18.5 Standing waves in air columns

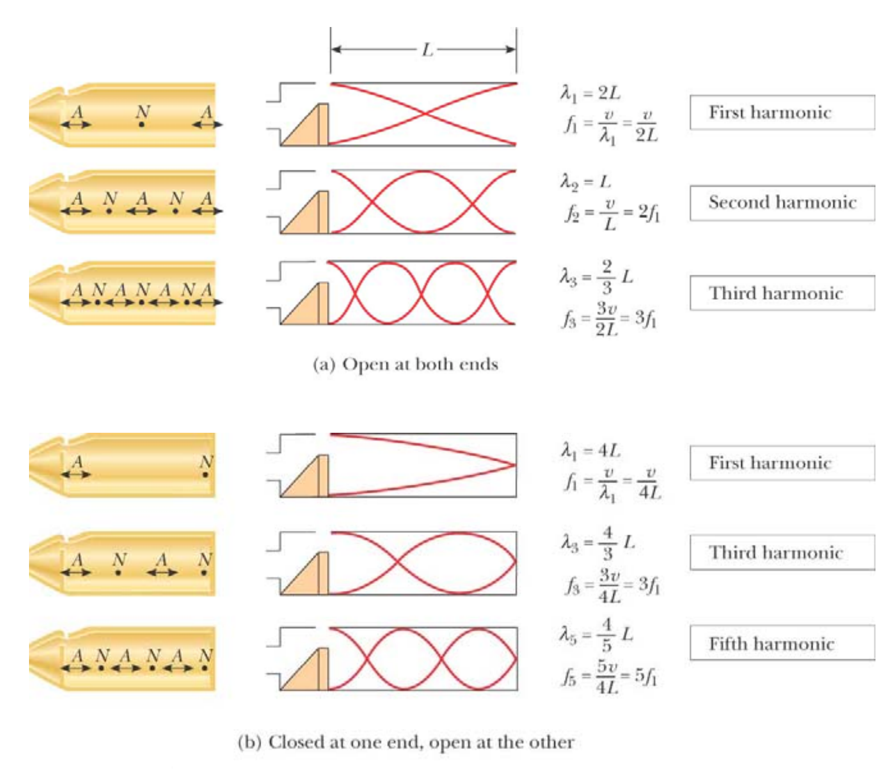
Natural frequencies of standing wave in pipes :

a) Both ends are open : any harmonic can exist.

$$f_n = n\,(v / 2L) \qquad \text{for } n = 1, 2, 3, \ldots$$

b) Only one end is open : only odd harmonics can exist.

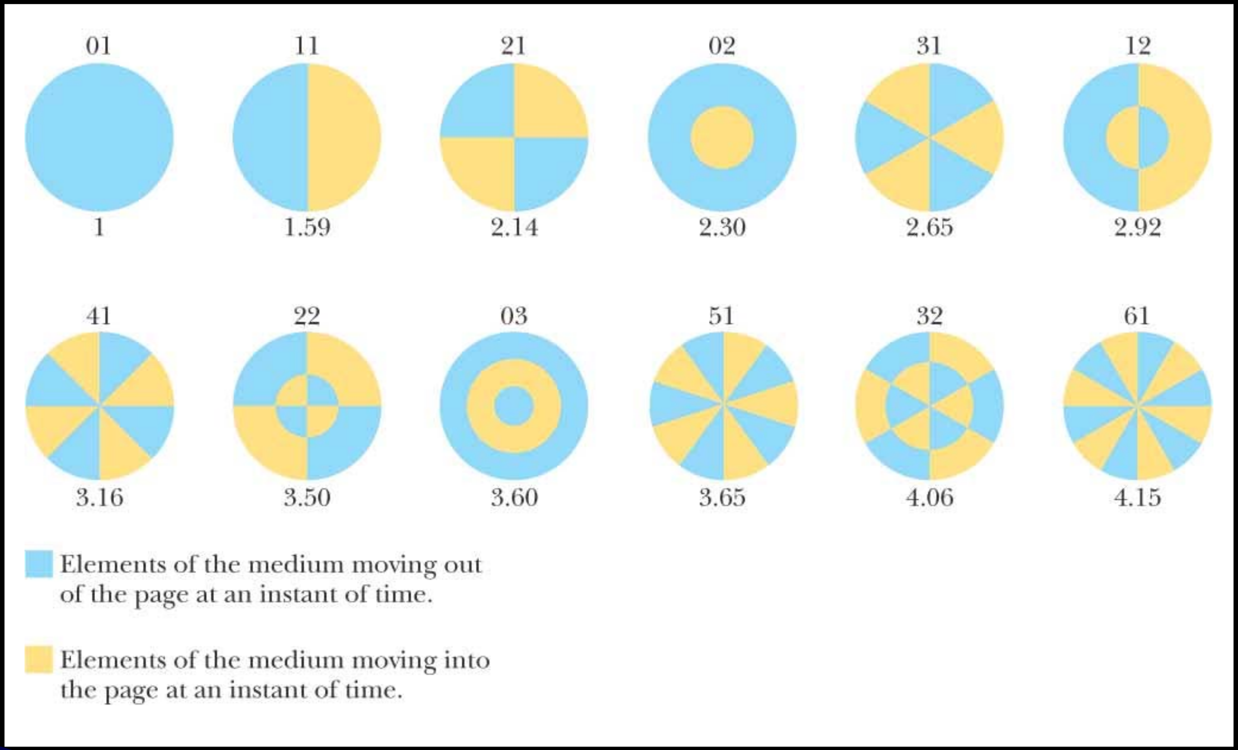
$$f_n = n\,(v / 4L) \qquad \text{for } n = 1, 3, 5, \ldots$$



