CSC 225 FALL 2012
ALGORITHMS AND DATA STRUCTURES I
MIDTERM EXAMINATION
UNIVERSITY OF VICTORIA

1. Student ID: _____
2. Name: _____
3. DATE: 19 October 2012
   DURATION: 45 MINUTES
   INSTRUCTOR: V. SRINIVASAN
4. THIS QUESTION PAPER HAS EIGHT PAGES (INCLUDING THE COVER PAGE).
5. THIS QUESTION PAPER HAS FOUR QUESTIONS.
6. ALL ANSWERS TO BE WRITTEN ON THIS EXAMINATION PAPER.
7. THIS IS A CLOSED BOOK EXAM. NO CALCULATORS ARE ALLOWED.
8. KEEP YOUR ANSWERS SHORT AND PRECISE.

Q1 (5)	
Q2 (5)	
Q3 (5)	
Q4 (5)	
TOTAL (20) =	

1. The two parts of Question 1 test the basics of **asymptotic analysis**. Logarithms in this question are to the base 2.

   (a) Order the following functions by order of growth starting with the slowest. Briefly explain why. [3 Marks]

   $5n$, $(\log n)^8$, $n^{0.01}$, 2^{100}, 2^{2^n}, $4^{\log n}$.

(b) Consider the following sum: $S(n) = \sum_{i=1}^{n} i \log i$. Give a simple function $f(n)$ so that the sum $S(n)$ is in $\Theta(f(n))$. Briefly explain why. [2 marks]

2. The two parts of Question 2 test our knowledge about solving **recurrence equations and proofs by induction**.

   (a) Solve the following recurrence equation to get a closed-formula for $T(n)$. Assume the n is a power of three [2.5 Marks].

$$\begin{aligned} T(n) &= 1 \text{ if } n = 1 \\ &= 2T\left(\frac{n}{3}\right) + n \text{ if } n \geq 2 \end{aligned}$$

(b) Consider the following recurrence equation

$$\begin{aligned} T(n) &= 4 \text{ if } n = 1 \\ &= T(n-1) + 4 \text{ if } n \geq 2 \end{aligned}$$

Using induction, prove that $T(n) = 4n$ for all $n \geq 1$ [2.5 Marks].

3. Question 3 is based on **basic data structures** such as Priority Queues and Hash Tables.

   (a) Suppose you are given an input sequence $S = [18\ 5\ 25\ 19\ 9\ 32]$. Show how a heap-sort algorithm runs on input S by constructing the heap at the end of each of the six insertion steps of the algorithm [2.5 Marks].

(b) Let $U = \{0, 1, 2, \ldots, 34\}$ denote the universe of all possible keys. Suppose that the hashing scheme stores a set $S \subseteq U$ of size 5 into a table $T[0, \ldots, 6]$ of size 7 using the hash function

$$h(k) = 2k + 3 \bmod 7.$$

Find a set S of size 5 that is mapped by h into a single slot $T[4]$ of the table [2.5 Marks].

4. Question 4 is based on **Quick Sort**.

   Show how Quick-Sort algorithm works on the following input sequence S using the quick-sort tree.

   Use the following pivot rule that picks the element in the "middle" - For an array $A[0, 1, \ldots, n-1]$ of size n, it uses the element in $A[n/2]$ as pivot if n is even and the element in $A[(n-1)/2]$ as pivot if n is odd [5 Marks].

$$S = [85\ 24\ 63\ 45\ 17\ 31\ 96\ 50]$$

END OF EXAM