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1 Give a brief description, not exceeding one page, of your number, including the characteristics that make it unique.

Function : Natural logarithm of 2 i.e. $ln_e 2$

Definitions :
Irrational Numbers - are the numbers that cannot be represented as ratio or a fraction.

Natural Logarithm - The natural logarithm of a number x is nothing but log to the base e of x. Here e has a approximate value of 2.718.
Natural logarithm is computing the time taken to reach the desired growth.
$log_e x$ can be written as ln x
ln is called the natural log.

Natural Logarithm of 2 - The project is based on the natural logrithm of 2 ie. $ln_e 2$.
The value of $ln_e 2 \approx 0.69314718056$ and it is an irrational number i.e cannot be expressed in fractional form.

The proof of $ln_e 2$ being irrational goes something like :
Let suppose, $ln_e 2$ is rational i.e. there exist a x,y integers > 0 and they can represent the natural log of 2.
Therefore it can be said :
$ln_e 2 = x/y$
Applying exponential to both LHS and RHS , we get:
$e^{ln_e 2} = e^{x/y}$
$2 = e^{x/y}$
$2^y = e^x$

Since we know e is a transcendental number and from the theorm mentioned the famous book - "Proofs from the book" [1],Page 45, e^r , where r is rational number not equal 0 , is irrational we can say that $ln_e 2$ is also an irrational number i.e. cannot be denoted as ratio of two integers with value > 0. The understanding of the proof was gathered from the website [2] - concept explained by Richard Morris, Maths tutor, doctorate in mathematics/computer science.

Application of natural logarithm of 2

The uniqueness of this number has been noticed in below concepts:

1. Half-life : Natural Logarithm of 2 plays a significant role in computing half life of a substance i.e computing the time taken by a substance to reduce to half of it initial value.This is concept is used in nuclear physics and biology.

2. Finance - The Rule 72 : Natural Logarithm of 2 is used in finance sector as a way to quickly compute annually computed interest and continously compounded interest. i.e. when we have to find the time taken (in years) to double the principle at a given interest rate, we have to divide 72 by interest rate(given). And this number 72 is calculated using natural logarithm of 2.

Reference

1. Aigner, Martin, and Günter M. Ziegler. Proofs from THE BOOK. Fourth ed.
2. "How Do I Prove ln2 Is Irrational?" Quora, www.quora.com/How-do-I-prove-ln2-is-irrational.