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# Physical Modelling of Musical Instruments for Real-Time Sound Synthesis Applications

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Ph.D. Dissertation  
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# Curriculum Vitae

Silvin Willemsen



Here is the CV text.

## Curriculum Vitae

# Abstract

English abstract

## Abstract

# Resumé

Danish Abstract

## Resumé







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## Contents

# Preface

As my background does (did) not lie in mathematics, physics or computer science, which – trust me – were three equally crucial components in creating the result of this project, I added a, say, more pedagogical section at the end of this thesis. These tutorials are a result of the things that I learned and (hopefully) explain topics such as *Energy Analysis*, *Stability Analysis*, etc. in a way so that others with the same background will be able to understand what is going on.

Silvin Willemsen  
Aalborg University, January 14, 2021

## Preface

## **Part I**

# **Introduction**





# Introduction

## 1 History of bowed strings

In static bow-string-interaction models, the friction force is defined as a function of the relative velocity between the bow and the string only. The first mathematical description of friction was proposed by Coulomb in 1773 [?] to which static friction, or *stiction*, was added by Morin in 1833 [?] and viscous friction, or velocity-dependent friction, by Reynolds in 1886 [?]. In 1902, Stribeck found a smooth transition between the static and the coulomb part of the friction curve now referred to as the Stribeck effect [?]. The latter is still the standard for static friction models today.

## 2 To do thingies

- Think about how to define real-time.
- Create an intuition for different parts of the equation
- Talk about input and output locations and how that affects frequency content (modes).

### 2.1 Intuition for the frequency dependent damping term

Take the frequency independent damping term  $-2\sigma_0\partial_t u$ . The more positive the velocity  $\partial_t u$  is, i.e., the string is moving upwards the more this term applies a negative, or downwards force (/effect) on the string. Vice versa, a more negative velocity will make this term apply a more positive force on the string. As for the frequency dependent damping term, apart from the obvious  $\sigma_1$ , the effect of the term increases with an increase of  $\partial_t\partial_x^2 u$  which describes the rate of change of the curvature of the string.

Let's first talk about positive and negative curvature, i.e., when  $\partial_x^2 u > 0$  or  $\partial_x^2 u < 0$ . Counterintuitively, in the positive case, the curve points downwards. Think about the function  $f(x) = x^2$ . It has a positive curvature (at any point),

but has a minimum. We can prove this by taking  $x = 0$  and setting grid spacing  $h = 1$ .

$$\begin{aligned}\delta_{xx}f(x) &= \frac{1}{h^2} (f(-1) - 2f(0) + f(1)), \\ &= \frac{1}{1^2} ((-1)^2 - 2 \cdot 0^2 + 1^2), \\ &= (1 - 0 + 1) = 2.\end{aligned}\tag{1}$$

In other words, the second derivative of the function  $f(x) = x^2$  around  $x = 0$  is positive.

As our term does not only include a second-order spatial derivative but also a first-order time derivative, we are now talking about a positive or negative *rate of change* of the curvature, i.e., when  $\partial_t \partial_x^2 u > 0$  or  $\partial_t \partial_x^2 u < 0$ . A positive rate of change of curvature means that the string either has a positive curvature and is getting more positive, i.e., the string gets more curved over time, or that the string has a negative curvature and is getting less negative, i.e., the string gets less curved or 'loosens up' over time. In the same way, a negative rate of change of curvature means that the string either has a negative curvature and is getting more negative, or that the string has a positive curvature and is getting less positive.

Let's see some examples. Take the same function described before, but now  $f$  changes over time, fx.  $f(x, t) = tx^2$ . When  $t$  increases over time, the curvature gets bigger. Repeating what we did above with  $x = 0$  and grid spacing  $h = 1$ , but now with  $t = 2$  and step size  $k = 1$ , but now with a backwards time derivative we get:

$$\begin{aligned}\delta_{t-} \delta_{xx} f(x, t) &= \frac{1}{kh^2} \left( f(-1, 2) - 2f(0, 2) + f(1, 2) \right. \\ &\quad \left. - (f(-1, 1) - 2f(0, 1) + f(1, 1)) \right), \\ &= \frac{1}{1 \cdot 1^2} \left( 2 \cdot (-1)^2 - 2 \cdot 2 \cdot (0)^2 + 2 \cdot 1^2 \right. \\ &\quad \left. - (1 \cdot (-1)^2 - 2 \cdot 1 \cdot (0) + 1 \cdot (1^2)) \right), \\ &= 2 + 2 - (1 + 1) = 2.\end{aligned}$$

So the rate of change of the curvature is positive, i.e., the already positively curved function  $x^2$  gets more curved over time.

