

22a Cache Examples – Part 1

CSC 230

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Problem 5.7 (from HVZ)

A computer uses a small direct-mapped cache between the main memory and the processor. The cache has **four 16-bit words**, and each word has an associated 13-bit tag, as shown in the figure below. When a miss occurs during a read operation, the requested word is read from the main memory and sent to the processor. At the same time, it is copied into the cache, and its block number is stored in the associated tag.

	13 bits	16 bits
	Tag	Content
address 0		
address 2		
address 4		
address 6		

Consider the following loop in a program where all instructions and operands are 16 bits long and the code starts at address **02EC**:

```
LOOP:  ADD    (R1)+,R0    02EC
        DECR   R2         02EE
        BNE    LOOP      02F0
```

Assume that, before this loop is entered, registers R0, R1 and R2 contain:
R0 = 0, R1 = 054E, R2 = 3. Also assume that main memory contains the data as shown below:

A03C	054E
05D9	0550
10D7	0552

Show the contents of the cache at the end of each pass through the loop.

ADD	02EC
DECR	02EE
BNE	02F0

. . . .

A03C	054E
05D9	0550
10D7	0552

LOOP:	ADD (R1)+,R0	02EC
	DECR R2	02EE
	BNE LOOP	02F0

translated to ARM code

LOOP:	LDR R3,[R1],#2
	ADD R0,R0,R3
	SUBS R2,R2,#1
	BNE LOOP

*Example will remain in language as the book
for consistency of the answers*

LOOP: ADD (R1)+,R0
 DECR R2
 BNE LOOP

02EC = 0000 0010 1110 1100 → 4
 02EE = 0000 0010 1110 1110 → 6
 02F0 = 0000 0010 1111 0000 → 0

tag

**3 bits = which
 block in cache**

A03C
05D9
10D7

054E = 0000 0101 0100 1110 → 6
 0550 = 0000 0101 0101 0000 → 0
 0552 = 0000 0101 0101 0010 → 2

**this tells us the cache
 block number where
 each data
 will be stored**

**this tells us the cache
 block number where
 each instruction
 will be stored**

```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2       02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0
```

A03C	054E = 0000 0101 0100 1110 ➔ 6	R0 = 0 R1 = 054E R2 = 3
05D9	0550 = 0000 0101 0101 0000 ➔ 0	
10D7	0552 = 0000 0101 0101 0010 ➔ 2	
	13 bits 16 bits	
	Tag Content	

address 0

address 2

address 4

address 6


```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    → 4
        DECR  R2       02EE = 0000 0010 1110 1110    → 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    → 0
```

A03C	054E = 0000 0101 0100 1110 → 6	R0 = 0 R1 = 054E R2 = 3
05D9	0550 = 0000 0101 0101 0000 → 0	
10D7	0552 = 0000 0101 0101 0010 → 2	
	13 bits 16 bits	
	Tag Content	

Fetch ADD (1 M)

address 0

address 2

address 4

address 6

	ADD

```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2       02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0
```

A03C	054E = 0000 0101 0100 1110 ➔ 6	<div>R0 = 0 R1 = 054E R2 = 3</div>
05D9	0550 = 0000 0101 0101 0000 ➔ 0	
10D7	0552 = 0000 0101 0101 0010 ➔ 2	
	13 bits 16 bits	
	Tag Content	

Fetch (R1) (1 M)

address 0

address 2

address 4

address 6

	ADD
	A03C


```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2       02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0
```

A03C	054E = 0000 0101 0100 1110 ➔ 6	R0 = 0 = A03C R1 = 054E = 0550 R2 = 3
05D9	0550 = 0000 0101 0101 0000 ➔ 0	
10D7	0552 = 0000 0101 0101 0010 ➔ 2	
	13 bits 16 bits	
	Tag Content	

*Increment R1 and
ADD to R0*

address 0

address 2

address 4

address 6

	<i>ADD</i>
	<i>A03C</i>

```

LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2      02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0

```

A03C
05D9
10D7

054E =	0000 0101 0100 1110	➔ 6
0550 =	0000 0101 0101 0000	➔ 0
0552 =	0000 0101 0101 0010	➔ 2

13 bits
16 bits

Tag
Content

R0 = A03C
R1 = 0550
R2 = 3 = 2

Fetch DECR (1M)

