Maths

1 (1) 3

2 (4) no solution

3 (1) 10 unit

4 (2) proportional

5 (2) $-\frac{1}{2}$

6 (1) x^3y^3

7 Given poly. $4x^2 - 2x + (k-4)$

here $a = 4$

$b = -2$

$c = k-4$

Sum of zeros $= \frac{-b}{a} = -\left(\frac{-2}{4}\right) = \frac{2}{4} = \frac{1}{2}$

Product $= \frac{c}{a} = \frac{k-4}{4}$

A/C to question

sum of zeros $= \frac{1}{2} \times$ product of zeros

$$\frac{1}{2} = \frac{1}{2} \times \left(\frac{k-4}{4}\right)$$

$$\frac{1}{2} = \frac{k-4}{8}$$

$$8 = 2k - 8$$

$$8 + 8 = 2k$$

$$16 = 2k$$

$$\boxed{k = 8}$$

8. Given

$x(x+3) = 10x + 75 = 7(2x-3)$ from the quadratical A.

To find: x

∵ It is mid point P.

∴ AP = PC = CA

Now,

$AP = x^2 = x$

$\therefore x^2 + (3)^2 = x^2 + y^2 \Rightarrow \sqrt{13} = x$

$\Rightarrow x + 3 = x + y^2 \Rightarrow x - y^2 = -3\sqrt{2}$

$\Rightarrow 13 = 13x \Rightarrow 3x = 3\sqrt{2}$

$\Rightarrow x = \sqrt{2}$

$x + x^2 = x + y^2 = \sqrt{2}$

$x = 2\sqrt{2}$

$\frac{x}{y} = \sqrt{2}$

$\Rightarrow x = \sqrt{2}$

$\Rightarrow \sqrt{2} = 1$

$\Rightarrow \frac{(\sqrt{2})^2}{\sqrt{2}} = 1$

$\Rightarrow \sqrt{2} = 1$

So $x = \sqrt{2}$

9. The equation = $4px + 3y^2 = 7$

and $2x + 3y = 8$

∴ ensure the equation has exactly one solution

and now $a_1 = 4p$, $b_1 = 3$, $c_1 = 7$

$a_2 = 2$, $b_2 = 3$, $c_2 = 8$

for unique solution

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

So $\frac{4p}{2} = \frac{3}{3}$

$2p = 3$

$p = 3$

So except $p = 3$ for all real values of p the given system of equations will have exactly one solution.

10.

Given $DE \parallel BC$

$\frac{AD}{DB} = \frac{CE}{EC}$

So ensure: $DE \parallel BC$

Proof: ∵ $DE \parallel BC$ given

Using Thales Theorem (BPT)

$\frac{AE}{EC} = \frac{AD}{DB}$ —(1)

$\frac{AE}{EC} = \frac{AD}{DC}$ (given) —(2)

from ① & ②

$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$ — ③

now → $\frac{PQ}{BC} = \frac{PR}{CR} = \frac{AQ}{CQ}$

hence converse of BPT

$PR \parallel BC$

hence proved

12 given: $\sin\theta = \frac{1}{4}$ to show: $\frac{1}{4}\sin\theta\cos\theta = \frac{1}{8}$

$\because \sin\theta = \frac{1}{4}$

let P be θ and $AB = 2x$

then by pythagoras theorem

$x^2 = AC^2 - P^2$

$AC = \sqrt{(2x)^2 - x^2}$

$AC = \sqrt{4x^2 - x^2}$

$AC = \sqrt{3x^2}$

$\therefore AC = x\sqrt{3}$ & $P = x$

Now $\sin\theta = \frac{P}{H} = \frac{x}{2x} = \frac{1}{2}$

$\cos\theta = \frac{AC}{AB} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

R.H.S $\frac{1}{4} \times \sin\theta \times \cos\theta$

$\frac{1}{4} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{8}\sqrt{3} = \frac{1}{8}$

now $\frac{1}{4} \times \sin\theta \times \cos\theta = \frac{1}{8}$

hence proved

E $\frac{257}{5000}$

now denominator is 5000

so factors of 5000 = $2^3 \times 5^4$

$\therefore$ the 2×5 pair of $10 \times 10 \times 10 \times 10$ will be required to make it terminating decimal

