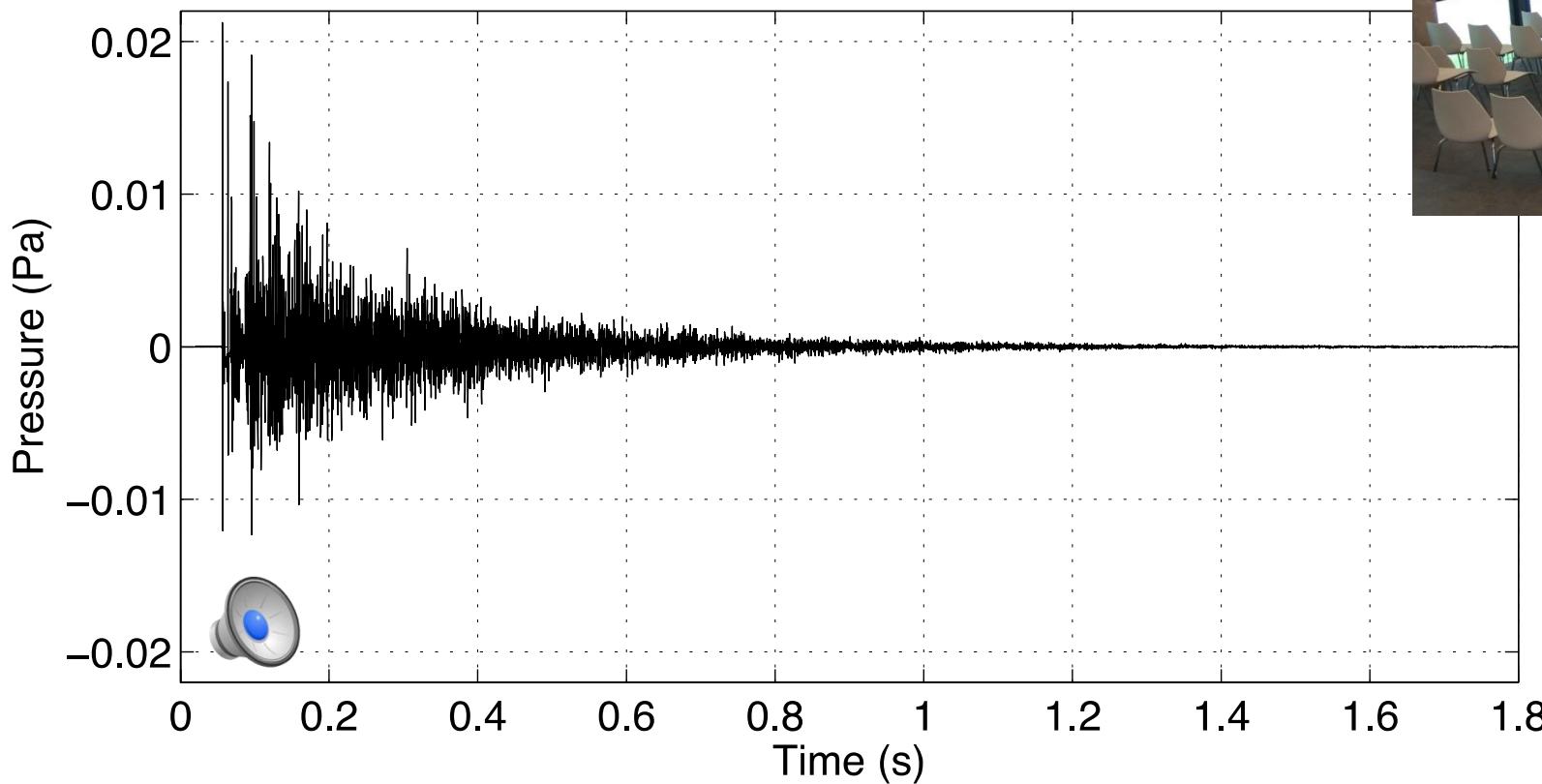


## Transient response

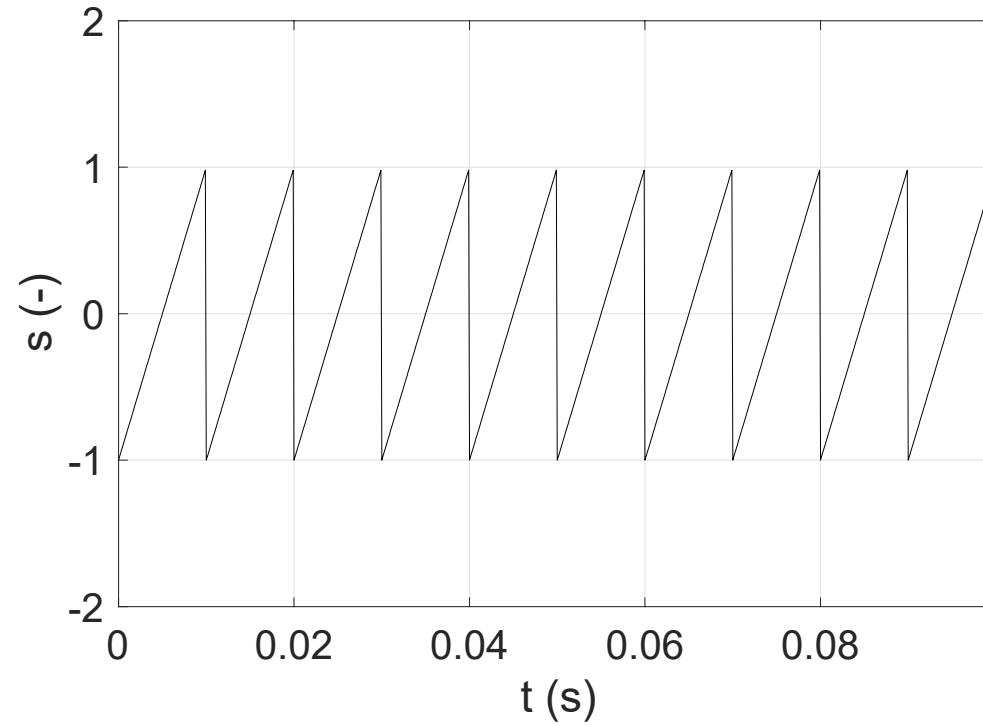


$$s(t) = \hat{s} e^{\delta t} e^{j(\omega t + \phi)}$$

## Transient response



## Periodic signals

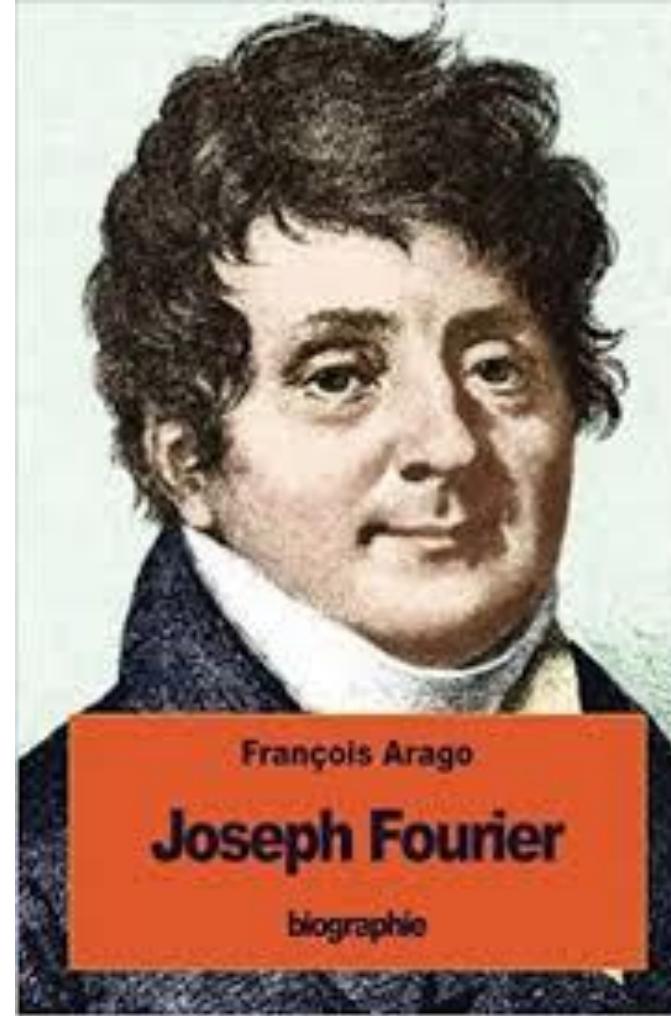


$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_0 n t}$$

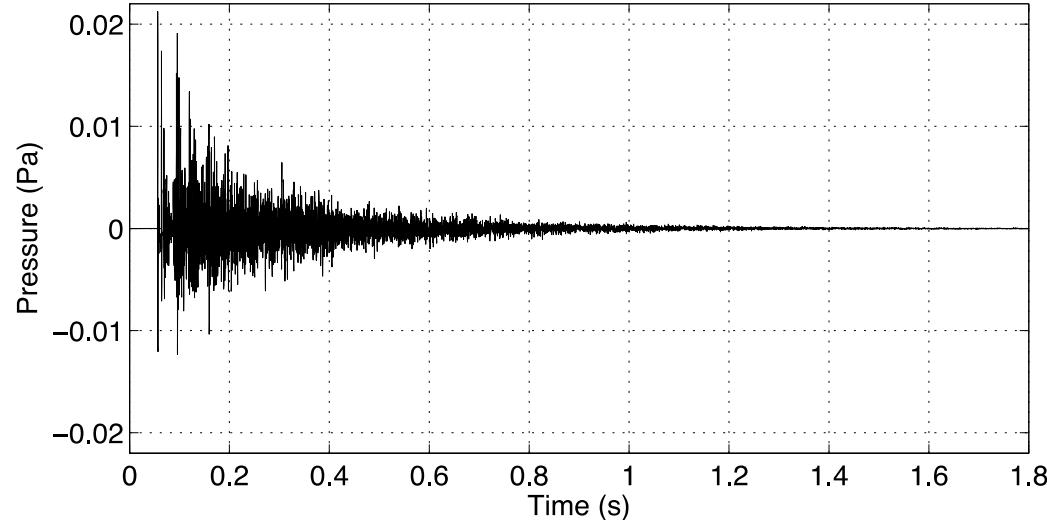
$C_n$  = Fourier coefficients

$$\omega_0 = \frac{2\pi}{T}$$

# Fourier Transform



## Non-periodic signals

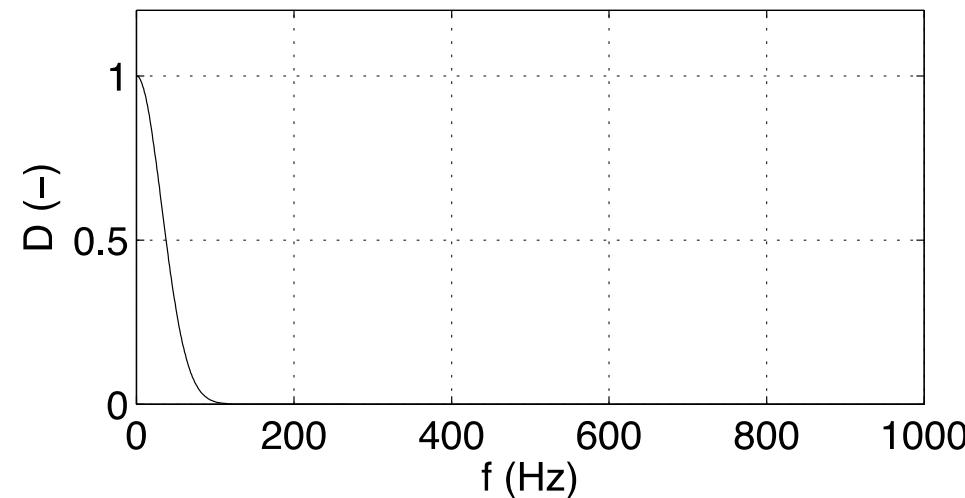
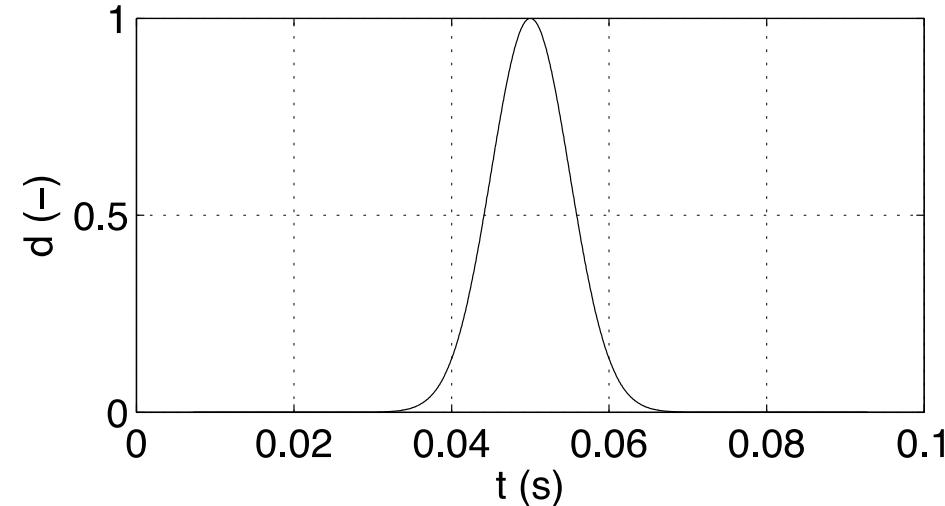


$$s(t) = \int_{-\infty}^{\infty} C(\omega) e^{j\omega t} d\omega$$

$$C(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt$$

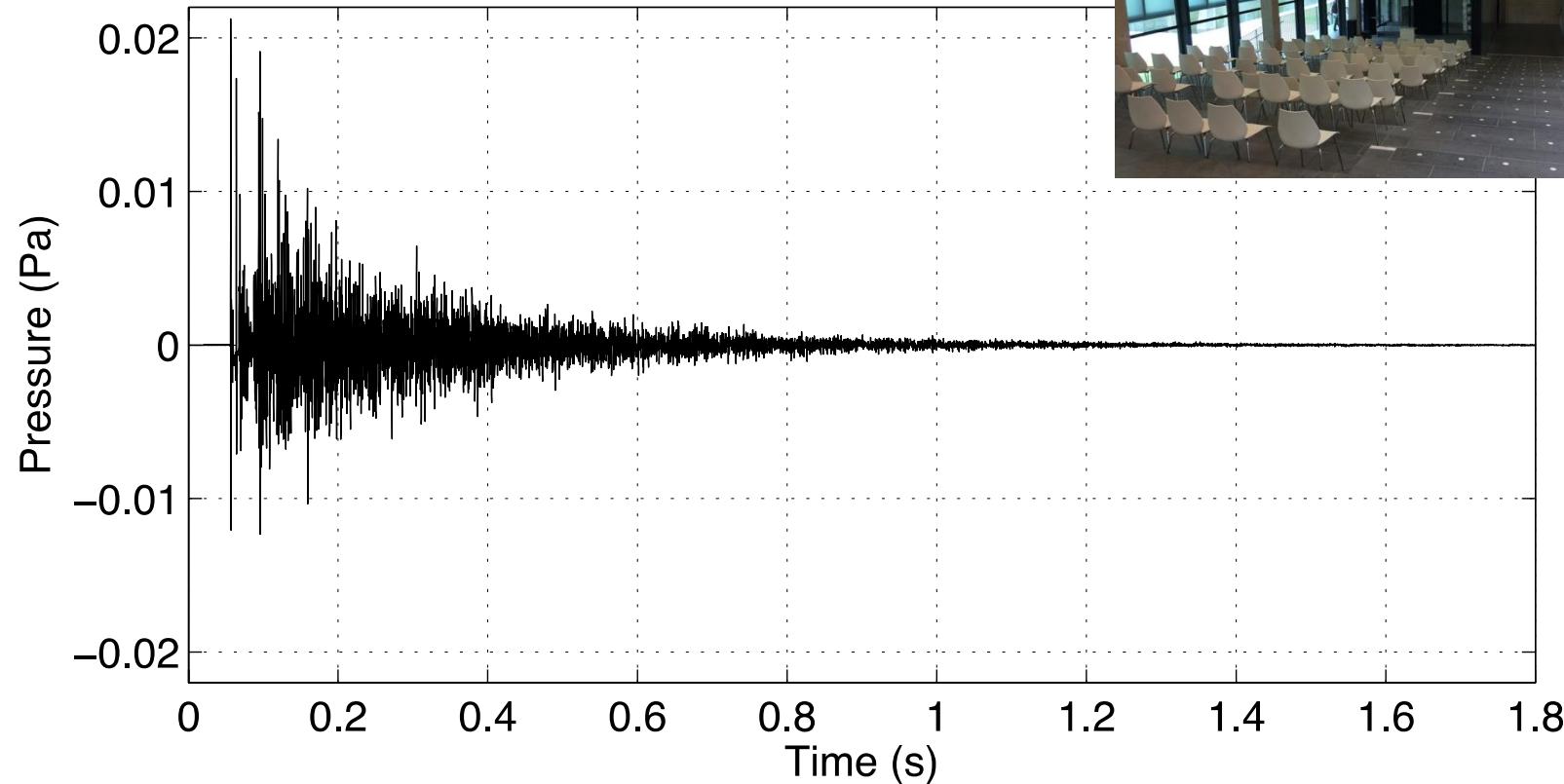
Fourier Transform

## Impulsive signal properties



## Sound pressure level

Time average of a sound signal is almost zero,  
how to calculate the time-averaged level?



## Sound pressure level

$$L = 20 \cdot \log_{10} \left( \frac{\tilde{p}}{p_b} \right)$$

$p_b$  Reference sound pressure,  $2 \cdot 10^{-5}$  Pa

$$\tilde{p} = \sqrt{\frac{1}{t_0} \int_0^{t_0} p(t)^2 dt} = \frac{\hat{p}}{\sqrt{2}}$$

Sound pressure level difference between two signals

$$\Delta L = 20 \cdot \log_{10} \left( \frac{\tilde{p}_1}{\tilde{p}_2} \right) dB$$

# Octave bands

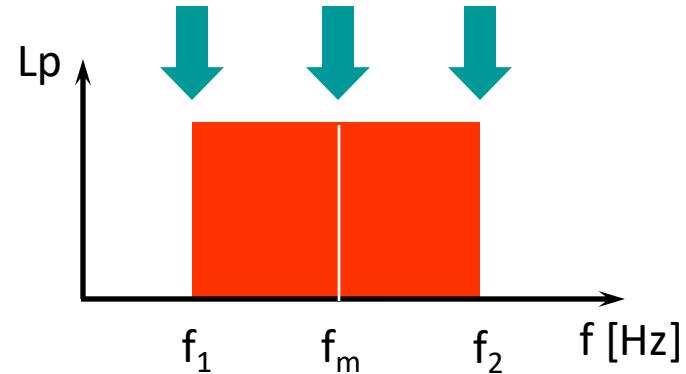
## Octave bands

Example  
1 kHz  
octave band

$$f_m = 1000 \text{ Hz}$$

$$f_1 = 707 \text{ Hz}$$

$$f_2 = 1414 \text{ Hz}$$



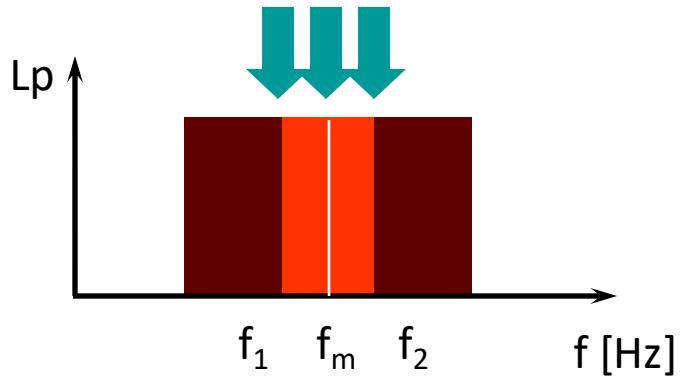
## 1/3 Octave bands

Example  
1 kHz  
1/3 octave band

$$f_m = 1000 \text{ Hz}$$

$$f_1 = 890 \text{ Hz}$$

$$f_2 = 1122 \text{ Hz}$$



## Octave bands

Lower Band Limit [Hz]	Centre Frequency [Hz]	Upper Band Limit [Hz]
44	63	88
88	125	177
177	250	355
355	500	710
710	1000	1420
1420	2000	2840
2840	4000	5680
5680	8000	11,360

Murphy, E., & King, E. (2014). *Environmental noise pollution: Noise mapping, public health, and policy*. Newnes.

## Octave bands

$$L_{p,1/3\text{octave},j} = 10 \log_{10} \frac{\sum_{i=1}^{N_j} p_{eff}^2(f_{j,i})}{p_{ref}^2} \quad [\text{dB}]$$

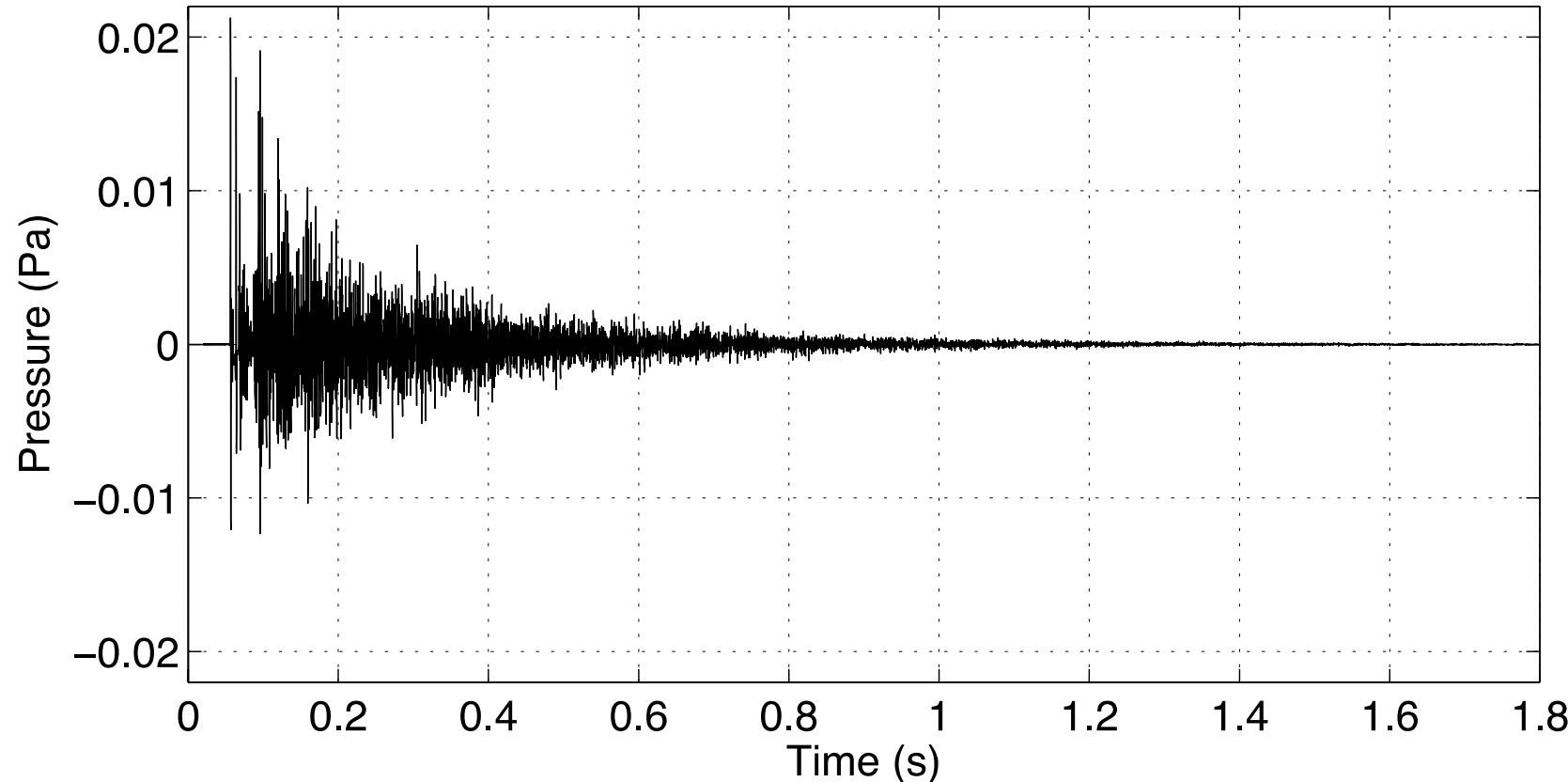
$$= 10 \log_{10} \frac{\sum_{i=1}^{N_j} L_p(f_{j,i})}{10^{-10}} \quad [\text{dB}]$$

$j$  = 1/3 octave band index [-]

$i$  = frequency number within 1/3 octave band [-]

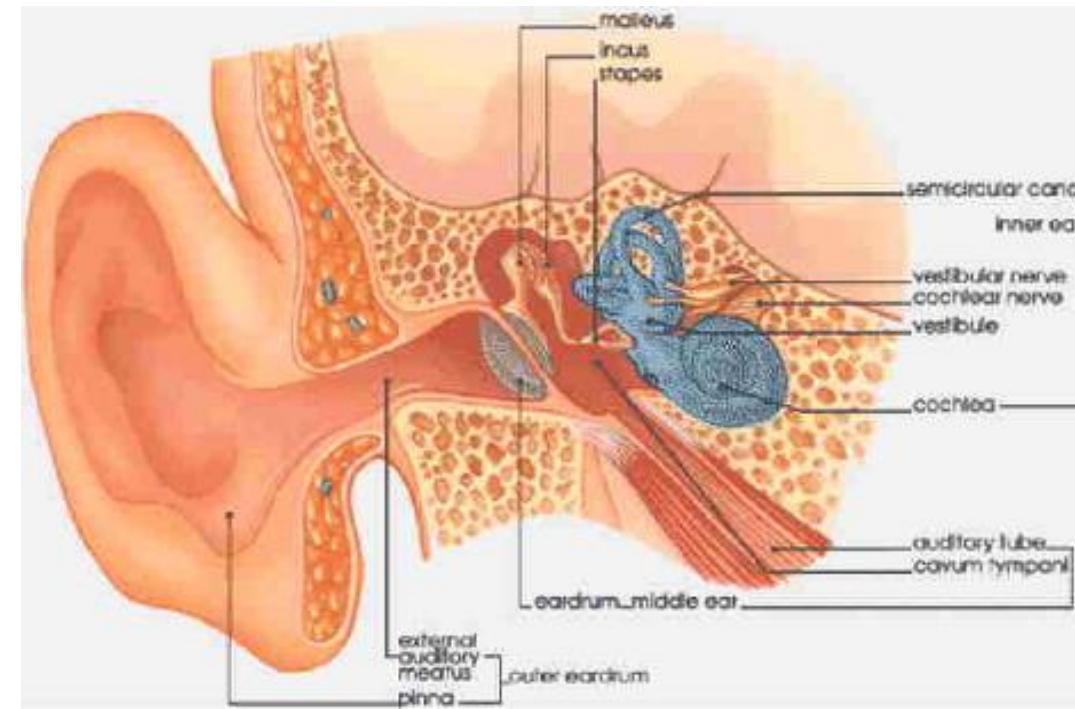
$N_j$  = number of frequencies in 1/3 octave band j [-]