If the curvature around a point along a string gets more positive (or less negative) over time, the force applied to that point will be positive. Vice versa, if the curvature around a point along a string gets more negative (or less positive) over time, the force applied will be negative.

The fact the frequency dependent term to be added rather than subtracted to the FDS, is caused by the fact that a positive curvature implies a negative

### 3. Notes

position in the string (think of the function  $x^2$  which has a positive curvature, but the function 'points' downwards). This translated to the force/effect this term has on the scheme means...

## 3 Notes

One over number  $\rightarrow$  reciprocal of number

Example: When the waveform consists entirely of harmonically related frequencies, it will be periodic, with a period equal to the reciprocal of the fundamental frequency (from An Introduction to the Mathematics of Digital Signal Processing Pt 2 by F. R. Moore)



# **Part II**

# **Models**



# Models

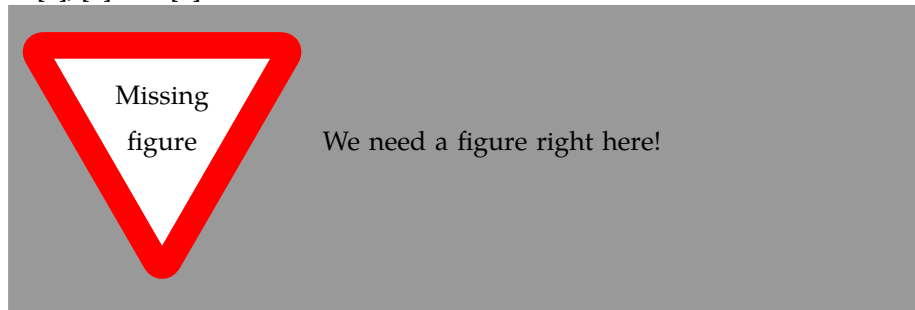
## 1 Bow Models

As opposed to less complex bow models, such as the hyperbolic [source] and exponential [source] models, the elasto-plastic bow model assumes that the friction between the bow and the string is caused by a large quantity of bristles, each of which contributes to the total amount of friction.

## 2 Section 2 name

Here is section 2. If you want to learn more about  $\text{\LaTeX}2_{\epsilon}$ , have a look at [2], [4] and [3].

I think this word is misspelled



### 2.1 Examples

You can also have examples in your document such as in example 2.1.

#### Example 2.1 (An Example of an Example)

Here is an example with some math

$$0 = \exp(i\pi) + 1 . \quad (2)$$

You can adjust the colour and the line width in the `macros.tex` file.

## 2.2 How does Subsections and Subsubsections Look?

Well, like this

**This is a Subsubsection**

and this.

**A Paragraph** You can also use paragraph titles which look like this.

**A Subparagraph** Moreover, you can also use subparagraph titles which look like this. They have a small indentation as opposed to the paragraph titles.

I think that a summary of this exciting chapter should be added.

## 3 Conclusion

In case you have questions, comments, suggestions or have found a bug, please do not hesitate to contact me. You can find my contact details below.

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Denmark

Is it possible to add a subsub-paragraph?



# **Part III**

## **Dynamic Grids**



# Dynamic Grids

## 1 Introduction

Simulating musical instruments using physical modelling – as mentioned in Part I– could potentially allow for manipulations that are impossible in the physical world. Examples of this are changes in material properties, size, etc.

check if is still true

There are examples in the physical world where certain aspects of the instrument are manipulated in real-time

Some even use this in performance [1]

## References

- [1] J. Gomm, 2011. [Online]. Available: <https://www.youtube.com/watch?v=nY7GnAq6Znw>
- [2] L. Madsen, "Introduktion til LaTeX," <http://www.imf.au.dk/system/latex/bog/>, 2010.
- [3] F. Mittelbach, *The LATEX companion*, 2nd ed. Addison-Wesley, 2005.
- [4] T. Oetiker, "The not so short a introduction to LaTeX2e," <http://tobi.oetiker.ch/lshort/lshort.pdf>, 2010.