Exercise 1

TMA4300 Computer Intensive Statistical Models

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Problem A: Stochastic simulation by the probability integral transform and bivariate techniques

1.

Let $X \sim \text{Exp}(\lambda)$, with the cdf
$$F_X(x) = 1 - e^{-\lambda x}, \quad x \geq 0.$$
Then the random variable $Y := F_X(X)$ has a Uniform$(0,1)$ distribution. The probability integral transform becomes

$$Y = 1 - e^{-\lambda X} \Leftrightarrow X = -\frac{1}{\lambda}\log(1-Y). \tag{1}$$

Thus, we sample Y from `runif()` and transform it using (1), to sample from the exponential distribution. Figure 1 shows one million samples drawn from the `generate_from_exp()` function defined in the code chunk below.

```r
set.seed(123)

generate_from_exp <- function(n, rate = 1) {
  Y <- runif(n)
  X <- -(1 / rate) * log(1 - Y)
  X
}

# sample
n <- 1000000  # One million samples
lambda <- 4.32
x <- generate_from_exp(n, rate = lambda)

# plot
hist(x,
  breaks      = 80,
  probability = TRUE,
  xlim        = c(0, 2)
)
curve(dexp(x, rate = lambda),
  add = TRUE,
  lwd = 2,
  col = "red"
)
```

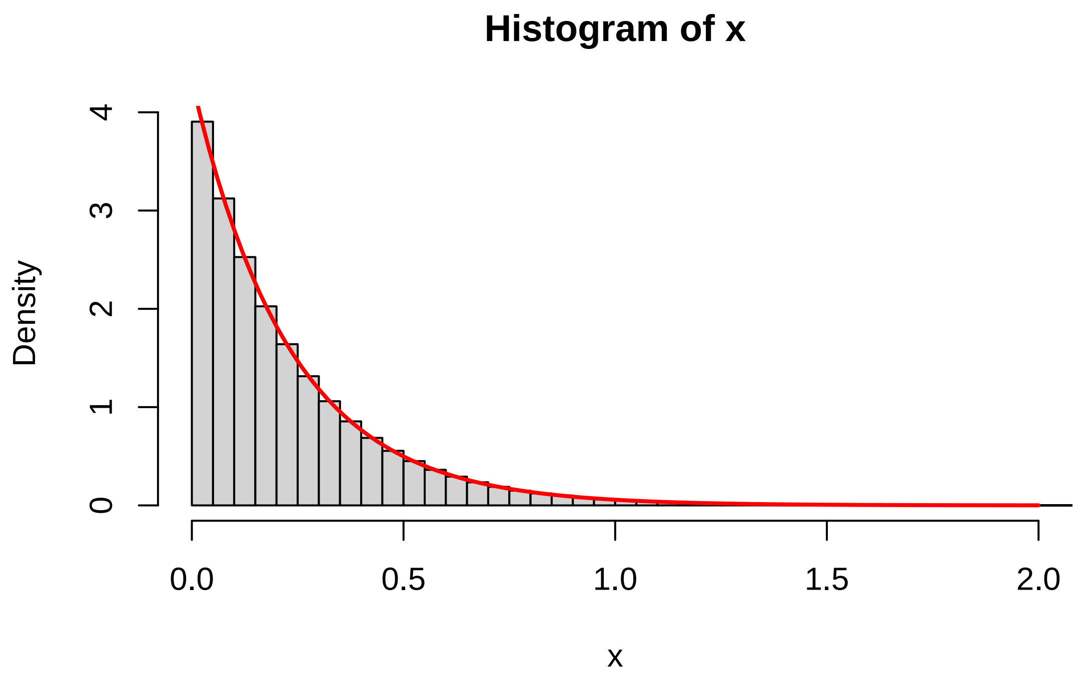

Figure 1: Normalized histogram of one million samples drawn from the exponential distribution, together with the theoretical pdf, with $\lambda = 4.32$.

2.

(a)

(b)

3.

(a)

(b)

(c)

4.

5

Problem B: The gamma distribution

1.

(a)

(b)

2.

(a)

(b)

3.

(a)

(b)

4.

5.

(a)

(b)

Problem C: Monte Carlo integration and variance reduction

1.

2.

3.

(a)

(b)

Problem D: Rejection sampling and importance sampling

1.

2.

3.

4.