now $\frac{257}{2^3 \times 5^4} = \frac{257}{2^3 \times 625} = \frac{257}{5000}$

Multiplying both sides by 2

$\frac{257 \times 2}{5000 \times 2} = \frac{514}{10000} = 0.0514$ Ans

B) To prove: $\sqrt{2}$ is irrational

let us suppose $\sqrt{2}$ is rational number

Then $\sqrt{2}$ can be written as $\frac{p}{q}$ where

p & q are integer of $q \neq 0$

$\sqrt{2} = \frac{p}{q}$ $\Rightarrow (\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$

p & q are co prime

squaring both sides

$(\sqrt{3})^2 = \left(\frac{p}{q}\right)^2$

$3 = \frac{p^2}{q^2}$

$3q^2 = p^2$ — (1)

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ 3 divides p — (2)

let $p = 3m$ for some true integer

from equation (1)

$3q^2 = (3m)^2$

$3q^2 = 9m^2$

$q^2 = 3m^2$

3 divides q^2

3 divides q — (3)

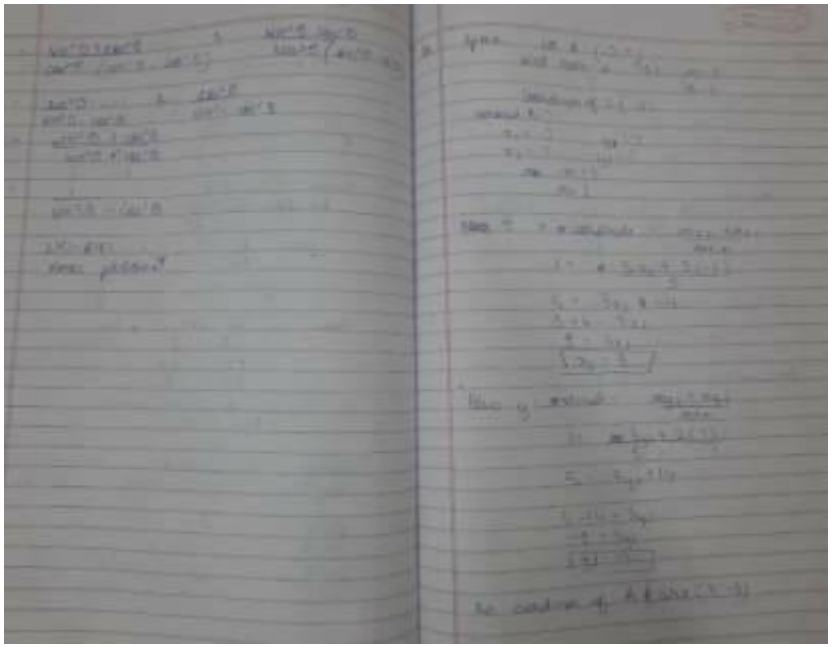
So from (2) & (3)

3 is common factor of p & q

But this is contradiction to our assumption

so. our assumption is wrong.

so $\sqrt{3}$ is an irrational number



19 Let the fraction be $\frac{x}{y}$

Ist case

$$\frac{x+2}{y+1} = 1$$

$$\frac{x}{y+1} = 1 \Rightarrow x = y+1 \quad \text{--- (1)}$$

IInd case

$$\frac{x+2}{2y} = \frac{1}{2} \quad \text{--- (2)}$$

$$2(x+2) = 2y$$

$$x+2 = y \quad \text{--- (2)}$$

Let equation (1) as equation (2)

$$x = y+1$$

$$x = y+1$$

Now putting x value of x in (2)

$$x = 2(y+1) = 2y$$

$$x+1 = 2y$$

$$x = 2y-1$$

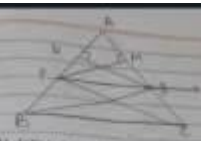
$$x = y$$

Now $x = y$ $\Rightarrow$ $\frac{x}{y} = \frac{y}{y} = 1$

We know if $x = y$, fraction is $\frac{x}{y}$

So fraction is $\frac{x}{y}$

20.



Construct: Draw a line through F parallel to the other side of A to intersect BC at M. Join F with the point of A as shown. Now we have a triangle ABC and a line parallel to side BC.

