

Supplemental Materials for “Multiple Axialities: A Computational Model of the Axial Age” in the *Journal of Cognition and Culture* (2018)

F. LeRon Shults, Wesley J. Wildman, Justin E. Lane, Christopher J. Lynch, and
Saikou Y. Diallo

A. Construction of MAXiM

Over the past few decades, a number of scholars in the field of social simulation have applied computer modeling and simulation techniques to recreate, represent, and analyze historical episodes (Epstein & Axtell, 1996; Sokolowski & Banks, 2009). Some have taken up themes similar to those of MAXiM. For example, computer simulations have been used to explicate the role of punishment in holding societies together (Axelrod, 1986). Agent-based simulation of behavioral synchrony suggests that the emergence of priesthoods, as well as the deification of historical figures, would have facilitated and enabled network coordination, which revolved around closeness to “True Information” (Dávid-Barrett & Carney, 2016). This is certainly consonant with a link between the ideological-political and cognitive-coalitional paths (though not the material-social path). Our model works out of this trajectory of historical research, employing a computational approach. MAXiM was developed using the AnyLogic multi-method simulation platform, version 8.1 (AnyLogic, 2018).

A system-dynamics model (SDM) expresses the “flow” of some quantity between “stocks” where the flow is governed by “flow rates” (Forrester, 1961; Sterman, 2000). Differential equations define the relationship between stocks and flow rates. MAXiM is a SDM that maps shifts back-and-forth between two ways of thinking within a population, axial and pre-axial (or “traditional”). Population flow is measured in terms of people who are tracked from their births through their deaths. Exposure to axial and pre-axial ways of thinking can cause shifts from one way of thinking to the other. It is thus a kind of conversion model, as conveyed in Figure 1.

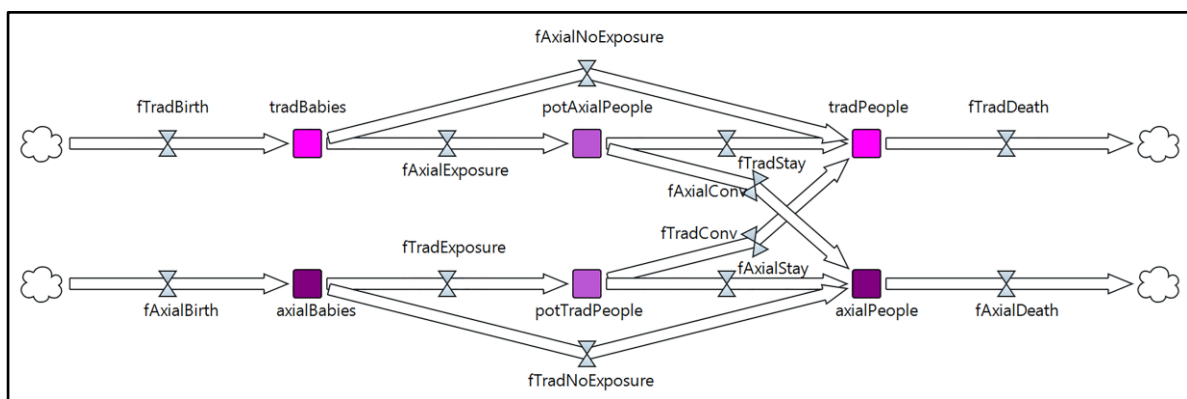


Figure 1. The basic structure of the conversion model: boxes are stocks, arrows are flows of people, and hourglasses are flowrates.

A SDM typically has feedback systems, control parameters, and key variables (Stermann, 2000). The conversion model with these features in place is depicted in Figure 2. The MAXiM model's key feedback loop is related to the interdependency between resource scarcity which impacts birth rates and the traditional and axial populations based on the model's carrying capacity. Six key variables are arrayed along the top of Figure 2: birth rates for traditional (tradBirthRate) and axial (tradAxialBirth) people; exposure rates that portray the exposure of both populations to the other population's way of thinking (tradExpToAxialRate and axialExpToTradRate); and conversions rates for switching exposed people to the other way of thinking (tradConvToAxialRate and axialConvToTradRate).

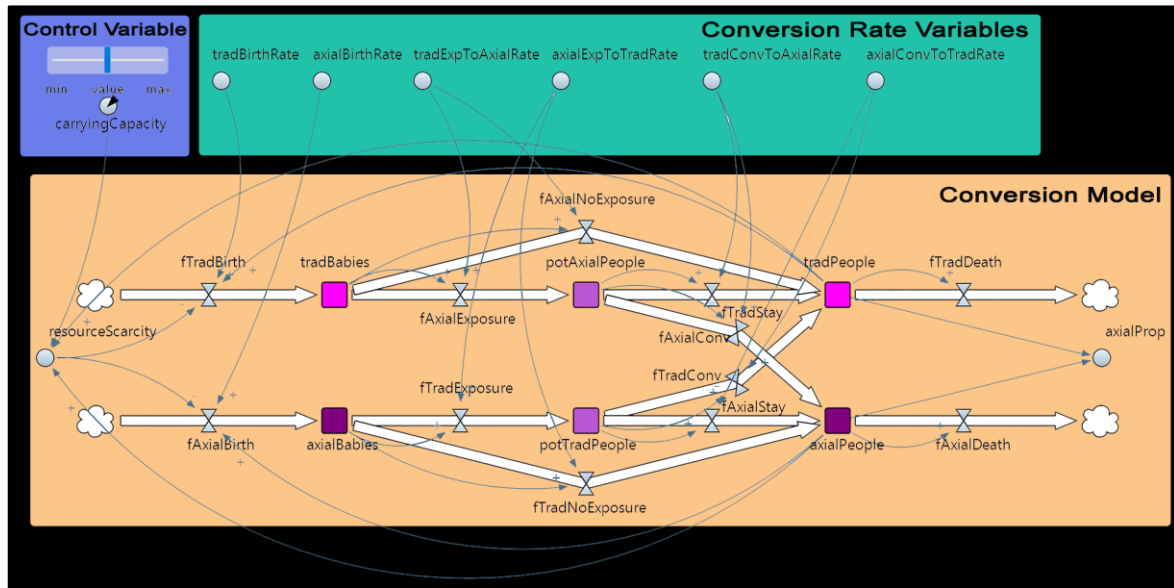


Figure 2. The conversion model in full detail, showing the role of six key variables related to birth rates of people with the two kinds of worldviews (traditional and axial), rates of exposure to people of the other kind, and rates of conversion for those exposed.

This conversion model is applicable to a host of situations where people can change their way of thinking or social affiliations. Historical context now drives the details for generating the above six key variables. The theoretical layout of the causal connections between variables implemented within MAXiM is depicted in Figure 3. This is the heart of the causal architecture that facilitates the transition from pre-axial to axial civilizations. Circles represent the variables and arrows represent the causal links between them. Each variables' value is determined by an intuitive mathematical formula expressing the effect placed on the variable by the variables that feed into it.