1 collision

address 0

address 2

address 4

address 6

	ADD
	A03C DECR

```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    → 4
        DECR  R2       02EE = 0000 0010 1110 1110    → 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    → 0
```

A03C	054E = 0000 0101 0100 1110 → 6	R0 = A03C R1 = 0550 R2 = 2
05D9	0550 = 0000 0101 0101 0000 → 0	
10D7	0552 = 0000 0101 0101 0010 → 2	
	13 bits 16 bits	
	Tag Content	

Fetch BNE (1M)

address 0

BNE

address 2

address 4

ADD

address 6

~~*A03C*~~

DECR

```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2      02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0
```

A03C	054E = 0000 0101 0100 1110 ➔ 6	<div>R0 = A03C R1 = 0550 R2 = 2</div>
05D9	0550 = 0000 0101 0101 0000 ➔ 0	
10D7	0552 = 0000 0101 0101 0010 ➔ 2	

13 bits

16 bits

After 1st pass in loop

Tag

Content

4 M

address 0

BNE

address 2

address 4

ADD

address 6

DECR

2nd pass

LOOP: ADD (R1)+,R0 02EC = 0000 0010 1110 1100 → 4
 DECR R2 02EE = 0000 0010 1110 1110 → 6
 BNE LOOP 02F0 = 0000 0010 1111 0000 → 0

A03C	054E =	0000 0101 0100 1110	→ 6	<div>R0 = 0</div> <div>R1 = 0550</div> <div>R2 = 2</div>
05D9	0550 =	0000 0101 0101 0000	→ 0	
10D7	0552 =	0000 0101 0101 0010	→ 2	
		13 bits	16 bits	
		Tag	Content	

Fetch ADD (1 C)

address 0

BNE

address 2

address 4

ADD

address 6

DECR

```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2       02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0
```

A03C	054E = 0000 0101 0100 1110 ➔ 6	<div>R0 = 0 R1 = 0550 R2 = 2</div>
05D9	0550 = 0000 0101 0101 0000 ➔ 0	
10D7	0552 = 0000 0101 0101 0010 ➔ 2	
	13 bits 16 bits	
	Tag Content	

Fetch (R1) (1 M)
1 collision

address 0

~~BNE~~ 05D9

address 2

address 4

ADD

address 6

DECR

LOOP: ADD (R1)+,R0	02EC =	0000 0010 1110 1100	➔	4
DECR R2	02EE =	0000 0010 1110 1110	➔	6
BNE LOOP	02F0 =	0000 0010 1111 0000	➔	0

A03C	054E =	0000 0101 0100 1110	➔ 6	<div> R0 = 0 = A615 R1 = 0550 = 0552 R2 = 2 </div>
05D9	0550 =	0000 0101 0101 0000	➔ 0	
10D7	0552 =	0000 0101 0101 0010	➔ 2	
		13 bits	16 bits	
		Tag	Content	

Increment R1 and ADD to R0

address 6

	<i>05D9</i>
	<i>ADD</i>
	<i>DECR</i>

LOOP: ADD (R1)+,R0	02EC =	0000 0010 1110 1100	→	4
DECR R2	02EE =	0000 0010 1110 1110	→	6
BNE LOOP	02F0 =	0000 0010 1111 0000	→	0

A03C	054E =	0000 0101 0100 1110	→ 6	<div>R0 = A615</div> <div>R1 = 0552</div> <div>R2 = 2 = 1</div>
05D9	0550 =	0000 0101 0101 0000	→ 0	
10D7	0552 =	0000 0101 0101 0010	→ 2	
		13 bits	16 bits	
		Tag	Content	

Fetch DECR (1C)

address 0

05D9

address 2

address 4

ADD

address 6

DECR


```

LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2      02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0

```

A03C
05D9
10D7

054E = 0000 0101 0100 1110 ➔ 6 0550 = 0000 0101 0101 0000 ➔ 0 0552 = 0000 0101 0101 0010 ➔ 2	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> R0 = A615 R1 = 0552 R2 = 1 </div>
--	---

13 bits
16 bits
Fetch BNE (1M)

