Decision Trees

Administrivia

- Labs
  - Seems many of you cannot make the 11:30 or 12:30 labs, and the 1:30 lab is full.
  - They have opened a new lab at 2:30 (B04)
- Project Pitches Sept 22
  - If you can't make your lab for your pitch, contact Caleb cschortt@uvic.ca

Last Class

(Adapted from Leslie Kaelbling's example in the MIT courseware)

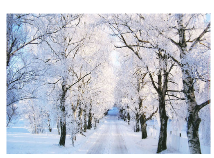
- Imagine I'm trying predict whether my neighbor is going to drive into work, so I can ask for a ride.
- Whether she drives into work seems to depend on the following attributes of the day:
  - temperature
  - expected precipitation
  - day of the week
  - what she's wearing

Memory

- Now, we find ourselves on a snowy "-5" degree Monday, and the neighbor is wearing casual clothes.
- Do you think she's going to drive?

Temp	Precip	Day	Clothes	
25	None	Sat	Casual	**Walk**
-5	Snow	Mon	Casual	**Drive**
15	Snow	Mon	Casual	**Walk**
-5	Snow	Mon	Casual	

Memory



- Standard answer in this case is "yes".
  - This day is just like one of the ones we've seen before, and so it seems like a good bet to predict "yes."
- This is the most rudimentary form of learning, which is just to memorize the things you've seen before.

Temp	Precip	Day	Clothes	
25	None	Sat	Casual	**Walk**
-5	Snow	Mon	Casual	**Drive**
15	Snow	Mon	Casual	**Walk**
-5	Snow	Mon	Casual	**Drive**

Noisy Data

- Things aren't always as easy as they were in the previous case. What if you get this set of noisy data?

Temp	Precip	Day	Clothes	
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Drive**
25	None	Sat	Casual	**Drive**
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	?

- We have certainly seen this case before, but the problem is that it has had different answers. Our neighbor is not entirely reliable.

Averaging

- One strategy would be to predict the majority outcome.

Temp	Precip	Day	Clothes	
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Drive**
25	None	Sat	Casual	**Drive**
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Walk**
25	None	Sat	Casual	**Walk**

Generalization

- Dealing with previously unseen cases
- Will she walk or drive?

Temp	Precip	Day	Clothes	
22	None	Fri	Casual	**Walk**
3	None	Sun	Casual	**Walk**
10	Rain	Wed	Casual	**Walk**
30	None	Mon	Casual	**Drive**
20	None	Sat	Formal	**Drive**
25	None	Sat	Casual	**Drive**
-5	Snow	Mon	Casual	**Drive**
27	None	Tue	Casual	**Drive**
24	Rain	Mon	Casual	**?**

We might plausibly make any of the following arguments:

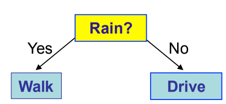
- She's going to walk because it's raining today and the only other time it rained, she walked.
- She's going to drive because she has always driven on Mondays…

Decision Trees

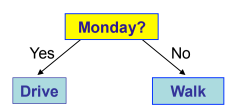
- Predict by splitting on attribute values

Temp	Precip	Day	Clothes	
22	None	Fri	Casual	**Walk**
3	None	Sun	Casual	**Walk**
10	Rain	Wed	Casual	**Walk**
30	None	Mon	Casual	**Drive**
20	None	Sat	Formal	**Drive**
25	None	Sat	Casual	**Drive**
-5	Snow	Mon	Casual	**Drive**
27	None	Tue	Casual	**Drive**
24	Rain	Mon	Casual	**?**

–She's going to walk because it's raining today and the only other time it rained, she walked.



–She's going to drive because she has always driven on Mondays…

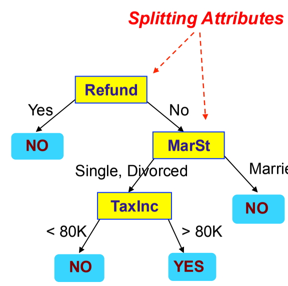


Example of a Decision Tree

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

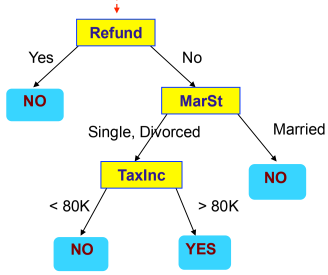
Training Data



Model: Decision Tree

Apply Model to Test Data

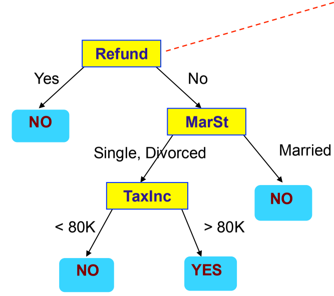
Start from the root of tree.



Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Apply Model to Test Data



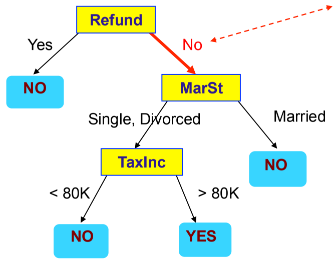
Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Apply Model to Test Data

Test Data

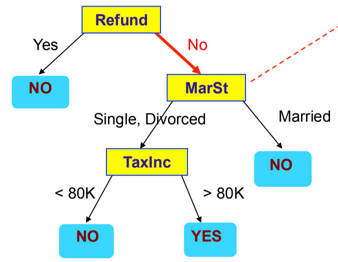
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data

Test Data

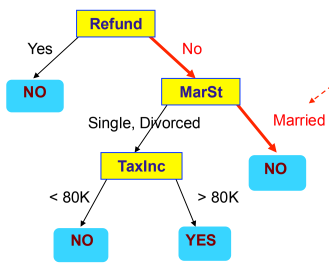
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data

Test Data

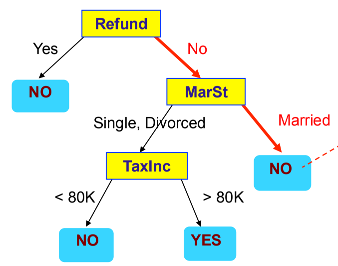
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Assign Cheat to "No"

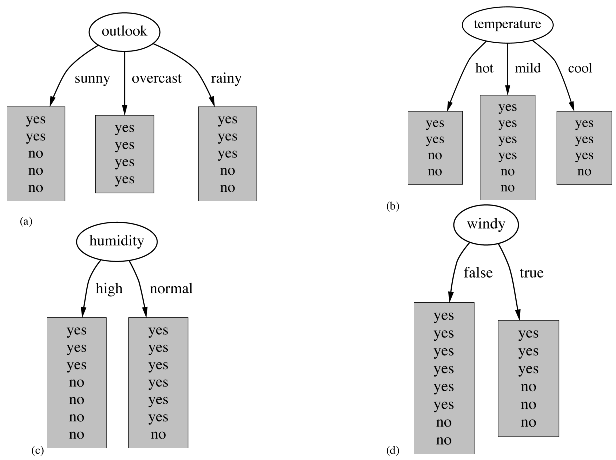
Constructing decision trees (ID3)

- Normal procedure: top down in a recursive **divide-and-conquer** fashion
  - First: an attribute is selected for the root node and a branch is created for each possible attribute value
  - Then: the instances are split into subsets (one for each branch extending from the node)
  - Finally: the same procedure is repeated recursively for each branch, using only instances that reach the branch
- Process stops if all instances have the same class

Weather data

Outlook	Temp	Humidity	Windy	Play
Sunny	Hot	High	False	No
Sunny	Hot	High	True	No
Overcast	Hot	High	False	Yes
Rainy	Mild	High	False	Yes
Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

Which attribute to select?



A criterion for attribute selection

- Which is the best attribute?
- The one which will result in the smallest tree
  - Heuristic: choose the attribute that produces the "purest" nodes
- Popular impurity criterion: entropy of nodes
  - **Lower the entropy, purer the node**.