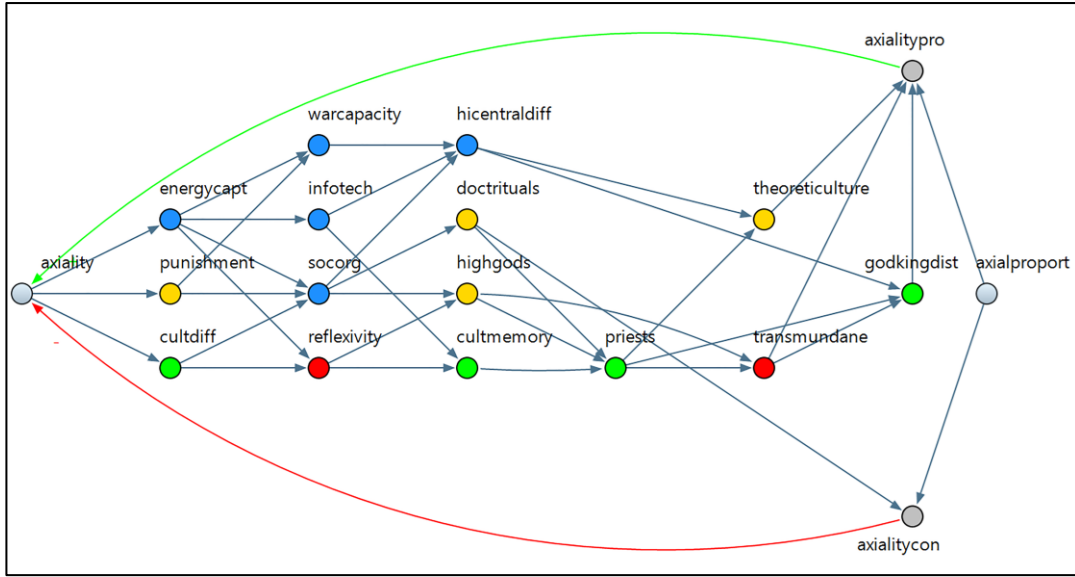


Figure 3. The theoretical integration expressed in terms of variables and causal connections among them.

With the initial causal architecture outlined, we introduce a few additional variables to the theoretical synthesis, including axiality, axiality pro, axiality con, and proportion of axiality people. Note that this structure constitutes a feedback loop that means everything affects everything else – one of the characteristics of complex adaptive systems and a feature of the theoretical debates surrounding the Axial Age transition. The definitions of each concept (variable) in the causal architecture are provided in Table 1 of Appendix B.

Control parameters are incorporated into the model to provide context to the model. These parameters express different circumstances in which an axial transmission may or may not occur. Collectively, they define a multidimensional space of possibilities that we will analyze to discover the model’s behavior. Figure 4 shows the control parameters that influence the causal architecture. The descriptions of the links between parameters and variables are provided in Table 3 of Appendix C.1.

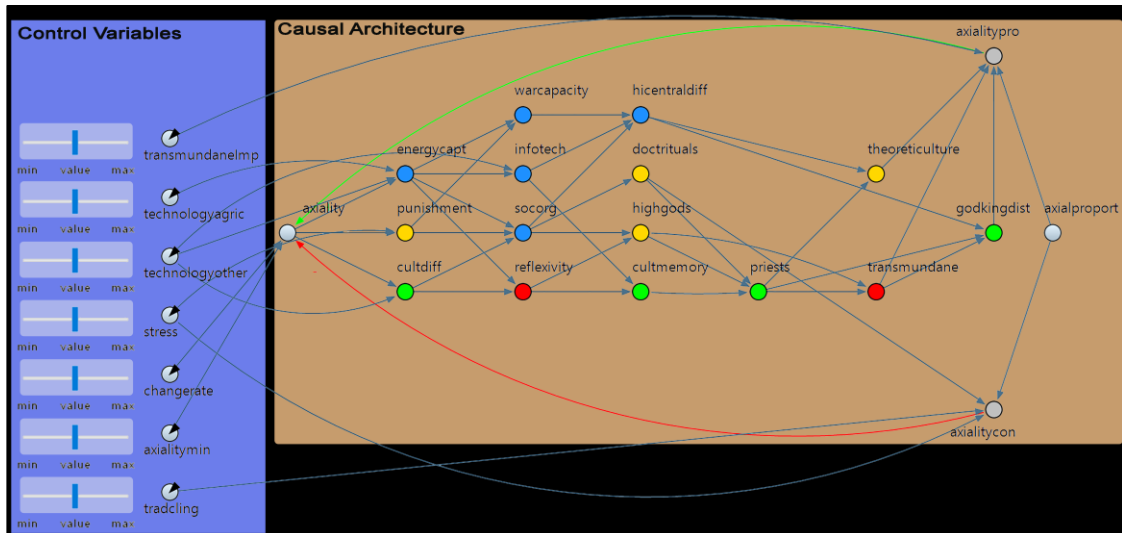


Figure 4. Control parameters impacting the causal architecture, and thereby model behavior.

The integration of the control parameters with the causal architecture must yield the six rates for birth, exposure, and conversion to drive the conversion model depicted in Figure 2. In response, the conversion model must furnish the proportion of Axial people (*axialproport*)

back into the causal architecture depicted in Figure 4. Figure 5 shows how these six rates are derived from the causal architecture. Note the introduction of three new control parameters. The interaction between the control parameters, the causal architecture, and the rate variables regulates MAxisM's behavior. Definitions and categorizations of these variables are provided in Table 2 of Appendix B.

When the causal architecture is combined with the conversion model, the causal architecture generates the six flow rates that regulate the conversion model, the conversion model generates the proportion of people with an axial mindset, and the proportion of axial people in turn helps to regulate the causal architecture. Figure 6 shows the full model.

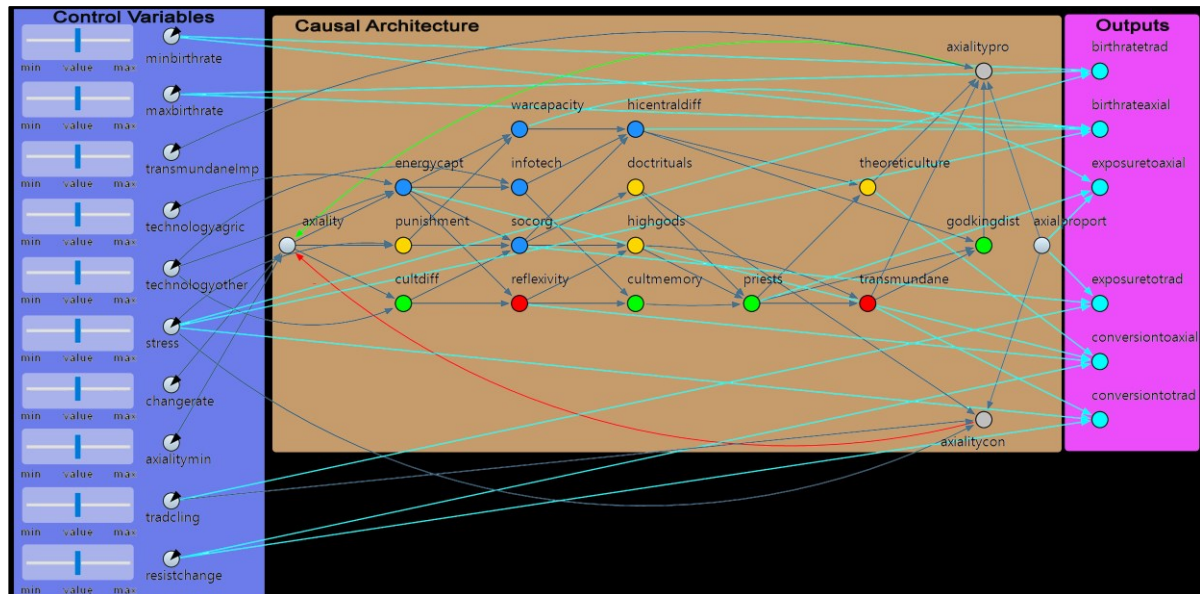


Figure 5. Deriving six flowrates from the causal architecture to feed into the conversion model.