Tag
Content
1 collision

address 0

address 2

address 4

address 6

	05D9 BNE
	ADD
	DECR

```

LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    → 4
        DECR  R2       02EE = 0000 0010 1110 1110    → 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    → 0

```

A03C	054E = 0000 0101 0100 1110 → 6	<div> R0 = A615 R1 = 0552 R2 = 1 </div>
05D9	0550 = 0000 0101 0101 0000 → 0	
10D7	0552 = 0000 0101 0101 0010 → 2	
	13 bits 16 bits	
	Tag Content	

After 2nd pass in
loop 2 M + 2C

address 0

address 2

address 4

address 6

	<i>BNE</i>
	<i>ADD</i>
	<i>DECR</i>

3rd pass

LOOP: ADD (R1)+,R0 02EC = 0000 0010 1110 1100 → 4
 DECR R2 02EE = 0000 0010 1110 1110 → 6
 BNE LOOP 02F0 = 0000 0010 1111 0000 → 0

A03C	054E =	0000 0101 0100 1110	→ 6	<div>R0 = A615</div> <div>R1 = 0552</div> <div>R2 = 1</div>
05D9	0550 =	0000 0101 0101 0000	→ 0	
10D7	0552 =	0000 0101 0101 0010	→ 2	
		13 bits	16 bits	
		Tag	Content	

Fetch ADD (1 C)

address 0

BNE

address 2

address 4

ADD

address 6

DECR

```

LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2      02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0

```

A03C
05D9
10D7

054E =	0000 0101 0100 1110	➔ 6
0550 =	0000 0101 0101 0000	➔ 0
0552 =	0000 0101 0101 0010	➔ 2

13 bits
16 bits

Tag
Content

R0 = A615
R1 = 0552
R2 = 1

Fetch (R1) (1 M)

address 0

BNE

address 2

10D7

address 4

ADD

address 6

DECR

```

LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2      02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0

```

A03C
05D9
10D7

054E =	0000 0101 0100 1110	➔ 6
0550 =	0000 0101 0101 0000	➔ 0
0552 =	0000 0101 0101 0010	➔ 2

13 bits
16 bits

Tag
Content

R0 = B6EC
R1 = 0552 = 0554
R2 = 1

*Increment R1 and
ADD to R0*

address 0

BNE

address 2

10D7

address 4

ADD

address 6

DECR

```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    ➔ 4
        DECR  R2       02EE = 0000 0010 1110 1110    ➔ 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    ➔ 0
```

A03C	054E = 0000 0101 0100 1110 ➔ 6	<div>R0 = B6EC R1 = 0554 R2 = 1 = 0</div>
05D9	0550 = 0000 0101 0101 0000 ➔ 0	
10D7	0552 = 0000 0101 0101 0010 ➔ 2	
	13 bits 16 bits	
	Tag Content	

Fetch DECR (1C)

address 0

BNE

address 2

10D7

address 4

ADD

address 6

DECR

LOOP: ADD (R1)+,R0	02EC =	0000 0010 1110 1100	→	4
DECR R2	02EE =	0000 0010 1110 1110	→	6
BNE LOOP	02F0 =	0000 0010 1111 0000	→	0

A03C	054E =	0000 0101 0100 1110	→ 6	<div> R0 = B6EC R1 = 0554 R2 = 0 </div>
05D9	0550 =	0000 0101 0101 0000	→ 0	
10D7	0552 =	0000 0101 0101 0010	→ 2	
		13 bits	16 bits	
		Tag	Content	

Fetch BNE (1C)

address 0

BNE

address 2

10D7

address 4

ADD

address 6

DECR

```
LOOP:  ADD (R1)+,R0    02EC = 0000 0010 1110 1100    → 4
        DECR  R2       02EE = 0000 0010 1110 1110    → 6
        BNE LOOP      02F0 = 0000 0010 1111 0000    → 0
```

A03C	054E = 0000 0101 0100 1110 → 6	R0 = B6EC R1 = 0554 R2 = 0
05D9	0550 = 0000 0101 0101 0000 → 0	
10D7	0552 = 0000 0101 0101 0010 → 2	
	13 bits 16 bits	
	Tag Content	

*After 3rd pass in
loop 1 M + 3C*

address 0	<i>BNE</i>
address 2	<i>10D7</i>
address 4	<i>ADD</i>
address 6	<i>DECR</i>

Assume that the access time of the main memory is 10 t and that of cache is 1 t. Calculate the execution time for each pass, ignoring the time taken by the processor between memory cycles.

