CENG461/ELEC514 Assignment 4

1. Consider a two-queue system as shown in Fig. 1. The packet service time of each server is exponentially distributed with the average service time equal to $1/\lambda$ seconds. When packet A arrives at the system, server 1 is serving packet B, and server 2 is idle. Packet A will be served by server 2 immediately. What is the probability that packet B leaves the system earlier than packet A? (Hint: Exponential distribution has the "memoryless" property.)

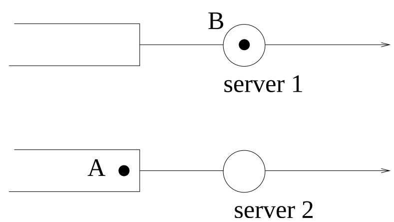

Figure 1: A two-queue system

Solution: Due to the memoryless property of exponential distribution, at the time packet B arrives at the queue, the remaining service time for packet A will still be exponentially distributed with the average time of $1/\lambda$ seconds. So both packet have the equal chance to leave the system earlier than the other one. The probability that packet B leaves earlier is $1/2$.

2. A bursty source produces data at an average rate of 5 Mbps and its maximum burst rate is 20 Mbps. Assume packet size is P bits. If a biased exponential distribution is used to model the inter-arrival time, estimate the position parameter and the shape parameter.

   Solution: Assume packet length is P bits. Please note that the average rate and burst rate above has the unit of "Mbps". We first convert them using unit of "packet/sec". Thus, $\sigma = 20\times10^6/P$ (packet/sec) and $\lambda = 5\times10^6/P$ (packet/sec).

   $a = 1/\sigma = 0.05P \times 10^{-6}$ s

   $T_a = 1/\lambda = 0.2P \times 10^{-6}$ s

   $b = \frac{\sigma\lambda}{\sigma-\lambda} = 6.667/P \times 10^6$ (per second)

3. Assume a 64 Kbps voice source which is modeled as an on-off source with an average duration of the active period $T_a = 0.4$ s and the average duration of the silent period $T_s = 0.6$ s.

   (a) Estimate the source parameters and the average data rate.

   Solution: Assume the duration of each time step is 0.01s.

   $T_a = 40$ steps

   $T_s = 60$ steps

   $a = 39/40$

   $s = 59/60$

   $\lambda_a = \lambda/(1 + T_s/T_a) = 25.6$ Kbps

   (b) Assume that there are 5 voice sources sharing a link. What is the probability that more than 3 voice sources are in the active state simultaneously. (Hint: Binomial distribution)

   Solution: Let p_i represent i voice sources are in the active state.

   $S_s = T_s/(T_s + T_a) = 0.6$

   $S_a = T_a/(T_s + T_a) = 0.4$

   $\Pr\{p_i > 3\} = \Pr\{p_i = 4\} + \Pr\{p_i = 5\}$

   $= \binom{5}{4} S_a^4 S_s + \binom{5}{5} S_a^5 = 0.087.$

   (c) Given the link capacity is $64 \times n$ Kbps, when there are more than n sources in the active state simultaneously, we call it an outage event. Given that the link capacity is 640 Kbps and 12 voice sources share the link, what is the probability of outage? (Hint: Prob$\{$outage$\}$ = Prob$\{n + 1$ sources in active state$\}$ + Prob$\{n + 2$ sources in active state$\}$ + ...)

   Solution:

   $n = 640/64 = 10$

   $\Pr\{outage\} = \sum_{k=n+1}^{12} \Pr\{p_i = k\}$

   $= \binom{12}{11} S_a^{11} S_s + \binom{12}{12} S_a^{12} = 0.000319.$