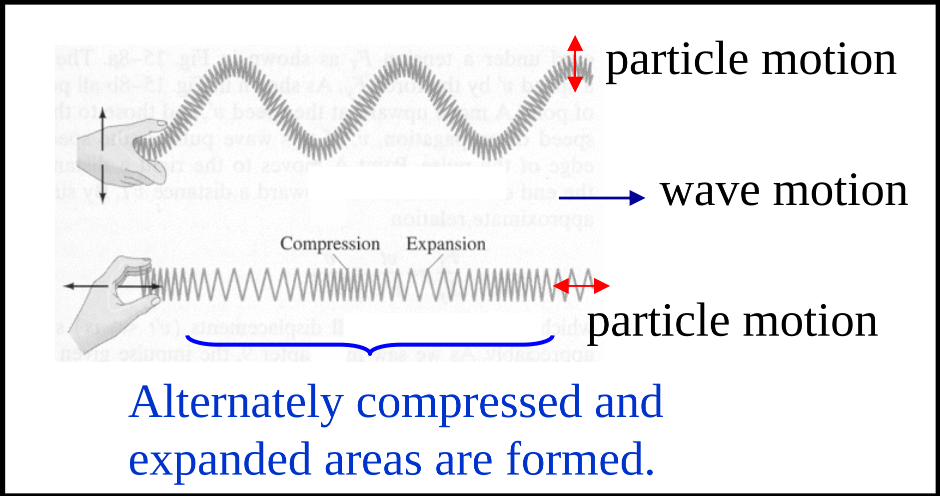
16.1. Propagation of a disturbance

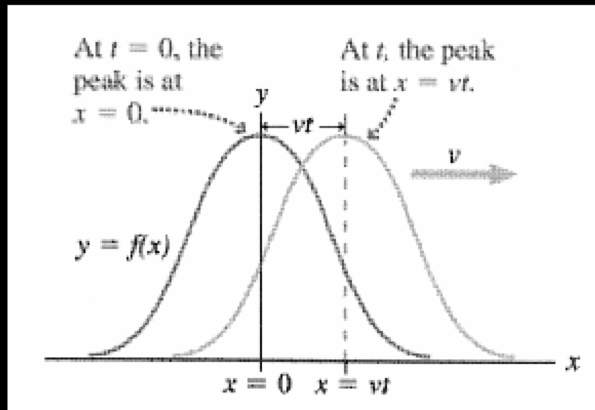
– Wave types : depending on the vibration of wave source

- ***Transverse wave*** : vibration of elements of medium ⊥ direction of wave motion.
- ***Longitudinal wave*** : vibration of elements // direction of wave motion.

(Ch. 17 : Sound Waves)



16.2. Mathematical description of traveling wave – *General Form*



Snapshot of the traveling wave at $t = 0$ and at a later time t. During the time interval t, the entire wave shifts distance vt to the right, but its shape stays the same.

General form of traveling wave :

$$\boxed{y\,(x, t) = f\,(x \pm vt)}$$

Note : All waves in which the variables x and t enter in the combination ($x \pm vt$) are traveling waves.

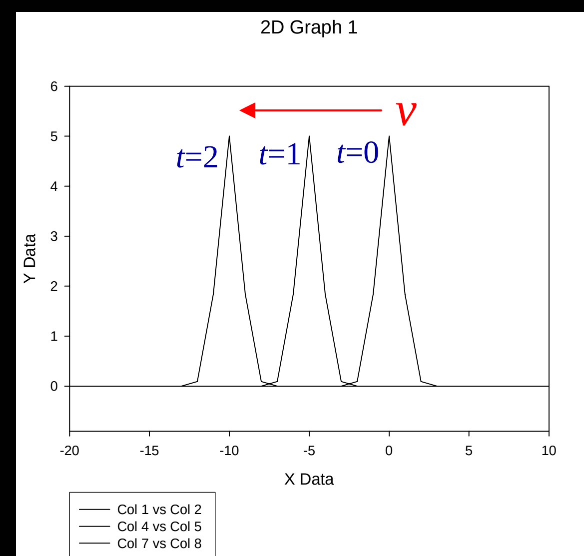
where f represents any function, *e.g.* sine function.

Problem :

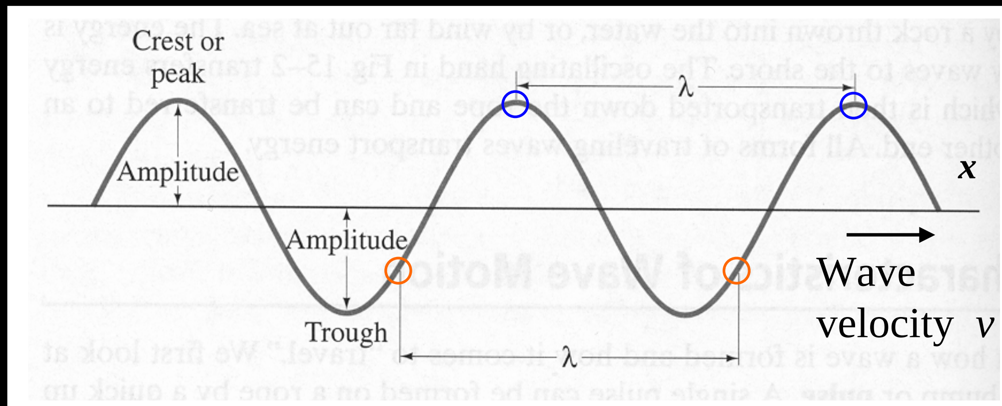
A wave moving along the x axis is described by

$$y(x, t) = 5\exp[-(x + 5t)^2]$$

where x is in meters and t is in seconds. Determine (a) the direction of the wave motion, and (b) the speed of the wave.