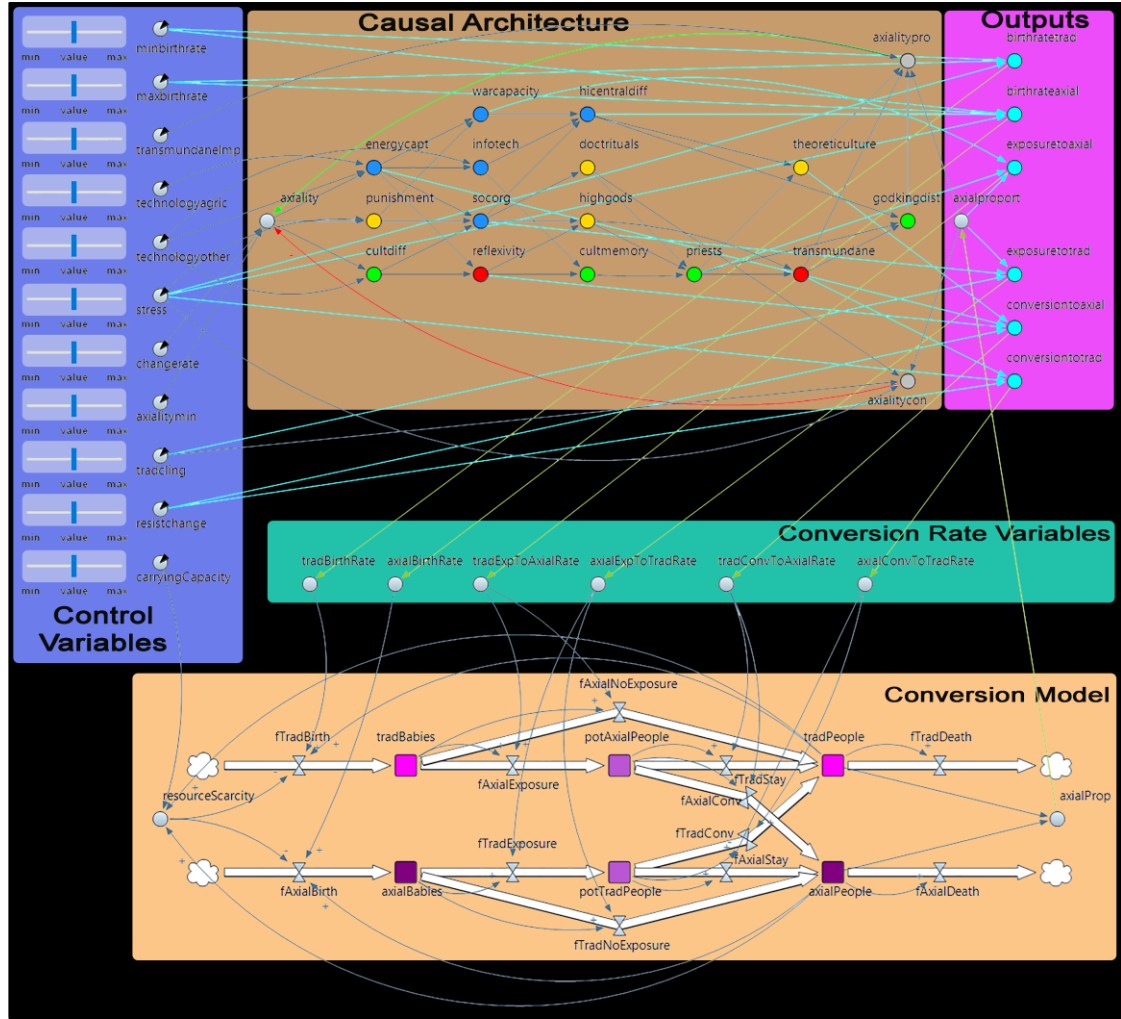


Figure 6. The complete MAXiM model containing both the causal architecture and the conversion model.

B. Definitions of Model Parameters

This section describes and categorizes the parameters within MAXiM. Table 1 provides a list of the parameters contained within MAXiM, including the name of the parameter, its corresponding abbreviated name within the model, the category containing the parameter, and a description. The colors of the text within Column 3 of Table 1 and Table 2 correspond to the colors of the variables contained within the causal architecture of Appendix A.

Table 1. Model parameters, their categorizations, and their descriptions.

Parameter	Name within Model	Category	Description
Energy Capture	energycapt	Environmental	the society's level (complexity, extent) of capacity to capture and utilize the free energy from their environment, such as food, fuel and raw materials
Social Organization	socorg	Environmental	the society's level of capacity (complexity, extent) to organize its captured energy
Information Technology	infotech	Environmental	the society's level of capacity (complexity, extent) to process and communicate prodigious amounts of information

Parameter	Name within Model	Category	Description
War Making Capacity	warcapacity	Environmental	the level (complexity, extent) of the capacity of a group of humans in this model to wage war on other groups
High-End State Centralization	hicentraldiff	Environmental	the extent to which a state is organized by leaders who centralize power in ways that utilize increasingly differentiated bureaucrats and militia
Cultural Differentiation	cultdiff	Network, Group Structure, Communication	the extent to which a society is characterized by distinct and relatively stable sodalities, guilds, ethnic groups or classes
Cultural Memory	cultmemory	Network, Group Structure, Communication	the extent to which a society comes to rely on external systems of notation, such as writing, for communicating memories across generations
Competitive Intellectual / Priestly Coalitions	priests	Network, Group Structure, Communication	the extent to which a society is characterized or influenced by competing coalitions of priestly-intellectual elites distinct from the political elites
God / King Distinction	godkingdist	Network, Group Structure, Communication	the extent to which people accept a distinction between the highest God and the leader (despot) of the state; that is, the ruler is ultimately accountable to God or a divine law
Axiological Reflexivity	reflexivity	Normative Beliefs	the extent to which the population's members critically reflect on their accepted norms and in-group values
Transmundane Soteriology	transmundane	Normative Beliefs	the extent to which people believe in a higher transcendental moral or metaphysical order that is beyond any given reality, and which impinges on this-worldly organization shaped by a "global" vision of a religious utopia
Broader Supernatural Punishment	punishment	Supernatural Beliefs	the extent to which people are open to believing in a wider range of supernatural punishers, beyond those who are interested only in their in-group (such as ancestor-ghosts)
Doctrinal Rituals	doctrinals	Supernatural Beliefs	the extent to which people are comfortable participating in rituals that are more doctrinal (low-arousal, high-frequency, etc.) than imagistic (high-arousal, low-frequency, etc.)
Moralizing High Gods	highgods	Supernatural Beliefs	the extent to which people share imaginative engagement with bigger gods, that is, supernatural agents whose knowledge and power extend over bigger groups
Theoretic Culture	theoreticulture	Supernatural Beliefs	the extent to which people rely less on mimetic or mythical modes of memory and social governance, and more on "theoretic" modes built on symbolic uses of external graphic memory devices and foster analysis and more principled reflection

Parameter	Name within Model	Category	Description
Axiality	axiality	Societal	the degree to which a society's institutional form embraces a highly centralized, differentiated, "theoretic" culture characterized by a distinction between the king and God (or ultimate reality) and a "global" vision of a religious utopia. A high axiality value indicates an axial civilization while a low value indicates a traditional (pre-axial) civilizational form
Axiality Pro	axialitypro	Societal	an incrementing mechanism by which axiality is increased in the population; it combines the impact of all variables capable of increasing Axiality
Axiality Con	axialitycon	Societal	a decrementing mechanism by which axiality is decreased in the population; it combines the impact of all variables capable of decreasing Axiality
Proportion of Axiality People	axialproport	Societal	the percentage of the population with axial worldviews. The variables axialitypro and axialitycon are amplified or suppressed by the proportion of people with traditional and axial worldviews

Table 2. Model parameters that are responsible for regulating model behaviors, their categorizations, and corresponding descriptions.

