Maxwell's equations:

$$\begin{cases} \text{rot E} = i\ \omega\ \mu\ \mu_0\ \text{H} + \text{K}\ \delta\ (\text{M-M}_0) \\ \text{rot H} = -\ i\ \omega\ \varepsilon\ \varepsilon_0\ \text{E} + \text{J}\ \delta\ (\text{M-M}_0) \end{cases}$$

μ_r and ε_r are symmetric tensors

$$\text{E} = \text{E}\exp(\text{i}\vec{\text{k}}.\vec{\text{u}})\exp(\text{-}i\omega t)$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = c\mu_0 = \frac{1}{c\varepsilon_0}$$

If we divide the first equation by $\sqrt{Z_0}$, and we multiply the second by $\text{i}\sqrt{Z_0}$, we obtain

$$\begin{cases} \text{rot}\ (\text{E}\ Z_0^{-\frac{1}{2}}) = \frac{\omega}{c}\ \mu_\text{r}\ (i\text{H}\ Z_0^{\frac{1}{2}}) + (\text{J}_\text{m} Z_0^{-\frac{1}{2}})\ \delta\ (\text{M-M}_0) \\ \text{rot}\ (i\text{H}\ Z_0^{\frac{1}{2}}) = \frac{\omega}{c} \varepsilon_r\ (\text{E}\ Z_0^{-\frac{1}{2}}) + (i\text{J}_\text{e} Z_0^{\frac{1}{2}})\ \delta\ (\text{M-M}_0) \end{cases}$$

These transformations lead to perfectly symmetric Maxwell equations.

Symmetrical Maxwell's Equations:

$$\begin{cases} \text{rot u} = \text{k}_0 \text{q v} + \text{v}^\text{s}\ \delta_{(\text{M}-\text{M}_0)} \\ \text{rot v} = \text{k}_0 \text{p u} + \text{u}^\text{s}\ \delta_{(\text{M}-\text{M}_0)} \end{cases}$$

EM wave equation:

$$\nabla\times(\text{q}^{-1}\nabla\times\text{u}) - \text{k}_0^2\text{pu} = \nabla\times(\text{q}^{-1}.\text{v}^\text{s}\delta) + \text{k}_0\text{u}^\text{s}\delta$$

	p	q	u	v	u^s	v^s
Electric formulation	ε_r	μ_r	$\frac{\mathcal{E}}{\sqrt{Z_0}}$	$\text{i}\,\mathcal{H}\sqrt{Z_0}$	$\text{i}\,\mathcal{J}_e\sqrt{Z_0}$	$\frac{\mathcal{J}_m}{\sqrt{Z_0}}$
Magnetic formulation	μ_r	ε_r	$\text{i}\,\mathcal{H}\sqrt{Z_0}$	$\frac{\mathcal{E}}{\sqrt{Z_0}}$	$\frac{\mathcal{J}_m}{\sqrt{Z_0}}$	$\text{i}\,\mathcal{J}_e\sqrt{Z_0}$

TE case:

$$\begin{cases} \dfrac{\partial \mathrm{E}^z}{\partial y} = k_0 \mathrm{B}^x \\ \dfrac{\partial \mathrm{H}^x}{\partial y} = -k_0 \varepsilon^z(x)\,\mathrm{E}^z + \dfrac{\partial \mathrm{H}^y}{\partial x} \end{cases} \quad \text{with} \quad \begin{cases} \mathrm{B}^x = \mu^x(x)\,\mathrm{H}^x \\ \mathrm{H}^y = \dfrac{\mathrm{B}^y}{\mu^y(x)} \\ k_0 \mathrm{B}^y = -\dfrac{\partial \mathrm{E}^z}{\partial x} \end{cases}$$

In Fourier space:

$$\begin{cases} \dfrac{d\hat{E}^z}{dy} = k_0 \hat{B}^x \\ \dfrac{d\hat{H}^x}{dy} = -k_0 \hat{D}^z + i\,\beta\,\hat{H}^y \end{cases} \quad \text{and} \quad \begin{cases} \hat{B}^x = \left[\dfrac{1}{\mu^x}\right]^{-1} \hat{H}^x \\ \hat{D}^z = \left[\,\varepsilon^z\,\right] \hat{E}^z \\ \hat{H}^y = \left[\,\mu^y\,\right]^{-1} \hat{B}^y \\ k_0 \hat{B}^y = -i\,\beta\,\hat{E}^z \end{cases}$$

$$\boxed{\frac{d}{dy}\begin{pmatrix} \hat{E}^z \\ \hat{H}^x \end{pmatrix} = \begin{pmatrix} 0 & k_0\left[\dfrac{1}{\mu^x}\right]^{-1} \\ -k_0\left[\varepsilon^z\right] + \dfrac{1}{k_0}\beta\left[\mu^y\right]^{-1}\beta & 0 \end{pmatrix}\begin{pmatrix} \hat{E}^z \\ \hat{H}^x \end{pmatrix} = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}\begin{pmatrix} \hat{E}^z \\ \hat{H}^x \end{pmatrix} = M \times \begin{pmatrix} \hat{E}^z \\ \hat{H}^x \end{pmatrix}}$$