## Non-periodic signals

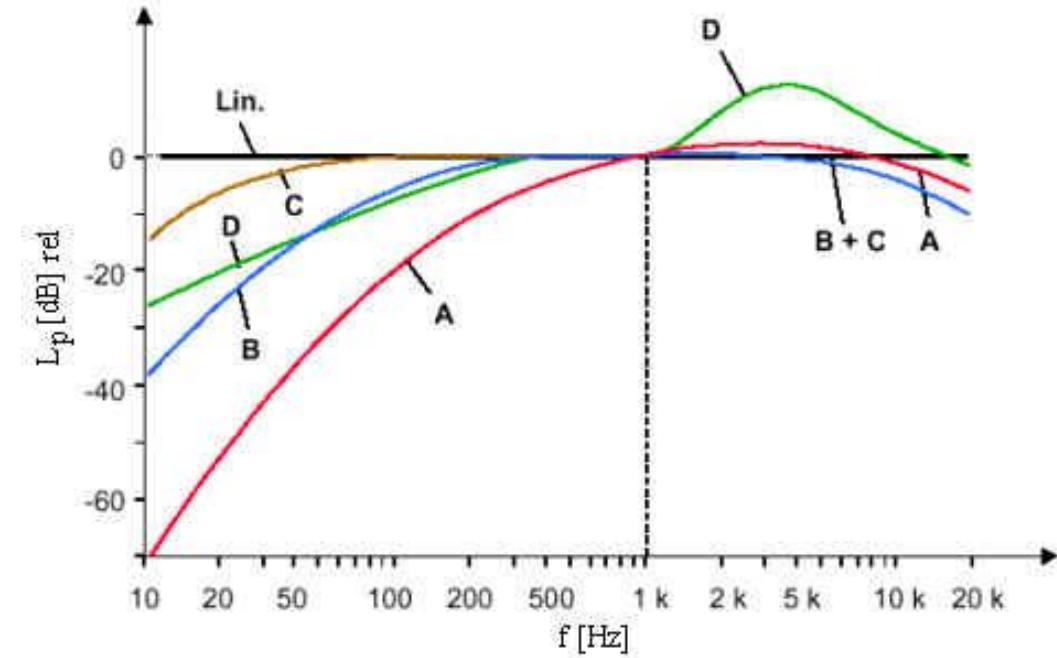
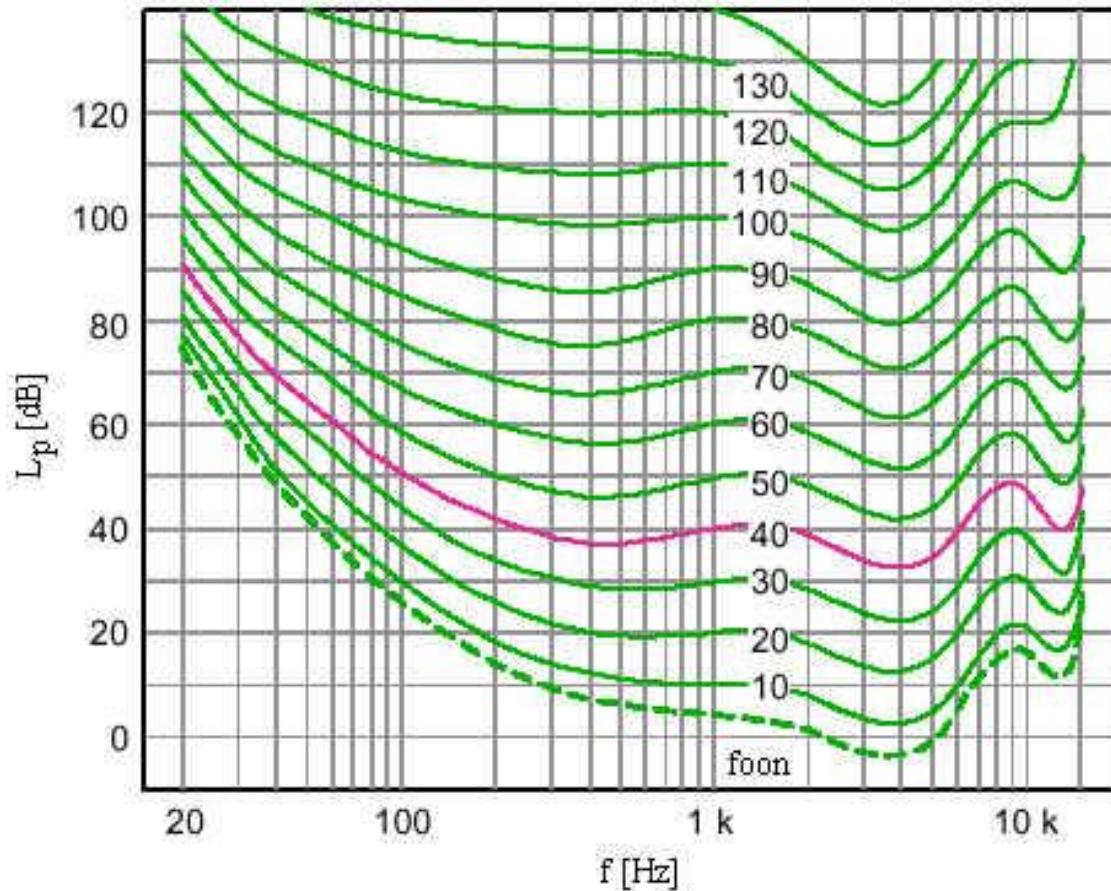


## Our hearing

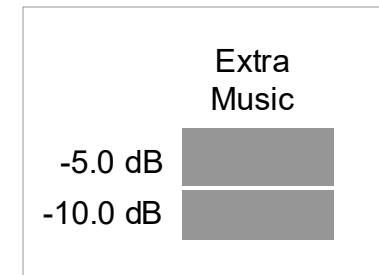
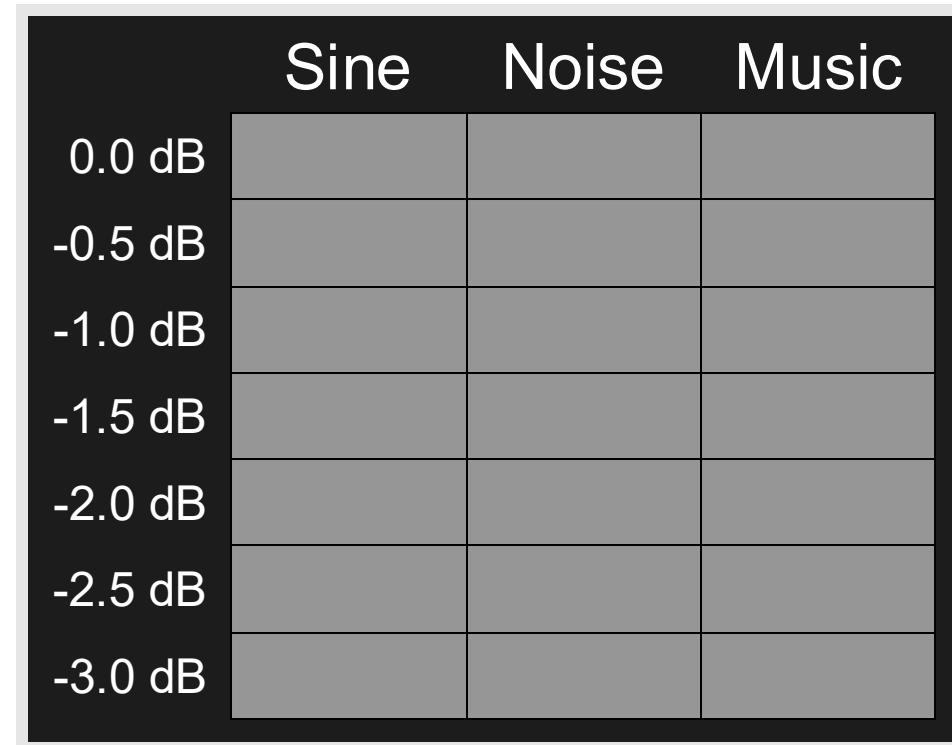
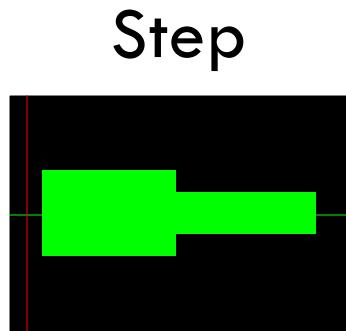
- 20 - 20000 Hz
- Averages fluctuating sound pressure levels
- Can distinguish time patterns
- Not equally sensitive to all frequencies
- Resolution in time and angle (1 degree)?



## Isophone curves and weighting

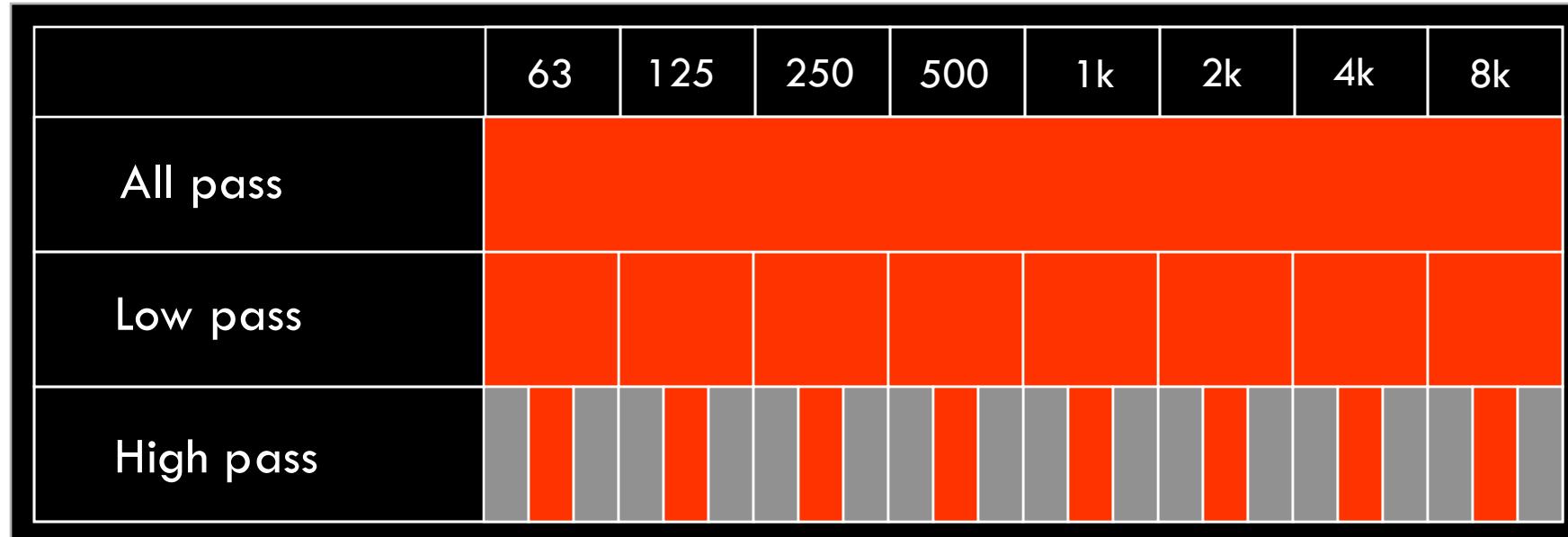


## Audible dB differences

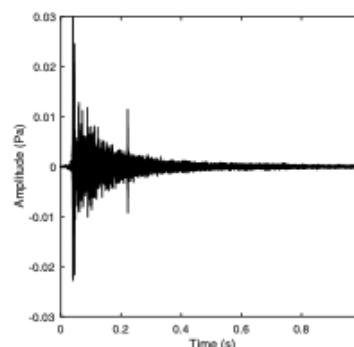
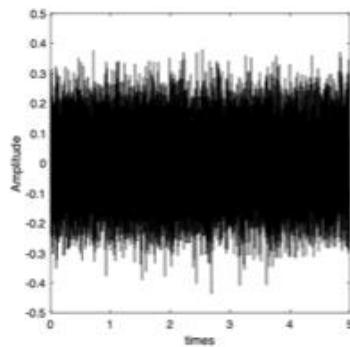




## High and low pass filtering

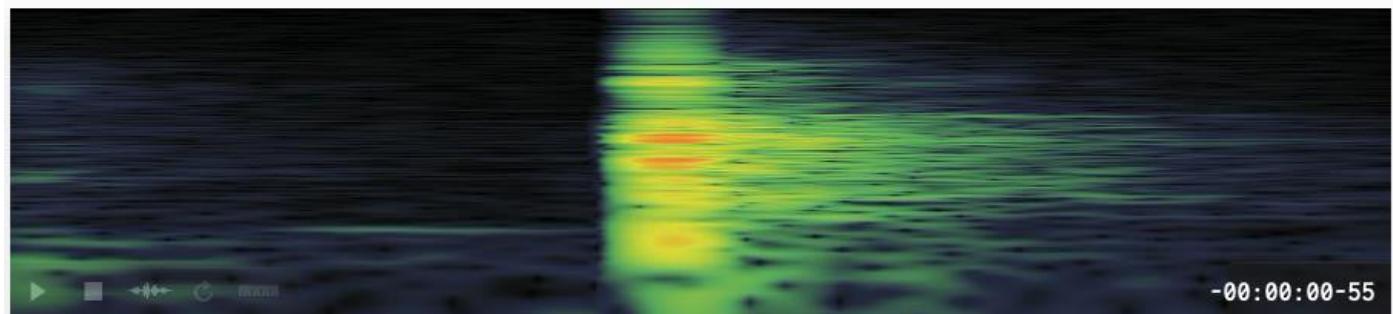
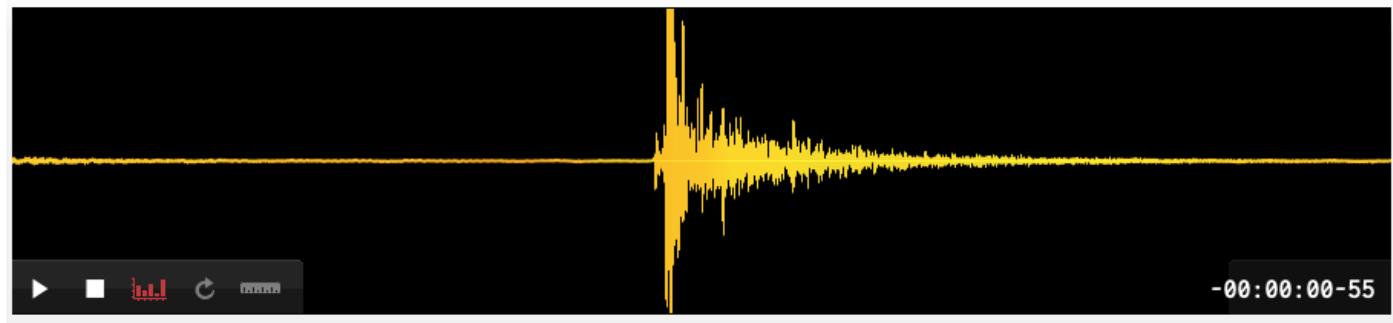


## Influence of binaural hearing on signals



# Influence of binaural hearing on signals

**Hand clap**



## Binaural hearing

Binaural hearing uses two main acoustic cues: interaural time difference (ITD) and interaural level difference (ILD)

- ITD is the delay between both ears. It is efficient for low frequencies (below 850 Hz). It is due to the envelope of the signal reaching the two ears. It can be reminded that a sound coming from the side at 90° has an ITD of 0.6 ms. When the source is situated in the front (azimuth 0°), the so-called front target, the ITD is 0 ms.
- ILD is related to the intensity reaching the two ears. The signal is more or less attenuated by the head shadow. This effect is mostly perceptible with high frequencies (above 3 kHz). ILD is 0 for the front target.

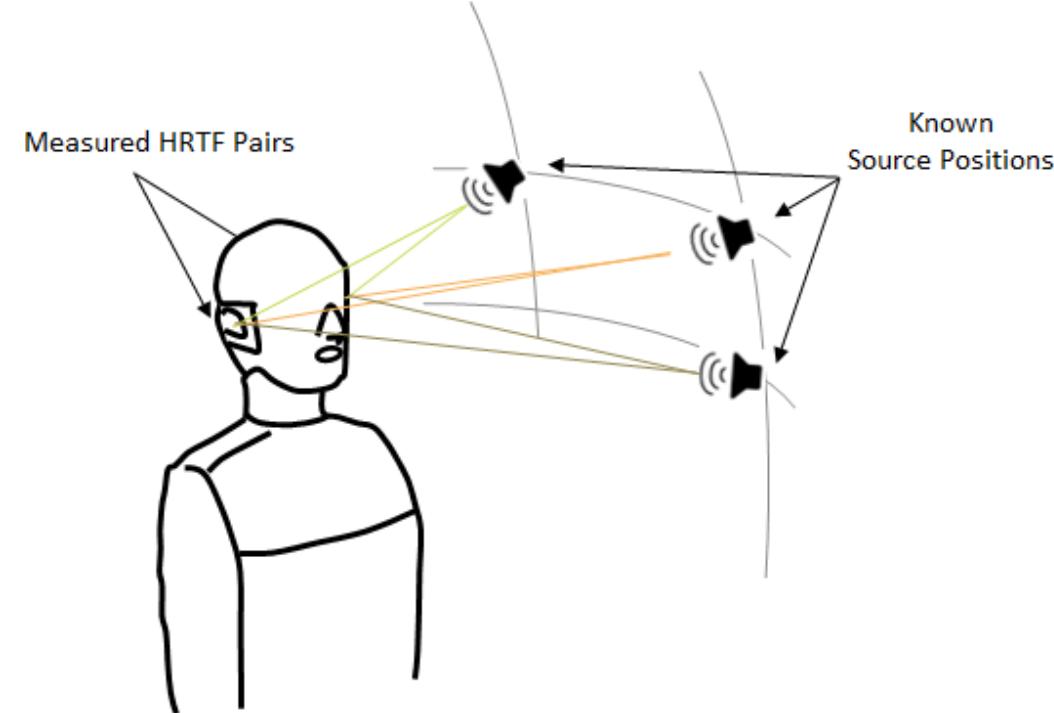
(Doerbecker and Ernst, 1996; Francart et al., 2011):

<https://www.sciencedirect.com/topics/medicine-and-dentistry/binaural-hearing>

## Binaural hearing

Head Related Impulse Response (or Transfer Function), HRIR (or HRTF)

- Frequency dependency
- Directivity



<https://www.mathworks.com/help/audio/ref/interpolatehrtf.html>

## Binaural hearing

HRTF spectrum

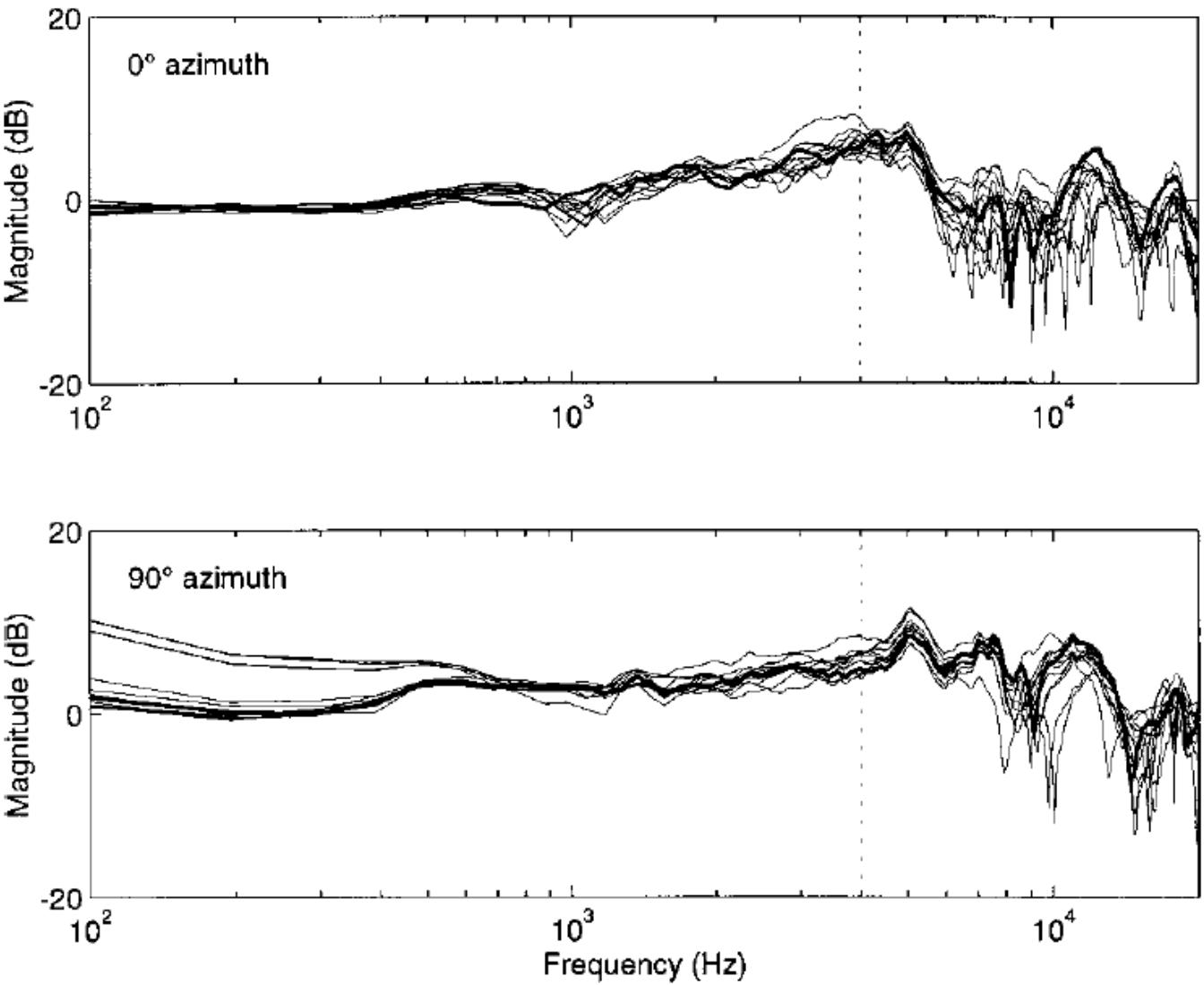
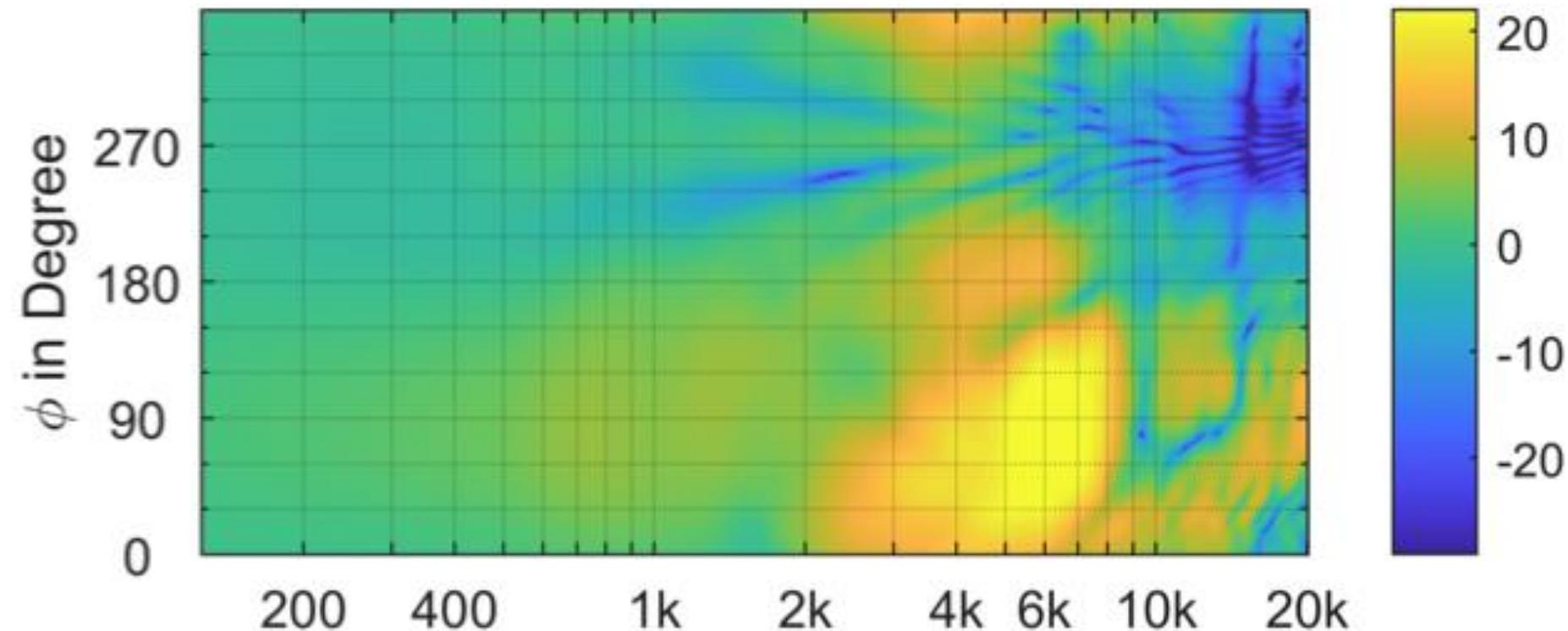


FIG. 1. Examples of 12 individual HRTFs (thin lines) and the general HRTF (heavy line) for the right ear in two different azimuth angles. The dotted vertical line marks the upper frequency of 4 kHz employed in the present study.