Entropy

- H(X) = E(I(X))   Expected value of the information in X
- Expected value:

$$E(f(X))=\sum_i P(x_i) * f(x_i)$$

- Information:

$$I(x_i) = -\log_2 P(x_i)$$

- Entropy:

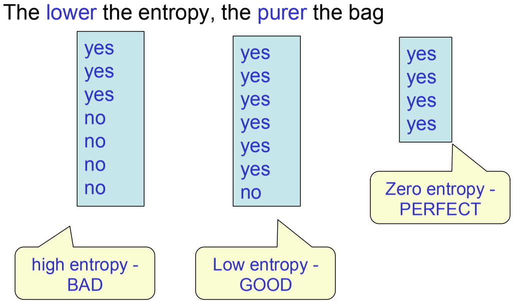
$$H(X) = E(I(X)) = \sum_i P(x_i) I(x_i) = -\sum_i P(x_i)\log_2 P(x_i)$$

- **Strategy: choose attribute that results in lowest entropy of the children nodes.**

Why low entropy?

Measuring Purity with Entropy

- Entropy is a measure of **disorder.** Also called **amount of information**.
  - The higher the entropy, the messier the bag
  - The lower the entropy, the purer the bag



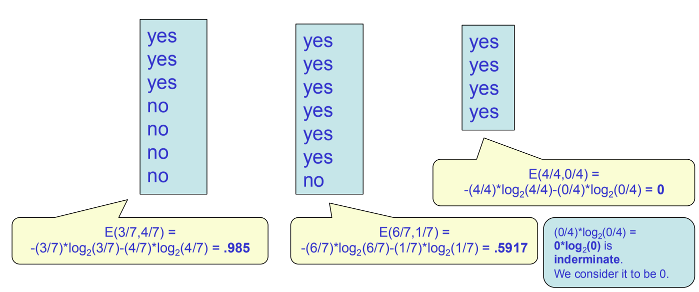
E(a/d, b/d) = -(a/d)*$\log_2$(a/d) - (b/d)*$\log_2$(b/d)

where:

d=total # of rows

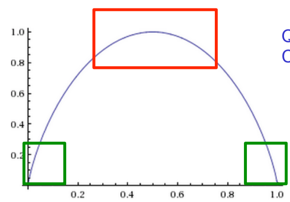
a = # yes

b = # no



Entropy Chart

- In the entropy formula: a/d + b/d = 1
- Denote
  a/d with x
  b/d with 1-x.
- $E(a/d, b/d) = -(a/d)*\log_2(a/d) - (b/d)*\log_2(b/d) =$
  $-x*\log_2(x) - (1-x)*\log_2(1-x)$



Question to think about:
Can entropy be larger than 1?

Entropy for more than two class values

For three class values:
$E(a/d, b/d, c/d) = -(a/d)*\log_2(a/d) - (b/d)*\log_2(b/d) - (c/d)*\log_2(c/d)$

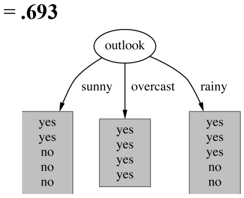
a/d + b/d + c/d = 1

For more class values:
$E(a_1/d,\ldots, a_n/d) = -(a_1/d)*\log_2(a_1/d) - \ldots - (a_n/d)*\log_2(a_n/d)$

$a_1/d + \ldots + a_n/d = 1$

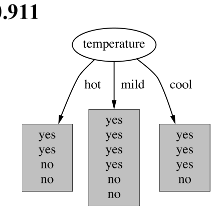
Attribute "Outlook"

outlook=sunny
entropy(2/5,3/5) = -2/5*log2(2/5) -3/5*log2(3/5) = .971
outlook=overcast
entropy(4/4,0/4) = -1*log2(1) -0*log2(0) = 0
outlook=rainy
entropy(3/5,2/5) = -3/5*log2(3/5)-2/5*log2(2/5) = .971
Expected info:
AE = .971*(5/14) + 0*(4/14) + .971*(5/14) = **.693**



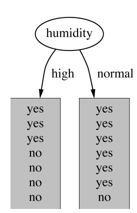
Attribute "Temperature"

temperature=hot
entropy(2/4,2/4) = -2/4*log2(2/4) -2/4*log2(2/4) = 1
temperature=mild
entropy(4/6,2/6) = -4/6*log2(4/6) -2/6*log2(2/6) = .918
temperature=cool
entropy(3/4,1/4) = -3/4*log2(3/4)-1/4*log2(1/4) = .811
Expected info:
AE = 1*(4/14) + .918*(6/14) + .811*(4/14) = **0.911**