Parameter	Name within Model	Control Category	Description
Minimum Birth Rate	minbirthrate	Societal	the minimum birth rate for the population
Maximum Birth Rate	maxbirthrate	Societal	the maximum birth rate for the population
Importance of the Transmundane	transmundaneimp	Societal	the extent to which people value a higher transcendental moral or metaphysical order beyond any given reality, and which impinges on this-worldly organization shaped by a "global" vision of a religious utopia. This parameter expresses the fact that some civilizations weigh transmundane soteriology more highly than others
Agricultural technology	technologyagric	Environmental	the population's level (complexity, quantity, quality, entanglement) of agricultural advancement
Other technology	technologyother	Environmental	the population's level (complexity, quantity, quality, entanglement) of technological advancement that is non-agricultural
Stress	stress	Environmental	the level of anxiety-producing or survival-threatening change in the population's natural environment

Parameter	Name within Model	Control Category	Description
Change Rate	changerate	Societal	the rate at which axialPro increments and axialCon decrements axiality
Axiality Minimum	axialitymin	Societal	expresses the irreversibility of some degree of axialization
Cling to Tradition	tradcling	Societal	the tendency to prefer the familiar, traditional worldview rather than the new axial way of thinking
Resistance to Change	resistchange	Societal	the extent to which the members of the population are resistant to change

C. Links and Output Variables

C.1 Links from Parameters that Regulate Model Behaviors

This section describes the links between the parameters presented in Table 2 of Appendix B. The connections between these links are provided in Appendix C.2.

Table 3. Structure and description of parameter links within the model which pertain to the parameters which are intended to regulate model behaviors.

Parameter	Links To (from Column 1)	Description
<i>minbirthrate</i>	birthratelow	direct causal relationship
	birthratehigh	direct causal relationship
<i>maxbirthrate</i>	birthratelow	direct causal relationship
	birthratehigh	direct causal relationship
<i>technology</i>	energycapt	increased technological advancement raises the capacity of a population to capture energy
<i>changerate</i>	axiality	higher rates of change increase and decrease the speed of change of Axiality
<i>stress</i>	energycapt	moderate levels of stress (anxiety-producing or survival-threatening conditions in the natural environment) put pressure on individuals to develop new ways of capturing energy. Extremely high levels of stress overwhelm the system so that people default to old modes of capturing (enough) energy
	birthrateaxial	populations with axial lifestyles are better able to weather moderate stress (anxiety-producing or survival-threatening conditions in the natural environment) and so their birth rates are higher. Extreme levels of stress push birthrateaxial down toward birthratetrad and also close to minbirthrate
	conversionaxial	individuals with traditional (pre-axial) lifestyles are attracted to axial lifestyles when confronted with moderately high levels of stress (anxiety-producing or survival-threatening conditions in the natural environment).
	conversiontrad	extremely high levels of stress unravel the fabric of axial lifestyles and people default to traditional lifestyles
<i>resistchange</i>	conversionaxial	the more resistant people are to change, the less likely they will convert
	conversiontrad	the more resistant people are to change, the less likely they will convert

C.2 Links among Model Variables

This section describes the links between the parameters presented in Table 1 of Appendix B based on MAXiM's causal architecture laid out in Figure 4 of Appendix A. Refer to Appendix B for descriptions of each parameter.

Table 4. MAXiM model's links among parameters.

Variable	Link Type	Link	Algorithm and Description
<i>axiality</i>	In	<i>axiality</i>	$Axiality = \min(\max(axialitymin, axialityOldVal + changerate * (axialitypro - axialitycon)), 0.99)$ <p>axiality is the only variable that increments (via axialpro) or decrements (via axialcon) its own old value. Thus, all of the model's reinforcing loops involve the link from axialpro to <i>axiality</i> (+) and all valancing loops involve the link from axialcon to <i>axiality</i> (-)</p>
	In	<i>axialitymin</i>	
	In	<i>changerate</i>	
	In	<i>axialitypro</i>	
	In	<i>axialitycon</i>	
	Out	<i>energycapt</i>	as people become more willing to participate in axial lifestyles, they will be more willing and motivated to participate in more complex forms of energy capture
	Out	<i>punishment</i>	as people become more willing to participate in axial lifestyles, they will be confronted with and become more open to accepting a plurality of beliefs in punitive supernatural agents, imaginative engagement with whom facilitates their inclusion in more competitive (cooperative, committed) groups
	Out	<i>cultdiff</i>	as people become more willing to participate in axial lifestyles, they will be confronted by and become more open to accepting the differentiated structures of the larger society
<i>punishment</i>	In	<i>axiality</i>	$punishment = \frac{(axiality + stress)}{2}$ <p>the tendency of a population to broaden its belief in and shared imaginative engagement with a wide range of punitive supernatural agents increases with the adoption of axial lifestyles</p>
	In	<i>stress</i>	
	Out	<i>warcapacity</i>	belief in a broad range of punitive gods increases the war capacity of a society because it implicitly motivates cooperation and commitment within larger groups
	Out	<i>socorg</i>	belief in a broad range of punitive gods increases the capacity of a society to organize the energy it captures in more complex ways because it implicitly motivates cooperation and commitment within larger groups
<i>cultdiff</i>	In	<i>axiality</i>	$cultdiff = \frac{(axiality + technologyother)}{2}$ <p>the extent to which a society is characterized by distinct and relatively stable sodalities, guilds, ethnic groups or classes increases with the adoption of axial lifestyles</p>
	In	<i>technologyother</i>	
	Out	<i>socorg</i>	the extent to which a society is characterized by distinct and relatively stable sodalities, guilds, ethnic groups or classes, increases its capacity to organize the energy it captures in more complex ways