Drullman, R. and Bronkhorst, A.W., 2000. Multichannel speech intelligibility and talker recognition using monaural, binaural, and three-dimensional auditory presentation. *The Journal of the Acoustical Society of America*, 107(4), pp.2224-2235.

## Binaural hearing

Measured left ear horizontal plane HRTF



## Tutorial time

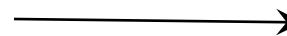
**Record and analyse your sounds with a Matlab mlx script**

**Try different sounds**

- An impulsive signal (hand clap or tongue click)
- A sustained vowel
- The background sound in the room

**Inspect the time signals, spectra with your neighbour and discuss**

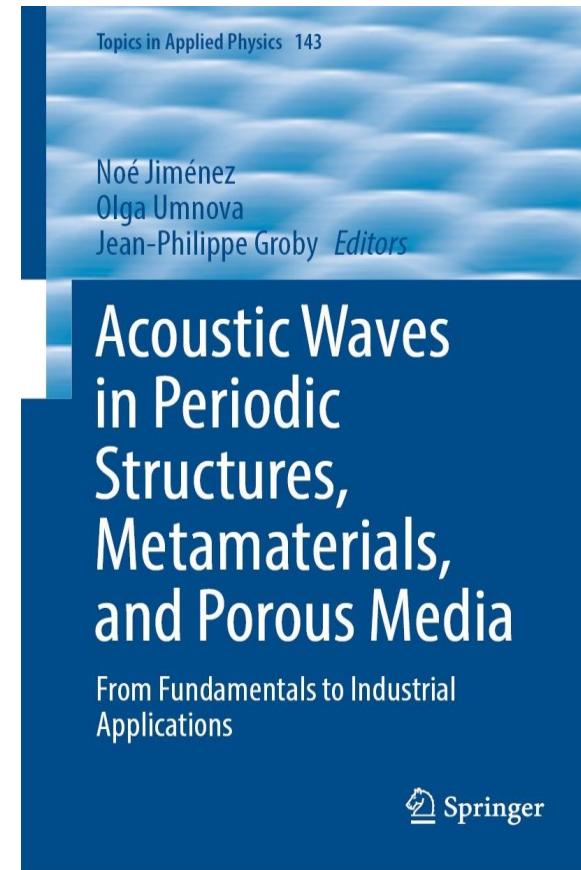
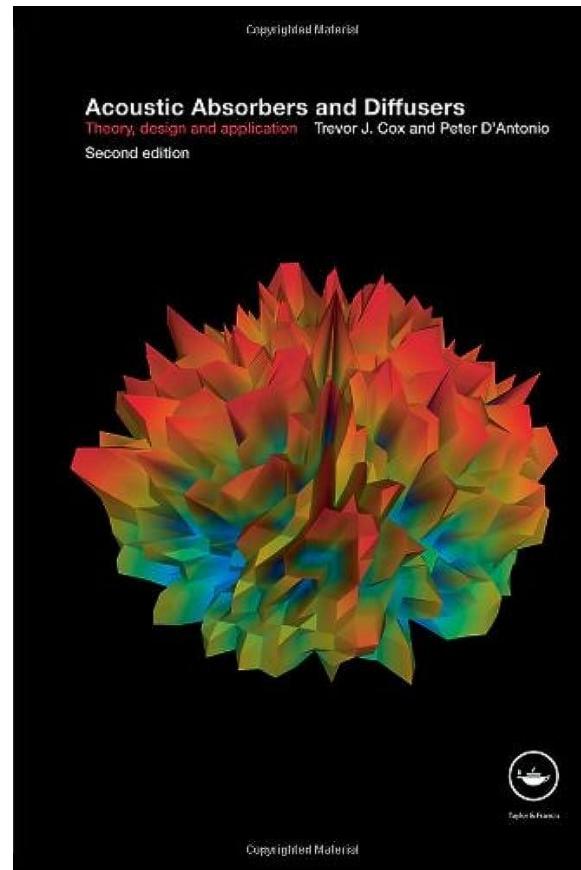
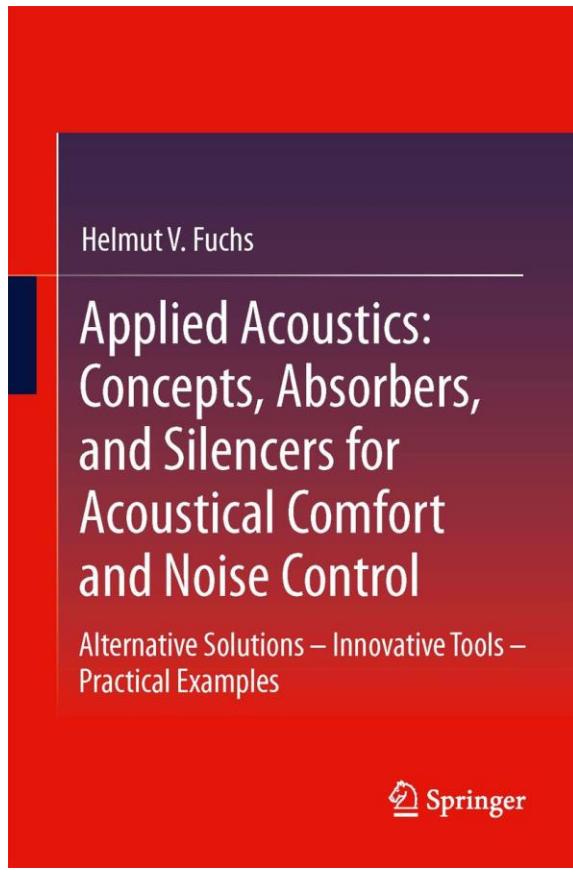
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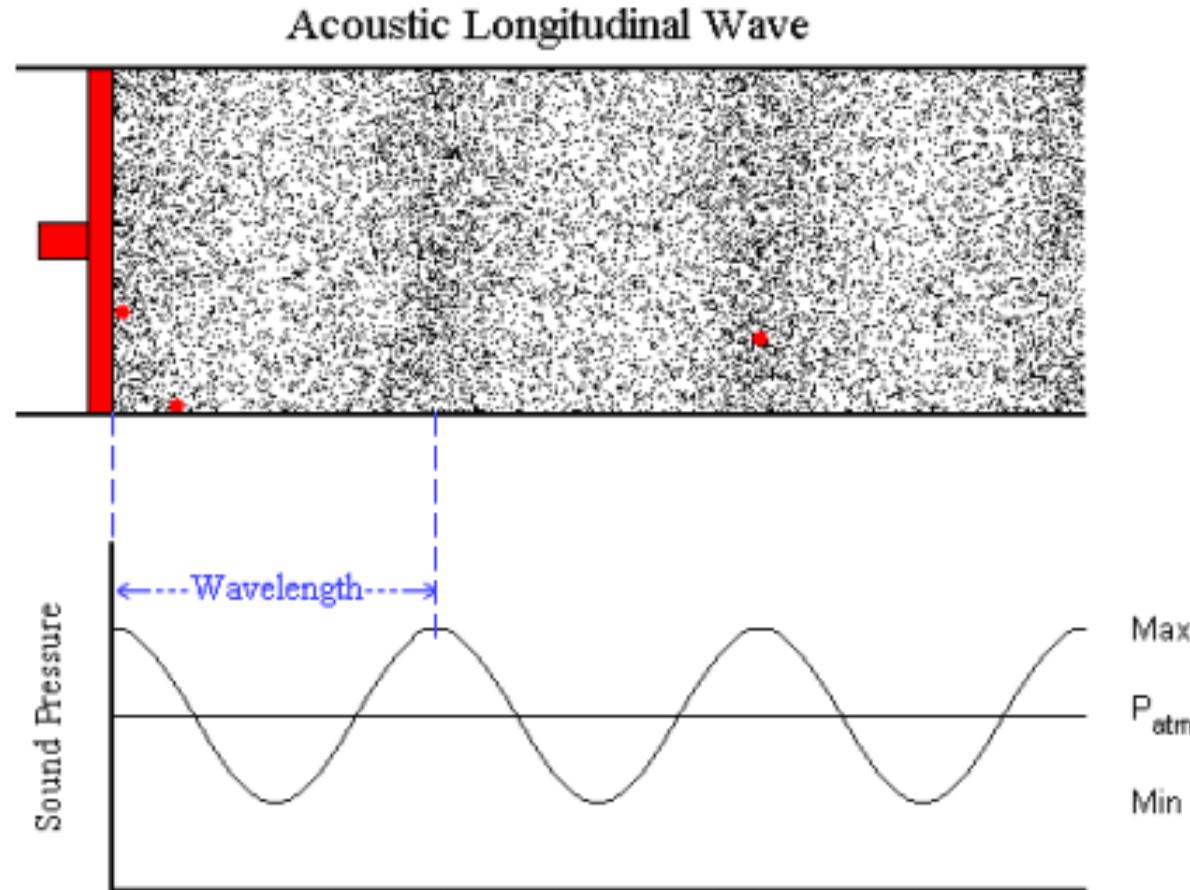
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## Books on acoustic materials



# One dimensional sound propagation



*isvr*

## One dimensional sound propagation



# 1D Wave Equation in fluids (as air)

## Assumptions

- No viscous effects
- No mean flow effects
- Temperature fluctuations but no heat flow (adiabatic conditions)
- Amplitudes of acoustic variables are small (pressure amplitude < 1 Pa) such that linearization applies
- We consider the molecular macro scale, i.e. not at molecule level
- + relation between pressure and density

$$p = c^2 \rho$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

## Harmonic waves: solution to 1D wave equation

$$p(x, t) = A e^{j(\omega t - kx)}$$

There is also a running wave in the other direction

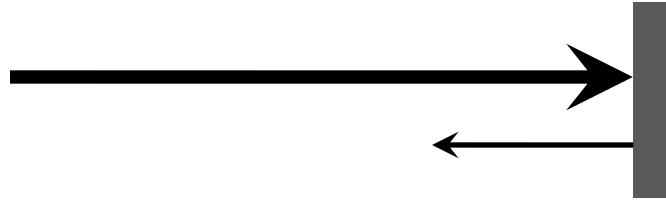
$$k = \frac{\omega}{c} \quad \text{wavenumber}$$

$$v_x(x, t) = B e^{j(\omega t - kx)}$$

Impedance of air

$$Z_0 = \rho_0 c = \frac{p}{v_x}$$

## Solution to 1D wave equation with boundary



$$p(x, t) = p_i + p_r = A(e^{j(\omega t - kx)} + R e^{j(\omega t + kx)})$$

With  $R$  the complex reflection coefficient of the boundary

$$R(f) = \frac{Z_n(f) - 1}{Z_n(f) + 1}$$

$Z_n$  = normalized surface impedance, depends on material

## Solution to 1D wave equation with boundary

$R$  can be a complex number (value between -1 and 1 real and imaginary), which leads to amplitude and phase effects of the reflected sound

**Demo** with sound field in front of boundary changing the amplitude and phase of the reflection coefficient

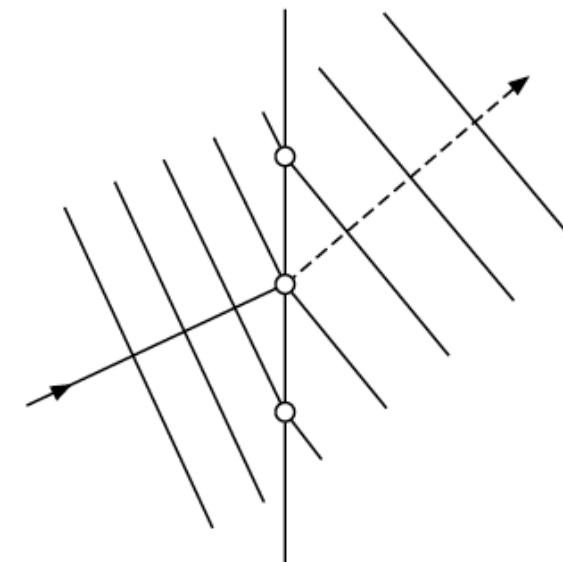
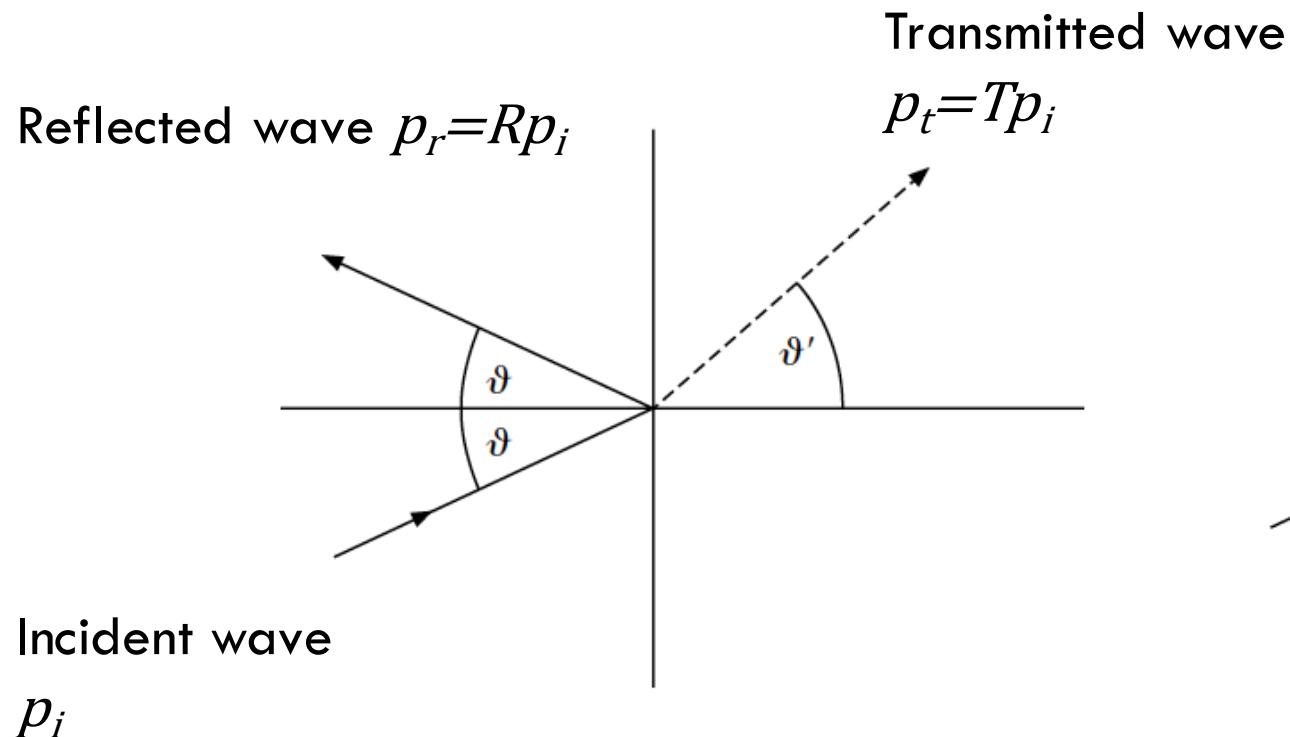
Demo

**Demo** with sound field in front of boundary changing the amplitude of the surface impedance

Demo

## Reflection in 2D: plane waves

Plane wave reflection coefficient



## Reflection in 2D (plane surface)

Plane wave reflection coefficient

$$R(f) = \frac{Z_n(f) \cos(\theta) - \cos(\theta')}{Z_n(f) \cos(\theta) + \cos(\theta')}$$

$$\frac{c}{\sin(\theta)} = \frac{c'}{\sin(\theta')}$$

$$\alpha = 1 - |R|^2$$

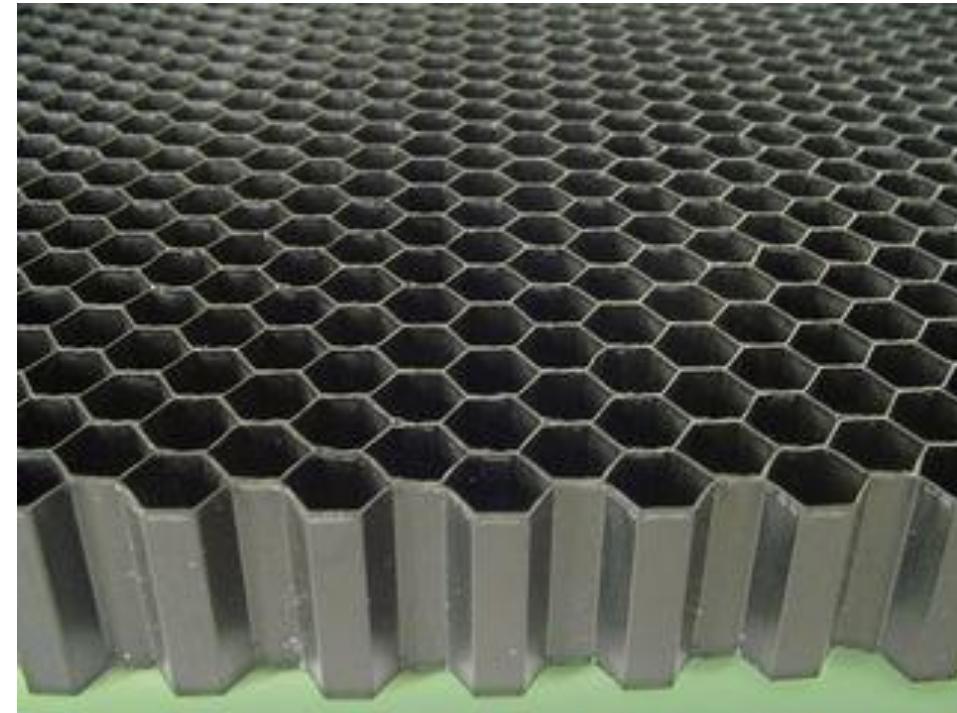
Absorption coefficient  $\alpha$  :

- is real value between 0 a 1
- does not contain phase information of the reflected sound

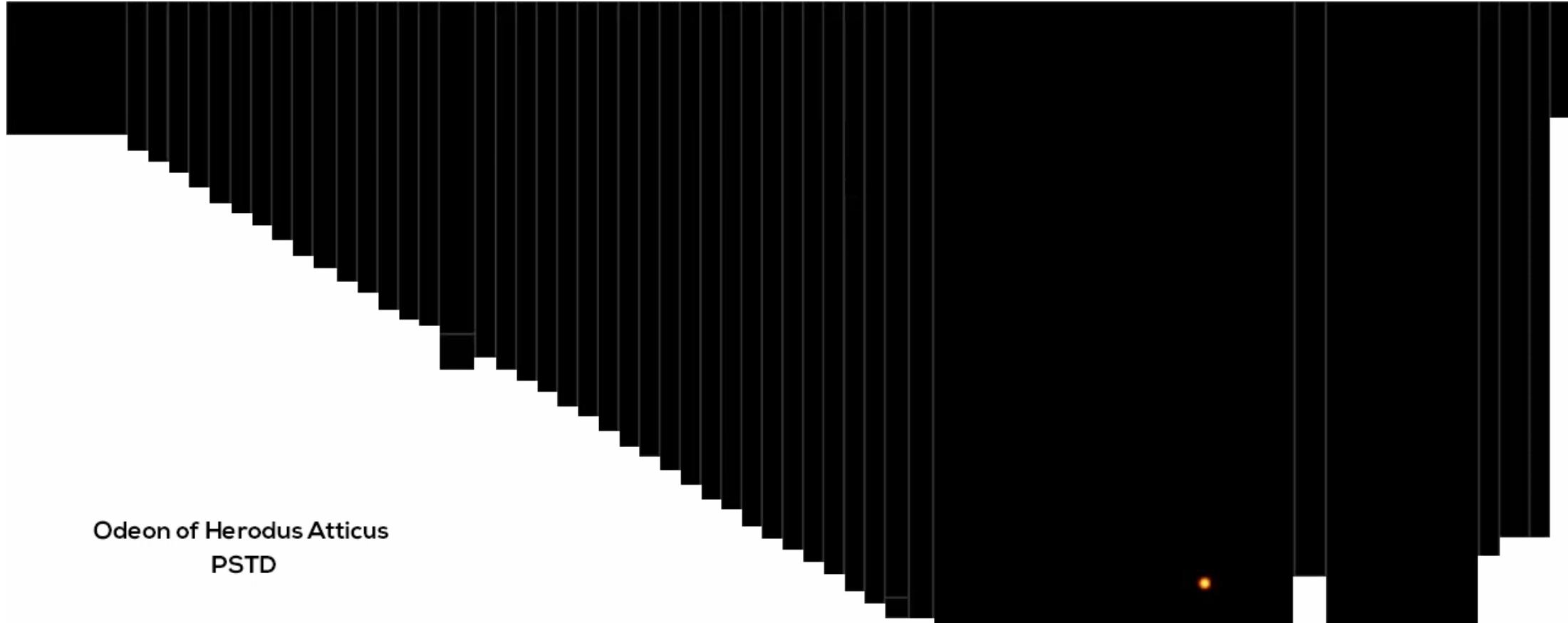
## Reflection in 2D (plane surface)

Locally reacting medium

$$R(f) = \frac{Z_n(f) \cos(\theta) - 1}{Z_n(f) \cos(\theta) + 1}$$



## Reflection in 2D: cylindrical waves



# Cylindrical and spherical wave reflection coeff.

## Cylindrical wave reflection coefficient can be found in

F. P. Mechel, A line source above a plane absorber, *Acoust. Acta Acoust.* 86 (2000) 203–215.

## Spherical wave reflection coefficient

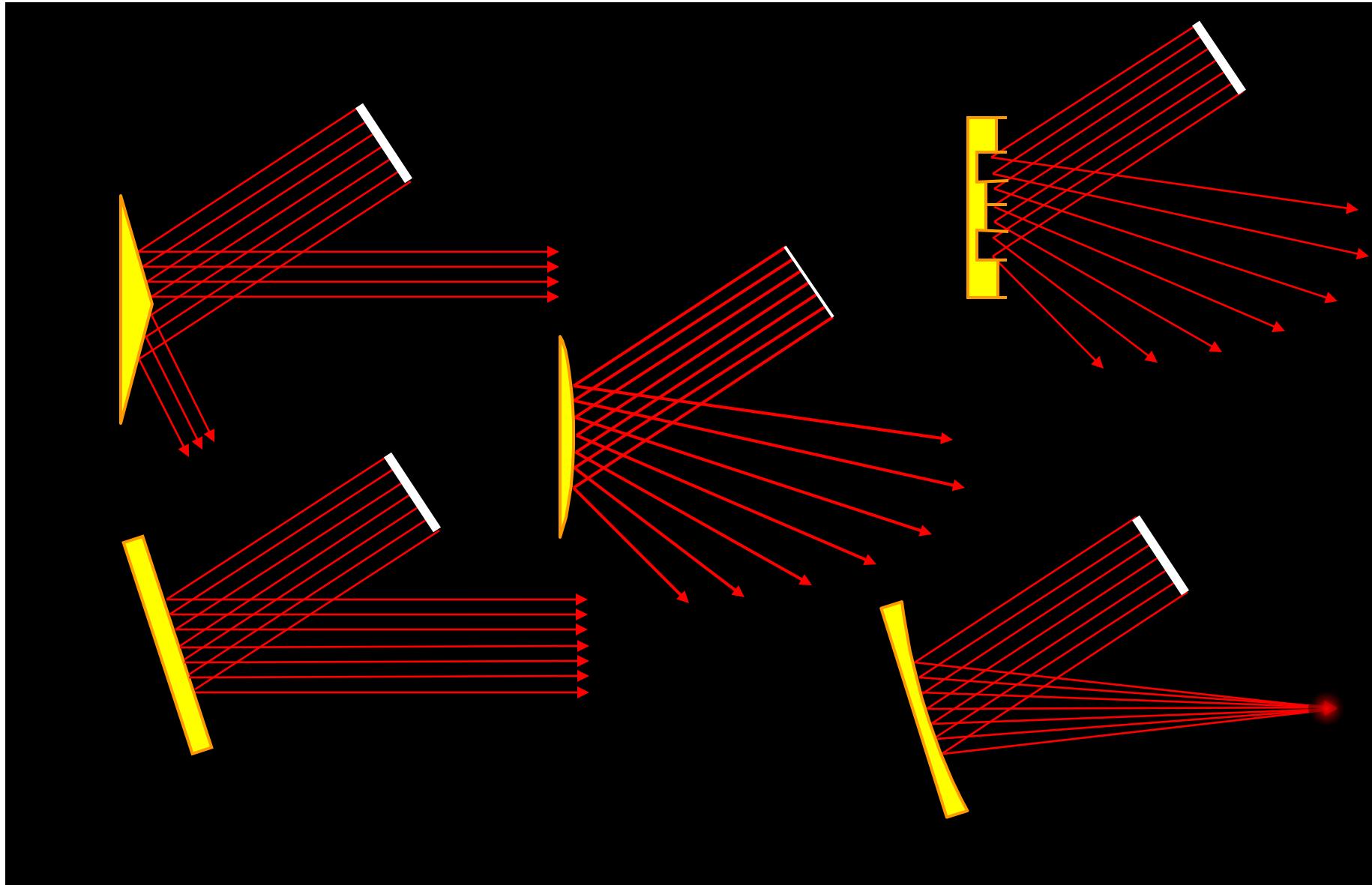
$$R_{\text{sph}} = R + (1 - R)F(w) \quad \text{sphe}$$

$$F(w) = 1 - j\sqrt{\pi}e^{-w^2} \operatorname{erfc}(jw) \quad 1$$

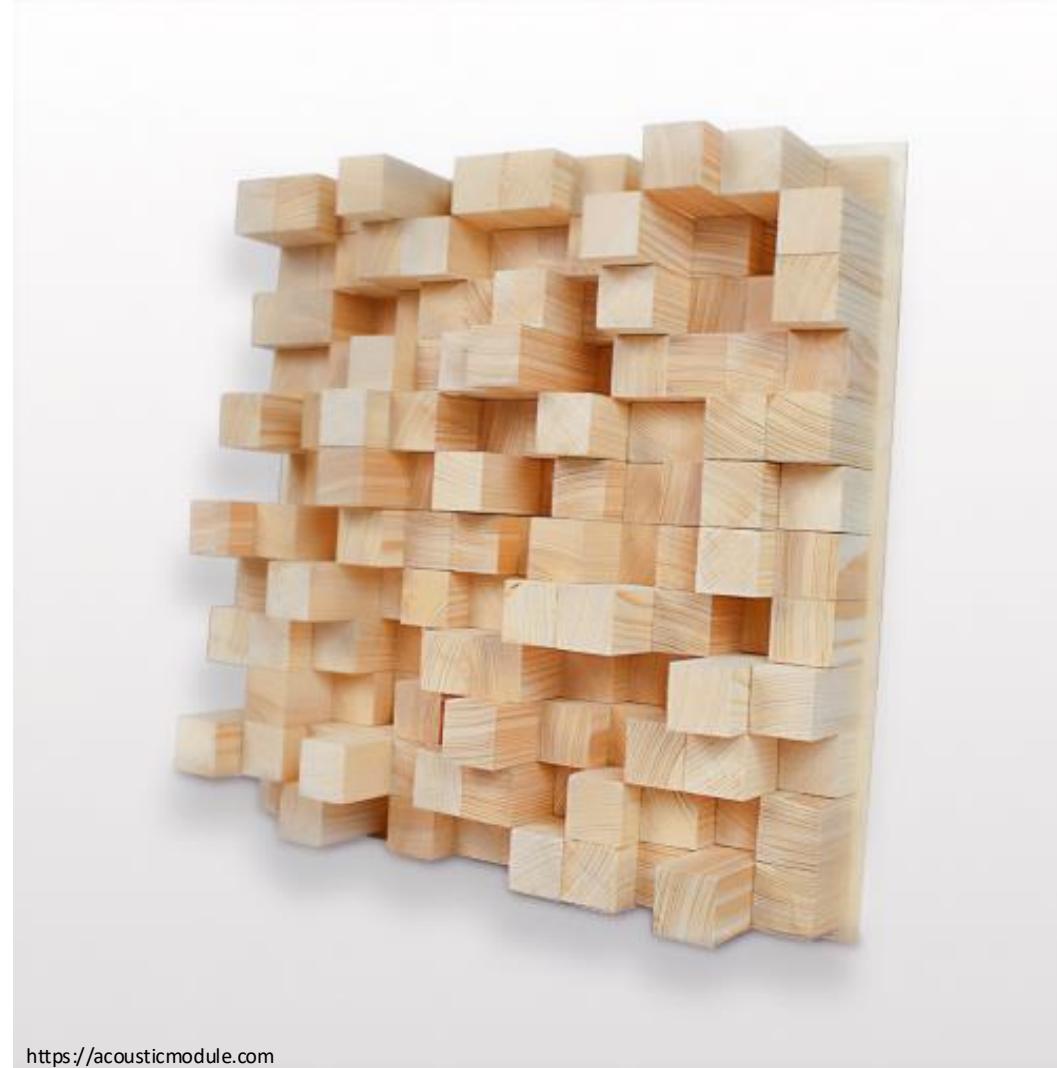
$$w \approx \frac{1}{2}(1 - j)\sqrt{kr_2}(\cos(\theta) + \frac{1}{Z})$$

$$R \approx R_{\text{sph}} \quad \text{if} \quad |w| > 4$$

## Surface types



## Surface types



## Porous material

Important for a porous material:

