Algorithm Ford-Fulkerson(G, s, t)

for each edge $(u,v) \in E$ **do** \\initialize

$f(u,v) \leftarrow 0; f(v,u) \leftarrow 0$

while there exists a path p from s to t in G_f **do**
\\find augmenting path

compute $c_f(p)$

for each edge (u,v) in p **do**

if $(u,v) \in E$ **then** $f(u, v) \leftarrow f(u,v) + c_f(p)$

else $f(v, u) \leftarrow -f(u,v)$

An altnate pseudocode for Algorithm Ford-Fulkerson(G, s, t)

Initialize f as zero-flow and residual network G_f with G

while there exists a path p from s to t in G_f **do**

Augment f using p

Update G_f

return f

Running time of Ford-Fulkerson

- Building the residual network
- Finding an augmenting path in the residual network
- How many augmenting paths can be found in the worst case?
    - value of maximum flow many (no more since the augmenting path has at least capacity 1)

Theorem: Ford-Fulkerson indeed computes a maximum flow

- To show this, we prove the Maxflow-mincut Theorem

st-cuts (continued)

- Recall: A *cut* in a (directed) graph is a partition of the vertices into two disjoint subsets.
- The *cut edges* of a graph with a cut are the edges that have one endpoint in each subset of the partition.
- An *st-cut* is a cut that places vertex s in one of its subsets and vertex t in the other.

st-cuts

- *Capacity* of an st-cut in an st-network: sum of the capacities of the cut's edges from the subset containing s to the subset containing t

- *Flow across* an st-cut in an st-network: difference between the sum of the flows of cut's edges from the subset containing s to the subset containing t and the sum of the flows of cut's edges from the subset containing t to the subset containing s

minimum st-cut problem (or *mincat* problem)

- Given an st-network, find an st-cut such that the capacity of no other cut is smaller.

Properties of feasible st-flows in st-flow networks

1. For any st-flow, the flow across each st-cut is equal to the value of the flow

2. The outflow from s is equal to the inflow to t

3. No st-flow's value can exceed the capacity of any st-cut

4. Let f be an st-flow and let (S,T) be an st-cut whose capacity equals $|f|$. Then f is a maximum flow and (S,T) is a minimum cut.

Maxflow-Mincut Theorem

- Let f be an st-flow. The following three conditions are equivalent:

  A. there exists an st-cut whose capacity equals $|f|$

  B. f is a maximum flow

  C. there is no augmenting path with respect to f

EdmondsKarp(G, s, t)

Initialize f as zero-flow and residual network G_f with G

while there exists a path p from s to t in G_f **do**

Let p be a path from s to t in G_f

Augment f using p

Update G_f

return f