CSC 225 B01 - SUMMER 2014 - QUIZ 1

Name:

Student Number:

Question 1 [10 marks]

Give a mathematical proof of the identity

$$\sum_{i=1}^{n} 2i = n^2 + n$$

for all integers $n \geq 1$.

Basis

When $n = 1$, $\sum_{i=1}^{n} 2i = 2 = 1^2 + 1$, so the identity holds.

Induction Hypothesis

Suppose the identity holds for some $n \geq 1$.

Induction Step

Consider $n + 1$.

$$\begin{aligned}
\sum_{i=1}^{n+1} 2i &= 2 \cdot 1 + 2 \cdot 2 + \ldots + 2 \cdot n + 2 \cdot (n+1) \\
&= \left[\sum_{i=1}^{n} 2i\right] + 2(n+1) \\
&= n^2 + n + 2(n+1) \qquad \text{(By the induction hypothesis)} \\
&= n^2 + n + 2n + 2 \\
&= (n^2 + 2n + 1) + (n+1) \\
&= (n+1)^2 + (n+1)
\end{aligned}$$

Therefore, the identity holds for $n + 1$, and, by induction, for all $n \geq 1$.

Question 2 [10 marks]

Arrange the following 5 functions into ascending order by their asymptotic complexity (with asymptotically smaller functions first):

$$n \log_2(n), \quad 2^{100}, \quad 2^{50}n, \quad \sqrt{10n}, \quad \log_2(n^{10})$$

Write your answer in the spaces below.

$$2^{100} \quad \log_2(n^{10}) \quad \sqrt{10n} \quad 2^{50}n \quad n \log_2(n)$$