- Flow resistivity
- Thickness
- Porosity

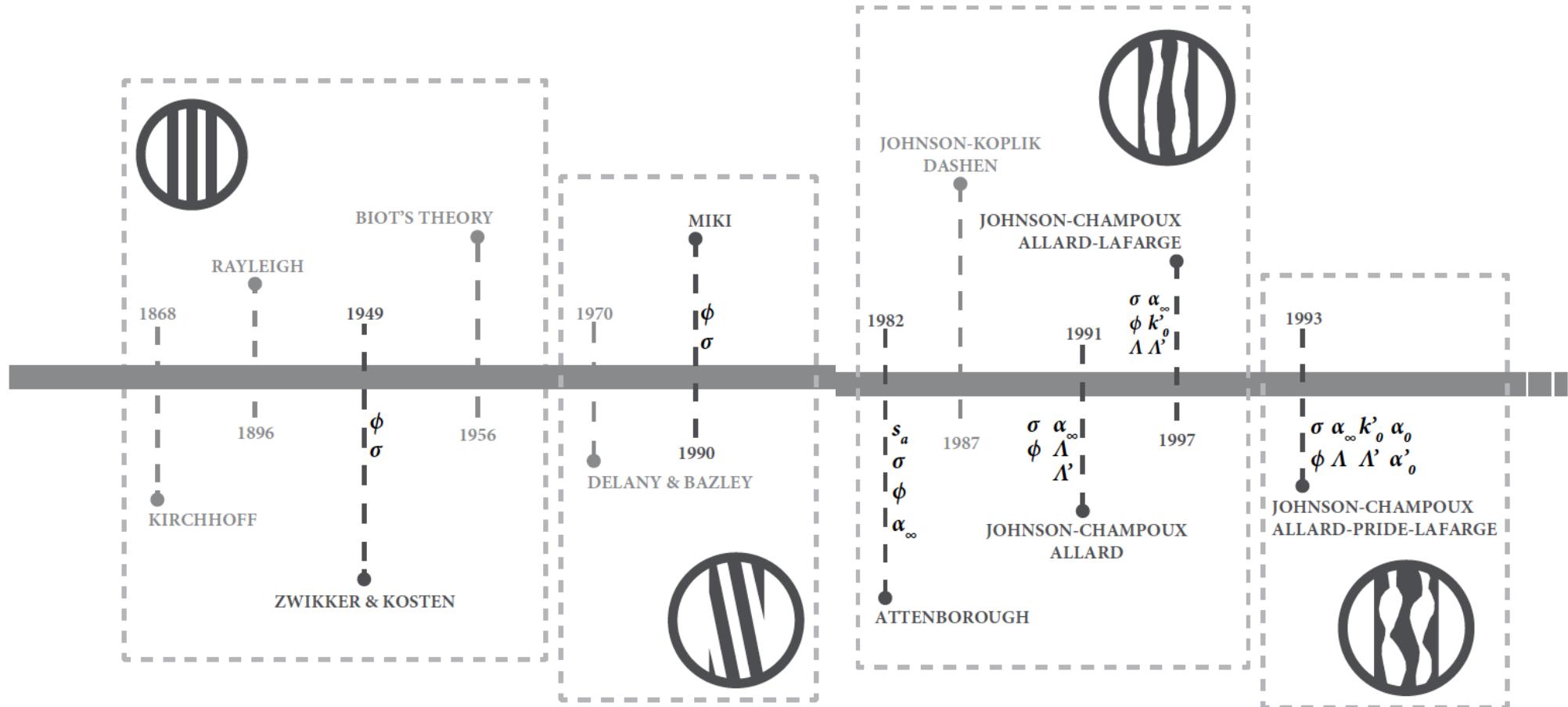


Wavenumber and specific impedance of a simplified porous material  
(according to Kuttruff, H. (2007). Acoustics: an introduction). Values within  
one pore

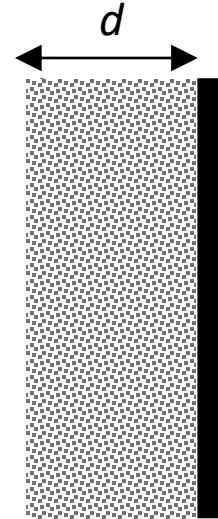
$$k' = \frac{\omega}{c} \sqrt{1 - \frac{j\sigma\Xi}{\rho_0\omega}}$$

$$Z' = Z_0 \sqrt{1 - \frac{j\sigma\Xi}{\rho_0\omega}}$$

# Porous materials



## Porous material on a hard boundary



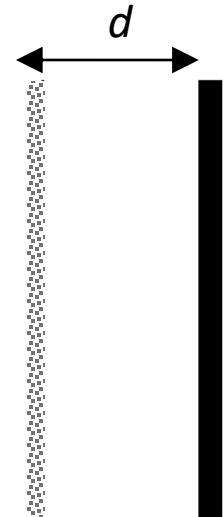
Porous material



$$Z = -jZ' \cot(k'd)$$

Impedance of porous layer with rigid backing at distance  $d$

## Porous sheet on a hard boundary



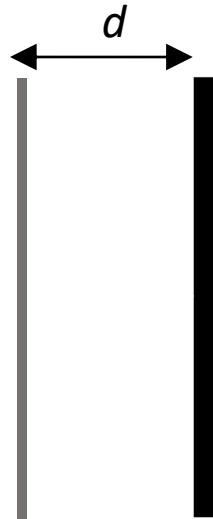
Porous sheet

$$Z = r_s - jZ_0 \cot(kd)$$

Impedance of porous sheet on air layer with rigid backing at  
distance d



## Solid sheet on a hard boundary



Solid sheet

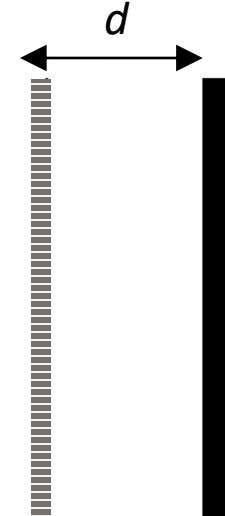


<http://www.melfoamacoustics.com>

$$Z = j\omega m' - jZ_0 \cot(kd)$$

Impedance of non-porous sheet on air layer with rigid backing at distance  $d$

## Perforated sheet on a hard boundary



Perforated sheet

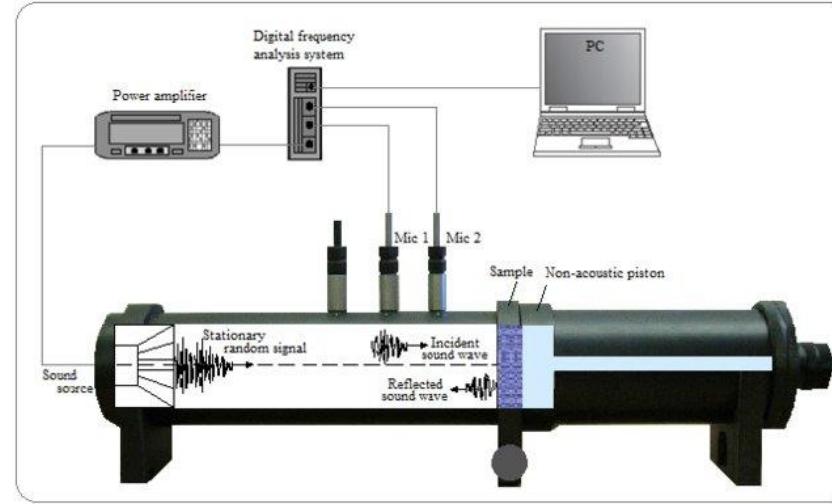


$$Z = r + j\omega m'_a - jZ_0 \cot(kd)$$

Impedance of perforated sheet on air layer with rigid backing at distance  $d$ , with  $m'_a$  the mass of the moving air in the pores and  $r$  the resistance due to friction from the movements in the pores

# Determining surface impedance/sound absorption

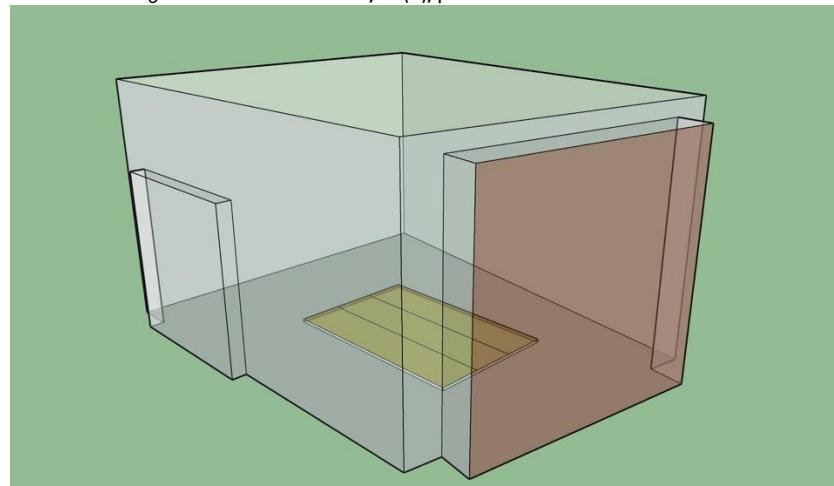
## Lab measurements



Çelikel, D.C. and Babaarslan, O., 2017. Effect of bicomponent fibers on sound absorption properties of multilayer nonwovens. *Journal of engineered fibers and fabrics*, 12(4), p.155892501701200403.

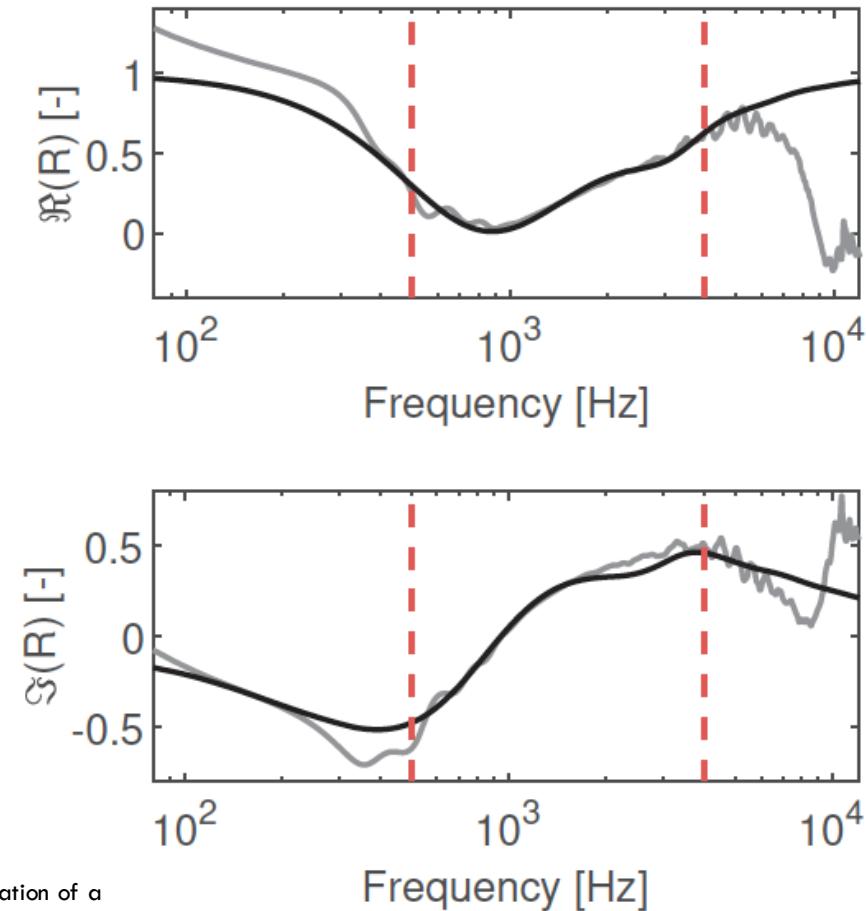
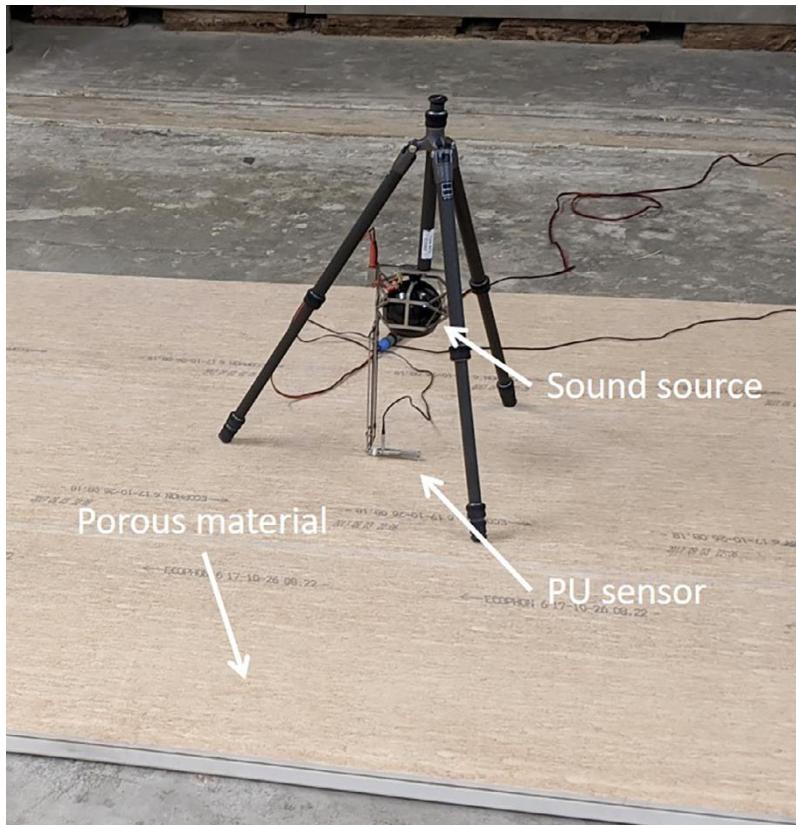


Baltazar Briere



# Determining surface impedance/sound absorption

## In-situ approach



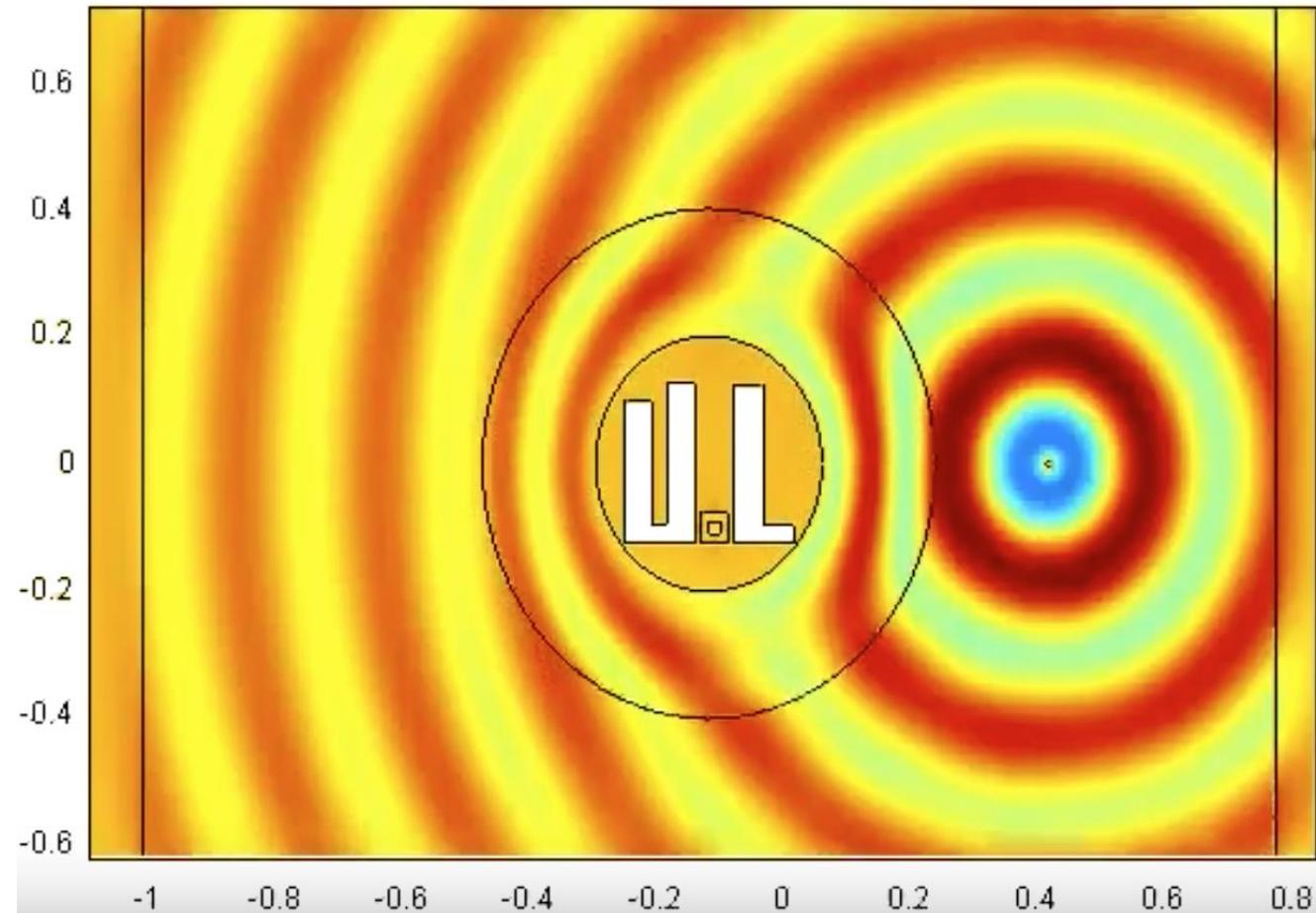
de La Hosseraye, B.B., Hornikx, M. and Yang, J., 2022. In situ acoustic characterization of a locally reacting porous material by means of PU measurement and model fitting. *Applied Acoustics*, 191, p.108669.

## Metamaterials as sound absorption



<https://www.youtube.com/watch?v=JUi3qK32mf0>

## Metamaterials as sound absorption



Sébastien Guenneau of Liverpool University, UK

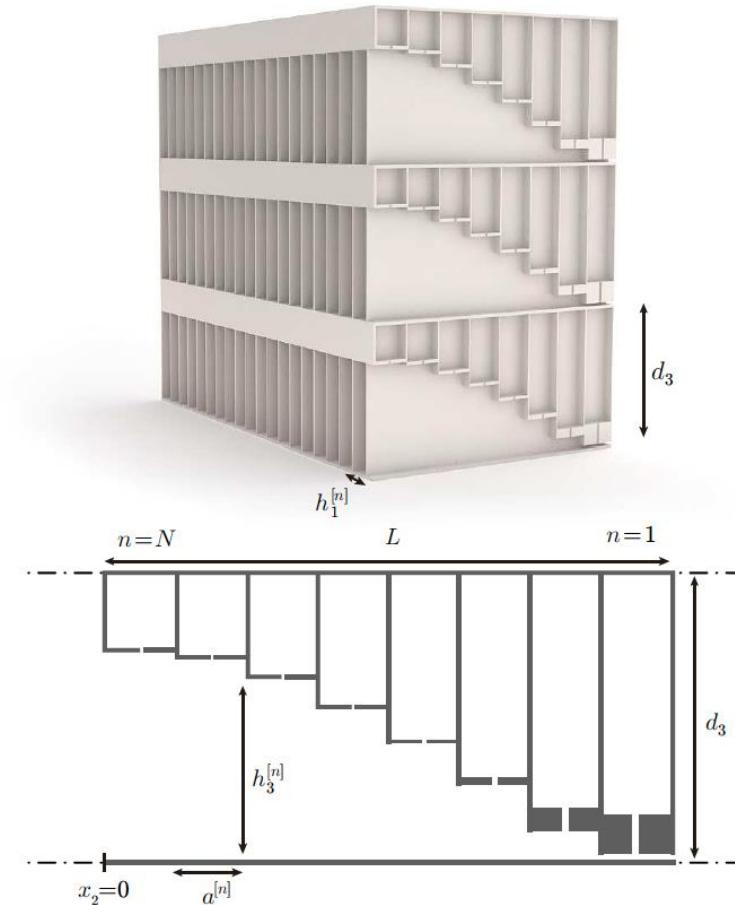
# Metamaterials as sound absorption

## Acoustic metamaterials

- Artificial material (not occurring in nature)
- Consisting of meta-atoms (typically periodic)
- Dimensions much smaller than wavelength
- Exhibiting behavior not encountered in nature
- ... materials designed to control, direct, and manipulate sound waves

# Metamaterials as sound absorption

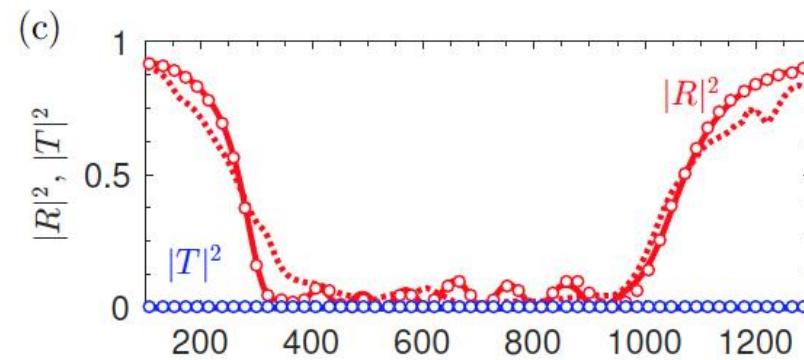
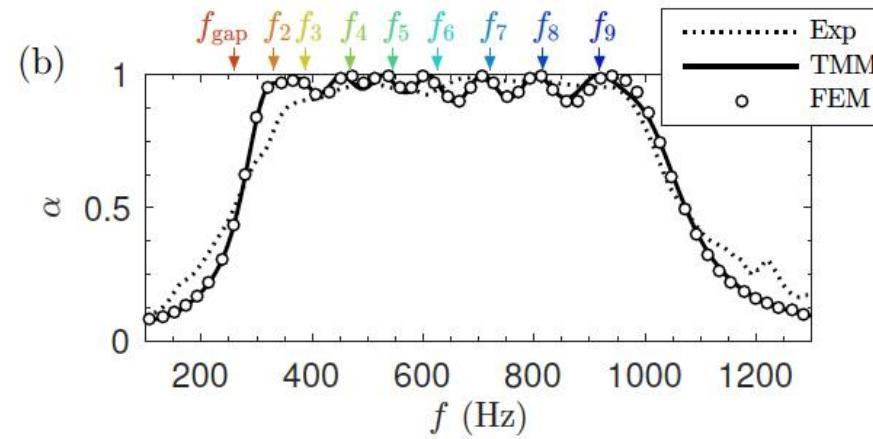
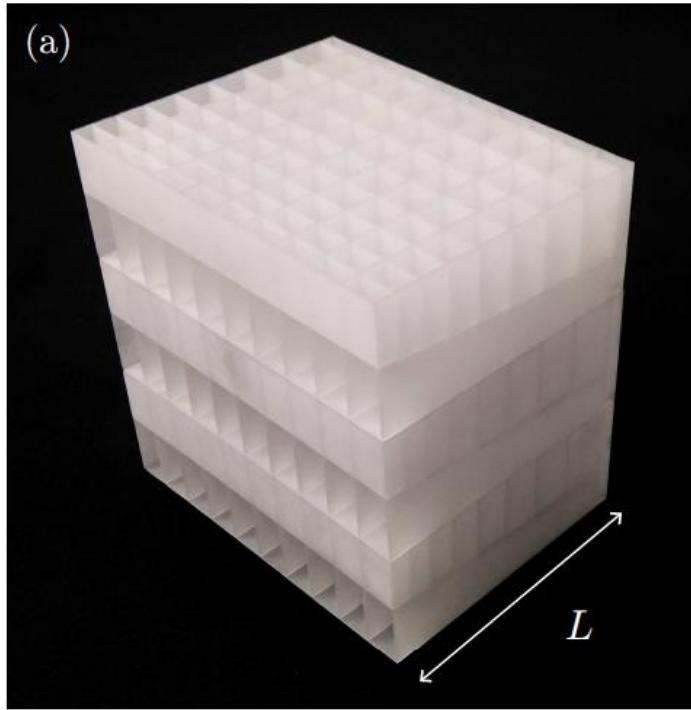
Example: Rainbow-trapping absorbers