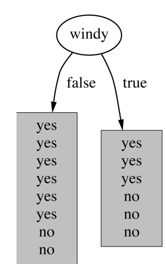
Attribute "Humidity"

humidity=high
entropy(3/7,4/7) = -3/7*log2(3/7) -4/7*log2(4/7) = .985
humidity=normal
entropy(6/7,1/7) = -6/7*log2(6/7) -1/7*log2(1/7) = .592
Expected info:
AE = .985*(7/14) + .592*(7/14) = **.789**



Attribute "Windy"

windy=false
entropy(6/8,2/8) = -6/8*log2(6/8) -2/8*log2(2/8) = .811
windy=true
entropy(3/6,3/6) = -3/6*log2(3/6) -3/6*log2(3/6) = 1
Expected info:
AE = .811*(8/14) + 1*(6/14) = **.892**



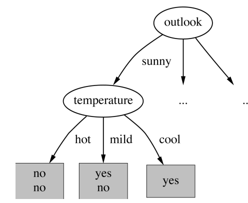
And the winner is...

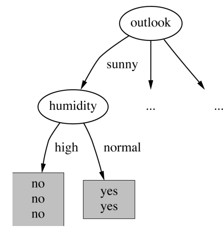
"Outlook"

...So, the root will be "Outlook"

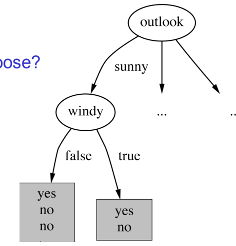
Continuing to split (for Outlook="Sunny")

Outlook	Temp	Humidity	Windy	Play
Sunny	Hot	High	False	No
Sunny	Hot	High	True	No
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Sunny	Mild	Normal	True	Yes





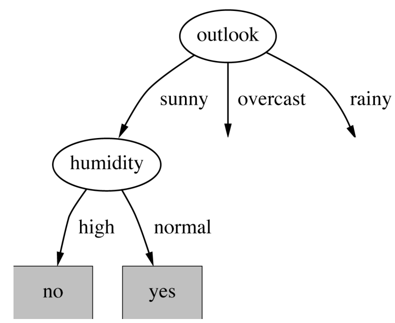
Which one to choose?



Continuing to split (for Outlook="Sunny")

temperature=hot: entropy(2/2,0/2) = 0
temperature=mild: entropy(1/2,1/2) = 1
temperature=cool: entropy(1/1,0/1) = 0
AE = 0*(2/5) + 1*(2/5) + 0*(1/5) = .4
humidity=high: entropy(3/3,0/3) = 0
humidity=normal: info(2/2,0/2) = 0
AE = 0
windy=false: entropy(1/3,2/3) =
-1/3*log2(1/3) -2/3*log2(2/3) = .918
windy=true: entropy(1/2,1/2) = 1
AE = .918*(3/5) + 1*(2/5) = .951
Winner is "humidity"

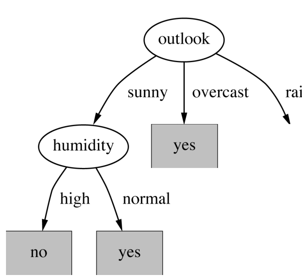
Tree so far



Continuing to split (for Outlook="Overcast")

Outlook	Temp	Humidity	Windy	Play
Overcast	Hot	High	False	Yes
Overcast	Cool	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes

- Nothing to split here, "play" is always "yes".



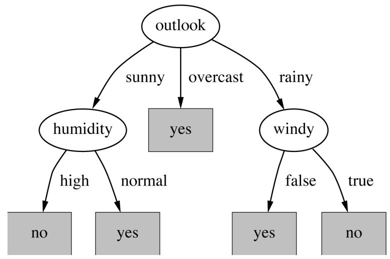
Tree so far

Continuing to split (for Outlook="Rainy")

Outlook	Temp	Humidity	Windy	Play
Rainy	Mild	High	False	Yes
Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Rainy	Mild	Normal	False	Yes
Rainy	Mild	High	True	No

- We can easily see that "Windy" is the one to choose. (Why?)

