Laboratory 1: Numerical and Data-Intensive Computing
Numerical Errors

1 Exercices

1. In mathematics, the associative property of addition means that the order in which the operations are performed does not matter as long as the sequence of the operands is not changed. However, this is not always true when it is implemented in a computer. This is due to the numerical floating-point representation. An example where the round-off error leads to not to fulfill the associative property of addition is the following and its value:

   let's consider the following infinite series:

   $$S = \sum_{i=1}^{\infty} \frac{1}{i^4} = \frac{\pi^4}{90}$$

   different results are obtained if the series is summed in ascending or descending order.

   (a) Why the associative property is not fulfilled?
   (b) What order is more accurate?. why?
   (c) Write a function called *sumSeriesA* with the number of terms to be taken into account as the argument. The output of the function should be the first n terms summed in ascending order.
   (d) Write a function called *sumSeriesD* but in this cases summing terms in descending order.
   (e) Write a program *sumSeries* in python using both functions to compare the results
   (f) Find out the n value from which different results are obtained. Explain why this n value is obtained.

   Hints:

   (a) Print results with 20 significant digits.
   (b) Plot the difference between the two values.
   (c) Plot the difference between each value and the real one.

2. Analytically equivalent expressions may not be when evaluated in a computer. As an example of this situation, let us consider the quadratic equation:

   $$ax^2 + bx + c = 0$$

   roots of this equation are given by the following equivalent expressions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1}$$

$$x = \frac{2c}{-b \pm \sqrt{b^2 - 4ac}} \tag{2}$$

In these equations the addition becomes delicate and round-off error-probe whenever $b > 0$ and under the condition $|ac| << b^2$. If either a or c or both is small, then one of the roots will involve the subtraction of b from a very nearly equal quantity (the discriminant). The correct way to compute the roots is

$$q = \frac{-1}{2}\left(b + sign(b)\sqrt{b^2 - 4ac}\right) \tag{3}$$

Then the two roots are

$$x_1 = \frac{q}{a}\ ,\ x_2 = \frac{c}{q} \tag{4}$$

(a) Show the equivalence between the three previous expressions

(b) Write functions *quadratic1*, *quadratic2* y *quadratic3* with parameters a, b and c as arguments and roots using the different expressions as output.

(c) Find out and example where the results are different. Explain the result.

(d) Which is the correct result?. Why?