# Metamaterials as sound absorption

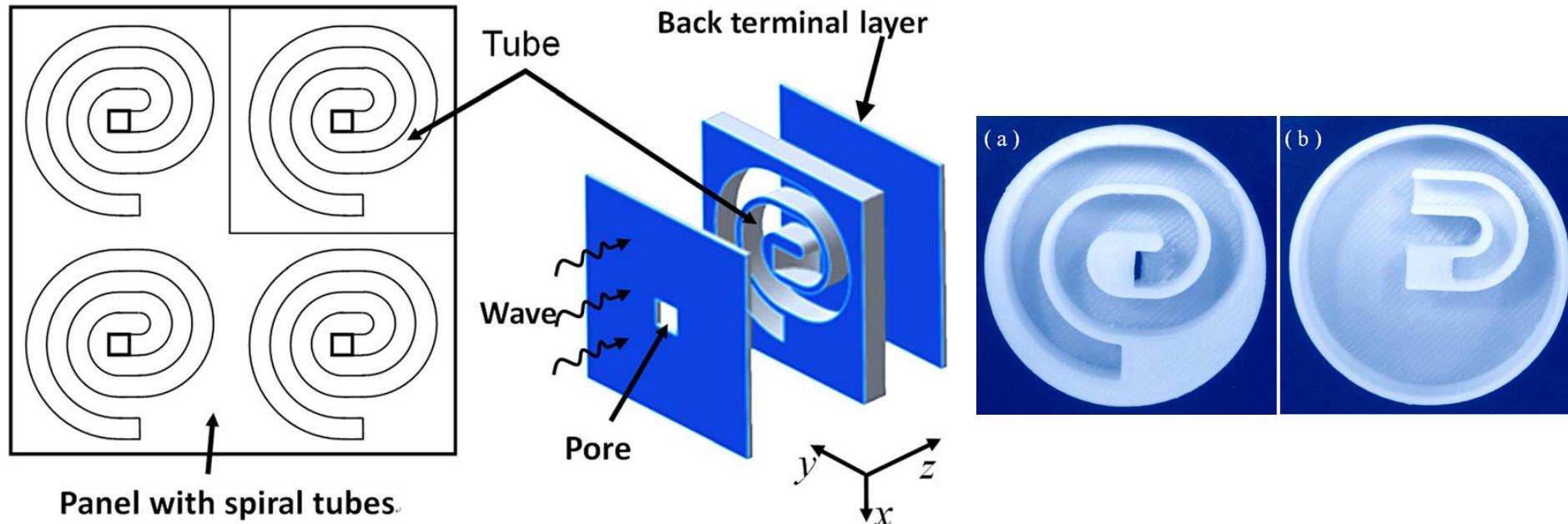
Example: Rainbow-trapping absorbers

Jiménez, N., Romero-García, V., Pagneux, V. and Groby, J.P., 2017. Rainbow-trapping absorbers: Broadband, perfect and asymmetric sound absorption by subwavelength panels with ventilation. *arXiv preprint arXiv:1708.03343*.



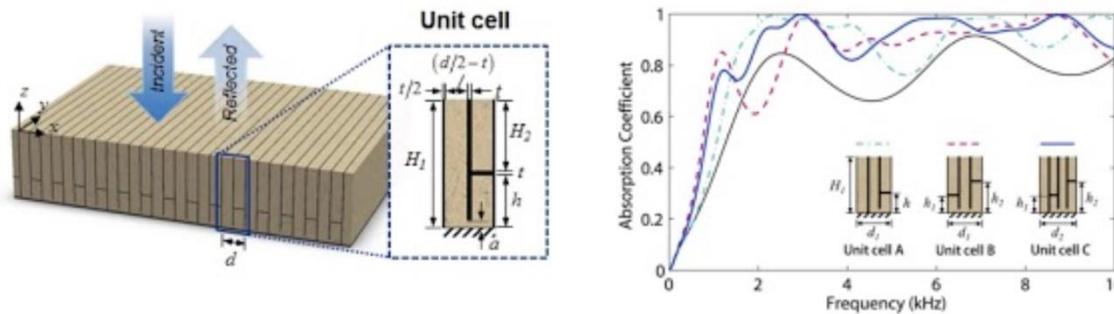
# Metamaterials as sound absorption

Example: Metaporous layer

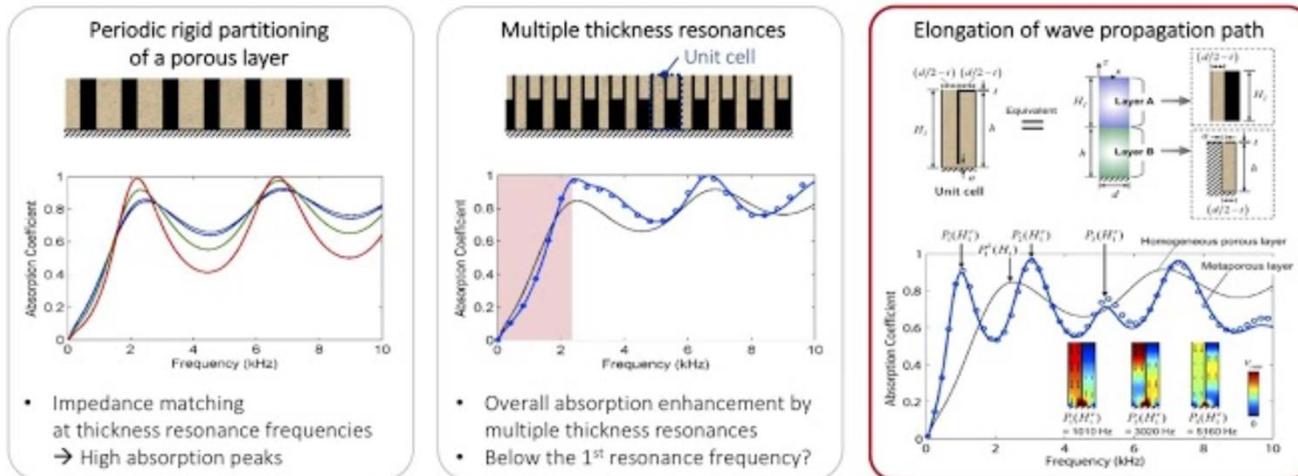


# Metamaterials as sound absorption

## Example: Metaporous layer

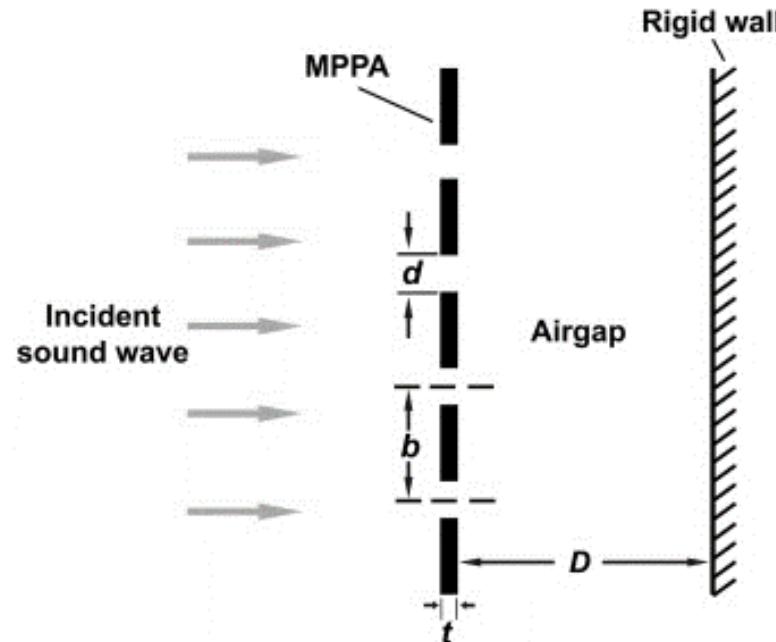
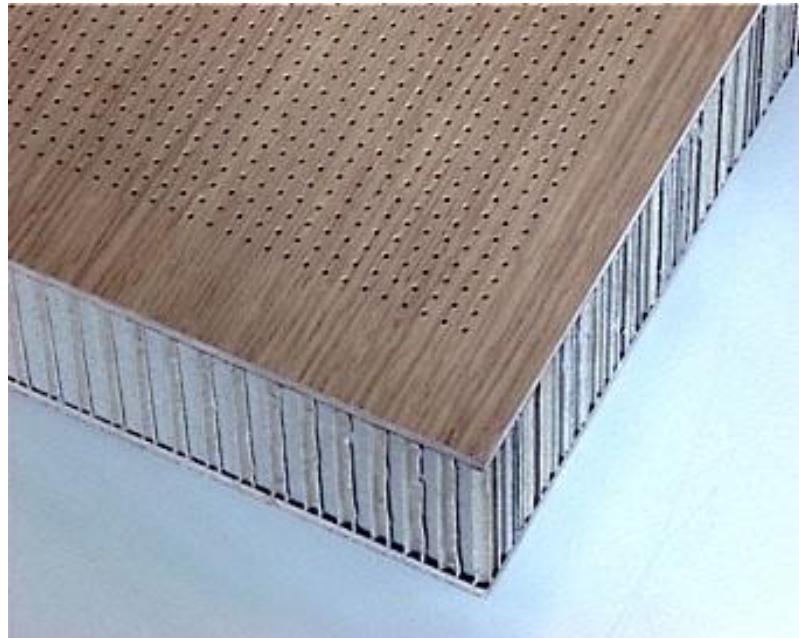


Broadband sound absorption is achieved by a combination of three different mechanisms



# Metamaterials as sound absorption

Example: Microperforated panel



# Metamaterials as sound absorption

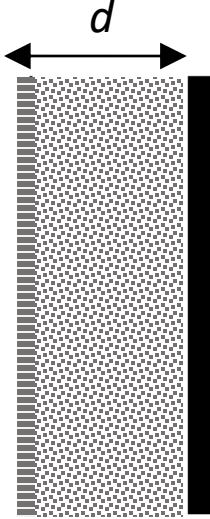
## General challenges

- Thin panel with broadband damping
- Combining mechanisms (porous material + metamaterial)

## Applications in terms of damping of sound

- Low frequency absorption in outdoor noise mitigation (screens, roofs)
- Damping modes in floors
- Reducing sound transmission of open windows

## Tutorial

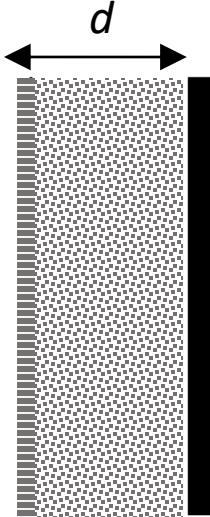


Perforated sheet

$$Z = r + j\omega m'_a - jZ_0 \cot(kd)$$

Impedance of perforated sheet on porous material layer with rigid backing at distance  $d$ , with  $m'_a$  the mass of the moving air in the pores and  $r$  the resistance due to friction from the movements in the pores.

## Tutorial



Perforated sheet



Change variables of this configuration and discuss the changes in reflection coefficient and absorption coefficient that occur.

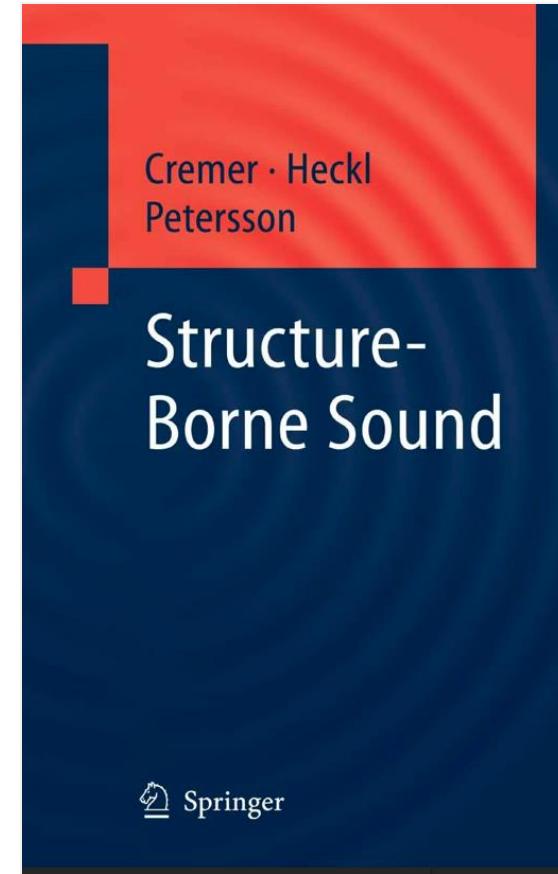
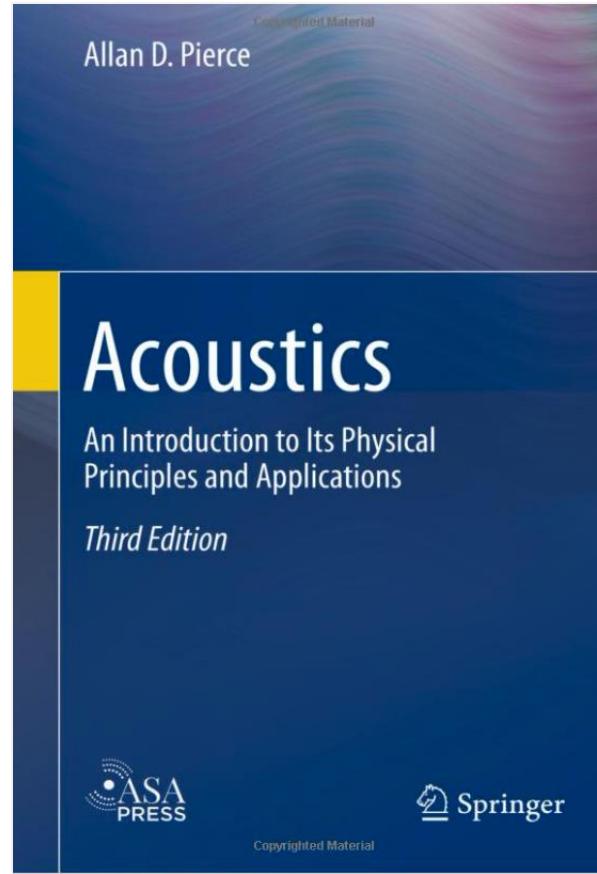
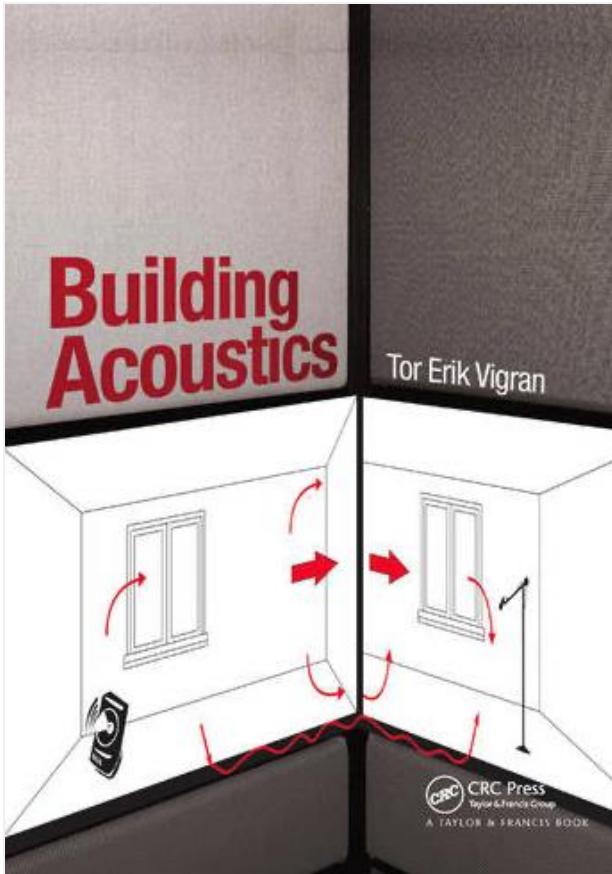
# Course outline

Time table / Lecture modules	TOPIC
<b>DAY 1 - 21/06/2024</b>	
8:30-9:00 h	Registration
9:00-10:30 h	<b>Lecture:</b> Introduction (why do we study acoustics) Basic signal properties: Period, wavelength and frequency. Definitions, and fundamental equations for propagation of sound.
10:30-11:00 h	Coffee break
11:00-12:00 h	<b>Lecture:</b> More advanced signal properties: Pure and complex sounds, frequency decomposition and sound spectrum. Acoustic quantities: levels and their manipulation, Octave bands and physiology of hearing, binaural hearing.
12:00-13:00 h	Plenary session I. Deep learning, AI and acoustics (Prof. Hamid Krim. North Carolina State University) ( <b>All the topics</b> )
13:00-14:00 h	Lunch
14:00-15:30 h	<b>Tutorial:</b> Tools for acoustic analysis Recording of recorded sound Manipulation of sound
15:30-16:00 h	Coffee break
16:00-17:00 h	<b>Lecture:</b> Wave-surface interactions, 1D wave equation solution including reflection coefficient Sound absorption, types of sound absorption materials

# Sound insulation

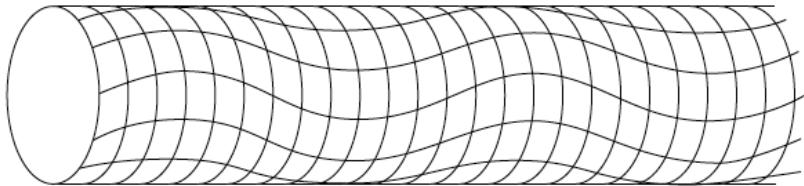


## Building Acoustics books

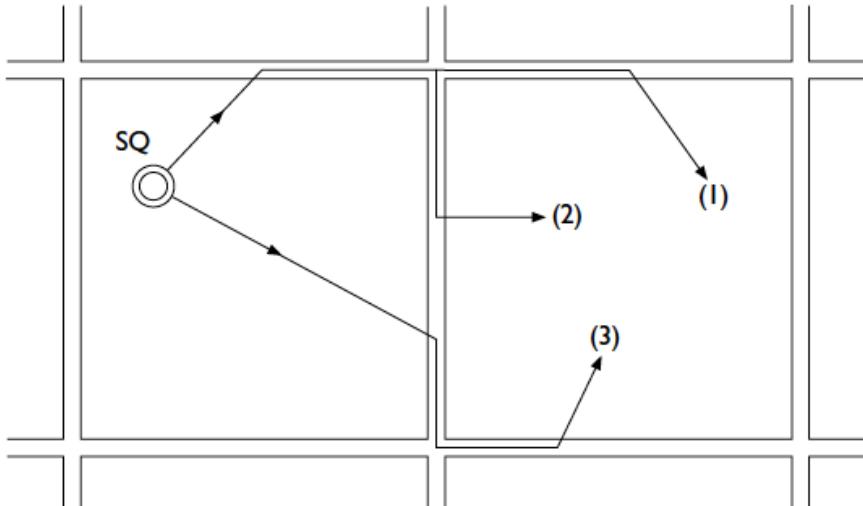


# Sound insulation lecture

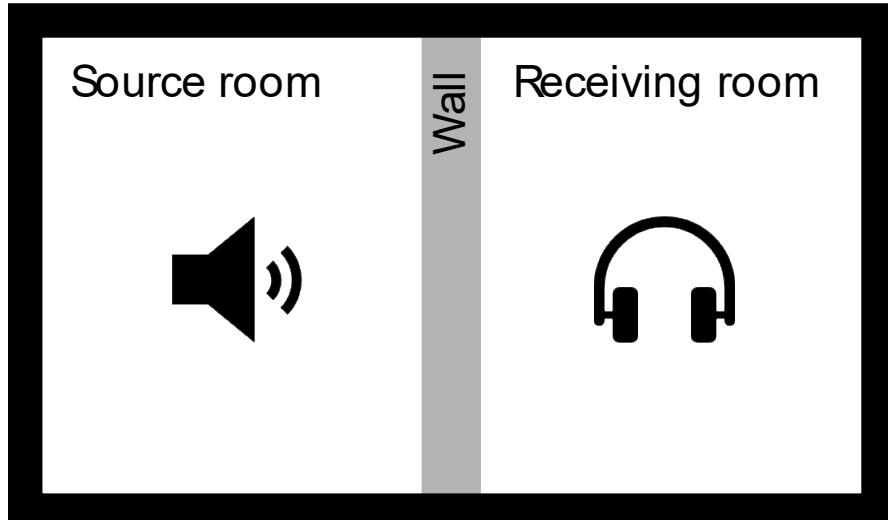
## Sound waves in isotropic solids



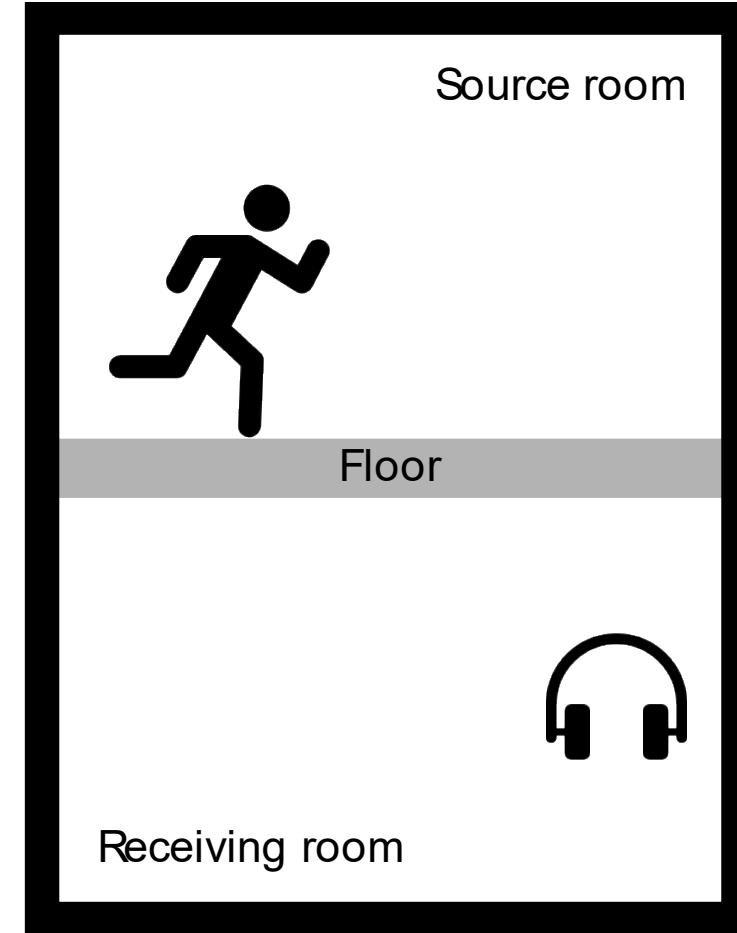
## Building Acoustics



## Sound insulation

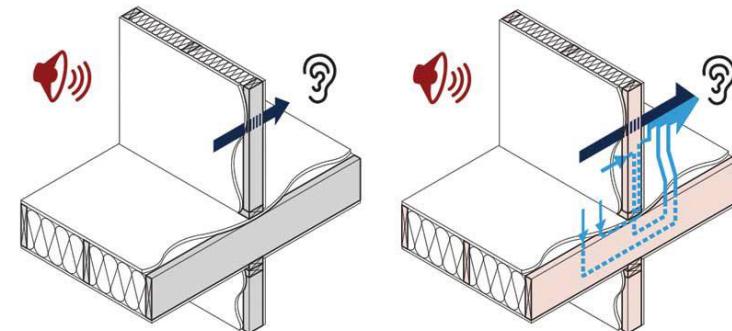
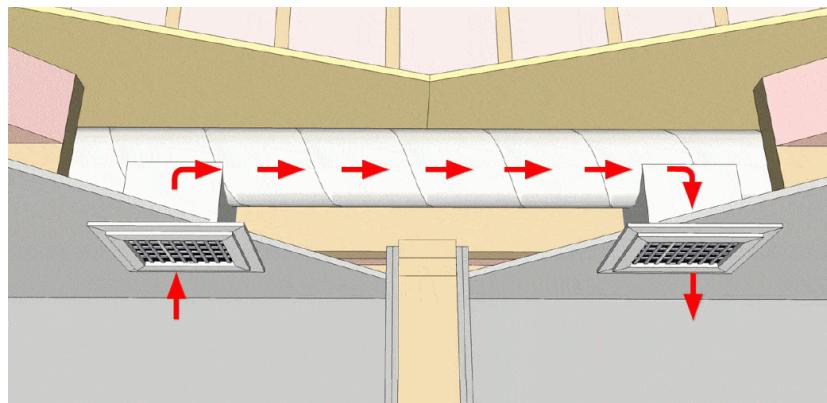
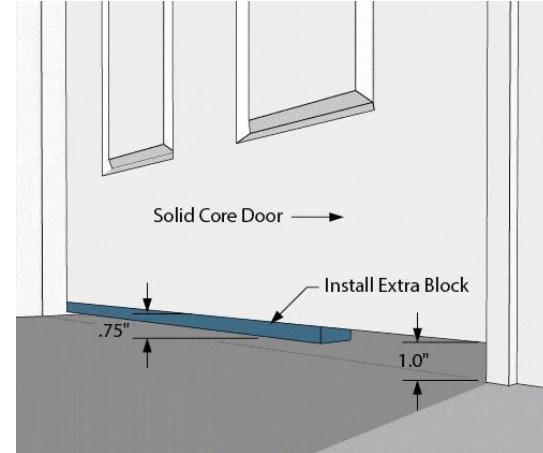


