(3) a) $P(D|\theta) = \theta^{\alpha_T}(1-\theta)^{\alpha_F}$ $\alpha_T = 7$ $\alpha_F = 3$

$\log P(D|\theta) = \alpha_T \log\theta + \alpha_F \log(1-\theta)$

$\frac{\partial}{\partial\theta} \log P(D|\theta) = \frac{\alpha_T}{\theta} - \frac{\alpha_F}{1-\theta} = 0$

$\frac{\alpha_T}{\theta} = \frac{\alpha_F}{1-\theta}$

$\frac{1-\theta}{\theta} = \frac{\alpha_F}{\alpha_T}$

$\frac{1}{\theta} - 1 = \frac{\alpha_F}{\alpha_T}$

$\frac{1}{\theta} = \frac{\alpha_F + \alpha_T}{\alpha_T} \Rightarrow \theta = \frac{\alpha_T}{\alpha_F + \alpha_T}$

$\Rightarrow \theta = \frac{7}{10}$

b) $P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)} \sim P(D|\theta)P(\theta)$

$\sim \theta^{\alpha_T}(1-\theta)^{\alpha_F}\,\theta^{\beta_1-1}(1-\theta)^{\beta_2-1}$

$P(X=T) = \frac{(\beta_1-1)}{[(\beta_1-1)+(\beta_2-1)]} = \frac{1}{2}$

$\frac{(4-1)}{[(4-1)+\beta_2-1]} = 1/2$

$\frac{3}{2+\beta_2} = \frac{1}{2} \Rightarrow 6 = 2+\beta_2$

$\beta_2 = 4$

$$P(\theta|D) = \theta^7(1-\theta)^3 \; \theta^{4-1}(1-\theta)^{4-1}$$

$$= \theta^{10}(1-\theta)^6$$

$$\log P(\theta|D) = 10 \log \theta + 6 \log(1-\theta)$$

$$\frac{\partial}{\partial \theta} \log P(\theta|D) = \frac{10}{\theta} - \frac{6}{1-\theta} = 0$$

$$\frac{10}{\theta} = \frac{6}{1-\theta}$$

$$\frac{1-\theta}{\theta} = \frac{6}{10} \Rightarrow \frac{1}{\theta} = \frac{6}{10} + \frac{10}{10}$$

$$\frac{1}{\theta} = \frac{16}{10}$$

$$\theta = \frac{10}{16}$$

(4) A) $$E(x) = \frac{1}{2N} \sum_{i=1}^{N} \left(y_i - w_0 - w_1 x_{i,1} - w_2 x_{i,2}^2\right)^2$$

B) $$\frac{\partial}{\partial w_0} E(x) = -\frac{1}{N} \sum_{i=1}^{N} \left(y_i - w_0 - w_1 x_{i,1} - w_2 x_{i,2}^2\right)$$

$$w_0 = w_0 + K \frac{\partial}{\partial w_0} E(x)$$

C) $$\frac{\partial}{\partial w_1} E(x) = -\frac{1}{N} \sum_{i=1}^{N} \left(y_i - w_0 - w_1 x_{i,1} - w_2 x_{i,2}^2\right) x_{i,1}$$

$$w_1 = w_1 - K \frac{\partial}{\partial w_1} E(x)$$

D) $\frac{\partial}{\partial w_2} E(x) = \frac{-1}{N} \sum_{i=1}^{N} (y_i - w_0 - w_1 x_{i,1} - w_2 x_{i,2}^2) x_{i,2}^2$

$w_2 = w_2 + K \frac{\partial}{\partial w_2} E(x)$