In a pipe open at both ends, the harmonic series created consists of all integer multiples of the fundamental frequency: f_1, $2f_1$, $3f_1$, ...

In a pipe closed at one end and open at the other, the harmonic series created consists of only odd-integer multiples of the fundamental frequency: f_1, $3f_1$, $5f_1$, ...

18.6 Standing waves in membranes (*descriptive*)



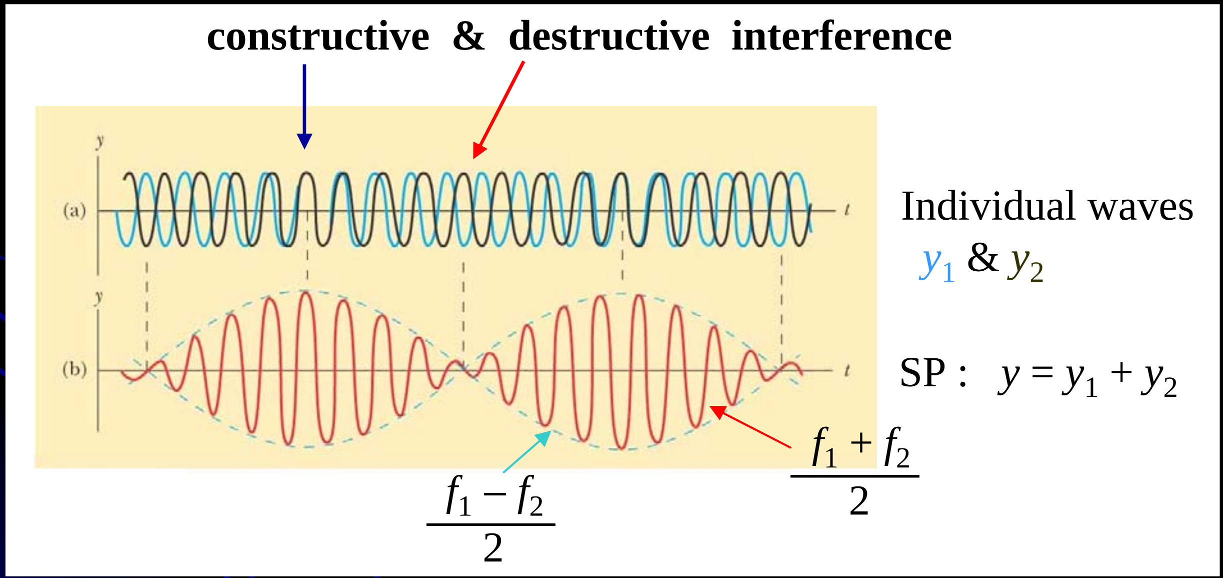
Representation of some of the normal modes possible in a circular membrane fixed at its perimeter. The pair of numbers above each pattern corresponds to the number of *radial nodes* and the number of *circular nodes*. In each diagram, elements of the membrane on either side of a nodal line move in opposite directions, as indicated by the colors.

