

Horn Minimization

An overview of some existing algorithms

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Let's discuss

1.1 - Toy example

List of film genres :

- ▶ *sci-fi, fantastic, fantasy, cyber-punk, post-apo, documentary, drama, romance, ...*

We can *derive* genres from others, e.g :

- ▶ *cyber-punk* leads probably to *sci-fi*,
- ▶ a *documentary* may not be *fantastic*,
- ▶ ...

Implications at first sight

1.1 - Toy example



Rewriting of previous examples :

- ▶ *cyber-punk* \longrightarrow *sci-fi*,
- ▶ *documentary, war* \longrightarrow *historical*,

Closure operator and systems

2.1 - Elements of set theory

Set Σ of attributes. A map $\varphi : 2^\Sigma \longrightarrow 2^\Sigma$ is a *closure operator* if,
 $\forall X, Y, Z \subseteq \Sigma$:

- ▶ $X \subseteq \varphi(X)$ (*increasing*)
- ▶ $X \subseteq Y \longrightarrow \varphi(X) \subseteq \varphi(Y)$ (*isotone*)
- ▶ $\varphi(\varphi(X)) = \varphi(X)$ (*idempotent*)

Some details :

- ▶ X is *closed* if $X = \varphi(X)$,
- ▶ Σ^φ set of closed sets : *closure system*.
- ▶ Σ^φ is closed under *intersection*, contains Σ .

Closure Example

2.1 - Elements of set theory

- ▶ Directed graph $G = (V, E)$.
- ▶ *Closure* $\varphi(X)$ of $X \subseteq V$: all the reachable vertices starting from X .
- ▶ $\varphi(\{A, B\}) = \{A, B, C, D\}$.
- ▶ $\varphi(\{F\}) = \{F\}$, $\{F\}$ is *closed*.

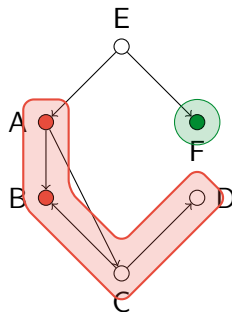


FIGURE – Closure of a vertex in a directed graph

Implications

2.1 - Elements of set theory

$A, B \subseteq \Sigma$. An *implication* is :

- ▶ $A \longrightarrow B$, A *premise*, B *conclusion*.
- ▶ *cause/consequence* relation : "If we have A , we have B ".
- ▶ $M \subseteq \Sigma$ *model* of $A \longrightarrow B$ if

$$B \subseteq M \vee A \not\subseteq M$$

denoted $M \models A \longrightarrow B$, $A \longrightarrow B$ *follows* from M .

Set of implications \mathcal{L} : *implication system*.

- ▶ $M \models \mathcal{L}$ if each element of \mathcal{L} follows from M ,
- ▶ $\mathcal{L} \models A \longrightarrow B$: all models of \mathcal{L} are models of $A \longrightarrow B$.

Implications and closure

2.1 - Elements of set theory

Given \mathcal{L} an implication system :

- ▶ closure operator $\mathcal{L}(X)$: *smallest model* (inclusion wise) of \mathcal{L} containing X , $X \subseteq \Sigma$.
- ▶ Models of \mathcal{L} form a *closure system* :

$$\Sigma^{\mathcal{L}} = \{M \subseteq \Sigma \mid M = \mathcal{L}(M)\}$$

Important property :

- ▶ $\mathcal{L} \models A \longrightarrow B$ iff $B \subseteq \mathcal{L}(A)$

Small Example

2.1 - Elements of set theory

Let \mathcal{L} be an implication system :

- ▶ $\Sigma = \{a, b, c, d, e\},$
- ▶ $\mathcal{L} = \{ab \longrightarrow c, bd \longrightarrow a, ce \longrightarrow abd\}$

We have :

- ▶ $\mathcal{L}(b) = b$, b is *closed*, hence a *model* of \mathcal{L} ,
- ▶ $\mathcal{L}(bd) = abcd$, bd is *not* a model (*abcd is*).
- ▶ \emptyset is also a model.

Applications

2.1 - Elements of set theory



- ▶ Relational Databases,
- ▶ Formal Concept analysis,
- ▶ Conceptual exploration,
- ▶ Linguistics ?

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3.1 - early 80s



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