Horn Minimization

An overview of some existing algorithms

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Let's discuss

1.1 - Toy example

List of film genres :

- *sci-fi, fantastic, fantasy, cyber-punk, post-apo, documentary, drama, romance, . . .*

We can *derive* genres from others, e.g :

- *cyber-punk* leads probably to *sci-fi*,
- a *documentary* may not be *fantastic*,
- . . .

Implications at first sight

1.1 - Toy example

Rewritting of previous examples :

- *cyber-punk* $\longrightarrow$ *sci-fi*,
- *documentary, war* $\longrightarrow$ *historical*,

Closure operator and systems

2.1 - Elements of set theory

Set Σ of attributes. A map $\varphi : 2^{\Sigma} \longrightarrow 2^{\Sigma}$ is a *closure operator* if, $\forall X, Y, Z \subseteq \Sigma$:

- $X \subseteq \varphi(X)$ *(increasing)*
- $X \subseteq Y \longrightarrow \varphi(X) \subseteq \varphi(Y)$ *(isotone)*
- $\varphi(\varphi(X)) = \varphi(X)$ *(idempotent)*

Some details :

- X is *closed* if $X = \varphi(X)$,
- Σ^{φ} set of closed sets : *closure system*.
- Σ^{φ} is closed under *intersection*, contains Σ.

Closure Example

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- Directed graph $G = (V, E)$.
- *Closure* $\varphi(X)$ of $X \subseteq V$: every vertices reachable from X.
- $\varphi(\{A, B\}) = \{A, B, C, D\}$.
- $\varphi(\{F\}) = \{F\}$, $\{F\}$ is *closed*.

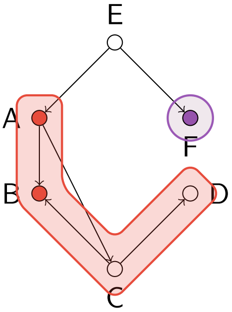

FIGURE – Closure of a vertex in a directed graph

Implications

2.1 - Elements of set theory

$A, B \subseteq \Sigma$. An *implication* is :

- $A \longrightarrow B$, A *premise*, B *conclusion*.
- relation : *"If we have A, we have B"*. (different from *causality*)
- $M \subseteq \Sigma$ *model* of $A \longrightarrow B$ if

$$B \subseteq M \vee A \nsubseteq M$$

denoted $M \models A \longrightarrow B$, $A \longrightarrow B$ *follows* from M.

Set of implications $\mathcal{L}$: *implication system*.

- $M \models \mathcal{L}$ if each element of $\mathcal{L}$ follows from M,
- $\mathcal{L} \models A \longrightarrow B$: all models of $\mathcal{L}$ are models of $A \longrightarrow B$.

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Given $\mathcal{L}$ an implication system :

- closure operator $\mathcal{L}(X)$: *smallest model* (inclusion wise) of $\mathcal{L}$ containing X, $X \subseteq \Sigma$.
- Models of $\mathcal{L}$ form a *closure system* :

$$\Sigma^{\mathcal{L}} = \{M \subseteq \Sigma \mid M = \mathcal{L}(M)\}$$

Important property :

- $\mathcal{L} \models A \longrightarrow B$ iff $B \subseteq \mathcal{L}(A)$

Small Example

2.1 - Elements of set theory

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Let $\mathcal{L}$ be an implication system :

- $\Sigma = \{a,\ b,\ c,\ d,\ e\}$,
- $\mathcal{L} = \{ab \longrightarrow c,\ bd \longrightarrow a, ce \longrightarrow abd\}$

We have :

- $\mathcal{L}(b) = b$, b is *closed*, hence a *model* of $\mathcal{L}$,
- $\mathcal{L}(bd) = abcd$, bd is *not* a model *($abcd$ is)*.
- $\emptyset$ is also a model.

Applications
2.1 - Elements of set theory

- Relational Databases,
- Formal Concept analysis,
- Conceptual exploration,
- Linguistics ?

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3.1 - early 80s

title
3.1 - early 80s

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4.1 - Pouf

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4.1 - Pouf