CSC 225 SPRING 2012
ALGORITHMS AND DATA STRUCTURES I
MIDTERM EXAMINATION
UNIVERSITY OF VICTORIA

1. Student ID: _____
2. Name: _____
3. DATE: 10 February 2012
   DURATION: 50 MINUTES
   INSTRUCTOR: V. SRINIVASAN
4. THIS QUESTION PAPER HAS EIGHT PAGES (INCLUDING THE COVER PAGE).
5. THIS QUESTION PAPER HAS FOUR QUESTIONS.
6. ALL ANSWERS TO BE WRITTEN ON THIS EXAMINATION PAPER.
7. THIS IS A CLOSED BOOK EXAM. NO AIDS ARE ALLOWED.
8. KEEP YOUR ANSWERS SHORT AND PRECISE.

Q1 (5)	
Q2 (5)	
Q3 (5)	
Q4 (5)	
TOTAL (20) =	

1. The three parts of Question 1 test the basics of **asymptotic analysis**.

   (a) State the definition of Big Oh. [1 Mark]

   (b) Order the following functions by order of growth starting with the slowest. [2 Marks]

   $5n$, $(\log n)^5$, n^5, 5, 5^n.

   (c) Consider the following sum: $S(n) = \sum_{i=1}^{n} \log i$. Give a simple function $f(n)$ so that the sum $S(n)$ is in $\Theta(f(n))$. Briefly explain why. [2 marks]

2. The two parts of Question 2 test our knowledge about solving **recurrence equations**.

   (a) Solve the following recurrence equation to get a closed-formula for $T(n)$. Assume the n is a power of two. [2.5 Marks]

$$\begin{aligned} T(n) &= 5 \text{ if } n = 1 \\ &= 2T(\frac{n}{2}) + 2n \text{ if } n \geq 2 \end{aligned}$$

(b) Using induction, prove that your closed formula from part (a) is correct. [2.5 Marks]

3. The three parts of Question 3 check if we understand **basic data structures** such as Priority Queues and Dictionaries.

   (a) The priority queue ADT has many possible implementations, each with its own running time for the two operations, insertItem and removeMin. Complete the following table [1.5 Marks].

Priority Queue Implementation	*Time for insertItem*	*Time for removeMin*
Unsorted Array		
Sorted Array		
Heaps		

   (b) Suppose you are given an input sequence S = [16 4 22 18 6 32]. Show how a heapsort algorithm runs on input S is built by constructing the heap at the end of each step of the algorithm. [2 Marks].

(c) Define the load factor α of a hash table with n keys and N slots. How is this related to the running time of an unsuccessful search when we use hashing with separate chaining [1.5 Marks]? Explain briefly.

4. Question 4 is based on **sorting algorithms**.

   (a) The following table compares the time and space requirements of sorting algorithms on an input of size N. [2 Marks]

Algorithm	*Worst-Case Running Time*	*Minimum Extra Space Needed*
Selection Sort		
Quick Sort		
Merge Sort		
Heap Sort		

   (b) Suppose that we are given an array A with n keys and k inversions. Here, an *inversion* is defined as a pair of entries that are out of order in the array. What is the running time of INSERTION SORT when it is used to sort A in Big Oh notation? Why? [1.5 Marks]

(b) Show how merge-sort algorithm works on the following input sequence using the merge-sort tree. [1.5 Marks]

$$S = [12\ 86\ 34\ 62\ 6\ 55\ 92\ 47]$$

END OF EXAM