Given: In $\triangle ABC$

$DE \parallel FG$ intersect AB at F & AC at G

To prove: $\frac{AF}{FB} = \frac{AG}{GC}$

Const: Let us draw FM $\perp$ BC & AN $\perp$ BC

Join BG & EF

Proof: $\frac{\text{area}(AEG)}{\text{area}(AGF)} = \frac{\frac{1}{2} \times AF \times AN}{\frac{1}{2} \times FG \times AN} = \frac{AF}{FG}$ --- (1)

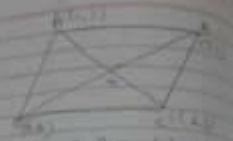
$\frac{\text{area}(AEG)}{\text{area}(FEG)} = \frac{\frac{1}{2} \times AG \times EM}{\frac{1}{2} \times GE \times FM} = \frac{AG}{GE}$ --- (2)

Now $\triangle EFG$ & $\triangle GBE$ are on same base & between same $\parallel$ lines $\Rightarrow$ ar($\triangle EFG$) = ar($\triangle GBE$) --- (3)

From equation (1), (2) & (3)

We have $\frac{AF}{FB} = \frac{AG}{GC}$

Hence proved



Q. In a rhombus of figure is
A(1,1)
B(1,3)
C(4,3)
D(x,y)

Sol. ACBD is diagonal of Rhombus
so O —
∴ Diagonal of figure must cross each
So mid point of AC is $\left(\frac{1+4}{2}, \frac{1+3}{2}\right)$

$\frac{5}{2}, 2$

Now coordinate of O are $\left(\frac{5}{2}, 2\right)$

Mid point of BD

$\left(\frac{1+x}{2}, \frac{3+y}{2}\right)$

So $\frac{1+x}{2} = \frac{5}{2}$ and $\frac{3+y}{2} = 2$

$4 + x = 10$ $\quad$ $3 + y = 4$
$x = 10 - 4$ $\quad$ $y = 4 - 3$
$x = 6$ $\quad$ $y = 1$