Air à **solid (wall)** à air  
*Airborne sound*



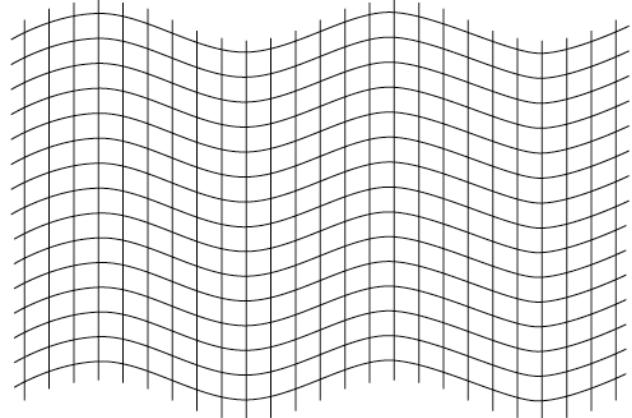
**Solid (floor)** à air  
*Structure-borne sound*

## Sound transfer paths

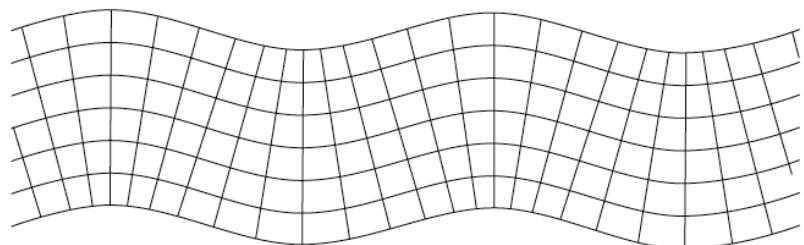


## Sound waves in isotropic solids

Sound waves in unbounded solids



Waves in plates and bars



## Wave propagation in solids

### Wave propagation in air

Velocity components  $v_x, v_y, v_z$  [m/s] vector  
 Pressure  $p$  [N/m<sup>2</sup>] scalar

### Wave propagation in solids

Displacement components  $\xi, \eta, \zeta$  [m] vector

### Elastic stresses

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \text{ N/m}^2 \text{ tensor}$$

## Wave propagation in solids

### Wave propagation in air

1D wave equation -> motion of particles in direction of wave propagation

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

### Wave propagation in solids

1D Wave equation -> motion of particles in direction of wave propagation

+ motion of particles perpendicular to wave propagation

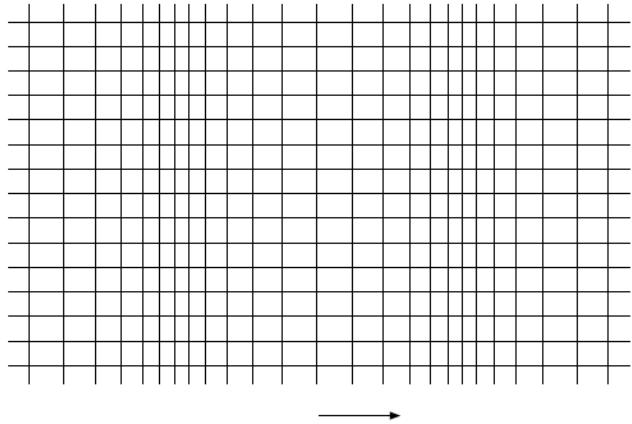
$$(2\mu + \lambda) \frac{\partial^2 \xi}{\partial x^2} = \rho_0 \frac{\partial^2 \xi}{\partial t^2} \quad \mu \text{ and } \lambda \text{ are material properties}$$

$$\mu \frac{\partial^2 \eta}{\partial x^2} = \rho_0 \frac{\partial^2 \eta}{\partial t^2}$$

$$\mu \frac{\partial^2 \zeta}{\partial x^2} = \rho_0 \frac{\partial^2 \zeta}{\partial t^2}$$

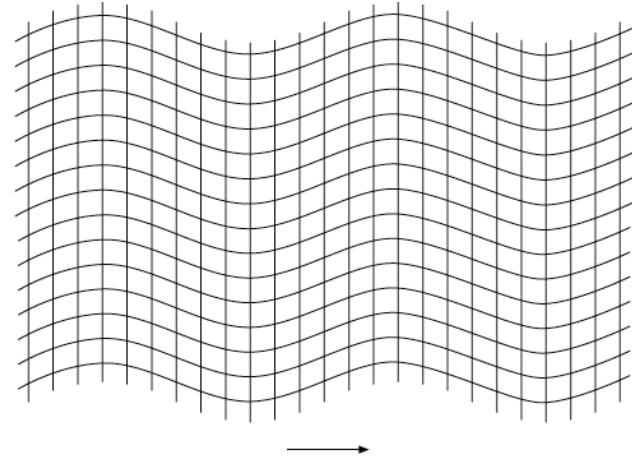
## Wave propagation in solids

Longitudinal wave / compressional wave



$$(2\mu + \lambda) \frac{\partial^2 \xi}{\partial x^2} = \rho_0 \frac{\partial^2 \xi}{\partial t^2}$$

Transverse wave ('string instrument') / shear wave

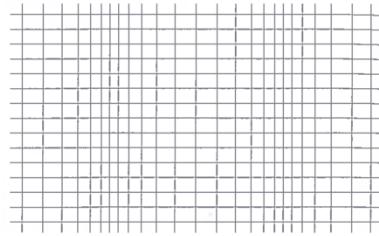


$$\mu \frac{\partial^2 \eta}{\partial x^2} = \rho_0 \frac{\partial^2 \eta}{\partial t^2}$$

$$\mu \frac{\partial^2 \zeta}{\partial x^2} = \rho_0 \frac{\partial^2 \zeta}{\partial t^2}$$

## Wave propagation in solids

Longitudinal wave

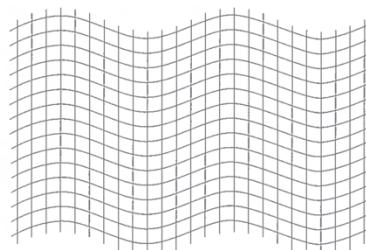


$$(2\mu + \lambda) \frac{\partial^2 \xi}{\partial x^2} = \rho_0 \frac{\partial^2 \xi}{\partial t^2}$$

Longitudinal wave speed  $c_L$

$$c_L = \sqrt{\frac{2\mu + \lambda}{\rho_0}}$$

Transverse wave



$$\mu \frac{\partial^2 \eta}{\partial x^2} = \rho_0 \frac{\partial^2 \eta}{\partial t^2}$$

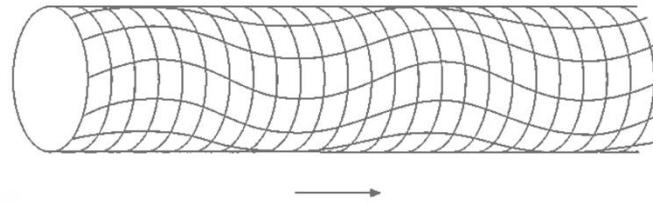
$$\mu \frac{\partial^2 \zeta}{\partial x^2} = \rho_0 \frac{\partial^2 \zeta}{\partial t^2}$$

Transverse wave speed  $c_T$

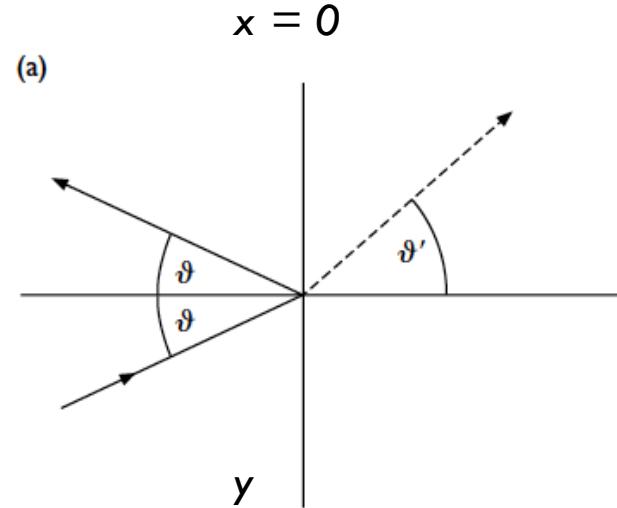
$$c_T = \sqrt{\frac{\mu}{\rho_0}}$$

# Wave propagation in solids

Special transversal wave: torsional wave



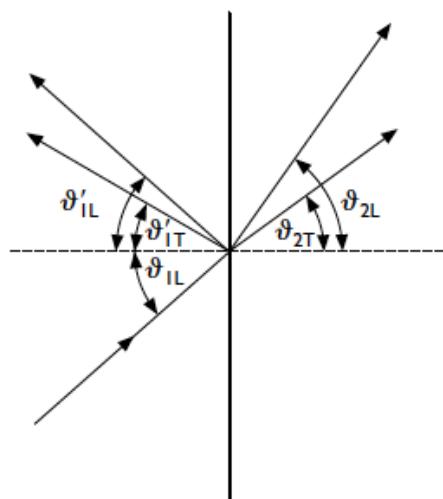
## Wave propagation in solids



### Interface of air with boundary:

Continuity of  $\rho$  and normal velocity component  $v_n$

Two additional waves to fulfill boundary conditions (BCs)  
Reflected and transmitted wave



### Interface of two solid media:

Incident longitudinal wave with  $x$  and  $y$  dependency

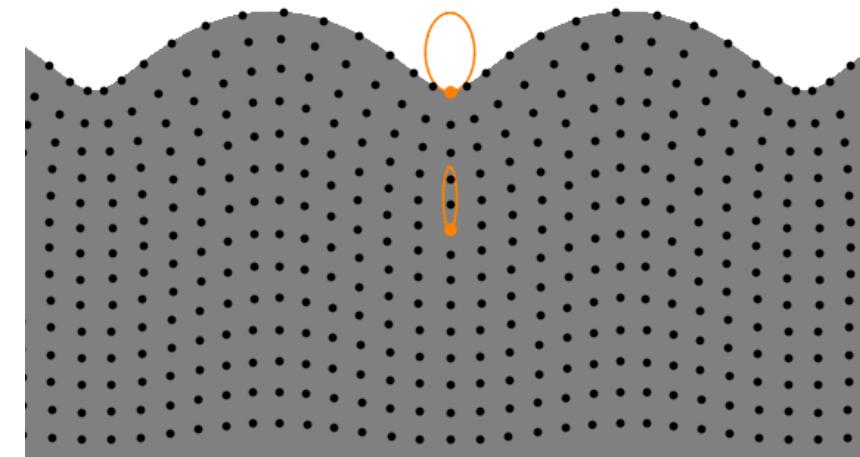
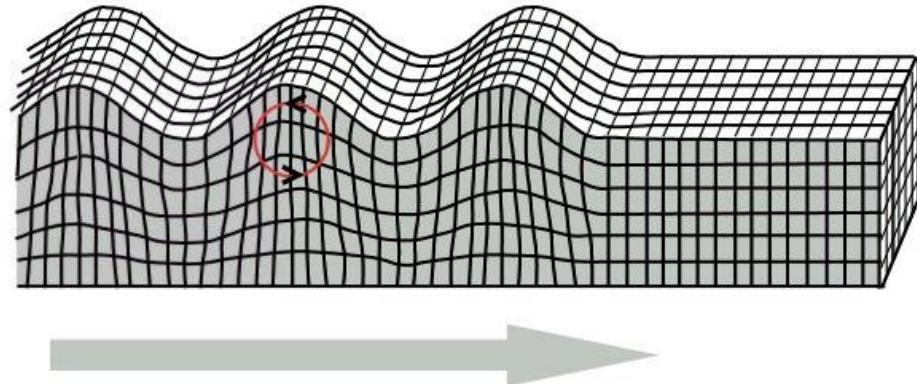
Continuity of normal displacement component and stresses  $\xi$ ,  $\eta$ ,  $\sigma_{xx}$  and  $\sigma_{xy}$

4 BCs, 4 additional waves -> wave conversion

## Wave propagation in solids: interface

**Rayleigh wave (surface wave)**, e.g. earthquake, water surface

Combination of longitudinal and transverse wave components, only close to surface.

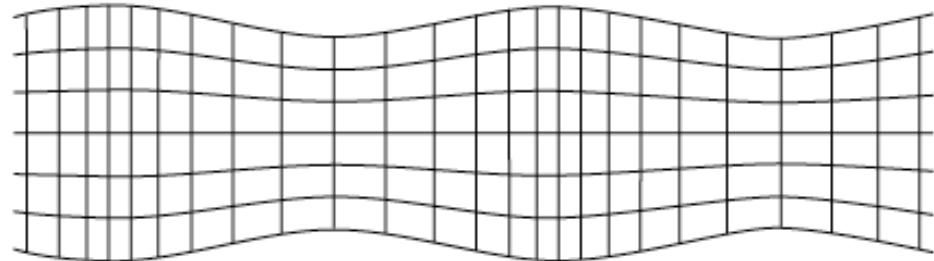


©2016, Dan Russell

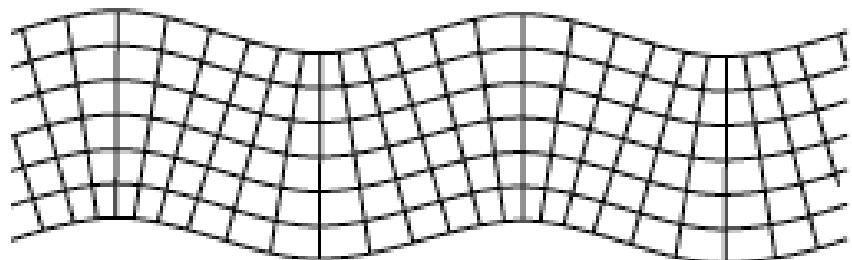
## Wave propagation in plates and bars

- Most practical solids have a finite dimension
- Boundary effects play a role
- More wave types exist

Extensional waves (quasi longitudinal waves)



Bending waves



## Wave propagation in plates and bars

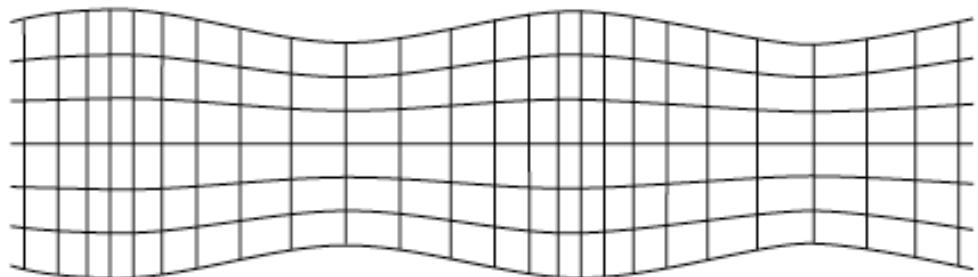
### Extensional waves in plates and bars

Assumption: thin plates and bars (thickness  $\ll l$ )

Wave equation

Y = Young's modulus

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{\rho_0}{Y} \frac{\partial^2 \xi}{\partial t^2}$$



Elastic deformation:  
extension and contraction

## Wave propagation in plates and bars

### Extensional waves in plates and bars

Extensional wave speed bar

$$c_{E1} = \sqrt{\frac{Y}{\rho_0}}$$

Extensional wave speed plate

$$c_{E2} = \sqrt{\frac{Y}{\rho_0(1 - \nu^2)}}$$

Value of  $\nu$  = Poisson's ratio ( $0 < \nu < 0.5$ )

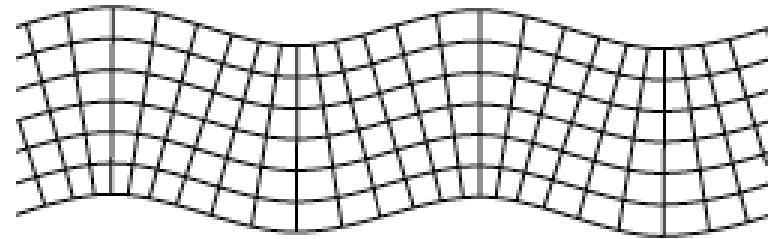
$$c_L > c_{E2} > c_{E1} > c_T$$

# Wave propagation in plates and bars

## Bending waves in plates and bars

Bending wave equation

$$\frac{\partial^4 \eta}{\partial x^4} + \frac{m'}{B} \frac{\partial^2 \eta}{\partial t^2} = 0$$



Bending stiffness B

$$B = \frac{d^3}{12} \cdot \frac{Y}{1 - \nu^2}$$

Solution

$$\eta(x, t) = \hat{\eta} \cdot e^{j(\omega t - k_B x)} \quad k_B^2 = \pm \omega \sqrt{m'/B}$$

## Wave propagation in plates and bars

### Bending waves in plates and bars

Two travelling waves

$$(k_B)_{1,2} = \pm \sqrt{\omega} \cdot \sqrt[4]{\frac{m'}{B}}$$

Bending wave speed, depending on frequency

$$c_B = \frac{\omega}{k_B} = \sqrt{\omega} \cdot \sqrt[4]{\frac{B}{m'}}$$

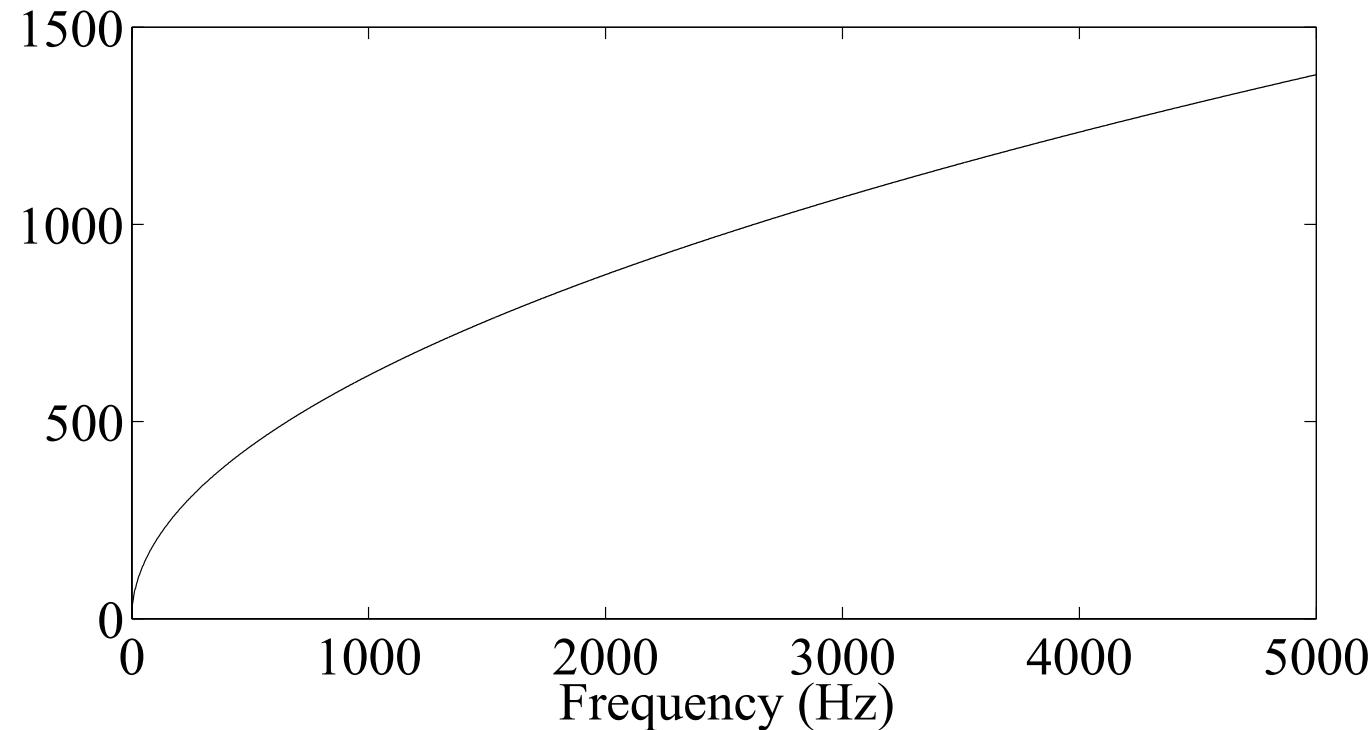
## Wave propagation in plates and bars

Bending wave speed, 0.1 m brickwork

$$B = 7 \cdot 10^5 \text{ Nm}^2$$

$$m' = 1900 \cdot 0.1 \text{ kg/m}^2$$

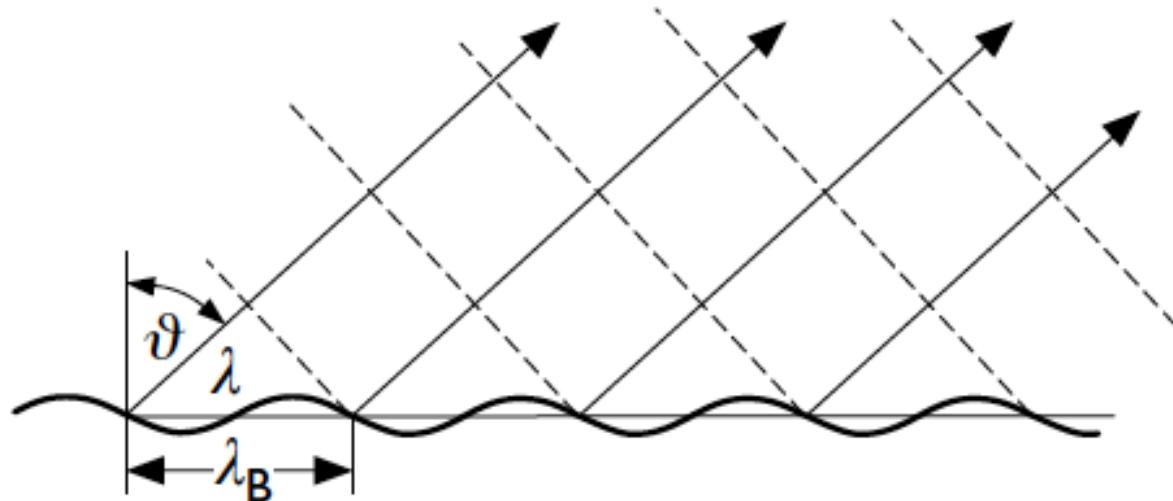
$$c_B = \frac{\omega}{k_B} = \sqrt{\omega} \cdot \sqrt[4]{\frac{B}{m'}}$$



