

# Math 101, Spring 2009

## Assignment 1

Due at the beginning of the class, January 21st, 2009.

No late assignments will be considered.

*Each question is worth 6 marks.*

1. (6 points) Find the volume of the solid of revolution obtained by rotating the region bounded by the curves  $y = 1 - x^2$  and  $y = 4 - 4x^2$  around
  - (a) (3 points) the  $x$ -axis. [Ans.:  $16\pi$ ]
  - (b) (3 points) the line  $y = -1$ . [Ans.:  $24\pi$ ]
2. (6 points) Evaluate using the cylindrical shell method, the volume of the solid of revolution obtained by rotating the region enclosed by the curves  $y = x^3$ ,  $y = -x$ , and  $y = 1$  around the line  $y = -2$ . [Ans.:  $137\pi/21$ ]
3. (6 points) Find the length of the curve  $24xy = x^4 + 48$  between  $(2, 4/3)$  and  $(3, 43/24)$ . [Ans.:  $9/8$ ]
4. (6 points) Consider a vertical cone of height  $h$  whose horizontal cross-section is an ellipse and whose base is the ellipse with major and minor semi-axes  $a$  and  $b$ . Verify that the volume of the cone is  $\frac{\pi abh}{3}$ . [Hint: The area of an ellipse with major and minor semi-axes  $\alpha$  and  $\beta$  is  $\pi\alpha\beta$ .]
5. (6 points) Let  $v = |t - 1|$  represent the velocity of a particle moving in a straight line where  $v$  is in m/s and  $t$  is given in seconds. Over the time interval  $[0, 2]$ , find the
  - (a) (3 points) net distance travelled. [Ans.: 1 m.]
  - (b) (3 points) total distance travelled. [Ans.: 1 m.]