So coordinate of D are $(6, 1)$

Q. $\sin^2\theta + \cos\theta = 1$ (to find)
$\sin^2\theta + \cos^2\theta = \cos^2\theta$

$1 - \cos^2\theta = \cos$

$\frac{1}{2} + 2\left(\frac{1}{2}\right) = \frac{1}{2} + 1$

$\frac{1}{2} + 1 + \left(\frac{\sqrt{3}}{2}\right)^2 + \sqrt{3}^2$

$\frac{1}{2} + \sqrt{3} + 3$

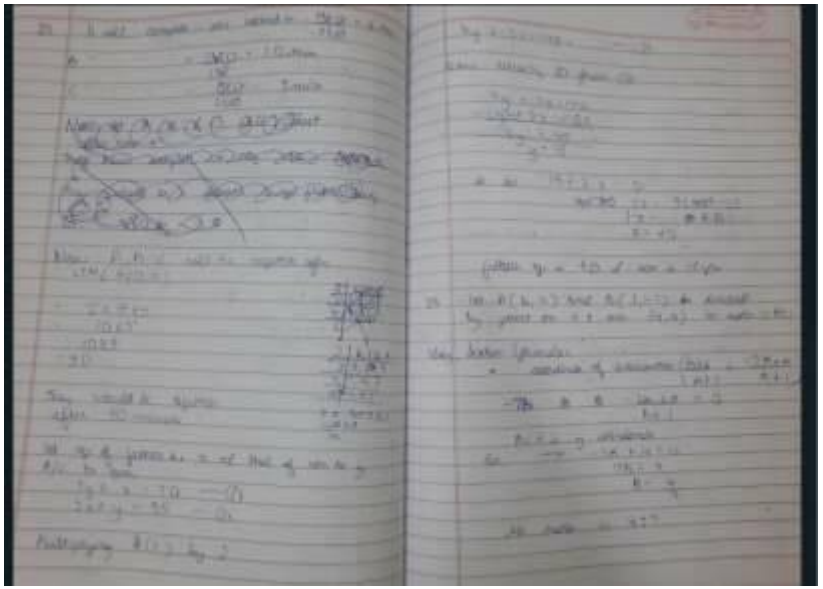
$\frac{1}{2} + \frac{3}{2} = 1$

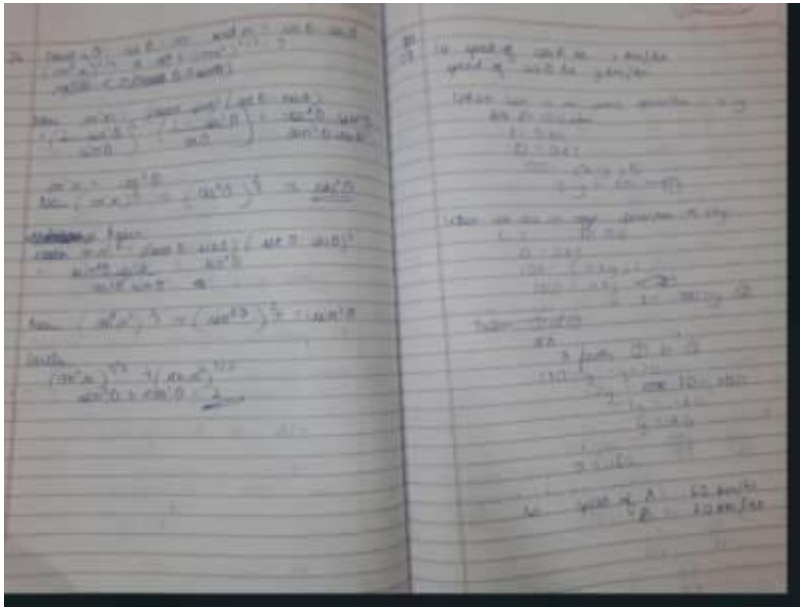
$\frac{1 + 3\sqrt{3} + 6}{2} = \frac{7 + 3\sqrt{3}}{2}$

