University of Victoria

Department of Mathematics and Statistics

MATH 101 (Calculus II), 2000 Spring Term
Final Examination - April 2000

Name: _____ Student number: _____

Maximum score: xx **Duration: 180 minutes**

Instructions:

1. The **Sharp EL-510R** scientific calculator is allowed. This is the **only** calculator permitted. **No other aids** such as books, notes, or scratch paper are permitted.

2. ALL questions are *multiple choice* questions. For questions requiring numerical answers, the choices are listed in numerically increasing order. Choose the value which is nearest your (unrounded) answer. In the special case that your (unrounded) answer is equidistant from the two nearest choices, choose the larger of these two choices. For verification purposes, show all calculations on your questions paper. Unverified answers may be disallowed. You may use the backs of pages if necessary.

3. Each of the multiple choice questions are worth X marks. The maximum score for this test is XX marks.

1. Please fill in the bubble (a) on your computer answer sheet as your "answer" for question number one.

2. Find the area bounded between the two graphs

$$y = 4x^2 \quad \text{and} \quad y = 4x$$

for $-1 \le x \le 1$. Pick the answer nearest to your result.

  (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

  (F) 5   (G) 6   (H) 7   (I) 8   (J) 9

3. Find the derivative of the function $y = \frac{x}{2} \tan^{-1} \frac{x}{2}$ and evaluate the derivative at the point where $x = 2.0$. Pick the answer nearest to your result.

  (A) 0.0   (B) 0.1   (C) 0.2   (D) 0.3   (E) 0.4

  (F) 0.5   (G) 0.6   (H) 0.7   (I) 0.8   (J) 0.9

4. Suppose that sand is flowing into a bin at $16 - 8t$ tonnes/min at time t (in minutes). Find how many tonnes of sand flow into the bin between $t = 1$ and $t = 2$ minutes. Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

5. Find the volume of the solid generated by revolving around the x-axis the region under the curve $y = \frac{2}{x}$ and above the x-axis from $x = 2$ to $x = 10$. Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

6. Use the method of cylindrical shells to find the volume of the solid generated by revolving the region inside the bounds

$$y = x^{-2}, \quad y = 0, \quad x = 1 \quad \text{and} \quad x = 2$$

about the y-axis. Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

7. Evaluate the integral

$$\int_1^\infty \frac{6\,\mathrm{d}x}{(3x+1)^2}.$$

Pick the answer nearest to your result.

(A) 0.0   (B) 0.1   (C) 0.2   (D) 0.3   (E) 0.4

(F) 0.5   (G) 0.6   (H) 0.7   (I) 0.8   (J) 0.9

8. Solve the differential equation
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{y^2}$$
with the initial condition $y(0) = 7$. Find the value of y when $x = 2$. Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

9. Find the volume of the solid generated by revolving around the y-axis the region in the first quadrant between the graphs
$$y^2 + 1 = 4x^2, \quad x = 0, \quad y = 0 \quad \text{and} \quad y = 2.$$
Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

10. Evaluate the integral
$$\int_0^{\frac{\pi}{4}} 2\sin^2 x \, dx.$$

Pick the answer nearest to your result.

(A) 0.0 (B) 0.1 (C) 0.2 (D) 0.3 (E) 0.4

(F) 0.5 (G) 0.6 (H) 0.7 (I) 0.8 (J) 0.9

11. Evaluate the integral
$$\int_0^{\ln 2} xe^x \, dx.$$

Pick the answer nearest to your result.

(A) 0.0 (B) 0.1 (C) 0.2 (D) 0.3 (E) 0.4

(F) 0.5 (G) 0.6 (H) 0.7 (I) 0.8 (J) 0.9

12. Evaluate the integral
$$\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx.$$

Pick the answer nearest to your result.

    (A) 0.0   (B) 0.1   (C) 0.2   (D) 0.3   (E) 0.4

    (F) 0.5   (G) 0.6   (H) 0.7   (I) 0.8   (J) 0.9

13. Consider the following partial fraction expansion:
$$\frac{1-3x}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

Find the value of $2A + B$. Pick the answer nearest to your result.

    (A) -4 (B) -3 (C) -2 (D) -1 (E) 0

    (F) 1   (G) 2   (H) 3   (I) 4   (J) 5

14. Find
$$\lim_{x \to 0} \frac{1 - \cos(2x^2)}{1 - \cos(3x^2)}.$$

Pick the answer nearest to your result.

(A) −0.4   (B) −0.3   (C) −0.2   (D) −0.1   (E) 0.0

(F) 0.1   (G) 0.2   (H) 0.3   (I) 0.4   (J) 0.5

15. Find
$$\lim_{x \to 0} (1 + 2x)^{\frac{1}{x}}.$$

Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

16. Evaluate the integral
$$\int_0^1 \sinh(4x)\,dx.$$

Pick the answer nearest to your result.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(F) 5 (G) 6 (H) 7 (I) 8 (J) 9

17. Given that the integral
$$\int \frac{dx}{(x^2 + a^2)^{\frac{1}{2}}} = \ln\left(x + \sqrt{a^2 + x^2}\right) + C,$$

compute the value of
$$\int_0^1 \frac{dx}{(144x^2 + 1)^{\frac{1}{2}}}.$$

Pick the answer nearest to your result.