1 st pass	→	4 memory	→	40t
2 nd pass	→	2 memory, 2 cache	→	22 t
3 rd pass	→	1 memory, 3 cache	→	13 t

Repeat process assuming separate instruction and data cache. Compute speed of execution. Compare values to the ones obtained here with a single cache. Analyze and comment.

Another example (HVZ Exercise 5.6)

How are the total bits organized in a direct-mapped cache with:

Main memory = 64K words

Cache size = 1K words

Block size = 128 words

✓ **Memory = 64 K words = $2^6 \times 2^{10}$ words = 2^{16} words**

✓ **Cache = 1 K words = 2^{10} words**

How is the cache organized exactly?

Since block size = 128 words = 2^7 words, then:

2^{10} words / 2^7 words = $2^3 \Rightarrow 8$ blocks in cache

thus cache has 8 blocks, each containing 128 words



Exercise 5.6

Memory = 64 K words = $2^6 \times 2^{10}$ words = 2^{16} words

Cache = 1 K words = 2^{10} words

READ ONLY



16-bit address

tag

block

word

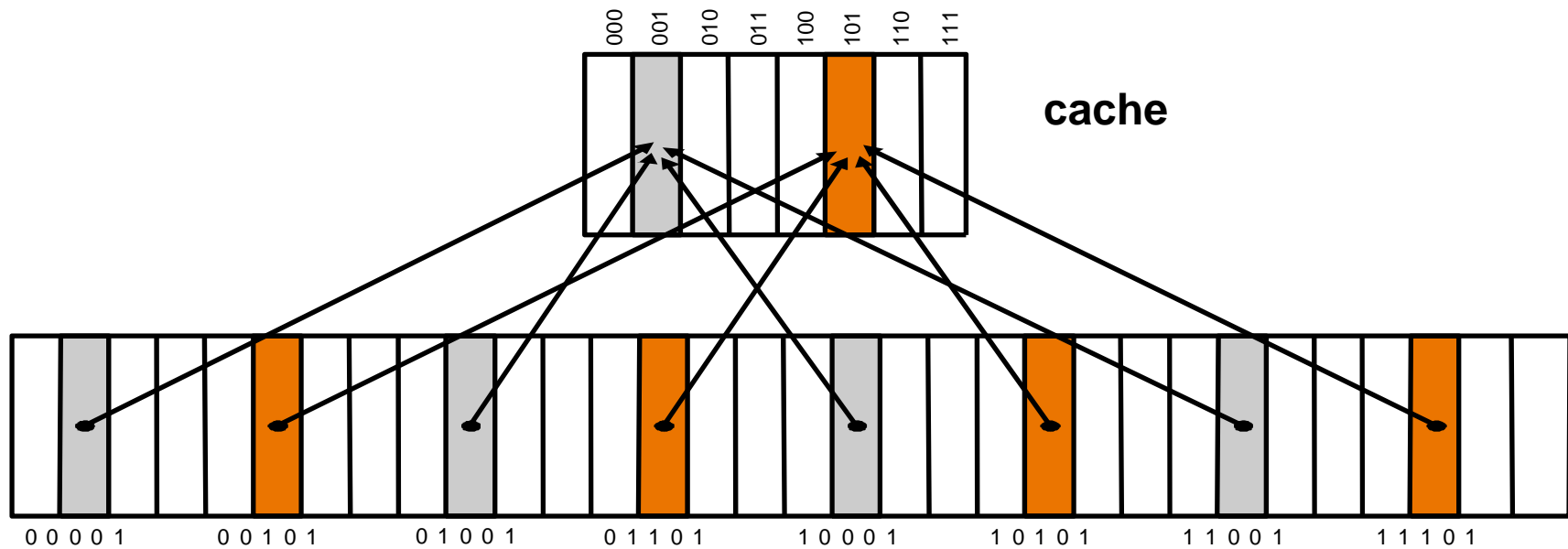
6 bits left for "tag"