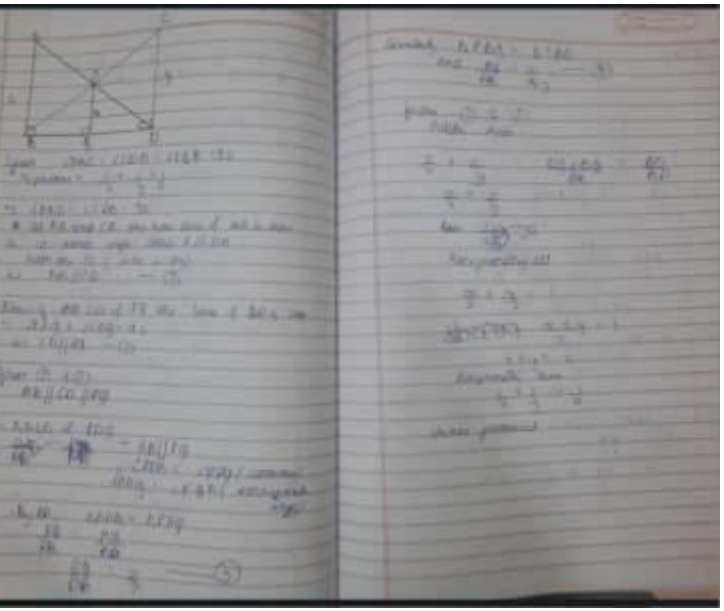
$\frac{2 + 3\sqrt{3}}{2}$

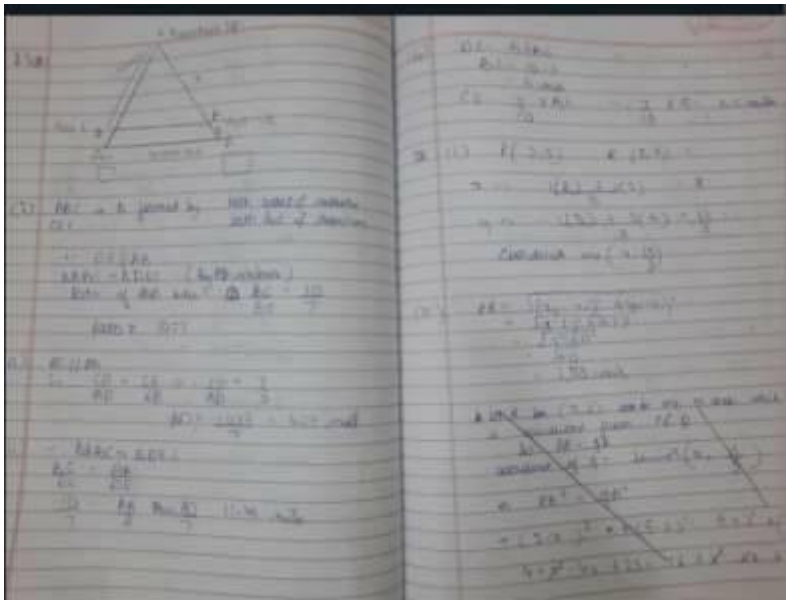
$\frac{7 + 3\sqrt{3}}{2} + \frac{4}{2}$

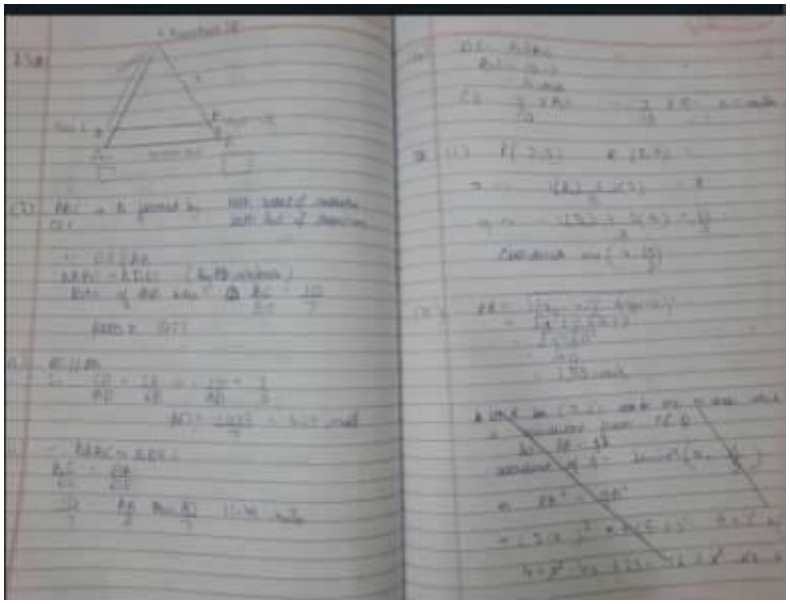
$\frac{7 + 4 + 3\sqrt{3}}{2}$











(iv) Mid point of PR

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\left(\frac{2+8}{2}, \frac{5+3}{2}\right)$$

$$\left(\frac{10}{2}, \frac{8}{2}\right)$$

$$(5, 4)$$

(iii) Let $A(x, 0)$ be point on x-axis which is equidistant from P & Q

so $PA = QA$

$PA^2 = QA^2$

$$(2-x)^2 + (5-0)^2 = (8-x)^2 + \left(\frac{13}{3} - 0\right)^2$$

$$4 + \cancel{x^2} - 4x + 25 = 16 + \cancel{x^2} - 8x + \frac{169}{9}$$

$$4x = 16 + \frac{169}{9} - 4 - 25$$

$$x = \frac{13}{9}$$

Hence the point is $\left(\frac{13}{9}, 0\right)$

given quadratic poly $p(x) = 2x^2 - 3x + 7$

here $a = 2$

$b = -3$

$c = 7$

$\alpha + \beta = -\frac{b}{a} = \frac{3}{2}$

$\alpha\beta = \frac{7}{2}$

$$\frac{1}{2\alpha - \beta} + \frac{1}{2\beta - 3}$$

$$\frac{(2\alpha - \beta) + (2\alpha - 3)}{(2\alpha - 3)(2\beta - 3)}$$

$$\frac{2\beta - 3 + 2\alpha - 3}{4\alpha\beta - 6\alpha - 6\beta + 9}$$

$$\frac{2\beta + 2\alpha - 6}{4\alpha\beta - 6(\alpha + \beta) + 9}$$

$$\frac{2(\alpha + \beta) - 6}{4\alpha\beta - 6(\alpha + \beta) + 9} = \cancel{\frac{2(\alpha + \beta) - 6}{4\alpha\beta - 6\alpha\beta}}$$

$$= \frac{2 \times \frac{3}{2} - 6}{4 \times \frac{7}{2} - 6 \times \frac{3}{2} + 9} = \frac{3 - 6}{14 - 9 + 3} = \frac{-3}{14}$$

so. $\frac{1}{2\alpha - 3} + \frac{1}{2\beta - 3} = \frac{-3}{14}$