(A) 0.0 (B) 0.1 (C) 0.2 (D) 0.3 (E) 0.4

(F) 0.5 (G) 0.6 (H) 0.7 (I) 0.8 (J) 0.9

18. Find the sum of the series
$$\sum_{n=0}^{\infty} \frac{8^n + (-8)^n}{10^n}.$$

Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

19. Find the radius of convergence of the power series
$$\sum_{n=1}^{\infty} \frac{(n+1)x^{2n}}{7^n}.$$

Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

20. Let the function $f(x)$ be defined by

$$f(x) = \frac{1}{1-x} + \sum_{n=1}^{\infty} \frac{x^n}{5^n}.$$

When the function $f(x)$ is expanded in a power series about $x = 0$, what will be the radius of convergence of the series? Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

21. Assume that the derivatives of a function $f(x)$ at $x = 0$ are $f^{(n)}(0) = 2n^2$ for every integer $n \geq 0$. Approximate $f(x)$ with a Taylor polynomial about $x = 0$ up to and including the quadratic terms. Use this approximation to compute $f(-1)$. Pick the answer nearest to your result.

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0

(F) 1   (G) 2   (H) 3   (I) 4   (J) 5

22. Let
$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{7n-2}.$$
Find the value of the third derivative of $f(x)$ at $x = 0$. Pick the answer nearest to your result.

(A) 0.0   (B) 0.1   (C) 0.2   (D) 0.3   (E) 0.4

(F) 0.5   (G) 0.6   (H) 0.7   (I) 0.8   (J) 0.9

23. Find the coefficient of the x^{11} term in the power series expansion of the function $f(x) = x\sin(-3x^2)$ about the point $x = 0$. Pick the answer nearest to your result.

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0

(F) 1   (G) 2   (H) 3   (I) 4   (J) 5

24. The function $f(x)$ has a Taylor series expansion

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

Find the value of a_5 if you know that $f^{(n)}(0) = 2n(-2)^n$ for every integer $n \geq 0$. Pick the answer nearest to your result.

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0

(F) 1 (G) 2 (H) 3 (I) 4 (J) 5

25. Find the limit of the sequence S_n given by

$$S_n = \sum_{i=1}^{n} \frac{4i}{2i + 1}.$$

Pick the answer nearest to your result.

(A) $-\infty$ (B) -3 (C) -2 (D) -1 (E) 0

(F) 1 (G) 2 (H) 3 (I) 4 (J) $+\infty$

26. Find the sum of the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{n(n+2)}.$$

Pick the answer nearest to your result.

(A) 1.0   (B) 1.1   (C) 1.2   (D) 1.3   (E) 1.4

(F) 1.5   (G) 1.6   (H) 1.7   (I) 1.8   (J) 1.9

27. Find the area of one leaf of the curve

$$r^2 = 3\sin 5\theta.$$

Pick the answer nearest to your result.

(A) 0.0   (B) 0.1   (C) 0.2   (D) 0.3   (E) 0.4

(F) 0.5   (G) 0.6   (H) 0.7   (I) 0.8   (J) 0.9

28. Consider the part of the graph of the polar coordinate formula $r = \sin 2\theta$ for which $\theta \in [0, \pi]$. This graph has leaf shaped patterns. In which quadrants does the curve have leaves? (Note the restriction on θ and the quadrants are labelled counterclockwise starting with $x > 0$ and $y > 0$.) Pick the correct answer.

   (A) I & II   (B) I & III   (C) I & IV   (D) II & III

   (F) II & IV   (G) III & IV   (H) I, II, III, & IV   (I) none of these

29. The point with rectangular coordinates $(1, -\sqrt{3})$ has polar coordinate representation (r, θ). Find the radian value of θ in the range $0 \le \theta < 2\pi$ when $r \ge 0$ and pick the answer nearest to your result.

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

   (F) 5   (G) 6   (H) 7   (I) 8   (J) 9

30. Find the slope of the tangent line to the parametric curve given by

$$x = t - t^2 \quad \text{and} \quad y = 2t^2 - t$$

at the point where $t = 1$. Pick the answer nearest to your result.

<div style="text-align:center">

(A) −4   (B) −3   (C) −2   (D) −1   (E) 0

(F) 1   (G) 2   (H) 3   (I) 4   (J) 5

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31. Find the area under the graph described by the parametric equations

$$x = t + e^t \quad \text{and} \quad y = 1 + e^{-t},$$

and above $y = 0$ over the interval $0 \le t \le 1$. Pick the answer nearest to your result.

<div style="text-align:center">

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

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32. Find the arc length of the curve

$$y = \frac{2}{3}x^{\frac{3}{2}}$$

from $x = 0$ to $x = 4$. Pick the answer nearest to your result.

      (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

      (F) 5   (G) 6   (H) 7   (I) 8   (J) 9

33. Compute the volume of the solid formed by rotating the first quadrant region bounded by

$$y = x - x^2 \quad \text{and} \quad y = 0$$

about the line $x = -1$. Pick the answer nearest to your result.

      (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

      (F) 5   (G) 6   (H) 7   (I) 8   (J) 9

34. Find the area inside the circle $r = 3 \sin \theta$ and outside the circle $r = 2 \sin \theta$. Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

35. Evaluate the integral
$$\int_1^4 x \ln x \, \mathrm{d}x.$$
Pick the answer nearest to your result.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

(F) 5   (G) 6   (H) 7   (I) 8   (J) 9

36. Evaluate the integral

$$\int_0^2 \frac{x}{x+2}\,\mathrm{d}x.$$

Pick the answer nearest to your result.

(A) 0.0 (B) 0.1 (C) 0.2 (D) 0.3 (E) 0.4

(F) 0.5 (G) 0.6 (H) 0.7 (I) 0.8 (J) 0.9