Variable	Link Type	Link	Algorithm and Description
	Out	<i>reflexivity</i>	the extent to which a society is characterized by distinct and relatively stable sodalities, guilds, ethnic groups or classes, increases the extent to which members of the population critically reflect on the accepted norms and values of their in-group
<i>energycapt</i>	In	<i>technologyagric</i>	$\text{energycapt} = ((\text{technologyagric} + \text{technologyother} + \text{axiality}) / 3)$ the capacity of a society to capture energy increases with both the increase in axiality and the increase in technology, to about the same degree.
	In	<i>technologyother</i>	
	In	<i>axiality</i>	
	Out	<i>warcapacity</i>	increased level of energy capture improves social networking and cooperation skills and imports technology and know-how, which raise the capacity to wage war
	Out	<i>infotech</i>	increased level of energy capture improves the ability of the population to develop more complex forms of managing information
	Out	<i>socorg</i>	increased level of energy capture improves the capacity of the population to develop more complex forms of social organization
	Out	<i>reflexivity</i>	increased level of energy capture increases the extent to which members of the population have time to critically reflect on the accepted norms and values of their in-group
	Out	<i>conversiontoaxial</i>	increased level of energy capture makes axial lifestyles seem more tempting, increasing conversion
<i>warcapacity</i>	In	<i>energycapt</i>	$\text{warcapacity} = \frac{(\text{energycapt} + \text{punishment})}{2}$ the capacity and willingness to wage war increases with improvements in energy capture
	In	<i>punishment</i>	
	Out	<i>exposuretoaxial</i>	increased capacity to wage war increases the chances that traditional people would encounter the war-making axial people
	Out	<i>hicentraldiff</i>	increased capacity to wage war stabilizes and intensifies highly-centralized states
<i>infotech</i>	In	<i>energycapt</i>	$\text{warcapacity} = \frac{(\text{energycapt} + \text{technologyother})}{2}$ the level of information technology increases with improvements in energy capture
	In	<i>technologyother</i>	
	Out	<i>cultmemory</i>	increased capacity to manage/disseminate information improves a state's ability to maintain and strengthen its use of external systems of notation, such as writing, for communicating memories across generations
	Out	<i>hicentraldiff</i>	increased capacity to manage/disseminate information improves a state's ability to maintain and strengthen its highly-centralized organizational structure
<i>socorg</i>	In	<i>energycapt</i>	$\text{socorg} = \frac{(\text{energycapt} + \text{punishment} + \text{cultdiff})}{3}$ social organization increases so long as energy capture and (supernatural) punishment increase
	In	<i>punishment</i>	
	In	<i>cultdiff</i>	
	Out	<i>hicentraldiff</i>	a society's capacity to organize the energy it captures improves its' ability to maintain and strengthen a highly-centralized organizational structure

Variable	Link Type	Link	Algorithm and Description
	Out	<i>exposuretotrad</i>	members of societies with a higher capacity to organize captured energy in complex ways are more likely to come into contact with members of neighboring and distant traditional societies
	Out	<i>doctrinals</i>	at high levels of complexity, the capacity of a society to organize the energy it captures drives the population toward doctrinal rituals
	Out	<i>highgods</i>	at high levels of complexity, the capacity of a society to organize the energy it captures drives the population toward belief in bigger (smarter, stronger) gods
<i>reflexivity</i>	In	<i>energycapt</i>	$reflexivity = \frac{(energycapt + cultdiff)}{2}$
	In	<i>cultdiff</i>	
	Out	<i>highgods</i>	the tendency of a population to reflect critically on the accepted norms and values of their in-group can enhance belief in bigger (smarter, stronger) gods
	Out	<i>cultmemory</i>	the tendency of a population to reflect critically on the accepted norms and values of their in-group can enhance reliance on external systems of notation, such as writing, for communicating memories across generations
	Out	<i>conversiontoaxial</i>	the tendency of a population to reflect critically on the accepted norms and values of their in-group can enhance the attractiveness of the lifestyle of its members
<i>hicentraldiff</i>	In	<i>warcapacity</i>	$hicentraldiff = \frac{(warcapacity + infotech + socorg)}{3}$
	In	<i>infotech</i>	
	In	<i>socorg</i>	
	Out	<i>birthrateaxial</i>	the ability of a state to construct, maintain and strengthen a highly-centralized and differentiated organizational structure increases the birth rate of its population
	Out	<i>theoreticulture</i>	the ability of a state to construct, maintain and strengthen a highly-centralized and differentiated organizational structure increases the tendency in the population to give up mythical and rely more heavily on theoretical modes of memory and social governance
	Out	<i>godkingdist</i>	the ability of a state to construct, maintain and strengthen a highly-centralized and differentiated organizational structure increases the extent to which people accept a distinction between the highest God and the leader (despot) of the state; that is, the ruler is ultimately accountable to God or a divine law
<i>doctrinals</i>	In	<i>socorg</i>	the tendency of a population to prefer and engage in doctrinal rituals increases with the tendency of a society to organize the energy it captures in more complex and differentiated ways
	Out	<i>axialitycon</i>	if doctrinal rituals become too tedious, that is, if they are not balanced with some imagistic-like religious experiences, some members of the population will convert from axial to traditional

Variable	Link Type	Link	Algorithm and Description
	Out	<i>priests</i>	as more of the population becomes accustomed to doctrinal rituals, this ratchets up the general willingness to support competing groups of priestly/intellectual elites
<i>highgods</i>	In	<i>socorg</i>	$highgods = \frac{(socorg + reflexivity)}{2}$ <p>The tendency of a population to believe in bigger (smarter, stronger) punitive gods increases with the tendency of a society to organize the energy it captures in more complex and differentiated ways</p>
	In	<i>reflexivity</i>	
	Out	<i>priests</i>	as the high gods (or God) imaginatively engaged within a society become more counter-intuitive, its members will rely more heavily on competing groups of priestly/intellectual elites
	Out	<i>conversiontotrad</i>	as the high gods (or God) imaginatively engaged within a society become too counter-intuitive, that is, as monotheistic belief pressures people to deny the existence or relevance of other supernatural agents, some axial members of the population will convert to traditional lifestyles
<i>cultmemory</i>	In	<i>infotech</i>	$cultmemory = \frac{(infotech + reflexivity)}{2}$ <p>The extent to which a society comes to rely on external systems of notation, such as writing, for communicating memories across generations increases the level of its information technology</p>
	In	<i>reflexivity</i>	
	Out	<i>priests</i>	higher reliance on external systems of notation and memory storage increases the relevance and need for competing coalitions of intellectual/priestly elites
<i>priests</i>	In	<i>cultmemory</i>	$priests = \frac{(cultmemory + highgods + doctrituals)}{3}$ <p>The extent to which a society is characterized or influenced by competing coalitions of priestly-intellectual elites distinct from political elites increases with axiological reflexivity, doctrinal rituals, high gods and cultural memory, about to the same degree</p>
	In	<i>highgods</i>	
	In	<i>doctrituals</i>	
	Out	<i>theoreticulture</i>	the growth of competing coalitions of priestly-intellectual elites increases the extent to which the population within a culture relies less on mimetic or mythical modes of memory and social governance, and more on “theoretic” modes
	Out	<i>godkingdist</i>	the growth of competing coalitions of priestly-intellectual elites increases the extent to which people accept a distinction between the highest God and the leader (despot) of the state; that is, the ruler is ultimately accountable to God or a divine law
	Out	<i>transmundane</i>	the growth of competing coalitions of priestly-intellectual elites increases the extent to which people believe in a higher transcendental moral or metaphysical order that is beyond any given this- or other-worldly reality
	Out	<i>exposuretoaxial</i>	the growth of competing coalitions of priestly-intellectual elites increases the extent to which traditional people will be exposed to axial lifestyles