TM case:

$$\begin{cases} \dfrac{\partial \mathrm{E}^x}{\partial y} = -k_0 \mu^z(x)\,\mathrm{H}^z + \dfrac{\partial \mathrm{E}^y}{\partial x} \\ \dfrac{\partial \mathrm{H}^z}{\partial y} = k_0 \mathrm{D}^x \end{cases} \quad \text{with} \quad \begin{cases} \mathrm{D}^x = \varepsilon^x(x)\,\mathrm{E}^x \\ \mathrm{E}^y = \dfrac{\mathrm{D}^y}{\varepsilon^y(x)} \\ k_0 \mathrm{D}^y = -\dfrac{\partial \mathrm{H}^z}{\partial x} \end{cases}$$

In Fourier space:

$$\begin{cases} \dfrac{d\hat{E}^x}{dy} = -k_0 \hat{B}^z + i\,\beta\,\hat{E}^y \\ \dfrac{d\hat{H}^z}{dy} = k_0 \hat{D}^x \end{cases} \quad \text{and} \quad \begin{cases} \hat{D}^x = \left[\dfrac{1}{\varepsilon^x}\right]^{-1} \hat{E}^x \\ \hat{B}^z = \left[\,\mu^z\,\right] \hat{H}^z \\ \hat{E}^y = \left[\,\varepsilon^y\,\right]^{-1} \hat{D}^y \\ k_0 \hat{D}^y = -i\,\beta\,\hat{H}^z \end{cases}$$

$$\boxed{\frac{d}{dy}\begin{pmatrix} \hat{E}^x \\ \hat{H}^z \end{pmatrix} = \begin{pmatrix} 0 & -k_0\left[\mu^z\right] + \dfrac{1}{k_0}\beta\left[\varepsilon^y\right]^{-1}\beta \\ k_0\left[\dfrac{1}{\varepsilon^x}\right]^{-1} & 0 \end{pmatrix}\begin{pmatrix} \hat{E}^x \\ \hat{H}^z \end{pmatrix} = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}\begin{pmatrix} \hat{E}^x \\ \hat{H}^z \end{pmatrix} = M \times \begin{pmatrix} \hat{E}^x \\ \hat{H}^z \end{pmatrix}}$$

Diagonalization of the final differential equation

$$\frac{d}{dy}\begin{pmatrix}\hat{E}\\ \hat{H}\end{pmatrix}=\begin{pmatrix}0 & A\\ B & 0\end{pmatrix}\begin{pmatrix}\hat{E}\\ \hat{H}\end{pmatrix} \quad \rightarrow \quad \frac{d^2}{dy^2}\hat{E}=(AB)\cdot\hat{E}$$

The associated eigen value problem is:

$$(AB)\times V=\upsilon_p^2\times V \qquad \text{with} \quad \upsilon_p=i\beta^y$$

with $\Re e\, \upsilon_p - \Im m\, \upsilon_p \le 0$ (Maystre's condition for upward waves)

$$\begin{pmatrix}\hat{E}\\ \hat{H}\end{pmatrix}_{(y+h)}=\begin{pmatrix}Q & Q\\ P & -P\end{pmatrix}\times\begin{pmatrix}\exp(\upsilon_p h) & 0\\ 0 & \exp(-\upsilon_p h)\end{pmatrix}\times\begin{pmatrix}\frac{Q^{-1}}{2} & \frac{P^{-1}}{2}\\ \frac{Q^{-1}}{2} & -\frac{P^{-1}}{2}\end{pmatrix}\begin{pmatrix}\hat{E}\\ \hat{H}\end{pmatrix}_{(y)}=T\begin{pmatrix}\hat{E}\\ \hat{H}\end{pmatrix}_{(y)}$$

Where $\begin{cases}Q=V\\ P=\dfrac{1}{\upsilon_p}BV\end{cases}$ or $\begin{cases}Q=V\upsilon_p\\ P=BV\end{cases}$

T matrix

$$\begin{pmatrix}\hat{E}_{(y+h)}\\ \hat{H}_{(y+h)}\end{pmatrix}=\begin{pmatrix}Q\,ch(\upsilon_p h)\,Q^{-1} & Q\,sh(\upsilon_p h)\,P^{-1}\\ P\,sh(\upsilon_p h)\,Q^{-1} & P\,ch(\upsilon_p h)\,P^{-1}\end{pmatrix}\begin{pmatrix}\hat{E}_{(y)}\\ \hat{H}_{(y)}\end{pmatrix}$$

G matrix

$$\boxed{\begin{pmatrix}\exp(-\upsilon_p h) & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\frac{Q^{-1}}{2} & \frac{P^{-1}}{2}\\ \frac{Q^{-1}}{2} & -\frac{P^{-1}}{2}\end{pmatrix}\begin{pmatrix}\hat{E}_{(y+h)}\\ \hat{H}_{(y+h)}\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & \exp(-\upsilon_p h)\end{pmatrix}\times\begin{pmatrix}\frac{Q^{-1}}{2} & \frac{P^{-1}}{2}\\ \frac{Q^{-1}}{2} & -\frac{P^{-1}}{2}\end{pmatrix}\begin{pmatrix}\hat{E}_{(y)}\\ \hat{H}_{(y)}\end{pmatrix}}$$