## Wave propagation in plates and bars

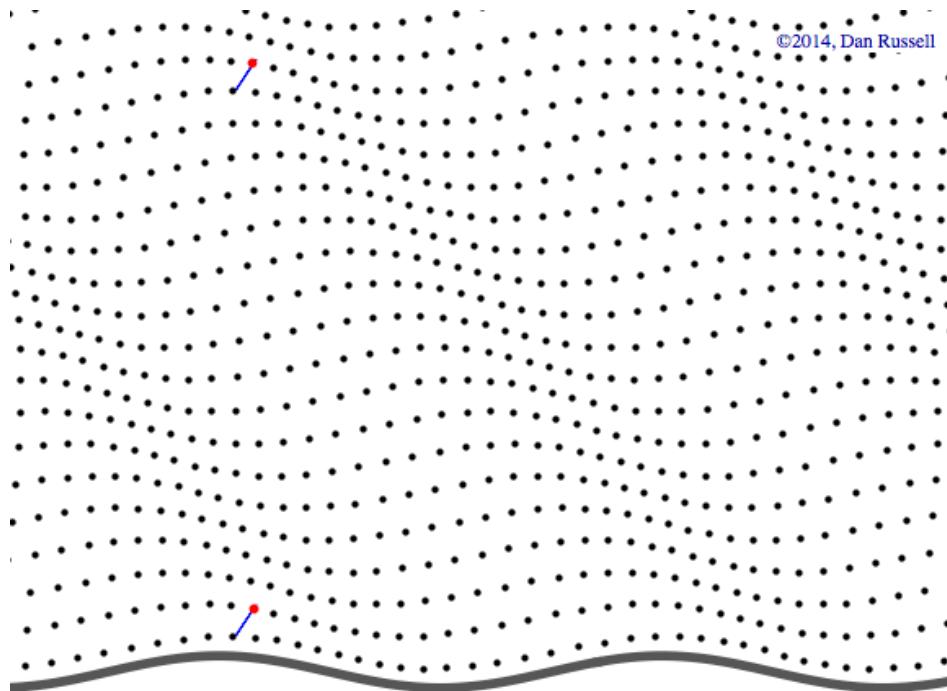
**Sound radiation: critical frequency is important!**

$$f_c = \frac{c^2}{2\pi} \sqrt{\frac{m'}{B}}$$



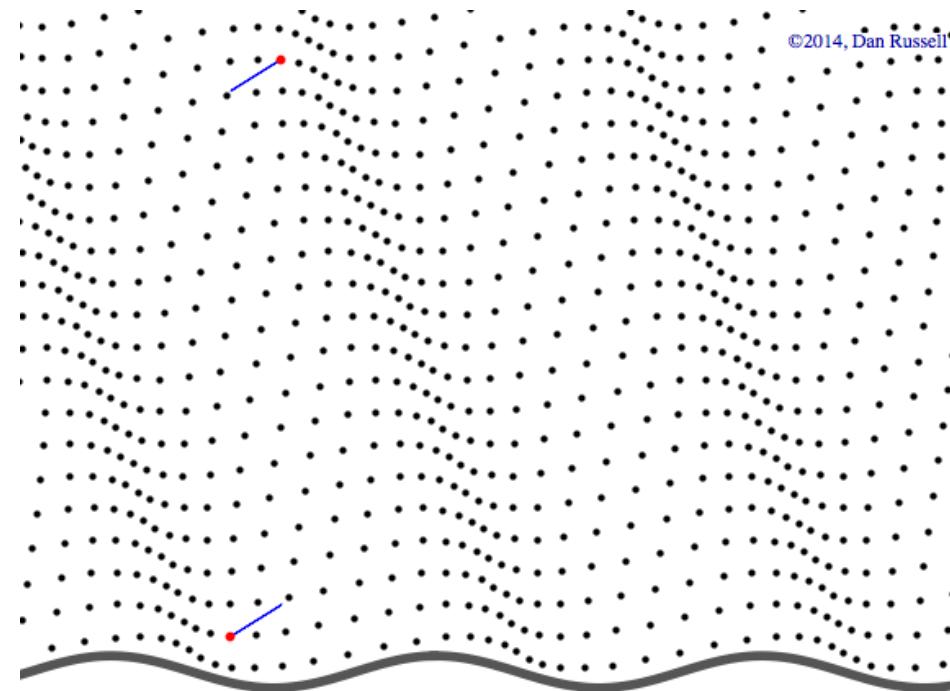
## Wave propagation in plates and bars

$$c_B = 1.5c$$



$$\theta = \arcsin \frac{1}{1.5} = 41.81^\circ$$

$$c_B = 1.1c$$



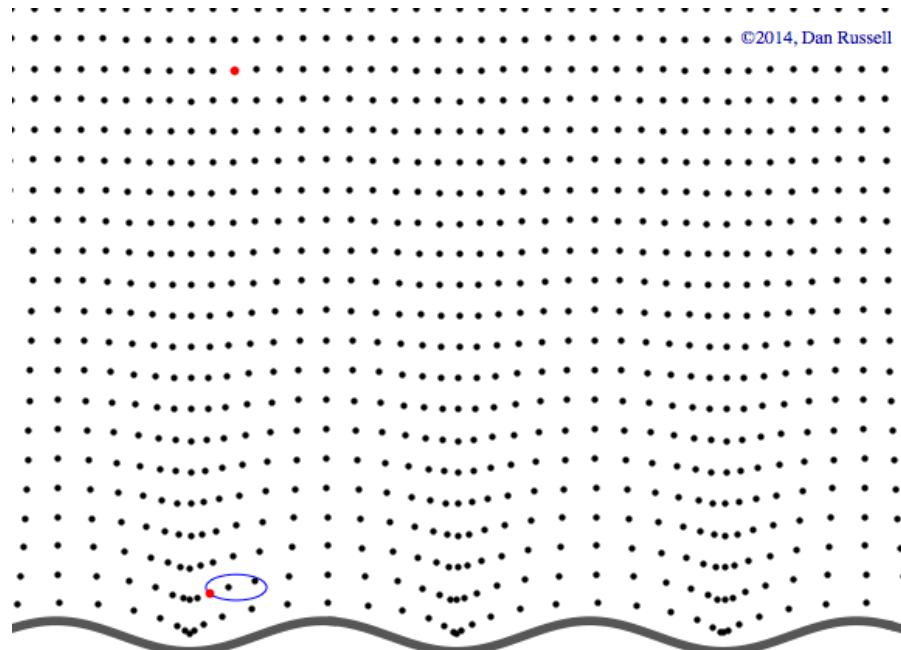
$$\theta = \arcsin \frac{1}{1.1} = 65.38^\circ$$

## Wave propagation in plates and bars

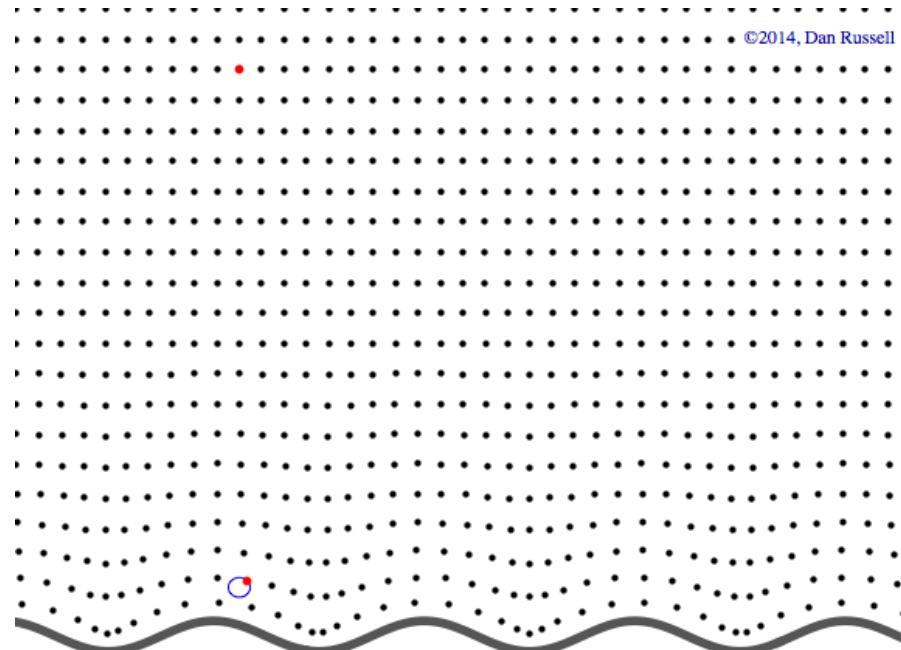
For  $c_B < c$  (or  $\lambda_B < \lambda$  or  $\omega < \omega_c$ ):

no radiation of sound

$$c_B = 0.95c$$



$$c_B = 0.75c$$



# Wave propagation in plates and bars

Application: ice thickness



## Wave propagation in plates and bars

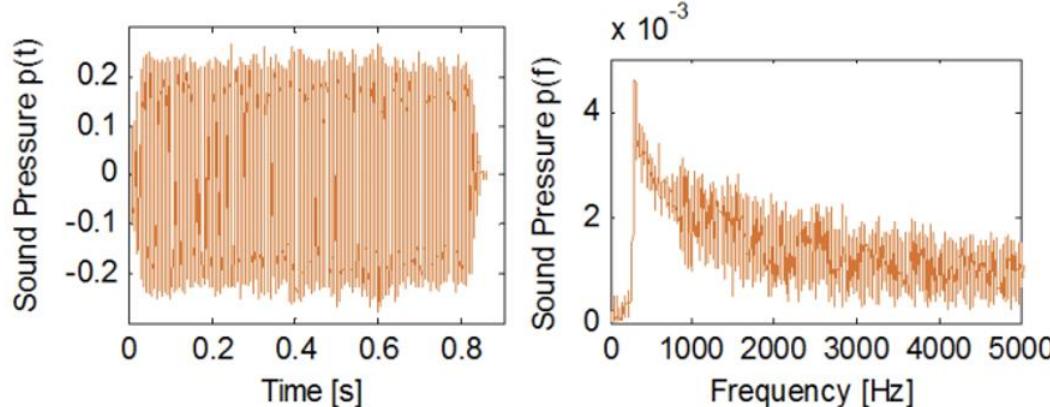


# Wave propagation in plates and bars

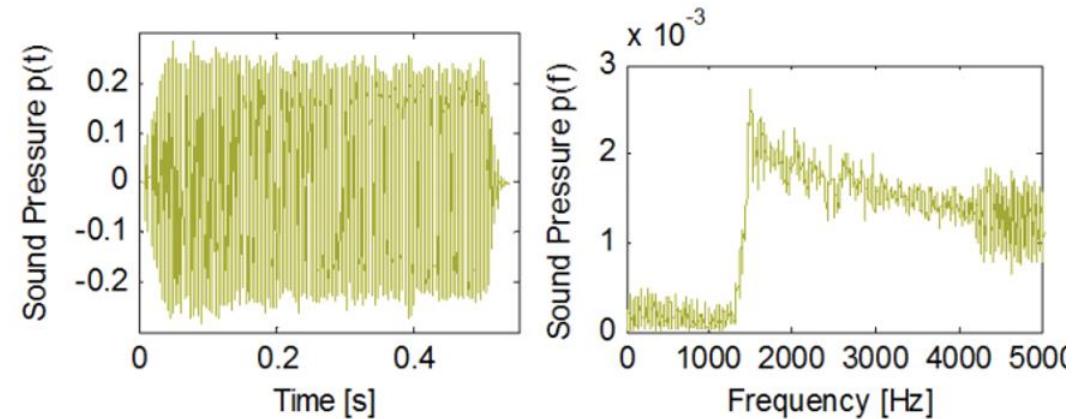
Application: ice thickness

**Q. Can you estimate the thicknesses from the measure data?**

Ice layer of thickness A



Ice layer of thickness B



Properties:  $Y = 9.1 \times 10^9 \text{ N/m}^2$

$$\rho_0 = 916.7 \text{ kg/m}^3$$

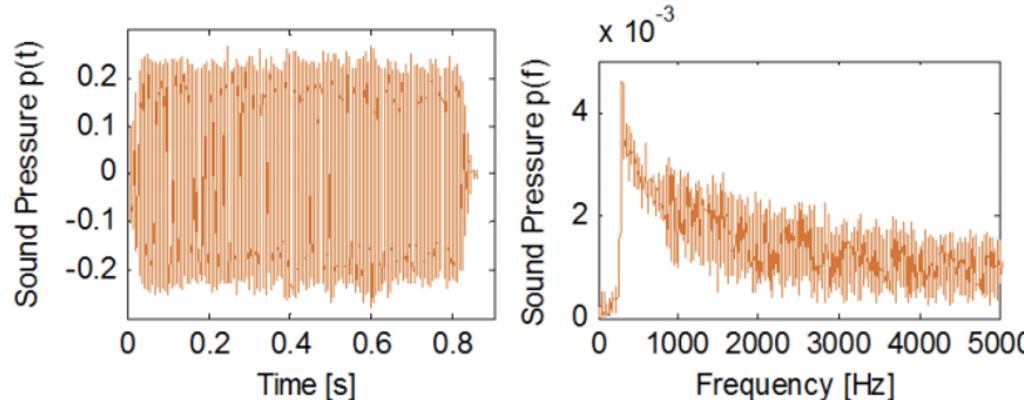
$$\nu = 0.33$$

$$c = 340 \text{ m/s}$$

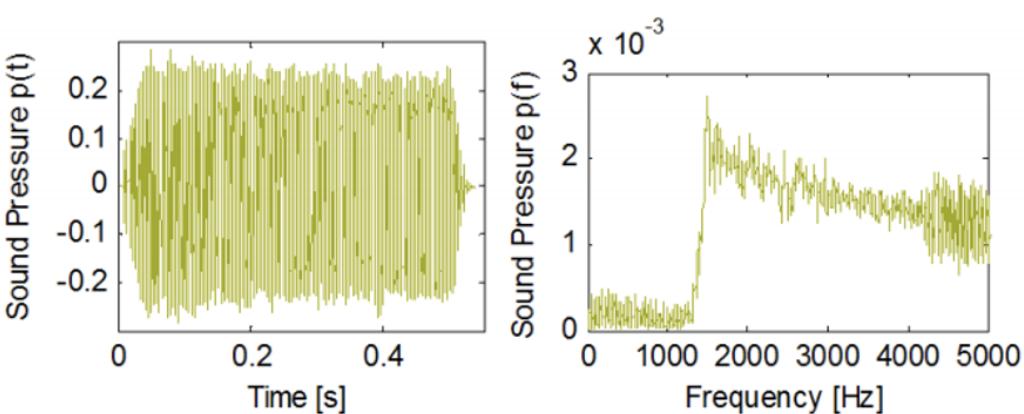
# Wave propagation in plates and bars

Application: ice thickness

Ice layer of thickness A



Ice layer of thickness B



Critical frequency of a plate

$$\omega_c = c^2 \sqrt{\frac{m'}{B}} \quad \text{where} \quad \begin{cases} m' = \rho_0 d \\ B = \frac{Yd^3}{12(1-\nu^2)} \end{cases}$$

$$\Rightarrow d = \frac{c^2}{\pi f_c} \sqrt{\frac{3(1-\nu^2)\rho_0}{Y}}$$

Estimation of the ice layer thickness

Thickness A

$$f_c = 250 \text{ Hz} \Rightarrow \text{thickness } d = 7.6 \text{ cm}$$

Thickness B

$$f_c = 1500 \text{ Hz} \Rightarrow \text{thickness } d = 1.3 \text{ cm}$$

# Building Acoustics

## Air-borne sound insulation

Measurements

Single leaf partitions

Double leaf partitions



# Measuring sound insulation

## Measurements

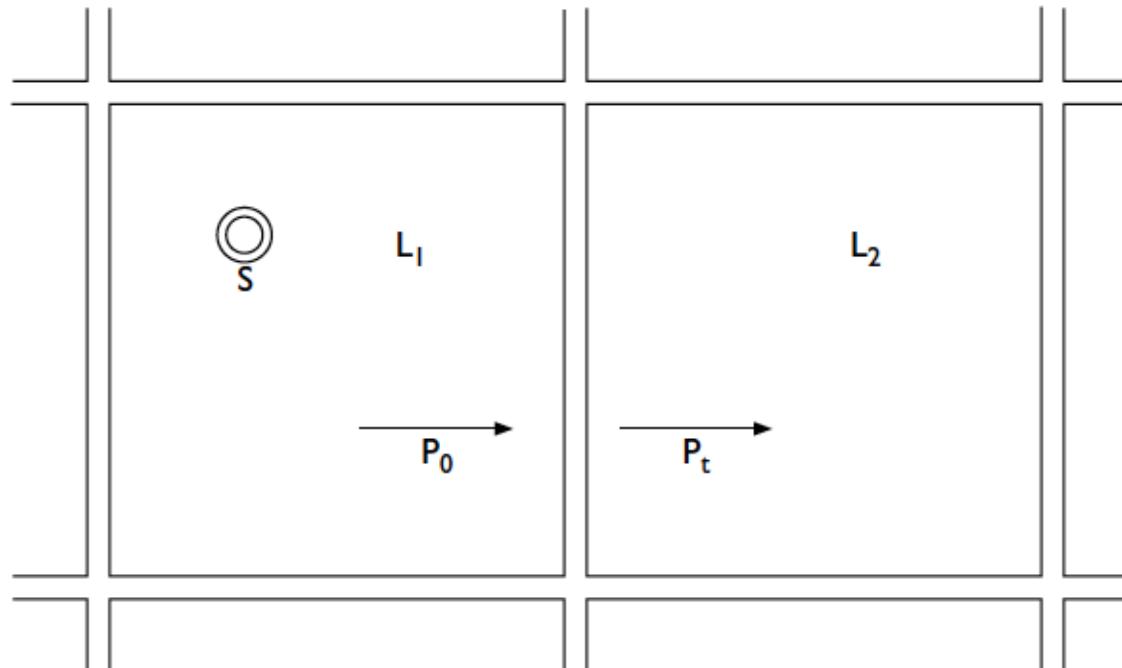
### Reduction index

$$R_A = 10 \log_{10} \left( \frac{I_0}{I_t} \right)$$

$$I_0 = P_0/S$$

$$I_t = P_t/S$$

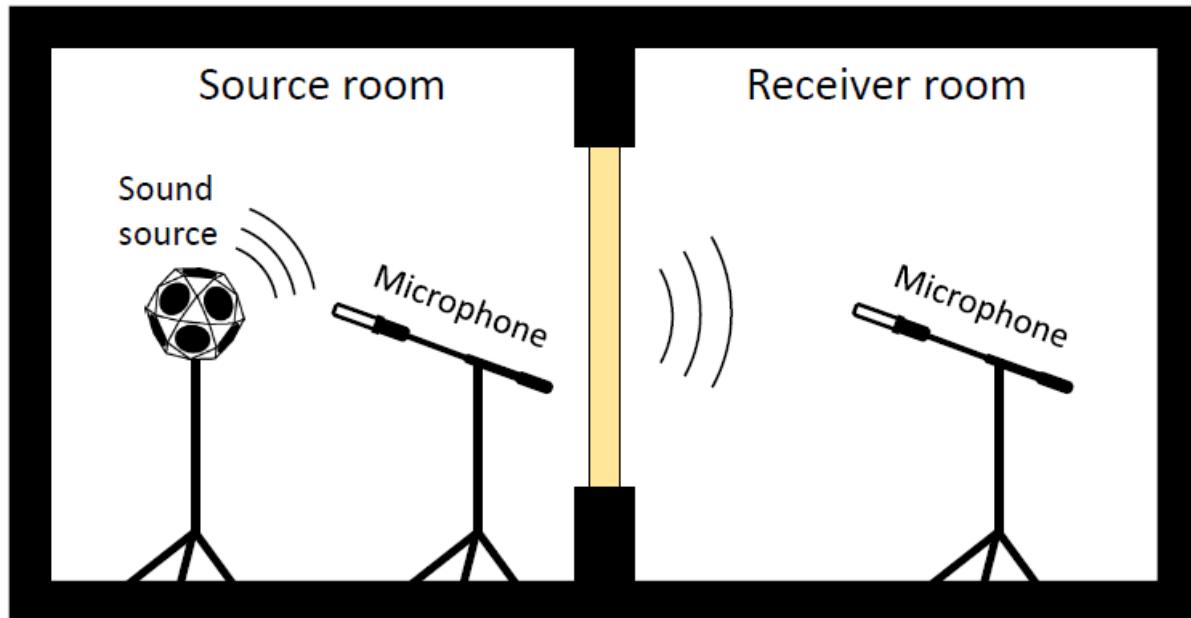
$$R_A = 10 \log_{10} \left( \frac{P_0}{P_t} \right)$$



# Measuring sound insulation

## Measurements

Diffuse sound field



# Measuring sound insulation

## Measurements

Diffuse sound field: acoustic energy density  $w$  [J/m<sup>3</sup>] is a constant

Incident power:

$$P_0 = \frac{c}{4} S \cdot w_1$$

Power in receiver room:

power radiated by wall = power absorbed by boundaries

$$P_t = \frac{c}{4} A \cdot w_2$$

# Measuring sound insulation

## Measurements

Using

$$R_A = 10 \log_{10} \left( \frac{P_0}{P_t} \right)$$

And relations from previous slide

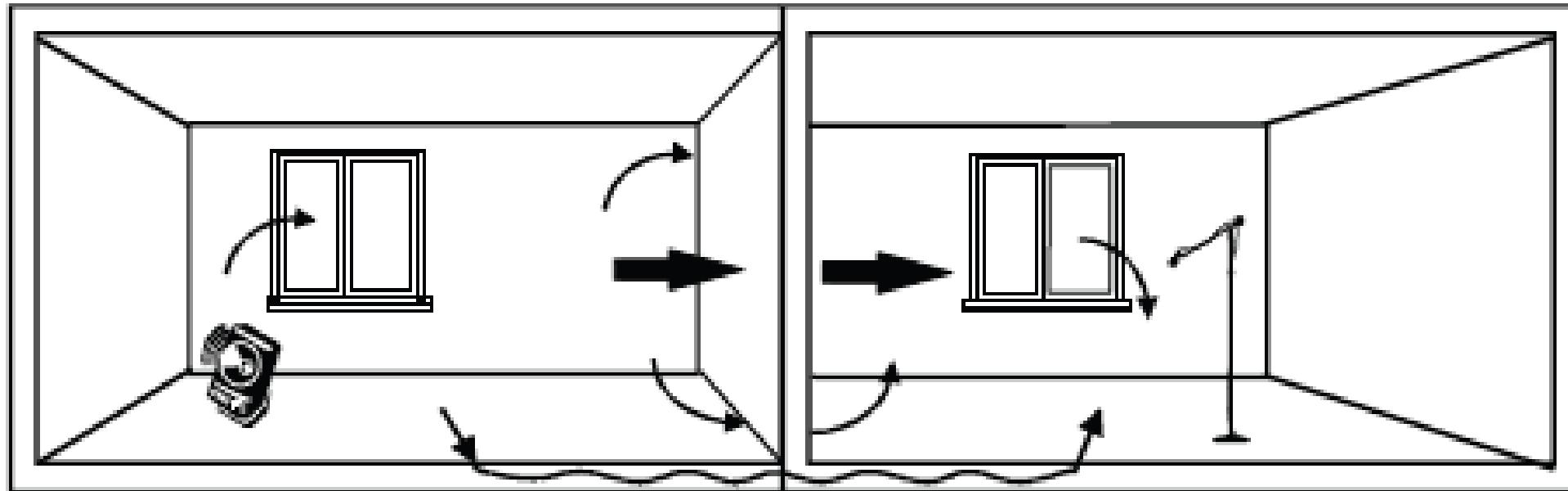
$$R_A = 10 \log_{10} \left( \frac{w_1}{w_2} \right) + 10 \log_{10} \left( \frac{S}{A} \right)$$

This leads to

$$R_A = L_1 - L_2 + 10 \log_{10} \left( \frac{S}{A} \right)$$

# Measuring sound insulation

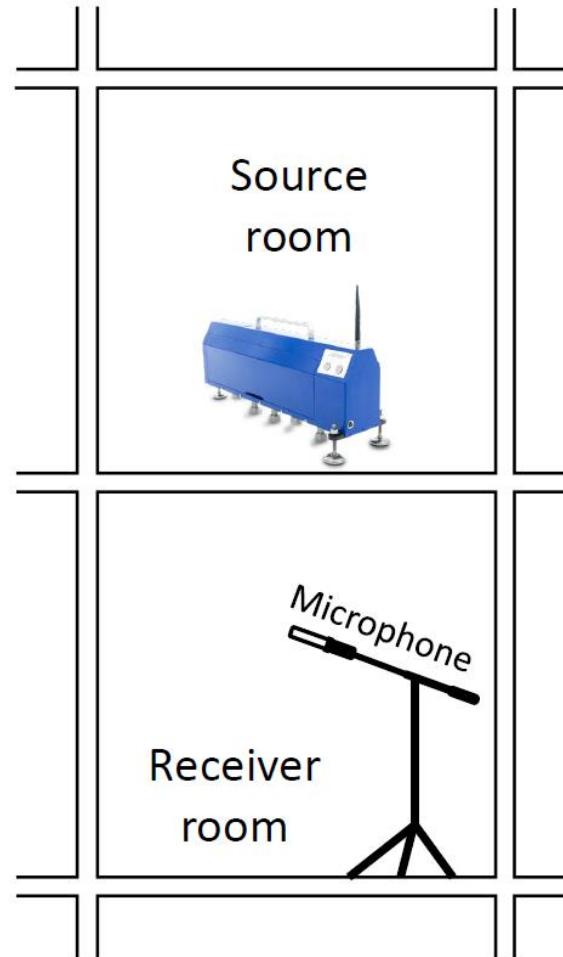
## Field measurements



Vigran, Building Acoustics

# Measuring sound insulation

Structure borne sound insulation



# Measuring sound insulation

## Flanking sound transmission

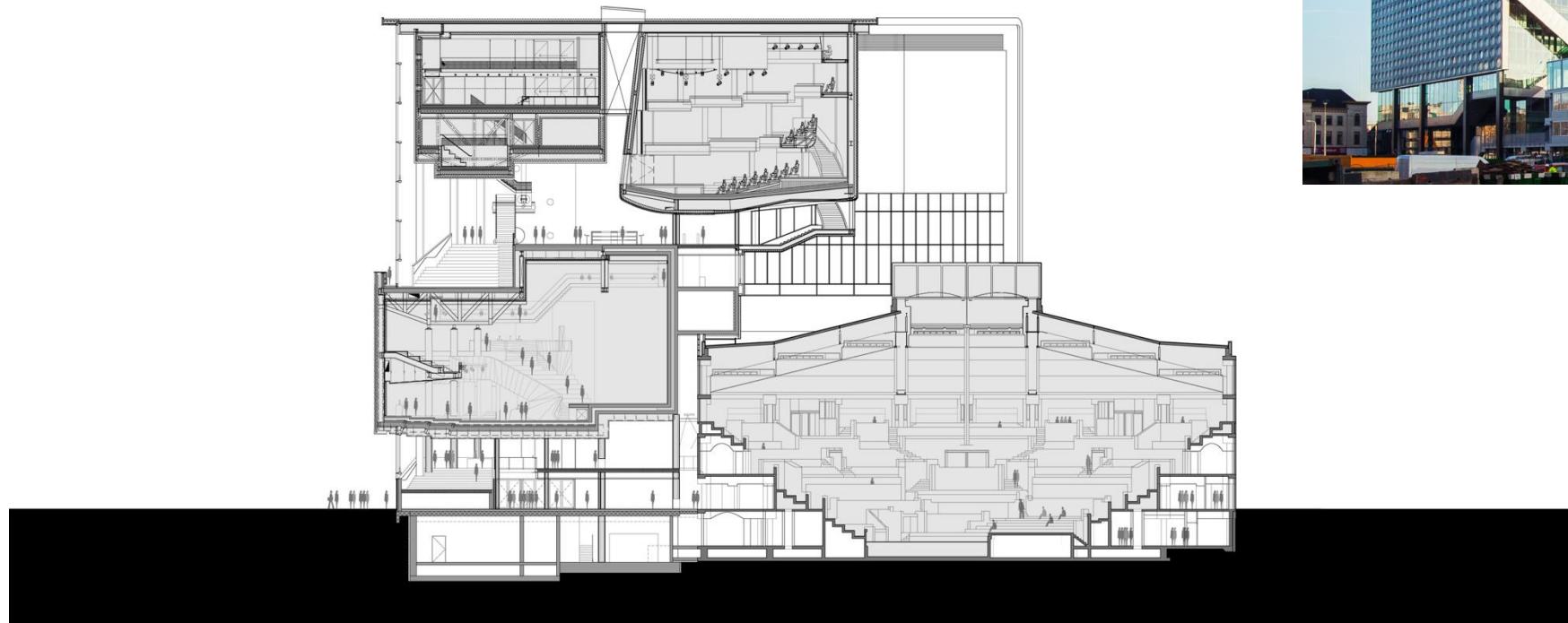


<https://acoustical-consultants.com/built-environment/noise-investigations/how-to-measure-astc-ratings-of-specific-sound-transmission-paths/>

