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Parameter Estimation Assignment

Q1 normal distribution mean $\to \theta_1$, variance $= \theta_2$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Joint density of $(x_1, x_2, x_3 \cdots x_n)$ is

$$L(\theta_1, \theta_2; x_1, x_2, x_3 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

taking log on both sides,

$$\ln[L(\theta_1, \theta_2)] = \ln\left((2\pi\theta_2)^{-n/2} \cdot e^{-\frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}}\right)$$

$$\ln[L(\theta_1, \theta_2)] = \frac{-n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \Sigma(x_i-\theta_1)^2$$

(θ_1) differentiate $\ln[L(\theta_1, \theta_2)]$ w.r.t to θ_1.

$$\cancel{\partial \log} \frac{\partial \ln L}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\boxed{\theta_1 = \frac{\Sigma x_i}{n}}$$ ← sample mean

(θ_2) differentiate $\ln[L(\theta_1, \theta_2)]$ w.r.t to θ_2

$$\frac{\partial \ln L}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \Sigma(x_i-\theta_1)^2 = 0$$

$$\frac{n}{\cancel{2\theta_2}} = \frac{1}{\cancel{2\theta_2^2}} \Sigma(x_i-\theta_1)^2$$

$$\boxed{\theta_2 = \frac{1}{n} \Sigma(x_i-\theta_1)^2}$$ ← variance

$\therefore$ M.L.E of θ_1 for is $\overline{x}$

M.L.E of θ_2 is var(x).

Q2. $B(m, \theta) \rightarrow$ binomial distribution

$m \rightarrow$ no. of trails $\qquad \theta = (0,1)$

$$f(x) = {}^mC_x \, p^x (1-p)^{m-x} \qquad \boxed{p = \theta}$$

Joint pro density,

$$L(\theta; x_1, x_2, \ldots x_m) = \prod_{i=1}^{m} P(x_i | m, p)$$

$$L(\theta) = \prod_{i=1}^{n} \left({}^nC_{x_i} \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i} \right)$$

Taking ~~log~~ ln on both sides.

$$\ln(L(\theta)) = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \sum_{i=1}^{n} x_i \log\theta + \sum_{i=1}^{n} (m-x_i) \log(1-\theta)$$

differentiate w.r.t. to θ.

$$\frac{\partial \ln(L)}{\partial(\theta)} = \frac{1}{\theta} \sum_{i=1}^{n} x_i + \frac{1}{1-\theta} \sum_{i=1}^{n} (m-x_i)(-1) = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{n} (m-x_i)$$

$$(1-\theta) \sum x_i = \theta \sum (m-x_i)$$

$$\sum x_i = \theta (\sum m)$$

$$\boxed{\theta = \frac{\sum x_i}{m}} \leftarrow \text{mean.}$$

M.L.E of θ for $B(m, \theta)$ is $\bar{x}$.