S matrix

$$\begin{pmatrix}\hat{E}_{(y+h)}\\ \hat{H}_{(y)}\end{pmatrix}=\begin{pmatrix}Q\dfrac{1}{ch(\upsilon_p h)}Q^{-1} & Q\,th(\upsilon_p h)\,P^{-1}\\ -P\,th(\upsilon_p h)\,Q^{-1} & P\dfrac{1}{ch(\upsilon_p h)}P^{-1}\end{pmatrix}\begin{pmatrix}\hat{E}_{(y)}\\ \hat{H}_{(y+h)}\end{pmatrix}$$

Note : $\Phi \otimes \Phi^* = \int \left(E \wedge H^* - E^* \wedge H\right) \bullet n\, dS = -2i \int \Im m \left(E^* \wedge H\right) = -4i \times Poynting\ Flux$

Normalization: Poynting Flux = 1

Homogeneous medium

TE:

$$\upsilon_p = \begin{pmatrix} \sqrt{\frac{\mu^x}{\mu^y}\beta_1^2 - k_0^2 \varepsilon^z \mu^x} & & \\ & \ddots & \\ & & \sqrt{\frac{\mu^x}{\mu^y}\beta_{N_x}^2 - k_0^2 \varepsilon^z \mu^x} \end{pmatrix}$$ and $A = k_0 \mu^x$

$$\mathrm{Q_m = V_m\ \upsilon_{p_m}} \qquad P_m = V_m \frac{\upsilon_{pm}^{\ 2}}{k_0 \mu^x}$$

Lorentz integral: $Lz_m = 2d\, V_m^2 \frac{\upsilon_{pm}^{\ 3}}{k_0 \mu^x} = -4i$ then: $V_m = \sqrt{\frac{-2ik_0\mu^x}{\upsilon_{pm}^{\ 3} d}}$

TM:

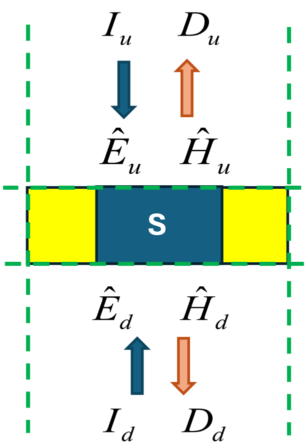
$$\upsilon_p = \begin{pmatrix} \sqrt{\frac{\varepsilon^x}{\varepsilon^y}\beta_1^2 - k_0^2 \mu^z \varepsilon^x} & & \\ & \ddots & \\ & & \sqrt{\frac{\varepsilon^x}{\varepsilon^y}\beta_{N_x}^2 - k_0^2 \mu^z \varepsilon^x} \end{pmatrix}$$ and $B = k_0 \varepsilon^x$

$$\mathrm{Q_m = V_m\ \upsilon_{p_m}} \qquad P_m = k_0 \varepsilon^x V_m$$

Lorentz integral: $Lz_m = 2d\, V_m^2\, k_0 \varepsilon^x \upsilon_{pm} = -4i$ then: $V_m = \sqrt{\frac{-2i}{k_0 \varepsilon^x \upsilon_{pm} d}}$

T and S matrix for up and down medium

Associate Fourier coefficients of fields E et H to incident and diffracted modes:



$$\begin{pmatrix} \hat{E}_d \\ D_d \end{pmatrix} = S_d \begin{pmatrix} I_d \\ \hat{H}_d \end{pmatrix} \qquad \begin{pmatrix} D_u \\ \hat{H}_u \end{pmatrix} = S_d \begin{pmatrix} \hat{E}_u \\ I \end{pmatrix}$$

$$\begin{pmatrix} \hat{E} \\ \hat{H} \end{pmatrix} = T_d \begin{pmatrix} I_d \\ D_d \end{pmatrix} \quad \Rightarrow \quad T_d = \begin{pmatrix} Q & Q \\ P & -P \end{pmatrix} \quad \Rightarrow \quad S_d = \begin{pmatrix} 2Q & -QP^{-1} \\ I & -P^{-1} \end{pmatrix}$$

$$\begin{pmatrix} D_u \\ I_u \end{pmatrix} = T_u \begin{pmatrix} \hat{E} \\ \hat{H} \end{pmatrix} \quad \Rightarrow \quad T_u = \begin{pmatrix} \frac{Q^{-1}}{2} & \frac{P^{-1}}{2} \\ \frac{Q^{-1}}{2} & -\frac{P^{-1}}{2} \end{pmatrix} \quad \Rightarrow \quad S_u = \begin{pmatrix} Q^{-1} & -I \\ PQ^{-1} & -2P \end{pmatrix}$$

Note : truncate S with only propagative waves