Variable	Link Type	Link	Algorithm and Description
<i>transmundane</i>	In	<i>priests</i>	$transmundane = \frac{(highgods + priests)}{2}$ <p>the more a society relies on and supports competing coalitions of intellectual/priestly elites, the more its population will believe in a higher transcendental moral or metaphysical order that is beyond any given this- or other-worldly reality</p>
	In	<i>highgods</i>	
	Out	<i>axialitypro</i>	the extent to which members of a population believe in a higher transcendental moral or metaphysical order that is beyond any given this- or other-worldly reality increases the rate at which axiality itself is ratcheted up in that society
	Out	<i>godkingdist</i>	the extent to which members of a population believe in a higher transcendental moral or metaphysical order that is beyond any given this- or other-worldly reality increases the extent to which people accept a distinction between the highest God and the leader (despot) of the state; that is, the ruler is ultimately accountable to God or a divine law
	Out	<i>conversiontotrad</i>	the extent to which members of a population believe in a higher transcendental moral or metaphysical order that is beyond any given this- or other-worldly reality increases the likelihood that some axial people will convert to traditional lifestyles
<i>theoreticulture</i>	In	<i>priests</i>	$theoreticulture = \frac{(hicaltraldiff + priests)}{2}$ <p>the growth of competing coalitions of priestly-intellectual elites increases the extent to which the population within a culture relies less on mimetic or mythical modes of memory and social governance, and more on “theoretic” modes</p>
	In	<i>hicaltraldiff</i>	
	Out	<i>axialitypro</i>	the success of societies that rely more on “theoretic” modes of analysis/organization that are built on the symbolic use of external graphic memory devices, and foster analysis and more principled reflection, increases the rate at which axiality itself is ratched up in that society
	Out	<i>conversiontoaxial</i>	the success of societies that rely more on “theoretic” modes of analysis/organization that are built on the symbolic use of external graphic memory devices, and foster analysis and more principled reflection, makes them more attractive to traditional people
<i>axialcon</i>	In	<i>tradcling</i>	$axialcon = \left(\frac{(doctrinals - 0.6)^2}{0.36} + (1 - 4 * stress * (1 - stress)) + tradcling \right) * \frac{1}{3} * (1 - axialproport)$ <p>two factors contribute to the decrease of axiality: extremely high levels of tedious doctrinal rituals, and either extremely low or extremely high stress. Their combined effect is amplified by the tendency to cling to pre-axial lifestyles. This effect rises quickly as the proportion of traditional individuals grows and then maxes out as traditional people become dominant</p>
	In	<i>stress</i>	
	In	<i>doctrinals</i>	
	In	<i>axialproport</i>	
	Out	<i>axiality (-)</i>	the decrementing mechanisms identified above lower overall axiality in the models’s population

Variable	Link Type	Link	Algorithm and Description
<i>axialpro</i>	In	<i>theoreticulture</i>	$axialpro = ((theoreticulture + godkingdist + transmundaneImp * transmundane) * axialproport) / (2 + transmundane)$ <p>three factors contribute to the increase of axiality: theoretic culture, the god-king distinction, and transmundane soteriology. This effect rises quickly as the proportion of axial individuals grows and then maxes out as axial people become dominant</p>
	In	<i>godkingdist</i>	
	In	<i>transmundaneImp</i>	
	In	<i>transmundane</i>	
	In	<i>axialproport</i>	
	Out	<i>axiality (+)</i>	the incrementing mechanisms identified above increase overall axiality in the models's population
<i>axialproport</i>	In	<i>axialProp</i>	$axialproport = axialProp$ $= \frac{axialPeople}{(axialPeople + tradPeople)}$ <p>this is taken directly from the conversion model; it is the proportion of the population living in axial social arrangements</p>
	Out	<i>exposuretoaxial</i>	the higher the percentage of the population is high-investment the more likely members of the two groups will be exposed to one another
	Out	<i>exposuretotrad</i>	the higher the percentage of the population is high-investment the more likely members of the two groups will be exposed to one another
	Out	<i>axialitycon</i>	the higher the percentage of the population is involved in traditional lifestyles the quicker axiality drops
	Out	<i>axialitypro</i>	the higher the percentage of the population is involved in axial lifestyles the quicker axiality itself is ratcheted up

D. Output Variables

This section describes the output variables presented in Appendix A. Table 5 provides the output variables contained within MxiM, including the name of the variable, its abbreviated name within the model, and a description.

Table 5. MAXiM output variables, algorithms for computing their values, and descriptions.

Variable	Name within Model	Algorithm	Description
Traditional Birth Rate	birthratetrad	$\text{birthratetrad} = \text{minbirthrate} + (\text{maxbirthrate} - \text{minbirthrate}) * (1 - \text{stress})$	in traditional societies, increasing stress drives birth rates from the maximum toward the minimum.
Axial Birth Rate	birthrateaxial	$\text{birthrateaxial} = \text{minbirthrate} + (\text{maxbirthrate} - \text{minbirthrate}) * \frac{(\text{hicaltraldiff} + (1 - \text{stress}))}{2}$	in axial societies, decreasing stress along with increases in highly-centralized, differentiated states drive birth rates from the minimum toward the maximum.
Exposure to Traditional Alternatives	exposuretotrad	$\text{exposuretotrad} = \text{socorg} * \text{tradcling} * (1 - \text{axialproport})$	Exposure to traditional worldviews occurs through the prevalence of traditional people, engagement with them through societies with complex organization, and the tendency to cling to traditional values.
Exposure to Axial Alternatives	exposuretoaxial	$\text{exposuretoaxial} = \left(\frac{\text{warcapacity} + \text{priests}}{2} \right) * \text{axialproport}$	Exposure to Axial worldviews occurs through the prevalence of axial people and either engagement with them through war or through priestly religion.
Conversion to Traditional Thinking	conversiontotrad	$\text{conversiontotrad} = \left(\frac{2 * \max(0, -1 + 2 * \text{stress})}{3} + \frac{\max(0, -1 + 2 * \text{transmundane})}{3} \right) * \text{resistchange}$	in axial societies, people are more likely to convert to traditional lifestyles under stressful conditions.
Conversion to Axial Thinking	conversiontoaxial	$\text{conversiontoaxial} = \left(\frac{\text{theoreticulture}}{3} + \frac{\text{reflexivity}}{3} + \frac{\text{energycapt}}{3} \right) * \text{resistchange}$	in traditional societies, people are more likely to convert to axial lifestyles as they are confronted with cultures that rely on theoretical reflection, are highly reflexive, and have higher levels of energy capture, moderated by their resistance to change.

E. Parameter Space

We ran a parameter variation experiment to explore MAXiM's behavior. Because the model has no stochastic elements, we ran each combination of parameter settings only once (see Table 6 for the details). A total of 2,304,000 runs of the simulation were made to create the parameter sweep's dataset. We used this dataset to determine the conditions under which the axiality transition occurs, and to run sensitivity analyses on parameters and key variables.

Table 6. Details of the parameter variation experiment.