18.7 Beats : Interference in time

- Spatial interference : SP of waves having same f. $\rightarrow$ Standing wave.
- Temporal interference : SP of waves having different f. $\rightarrow$ Beats.

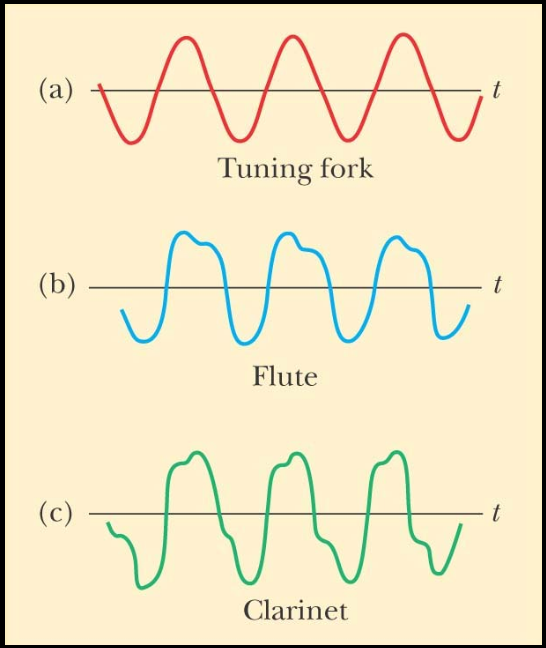
Definition of beating : Beating is the periodic variation in intensity at a given point due to the superposition of two waves having slightly different frequencies.

SP of two waves with slightly different f :



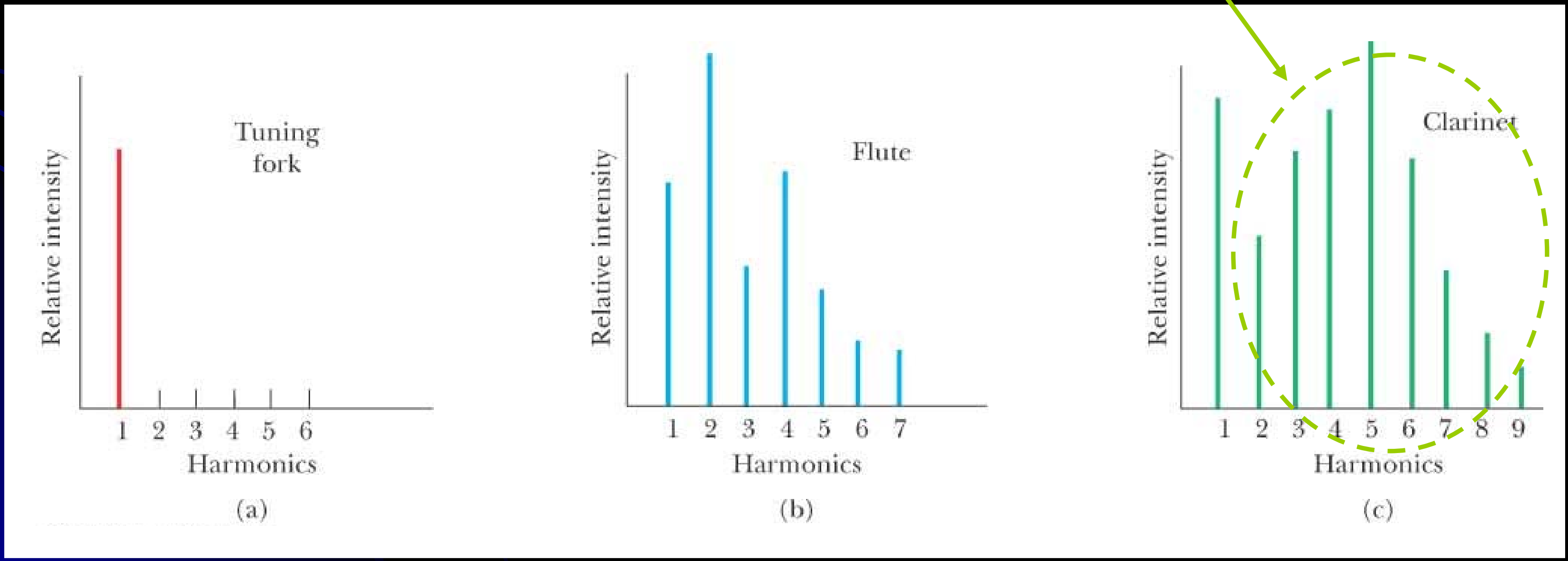
Beat frequency (*i.e.*, number of beats per second) : $\boxed{f_b = f_1 - f_2}$