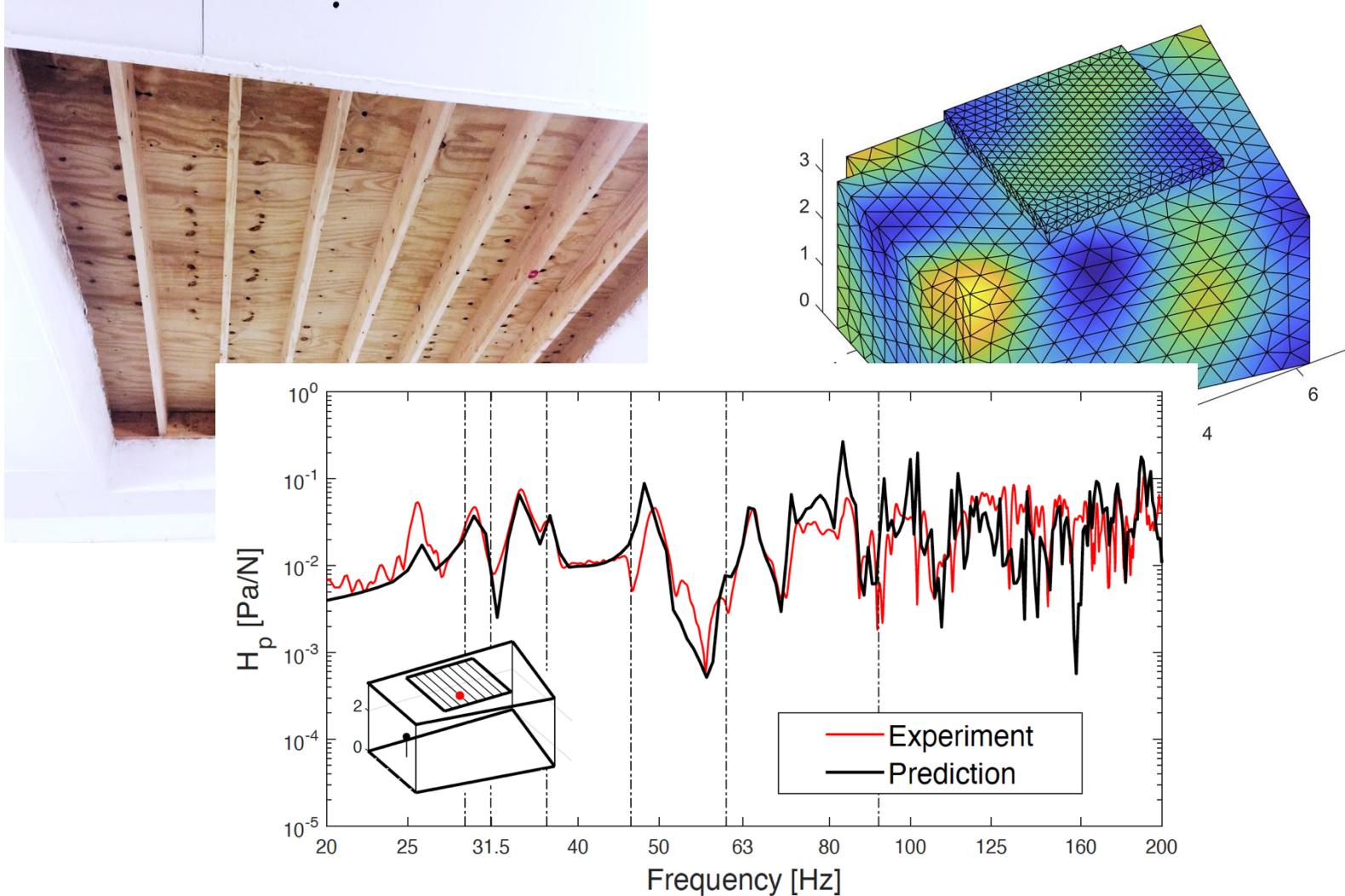


Mockup of CLT construction at University of Bologna

# Predicting sound insulation

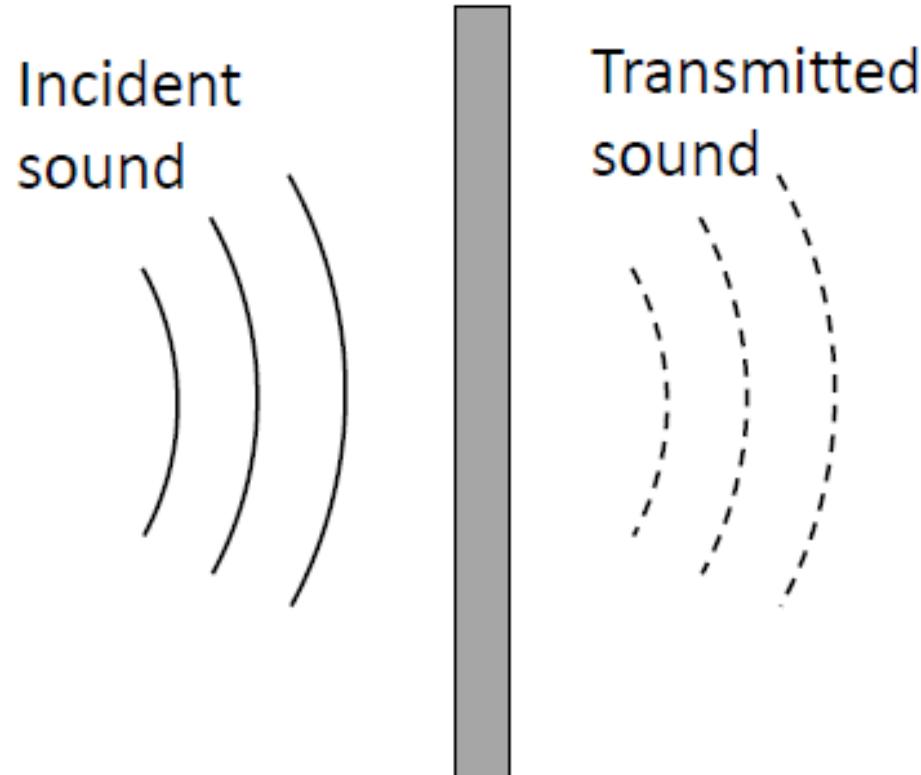


## Predicting sound insulation



# Building Acoustics: Air-borne sound insulation

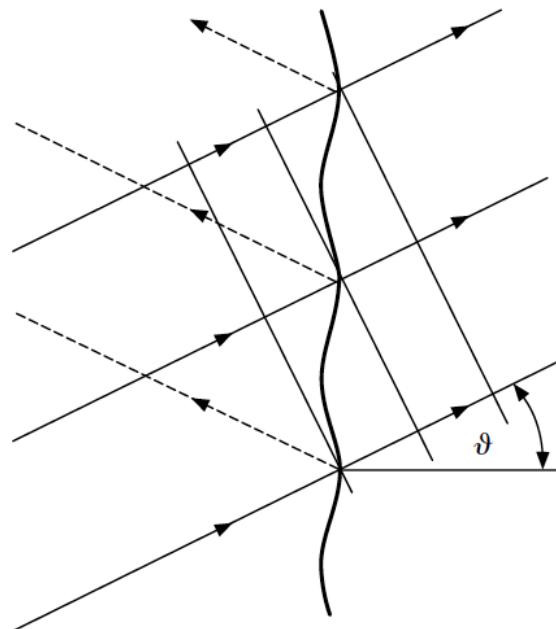
**Air-borne sound insulation:** Single leaf partitions



## Building Acoustics: Air-borne sound insulation

### Air-borne sound insulation: Single leaf partitions

Incident plane wave at thin wall      pressures at both sides of the wall



$$p_1(y) = \hat{p}(1 + R)e^{-jky \sin \vartheta} \cdot e^{j\omega t}$$

$$p_2(y) = \hat{p}T e^{-jky \sin \vartheta} \cdot e^{j\omega t}$$

displacement component in wall

$$\xi(y) = \hat{\xi} e^{-jky \sin \vartheta} \cdot e^{j\omega t}$$

## Building Acoustics: Air-borne sound insulation

**Air-borne sound insulation:** Single leaf partitions

Bending wave equation, imposed force is pressure difference over wall

$$m' \frac{\partial^2 \xi}{\partial t^2} + B \frac{\partial^4 \xi}{\partial y^4} = p_1 - p_2$$

$$p_1 - p_2 = (-\omega^2 m' + B k^4 \sin^4 \vartheta) \cdot \xi$$

To compute the sound reduction index, we need

$$R_A = 10 \log_{10} \left| \frac{p_1}{p_2} \right|^2$$

## Building Acoustics: Air-borne sound insulation

**Air-borne sound insulation:** Single leaf partitions

$$p_1 - p_2 = (-\omega^2 m' + B k^4 \sin^4 \vartheta) \cdot \xi \quad R_A = 10 \log_{10} \left| \frac{p_1}{p_2} \right|^2$$

Making use of  $p_1 = p_i + p_r$  and  $v_{x,i} + v_{x,r} = v_{x1} = v_{x2}$

$$R_A \square 10 \log_{10} (X^2)$$

$$X = \frac{\omega m'}{2Z_0} \cos(\theta) \boxed{1} - \frac{B \omega^2 \sin^4(\theta)}{m' c^4} \boxed{2}$$

# Building Acoustics: Air-borne sound insulation

**Air-borne sound insulation:** Single leaf partitions

$$\omega < \omega_c$$

$$R_A \square 10 \log_{10} (X^2)$$

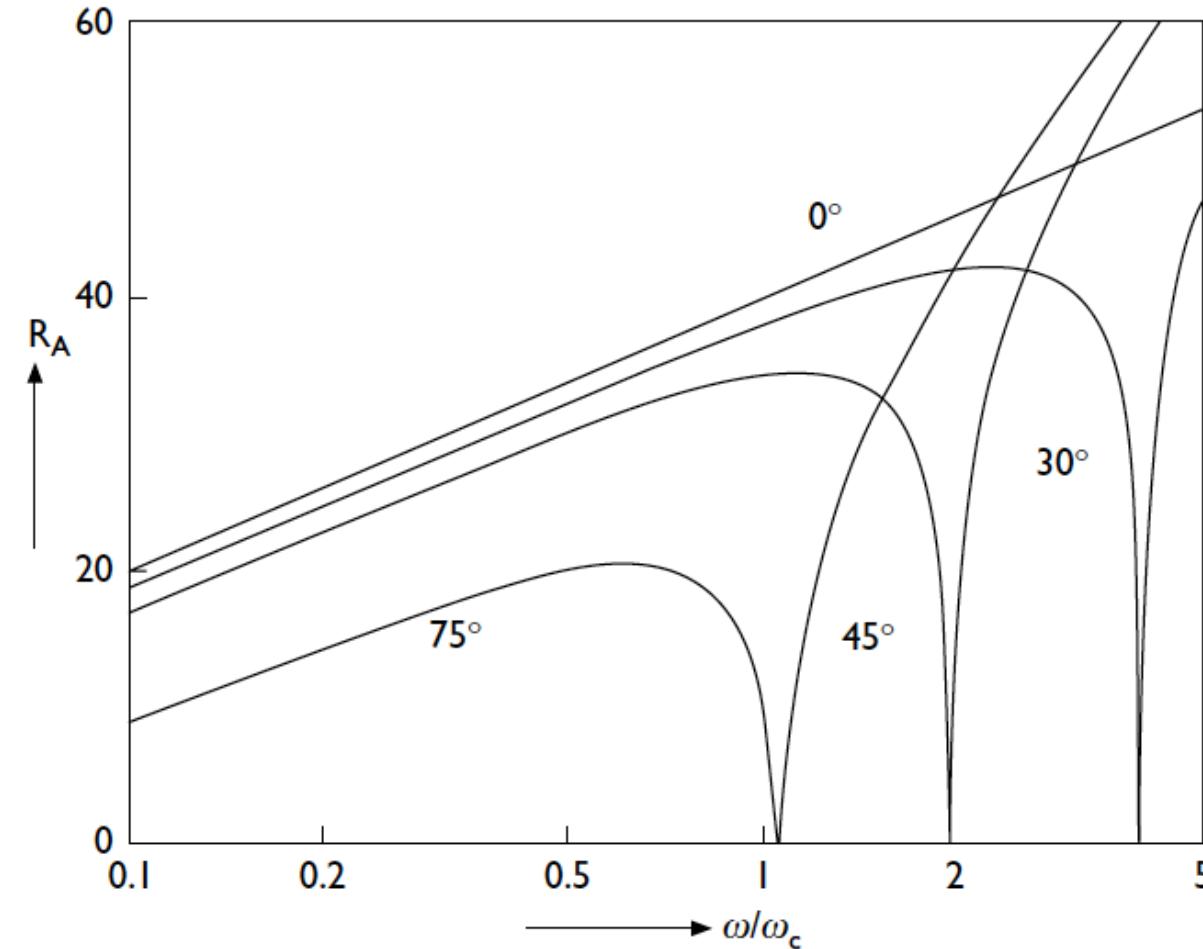
$$X = \frac{\omega m'}{2Z_0} \cos(\theta)$$

$$\omega > \omega_c$$

$$X = \cos(\theta) \frac{B\omega^3 \sin^4(\theta)}{2Z_0 c^4}$$

# Building Acoustics: Air-borne sound insulation

**Air-borne sound insulation:** Single leaf partitions

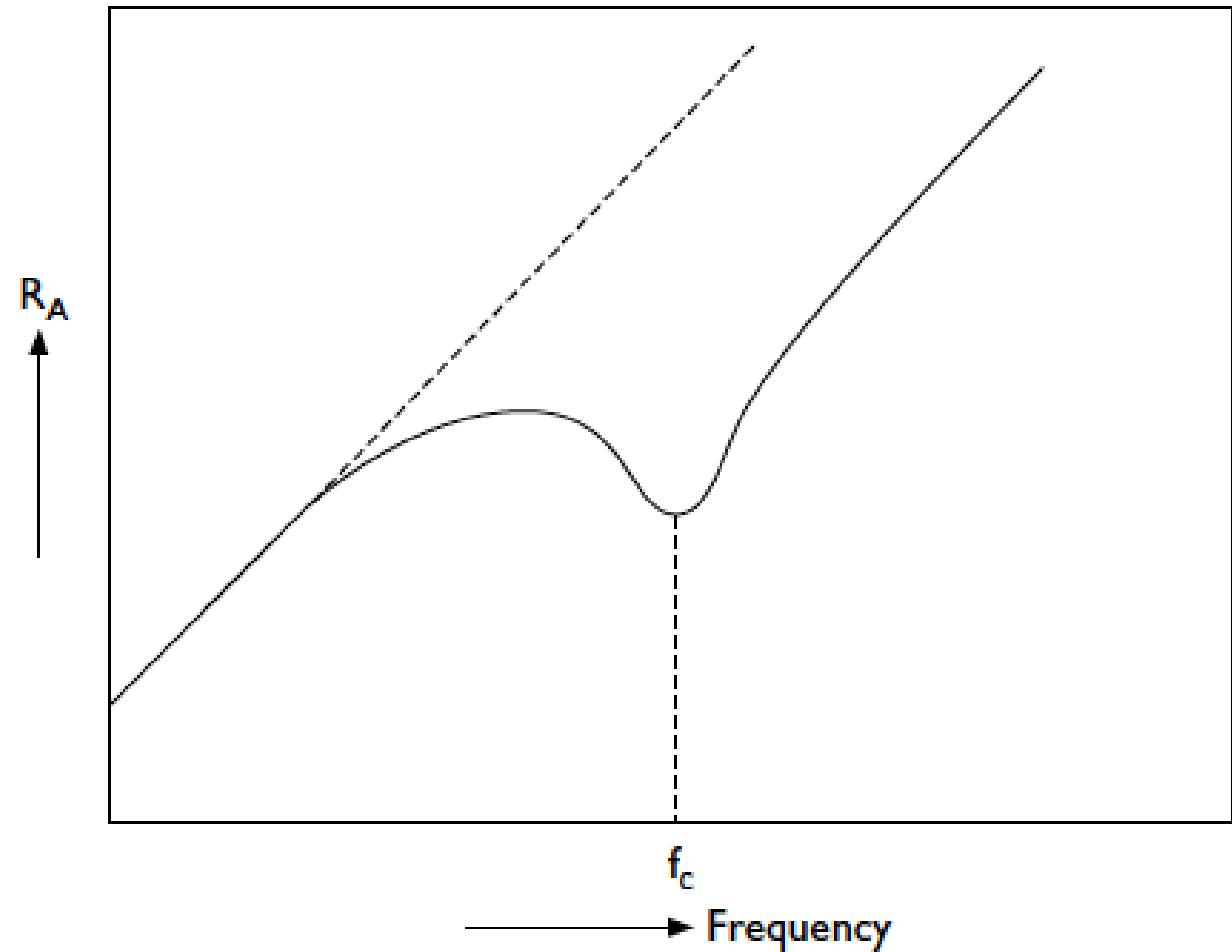


# Building Acoustics: Air-borne sound insulation

## Air-borne sound insulation: Single leaf partitions

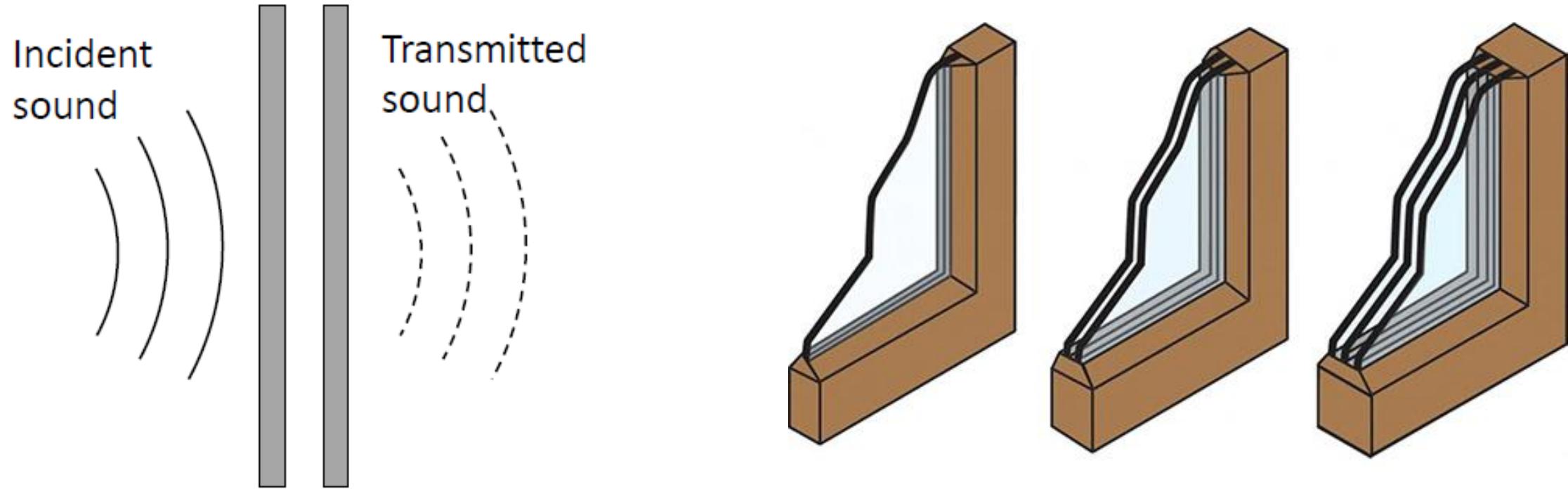
We also should include

- Damping of bending waves
- Finite dimensions of partitions
- Random incident sound waves



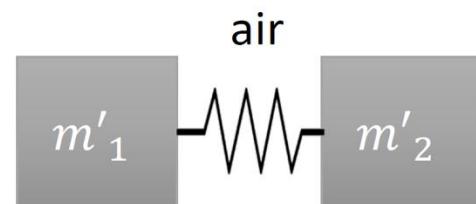
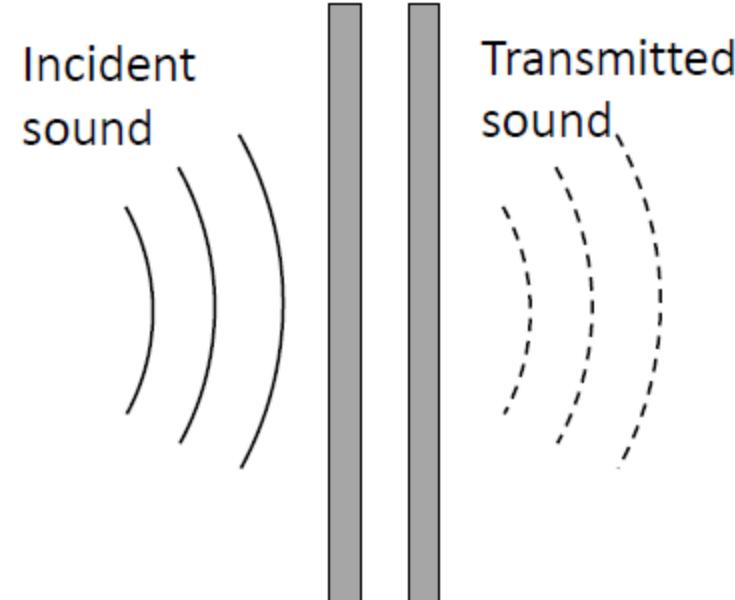
# Building Acoustics: Air-borne sound insulation

## Air-borne sound insulation: Double leaf partitions



# Building Acoustics: Air-borne sound insulation

## Double leaf partitions



## Building Acoustics: Air-borne sound insulation

**Air-borne sound insulation:** Double leaf partitions

For very low frequencies

The two leafs vibrate in equal phase

$$R_A \square 20 \log_{10} \left[ \frac{\omega(m'_1 + m'_2)}{2Z_0} \right]$$

The leafs vibrate in opposite phase at the mass-spring resonance frequency

$$\omega_0 = \sqrt{\frac{1}{n'} \left( \frac{1}{m'_1} + \frac{1}{m'_2} \right)}$$

## Building Acoustics: Air-borne sound insulation

### Air-borne sound insulation: Double leaf partitions

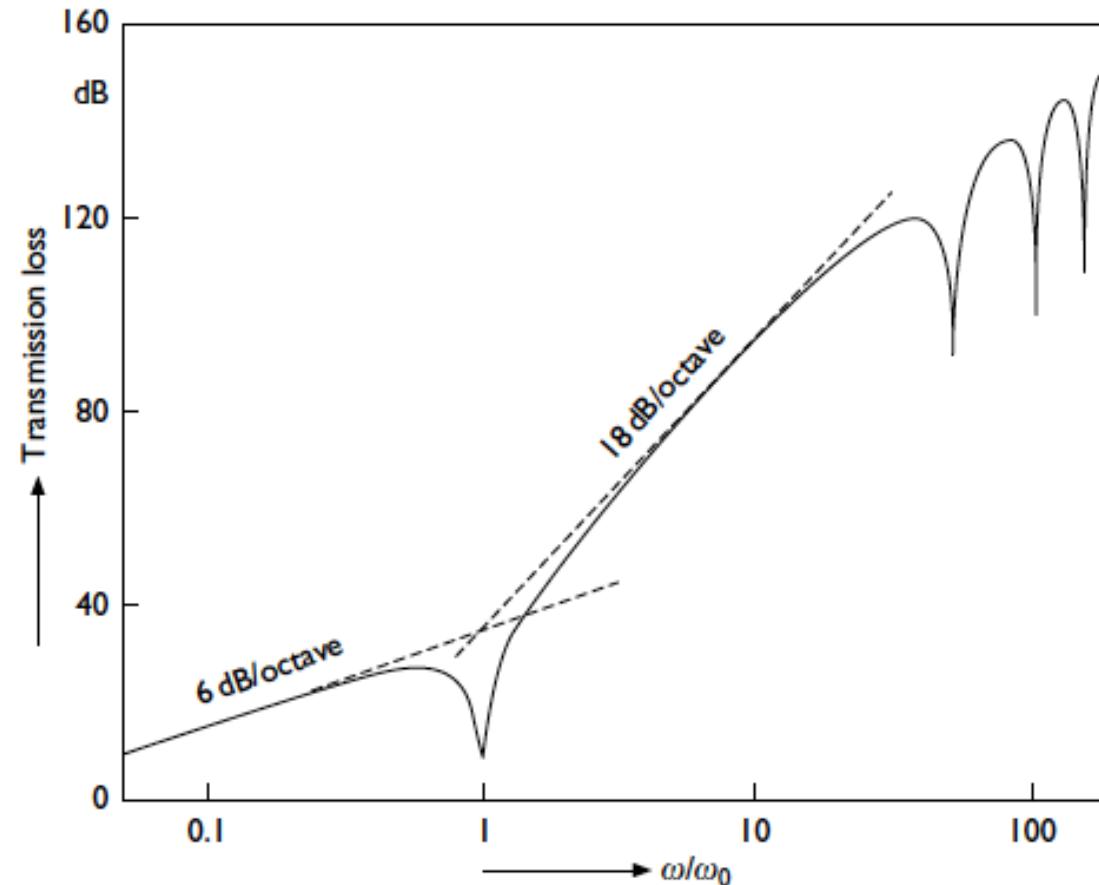
Above the mass spring resonance frequency

The two leafs vibrate in random phase

$$R_A \approx 20 \log_{10} \left( \frac{\omega^3 n' m'_1 m'_2}{2 Z_0} \right)$$

# Building Acoustics: Air-borne sound insulation

## Air-borne sound insulation: Double leaf partitions



# Building Acoustics: Air-borne sound insulation

## Air-borne sound insulation: Double leaf partitions

We also should include

- Damping of bending waves
- Finite dimensions of partitions (mechanical coupling of leafs)
- Random incident sound waves

## Tutorial Time

**Investigate the sound insulation of a wall as a function of frequency by changing:**

- Mass
- Thickness
- Angle of sound wave incidence

$$f_c = \frac{c^2}{2\pi} \sqrt{\frac{m'}{B}}$$



**Calculate the critical frequency of the wall**

Can you recognize this frequency in the sound insulation curve?