Parameter	Varied?	Possible Range	Sweep Range	Step	# Settings
minbirthrate	Yes	>0	0.8-0.9	0.1	2
maxbirthrate	Yes	>0	2.0-5.0	1.0	4
transmundaneimp	Yes	[0,1]	0.0-1.0	0.25	5
technologyagric	Yes	[0,1]	0.0-1.0	0.2	6
technologyother	Yes	[0,1]	0.0-1.0	0.2	6
stress	Yes	[0,1]	0.0-1.0	0.25	5
changerate	Yes	[0,1]	0.1-0.85	0.25	4
axialitymin	Yes	[0,0.1]	0.01-0.085	0.025	4
tradcling	Yes	[0,1]	0.0-1.0	0.25	5
resistchange	Yes	[0,1]	0.1-0.85	0.25	4
carryingCapacity	No	>0	Fixed at 10,000		1

We judged whether the axial transition occurs based on the value of the *axiality* variable: when *axiality* ≤ 0.1, the axial transition does not occur; when *axiality* ≥ 0.9, the axial transition does occur; and when 0.1 < *axiality* < 0.9, we have intermediate cases. Using this criterion, the axial transition occurred 1.012 million times out of the 2.304 million runs, or about 43.9% of the total runs. There are very few cases in the middle of the [0.01,0.99] range for the axiality variable: the axiality variable was between 0.1 and 0.9 in only 53 of the 2.304 million runs (0.000023%), and some of those cases were probably in transition when the cutoff number of iterations was reached. This is a clear indication that the *axiality* variable is unstable except at extreme values, as mentioned above.

Table 7 summarizes the characteristics of the two circumstances: when the axial transition does and does not occur. By comparing means, we can see which variables and parameters are most important for the axial transition to occur. Among parameters, *stress* ($\eta^2=0.569$), *technologyother* ($\eta^2=0.092$), and *technologyagric* ($\eta^2=0.018$) are the most important for distinguishing between whether or not the axial transition occurs. These three variables are related to energy capture: the two technology variables collectively impact the *energycapt* variable, which expresses the capture and organization of energy, and the stress variable reflects the difficulty of life circumstances for which greater energy capture and organization can make a difference. This is just as the proponents of the material-social pathway would predict.

Table 7. Conditions under which the axial transition occurs or does not occur, and intermediate cases.

	Axial transition does not occur (N=1,291,895)				Intermediate cases (N=53)				Axial transition does occur (N=1,012,052)			
	Mean	StdDev	Minimum	Maximum	Mean	StdDev	Minimum	Maximum	Mean	StdDev	Minimum	Maximum
Parameters												
changerate	0.475	0.280	0.100	0.850	0.322	0.262	0.100	0.850	0.475	0.280	0.100	0.850
axialitymin	0.047	0.028	0.010	0.085	0.046	0.027	0.010	0.085	0.048	0.028	0.010	0.085
technologyother	0.408	0.327	0.000	1.000	0.543	0.302	0.000	1.000	0.617	0.323	0.000	1.000
resistchange	0.454	0.278	0.100	0.850	0.525	0.258	0.100	0.850	0.501	0.279	0.100	0.850
maxbirthrate	3.516	1.116	2.000	5.000	2.943	0.929	2.000	5.000	3.480	1.121	2.000	5.000
minbirthrate	0.850	0.050	0.800	0.900	0.842	0.050	0.800	0.900	0.850	0.050	0.800	0.900
stress	0.264	0.241	0.000	1.000	0.580	0.161	0.250	0.750	0.801	0.219	0.000	1.000
tradcling	0.509	0.353	0.000	1.000	0.708	0.301	0.000	1.000	0.489	0.354	0.000	1.000
transmundaneImp	0.500	0.354	0.000	1.000	0.467	0.290	0.000	1.000	0.500	0.354	0.000	1.000
technologyagric	0.459	0.338	0.000	1.000	0.426	0.360	0.000	1.000	0.552	0.339	0.000	1.000
Variables												
axiality	0.047	0.028	0.010	0.097	0.352	0.215	0.100	0.819	0.990	0.000	0.915	0.990
axialproport	0.015	0.066	0.000	0.875	0.504	0.132	0.183	0.684	0.999	0.020	0.171	1.000
energycapt	0.305	0.153	0.003	0.695	0.338	0.114	0.145	0.603	0.720	0.154	0.162	0.997
punishment	0.155	0.121	0.005	0.543	0.313	0.083	0.126	0.417	0.896	0.110	0.255	0.995
cultdiff	0.227	0.164	0.005	0.543	0.294	0.151	0.005	0.529	0.804	0.161	0.042	0.995
warcapacity	0.230	0.080	0.004	0.431	0.325	0.034	0.269	0.389	0.808	0.072	0.289	0.996
infotech	0.356	0.222	0.002	0.848	0.441	0.182	0.143	0.772	0.669	0.221	0.165	0.998
socorg	0.229	0.091	0.004	0.468	0.315	0.048	0.239	0.409	0.806	0.086	0.240	0.996
reflexivity	0.266	0.145	0.004	0.619	0.316	0.111	0.157	0.531	0.762	0.144	0.152	0.996
hicentraldiff	0.272	0.121	0.003	0.582	0.360	0.077	0.242	0.506	0.761	0.117	0.237	0.997
doctrutials	0.229	0.091	0.004	0.468	0.315	0.048	0.239	0.409	0.806	0.086	0.240	0.996
highgods	0.248	0.115	0.004	0.544	0.316	0.079	0.198	0.466	0.784	0.113	0.196	0.996
cultmemory	0.311	0.183	0.003	0.733	0.379	0.146	0.150	0.652	0.715	0.182	0.166	0.997
priests	0.263	0.128	0.004	0.582	0.336	0.091	0.196	0.506	0.769	0.125	0.201	0.996
theoreticulture	0.267	0.124	0.004	0.582	0.348	0.084	0.219	0.506	0.765	0.121	0.219	0.996
transmundane	0.255	0.121	0.004	0.563	0.326	0.085	0.197	0.486	0.776	0.119	0.199	0.996
godkingdist	0.263	0.123	0.004	0.576	0.341	0.084	0.211	0.499	0.769	0.120	0.212	0.996

All variables are significantly different in the two cases of the axiality transition occurring or not occurring ($\eta^2 \geq 0.328$ in all cases and average $\eta^2 = 0.797$). This is because they are in a causal nexus with the *axiality* variable itself. The most notable contrast is for *axialproport* (the proportion of people with axial worldviews); this variable has the largest mean difference and the smallest variance. This supports the ideological-political pathway and the cognitive coalitional pathway, for which worldviews matter.

We can drill down into the dataset from the parameter sweep. Figure 14 depicts a faceted graph of a representative sample of runs from the parameter variation experiment. In each facet, the vertical axis is *axialproport* (the proportion of people with axial worldviews) and the horizontal axis is *axiality* (the degree to which the civilization is axial in form). Color corresponds to *energycapt*, with high-frequency colors (blue, violet) indicating high energy capture and low-frequency colors (red, orange) indicating low frequency colors. The size of the graph marker indicates the size of the *punishment* variable and the shape of the marker the level of *cultdiff* variable. The four facets simplify the diagram by dividing the cases based on the number of people with axial worldviews, from low to medium to high to very high.

Figure 7 makes clear that the axial transition in civilizational form occurs only when the *energycapt* variable is high. The axial transition rarely occurs unless enough people hold axial worldviews, and it almost always occurs when a lot of people hold axial worldviews. Also, when the axial transition occurs, *cultdiff* and *punishment* variables tend to be high (but we know from Table 2 that all variables are high under these circumstances). We see here, once again, the near-absence of intermediate values for the *axiality* variable, demonstrating the two-equilibrium-regime nature of MAXiM.

