Law and KNN

1 Setting

Let $X \subset R^n$ be a closed connected set of all possible cases, and $p(x) : X \to [0, 1]$ the fraction of people who believe the outcome of case x should be 1 (vs 0); we assume $p(\cdot)$ is Lipschitz.

Cases $x_t \sim uniform[X]$ arrive at each time t together with an opinion of a randomly sampled person $o_t \in \{0, 1\}$, and a decision d_t has to be made on a case at the time of arrival. The quality of history of decisions $h = \{x_t, d_t\}$ is evaluated by the number of people who disagree with them:

$$L(h) = \sum_{t=1}^{\infty} e^{-\delta t} |p(x_t) - d_t|,$$

where δ is the discount factor. We let $L^*(h)$ be the optimal value of $L(h)$ given sequence $\{x_t\}$, that is value of L when $d_t = [p(x_t)]$, where [] denotes rounding to closest integer.

2 Algorithms

2.1 Local precedents

Inputs: integer k, real D_{max}.

At every step the algorithm maintains the set of precedents S, elements of which are case-decision tuplets (x, d); S is initialized with empty set. For a case x_t, if there are at least k precedents within distance D of x_t, the decision d_t is made by the majority rule over k nearest precedents to x_t. If there are not k precedents within D, the decision is set to $d_t = o_t$ and a precedent (x_t, d_t) is added to S.

Theorem 2.1. *As $\delta \to 0$, $k \to \infty, D \to 0$, the loss converges to optimal $\frac{L(h)}{L^*(h)} \to 1$.*

Conjecture 2.2. *There exists an algorithm that uses finite memory while achieving the optimality condition above for $\delta \to -> 1$.*

2.2 Single juror

Set $d_t = o_t$.

Generalizes to multiple jurors when there are multiple samples o_t available for a single case.

2.3 Rulebook

Eg: As cases arrive, a decision tree is constructed and cases are settled according to that tree

3 Questions

1. When do local precedents out-compete single juror?
2. Is conjecture true?
3. What's a good algorithm when $p(x)$ changes over time?
4. (vague) how do costs of all algorithms compare as the dimension of X is increased?
5. Is there a tractable game-theoretic angle here?