3 bits for 8 blocks

7 bits for each 2^7
words in block

Index	V	Tag	Data
000	N		
001	N		
010	N		
011	N		
100	N		
101	N		
110	N		
111	N		

**Example of
hit/misses in
direct-mapped
cache**



Index	V	Tag	Data
000	N		
001	N		
010	N		
011	N		
100	N		
101	N		
110	N	10	Memory[10110]
111	N		

Binary address of reference	Hit or miss in cache	cache block (where)
10110	miss	110
11010	miss	010
10110	hit	110
11010	hit	010
10000	miss	000
00011	miss	011
10000	hit	000
10010	miss	010

Index	V	Tag	Data
000	N		
001	N		
010	N	11	Memory[11010]
011	N		
100	N		
101	N		
110	N	10	Memory[10110]
111	N		

Binary address of reference	Hit or miss in cache	cache block (where)
10110	miss	110
11010	miss	010
10110	hit	110
11010	hit	010
10000	miss	000
00011	miss	011
10000	hit	000
10010	miss	010

Index	V	Tag	Data
000	N		
001	N		
010	N	11	Memory[11010]
011	N		
100	N		
101	N		
110	N	10	Memory[10110]
111	N		

Binary address of reference	Hit or miss in cache	cache block (where)
10110	miss	110
11010	miss	010
10110	hit	110
11010	hit	010
10000	miss	000
00011	miss	011
10000	hit	000
10010	miss	010

Index	V	Tag	Data
000	N		
001	N		
010	N	11	Memory[11010]
011	N		
100	N		
101	N		
110	N	10	Memory[10110]
111	N		

Binary address of reference	Hit or miss in cache	cache block (where)
10110	miss	110
11010	miss	010
10110	hit	110
11010	hit	010
10000	miss	000
00011	miss	011
10000	hit	000
10010	miss	010

Index	V	Tag	Data
000	N	10	Memory[10000]
001	N		
010	N	11	Memory[11010]
011	N		
100	N		
101	N		
110	N	10	Memory[10110]
111	N		

Binary address of reference	Hit or miss in cache	cache block (where)
10110	miss	110
11010	miss	010
10110	hit	110
11010	hit	010
10000	miss	000
00011	miss	011
10000	hit	000
10010	miss	010

Index	V	Tag	Data
000	N	10	Memory[10000]
001	N		
010	N	11	Memory[11010]
011	N	00	Memory[00011]
100	N		
101	N		
110	N	10	Memory[10110]
111	N		

Binary address of reference	Hit or miss in cache	cache block (where)
10110	miss	110
11010	miss	010
10110	hit	110
11010	hit	010
10000	miss	000
00011	miss	011
10000	hit	000
10010	miss	010



Index	V	Tag	Data
000	N	10	Memory[10000]
001	N		
010	N	10	Memory[10010]
011	N	00	Memory[00011]
100	N		
101	N		
110	N	10	Memory[10110]
111	N		

Binary address of reference	Hit or miss in cache	cache block (where)
10110	miss	110
11010	miss	010
10110	hit	110
11010	hit	010
10000	miss	000
00011	miss	011
10000	hit	000
10010	miss	010



Think it through

How many total bits are required for a direct-mapped cache with 16KB of data and 4-word blocks, assuming a 32-bit address and word size = 4 bytes?

Think it through: read only

How many total bits are required for a direct-mapped cache with 16KB of data and 4-word blocks, assuming a 32-bit address and word size = 4 bytes?

$$16\text{KB} = 16 \times 1024 = 2^4 \times 2^{10} = \underset{\substack{\uparrow \\ \text{each of 4 bytes}}}{2^2} \times \underbrace{2^2 \times 2^{10}}_{\substack{\swarrow \\ \text{4K words}}}$$

2^{12} words \rightarrow with block size of 4 words $\rightarrow 2^{10}$ blocks

where each block has 4×32 bits = 128 bits, plus tag

tag = $32 - 10 - 4 = 18$ (10 is # blocks, 4 is #words in block)

Cache size = $2^{10} \times (128 + (18) + 1) = 2^{10} \times 147 = 147 \text{ K bits}$

or 18.4 KB for a 16 KB cache

\rightarrow 1.15 times the size of data \rightarrow 15% increase