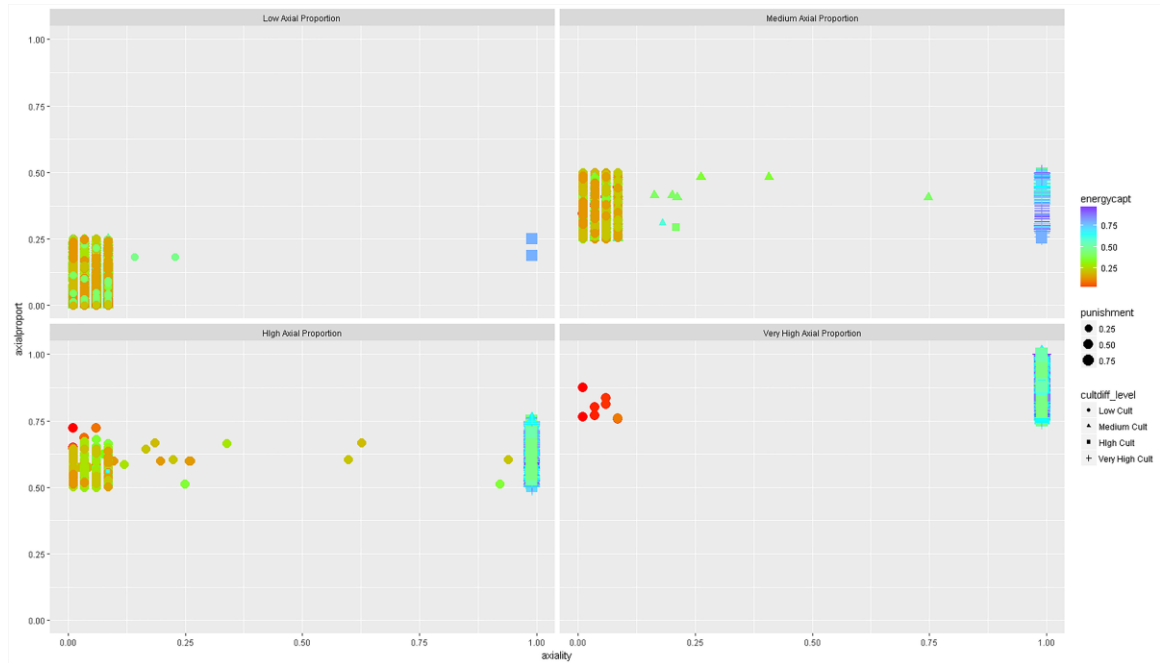


Figure 7. Results of a parameter variation experiment displaying five variables: axiality, axialproport, energycapt, punishment, and cultdiff.

This means that energy capture is a necessary condition for the axial transition, affirming the material-social theory; that the prevalence of axial worldviews and cultural differentiation are near-necessary conditions for the axial transition, affirming the ideological-political pathway; and that belief in supernatural punishment is a near-necessary condition for the axial transition, affirming the cognitive-coalitional path.

Figure 8 depicts further results from the same parameter variation experiment. The faceting and axes are the same as in Figure 7, as is energy capture. This time, the size of the graph marker indicates the magnitude of the *highgods* variable and the shape of the marker the level of *reflexivity* variable. This finding shows that belief in moralizing high gods and embracing axiological reflexivity are near-necessary conditions to cause the axial transition, further supporting the ideological-political and the cognitive-coalitional pathways.

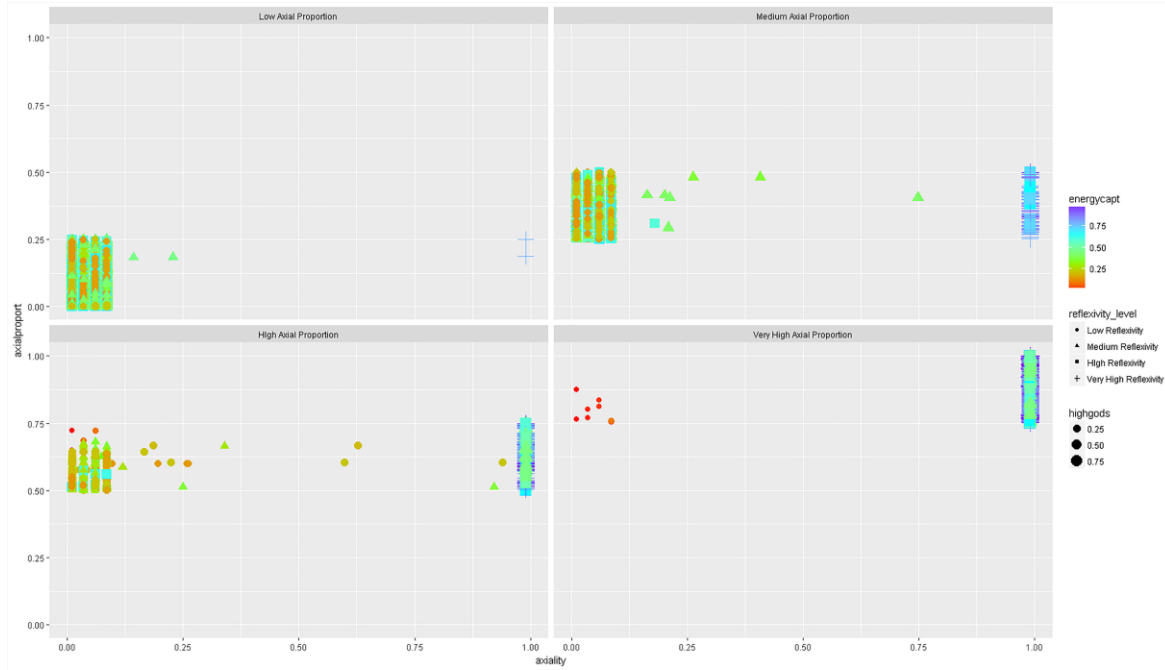


Figure 8. Results of a parameter variation experiment displaying five variables: axiality, axialproport, energycapt, reflexivity, and highgods.

This exploration of MAXiM’s parameter space offers additional support for the plausibility of our theoretical integration of the three main hypothesized pathways (ideological-political, material-social, and cognitive-coalitional) to an axial civilizational form. It also reinforces concerns about claims that any one theory is likely to provide a comprehensive explanation of the axial transition. At this stage of analysis, the model can be “face-validated” by comparing the results of the simulation experiments to what we know historically about the emergence of the axial age. A more detailed validation could involve comparing those findings to more specific archaeological, historical, demographic or other datasets.

F. Limitations

The main limitation of MAXiM is one it shares with other computer simulations: it cannot “prove” the correctness of a historical (or any other) theory. It only provides added warrant for accepting the plausibility of our synthesized theory (Barlas, 1996; Forrester & Senge, 1979; Naylor & Finger, 1967). A useful extension of MAXiM would be to link the micro- and macro-dynamics of changes in human systems, which is a topic of great interest in social theory. Peter Turchin, for example, uses the analogy of statistical physics, and argues for emphasizing the causal force of collectives. Yet he admits that he does “not know how to include individuals in the theoretical framework of cliodynamics, so we must, by necessity ignore their role. If somebody can figure out how to modify the theory to include individuals in it... I would be the first to applaud such a development” (Turchin 2007, 317). Such a development could be initiated, we believe, by the construction of an agent-based model to complement our system-dynamics model of the axial age.

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