• Mathematical description of traveling wave – *Sinusoidal Waves*



We describe the traveling wave by means of sinusoidal wave :

$$\boxed{y\,(x,\,t) = A\sin(kx \pm \omega\, t)}$$

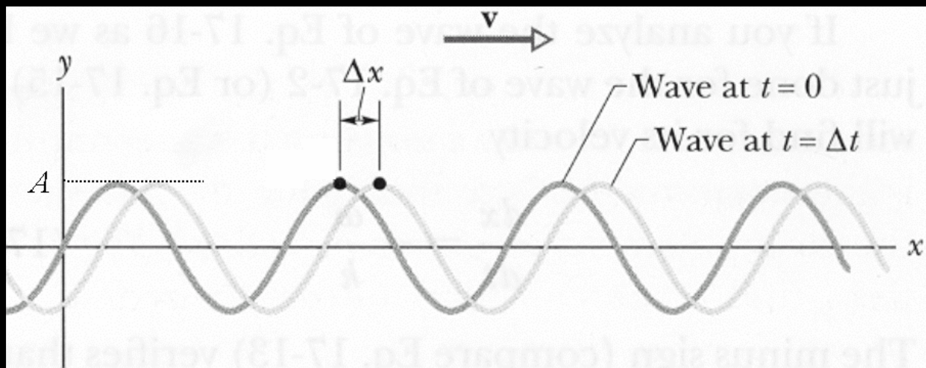
where A = amplitude, k = angular wave number, ω = angular frequency.

<u>Concepts of *k* and *ω* :</u>

$\boxed{k\ =\ 2\pi/\lambda}$ (rad/m) [angular wave number]
(not a spring constant k !)

$\boxed{\omega\ =\ 2\pi/T}$ (rad/s) [angular frequency]

Wave speed *v* of traveling wave :



Snapshot of the traveling wave at $t = 0$ and at a later time $t = \Delta t$. During the time interval Δt, the entire wave shifts distance Δx to the right.

$$\boxed{v = \omega/k = (2\pi/T)/(2\pi/\lambda) = \lambda/T = \lambda f}$$

Transverse speed v_y of wave :

While the wave travels in the *x* direction with wave speed *v*, each segment of the wave oscillates vertically (in the *y* direction).

$$v_y^{max} = \omega A$$

(identical as SHM, *see* 13-3 in lecture note)

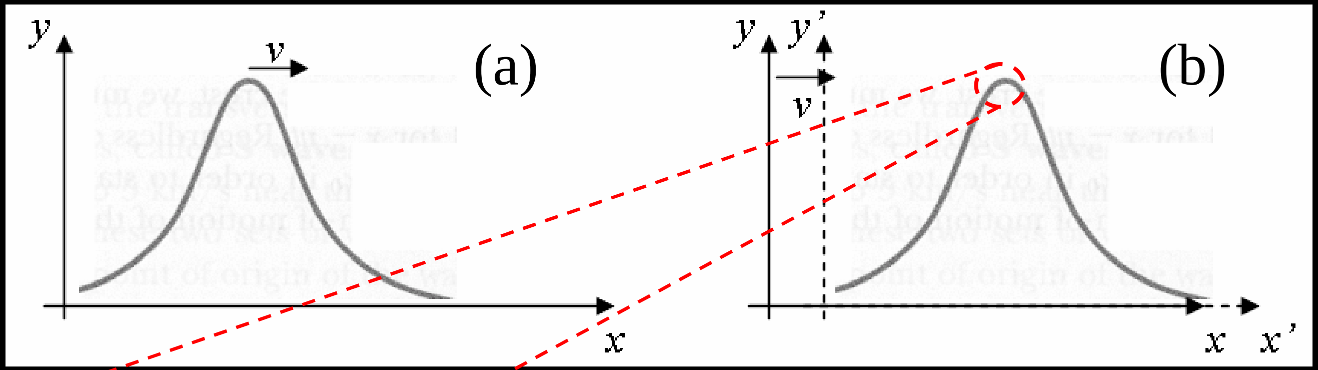
16.3. Speed of a wave on a stretched string

Find the speed of a traveling wave by

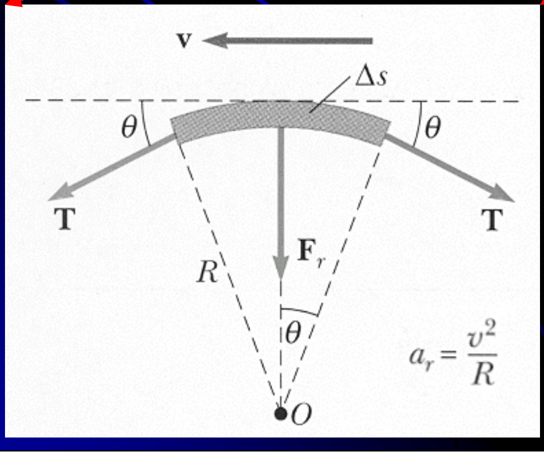
(*i*) using conceptual and dimensional analysis : → $v = C\sqrt{\dfrac{T}{\mu}}$

(*ii*) using mechanical analysis : considering a single symmetrical pulse.

(a) In a stationary frame of reference : (b) In a frame moving with the pulse :



- Consider a small segment of Δs, forming an arc of a circle of radius R



$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

$$\textit{Wave speed} = \sqrt{\frac{\textit{Force factor}}{\textit{Mass factor}}}$$