18.8 Nonsinusoidal Wave Patterns (*descriptive*)



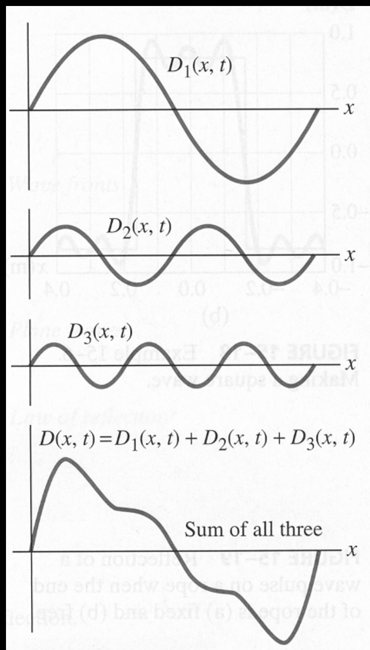
Sound wave patterns produced by (a) a tuning fork, (b) a flute, and (c) a clarinet, each at approximately the same frequency.

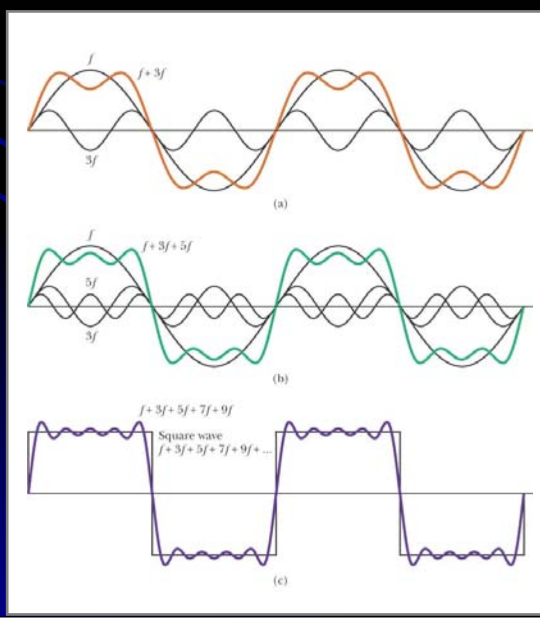
This mix of higher frequencies determines the instrument's unique sound.



- **Examples of the superposition principle :** Composite (*or* Complex) wave

(*A*)



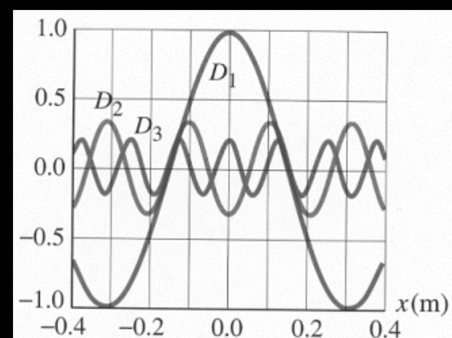


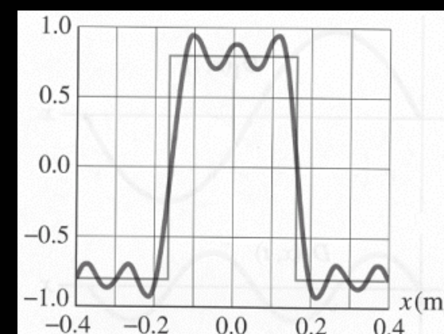
Fourier theorem :

$\Rightarrow$ Any complex wave can be considered as being composed of many simple sinusoidal waves of different amplitudes, wavelengths, and frequencies.

(*B*) Square wave :

$$y(x) = \cos(kx) + (1/3)\cos(3kx) + (1/5)\cos(5kx)$$





$\Leftarrow$ **Fourier synthesis of a square wave.**