The final decision tree



- **Note**: not all leaves need to be pure; sometimes identical instances have different classes

⇒ Splitting stops when data can't be split any further

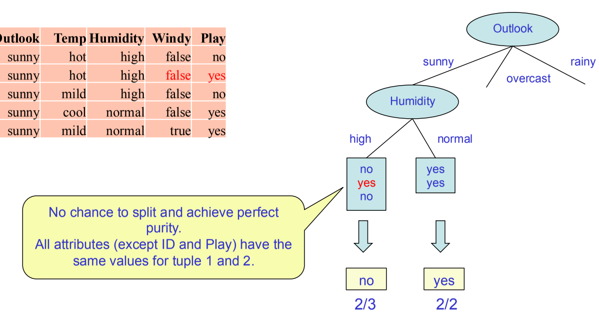
Algorithms

- Algorithm described so far is called
  "ID3" - **Iterative Dichotomiser**
  developed by **Ross Quinlan**
  at University of Sydney Australia
- Led to development of C4.5 (and its commercial version, C5.0, J48 in Java) which deals with
  - noisy data
  - missing values
  - numeric attributes
  - pruning the tree

Noisy data

- Not all leaves need to be pure; sometimes identical tuples have different class values
  – Splitting stops when data can't be split any further

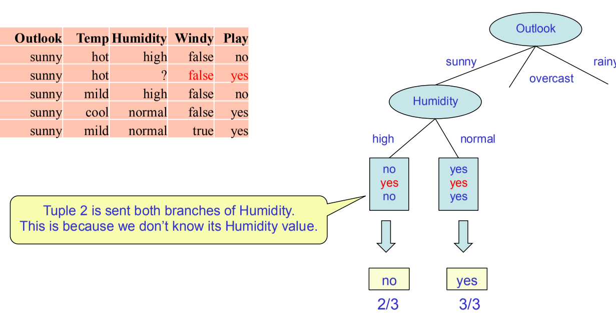
ID	Outlook	Temp	Humidity	Windy	Play
1	sunny	hot	high	false	no
2	sunny	hot	high	false	yes
8	sunny	mild	high	false	no
9	sunny	cool	normal	false	yes
11	sunny	mild	normal	true	yes



Missing data

- Sometimes, some attributes of some tuples have missing values

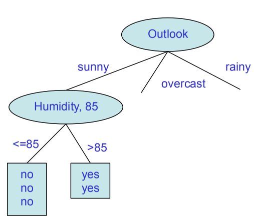
ID	Outlook	Temp	Humidity	Windy	Play
1	sunny	hot	high	false	no
2	sunny	hot	?	false	yes
8	sunny	mild	high	false	no
9	sunny	cool	normal	false	yes
11	sunny	mild	normal	true	yes



Continuous-valued attributes

- Some attributes can be numeric (continuous).
- No problem, we can have binary splits (≥v, <v), still use Entropy

ID	Outlook	Temp	Humidity	Windy	Play
1	sunny	85	85	false	no
2	sunny	80	90	true	no
3	overcast	83	86	false	yes
4	rainy	70	96	false	yes
5	rainy	68	80	false	yes
6	rainy	65	70	true	no
7	overcast	64	65	true	yes
8	sunny	72	95	false	no
9	sunny	69	70	false	yes
10	rainy	75	80	false	yes
11	sunny	75	70	true	yes
12	overcast	72	90	true	yes
13	overcast	81	75	false	yes
14	rainy	71	91	true	no



ID	Outlook	Temp	Humidity	Windy	Play
1	sunny	69	70	false	no
2	sunny	75	70	true	no
8	sunny	85	85	false	no
9	sunny	80	90	false	yes
11	sunny	72	95	true	yes

Pruning the tree

- Not always a good idea to grow the tree exhaustively
  – Saying goes:
    - "tree will **over fit** the training data"
    - "tree will **not abstract well** to classify new data"
- Solutions
  – **Pre-pruning**
  – **Post-pruning**

Decision Trees

- Pros:
  - Easy to visualize/interpret
  - Efficient to use
  - Handles discrete and continuous values
- Cons:
  - Can create overly-complex trees (pruning helps)
  - Unstable (small changes in data can give very different trees)
  - Finding optimal tree is NP complete