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# **A simple AAU Template for a Collection of Papers Ph.D. Thesis**

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Aalborg University  
Assistant PhD Supervisor: Assoc. Prof. XX  
Aalborg University  
PhD Committee: Prof. X, Y University  
Prof. X, Y University  
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# Curriculum Vitae

Author name



Here is the CV text.

## Curriculum Vitae

# Abstract

English abstract

## Abstract

# Resumé

Danish Abstract

## Resumé



# Contents

Curriculum Vitae	iii
Abstract	v
Resumé	vii
Preface	xiii
<b>I Introduction</b>	<b>1</b>
Introduction	3
1 History of bowed strings . . . . .	3
2 To do thingies . . . . .	3
2.1 Intuition for the frequency dependent damping term $2\sigma_1\partial_t\partial_x^2u$ . . . . .	3
3 Notes . . . . .	5
References . . . . .	5
<b>II Papers</b>	<b>7</b>
A Paper Errata	9

## Contents

# Todo list

## Contents

# Preface

As my background does (did) not lie in mathematics, physics or computer science, which – trust me – were three equally crucial components in creating the result of this project, I added a, say, more pedagogical section at the end of this thesis. These tutorials are a result of the things that I learned and (hopefully) explain topics such as *Energy Analysis*, *Stability Analysis*, etc. in a way so that others with the same background will be able to understand what is going on.

Name  
Aalborg University, October 14, 2020

## Preface

## **Part I**

# **Introduction**





# Introduction

## 1 History of bowed strings

In static bow-string-interaction models, the friction force is defined as a function of the relative velocity between the bow and the string only. The first mathematical description of friction was proposed by Coulomb in 1773 [?] to which static friction, or *stiction*, was added by Morin in 1833 [?] and viscous friction, or velocity-dependent friction, by Reynolds in 1886 [?]. In 1902, Stribeck found a smooth transition between the static and the coulomb part of the friction curve now referred to as the Stribeck effect [?]. The latter is still the standard for static friction models today.

## 2 To do thingies

- Think about how to define real-time.
- Create an intuition for different parts of the equation

### 2.1 Intuition for the frequency dependent damping term $2\sigma_1\partial_t\partial_x^2u$

Take the frequency independent damping term  $-2\sigma_0\partial_tu$ . The more positive the velocity  $\partial_tu$  is, i.e., the string is moving upwards the more this term applies a negative, or downwards force (/effect) on the string. Vice versa, a more negative velocity will make this term apply a more positive force on the string. As for the frequency dependent damping term, apart from the obvious  $\sigma_1$ , the effect of the term increases with an increase of  $\partial_t\partial_x^2u$  which describes the rate of change of the curvature of the string.

Let's first talk about positive and negative curvature, i.e., when  $\partial_x^2u > 0$  or  $\partial_x^2u < 0$ . Counterintuitively, in the positive case, the curve points downwards. Think about the function  $f(x) = x^2$ . It has a positive curvature (at any point), but has a minimum. We can prove this by taking  $x = 0$  and setting

grid spacing  $h = 1$ .

$$\begin{aligned}
 \delta_{xx}f(x) &= \frac{1}{h^2} (f(-1) - 2f(0) + f(1)), \\
 &= \frac{1}{1^2} \left( (-1)^2 - 2 \cdot 0^2 + 1^2 \right), \\
 &= (1 - 0 + 1) = 2.
 \end{aligned} \tag{1}$$

In other words, the second derivative of the function  $f(x) = x^2$  around  $x = 0$  is positive.

As our term does not only include a second-order spatial derivative but also a first-order time derivative, we are now talking about a positive or negative *rate of change* of the curvature, i.e., when  $\partial_t \partial_x^2 u > 0$  or  $\partial_t \partial_x^2 u < 0$ . A positive rate of change of curvature means that the string either has a positive curvature and is getting more positive, i.e., the string gets more curved over time, or that the string has a negative curvature and is getting less negative, i.e., the string gets less curved or 'loosens up' over time. In the same way, a negative rate of change of curvature means that the string either has a negative curvature and is getting more negative, or that the string has a positive curvature and is getting less positive.

Let's see some examples. Take the same function described before, but now  $f$  changes over time, fx.  $f(x, t) = tx^2$ . When  $t$  increases over time, the curvature gets bigger. Repeating what we did above with  $x = 0$  and grid spacing  $h = 1$ , but now with  $t = 2$  and step size  $k = 1$ , but now with a backwards time derivative we get:

$$\begin{aligned}
 \delta_t - \delta_{xx}f(x, t) &= \frac{1}{kh^2} \left( f(-1, 2) - 2f(0, 2) + f(1, 2) \right. \\
 &\quad \left. - \left( f(-1, 1) - 2f(0, 1) + f(1, 1) \right) \right), \\
 &= \frac{1}{1 \cdot 1^2} \left( 2 \cdot (-1)^2 - 2 \cdot 2 \cdot (0)^2 + 2 \cdot 1^2 \right. \\
 &\quad \left. - \left( 1 \cdot (-1)^2 - 2 \cdot 1 \cdot (0) + 1 \cdot (1^2) \right) \right), \\
 &= 2 + 2 - (1 + 1) = 2.
 \end{aligned}$$

So the rate of change of the curvature is positive, i.e., the already positively curved function  $x^2$  gets more curved over time.

If the curvature around a point along a string gets more positive (or less negative) over time, the force applied to that point will be positive. Vice versa, if the curvature around a point along a string gets more negative (or less positive) over time, the force applied will be negative.

### 3. Notes

The fact the frequency dependent term to be added rather than subtracted to the FDS, is caused by the fact that a positive curvature implies a negative position in the string (think of the function  $x^2$  which has a positive curvature, but the function 'points' downwards). This translated to the force/effect this term has on the scheme means...

## 3 Notes

One over number  $\rightarrow$  reciprocal of number

Example: When the waveform consists entirely of harmonically related frequencies, it will be periodic, with a period equal to the reciprocal of the fundamental frequency (from An Introduction to the Mathematics of Digital Signal Processing Pt 2 by F. R. Moore)

## References

## References

# **Part II**

# **Papers**



# Paper A

## Paper Errata

Here, some errors in the published papers will be listed

**Tromba marina paper:** The minus sign in Eq. (28) (and thus Eqs. (31) and (35)) should be a plus sign.

**DigiDrum:**  $\sigma_0$  and  $\sigma_1$  should be multiplied by  $\rho H$  in order for the stability condition to hold.

**DigiDrum:** Stability condition is wrong. Should be:

$$h \geq \sqrt{c^2 k^2 + 4\sigma_1 k} + \sqrt{(c^2 k^2 + 4\sigma_1 k)^2 + 16\kappa^2 k^2} \quad (\text{A.1})$$

**DigiDrum:** Unit for membrane tension is N/m.