CSC320 – Foundations of Computer Science
Sample Final

NAME: ______________________________ ID NO: ____________________________

INSTRUCTIONS

- Time allowed: 3 hours
- No calculators or other aids are allowed.
- Answer all questions in the space provided on the examination paper.
- This exam contains a cover page and 14 pages of questions. Please check now that all pages are present, and report any discrepancy to an invigilator immediately
- Scrap paper will be provided
- For questions 2, 4, 6 and 8, if you leave a part (a) question blank you will receive 2 marks for that part; if you leave a part (b) blank you will receive 1 mark for that part; and if you leave a part (c) blank you will receive $\frac{1}{2}$ mark for that part.
- Good luck!

	Out of	Mark
1	5	
2	10	
3	5	
4	10	
5	5	
6	10	
7	5	
8	10	
	60	

1. (5 MARKS) For each T/F question, circle the correct answer. For multiple choice questions, circle all the correct answers.

   (a) For any language L, L^* cannot be the empty set. T F

   (b) If M is a NFA with n states, then any DFA accepting $L(M)$ must have at least 2^n states. T F

   (c) If L is regular then $L' = \{w \mid wx \in L \text{ for some string } x\}$ must be regular. T F

   (d) If L is regular then $L' = \{ww \mid w \in L\}$ must be regular. T F

   (e) If L is regular then $L' = \{ww^R \mid w \in L\}$ must be regular. T F

   (f) If L is regular and $L' \subseteq L$, then L' must be regular. T F

   (g) If q_0 is the start state of an NFA with ϵ-transitions then the start state of an equivalent DFA is given by $E(q_0)$. T F

   (h) For any NFA M, there is an equivalent NFA M_0 with exactly one accept state. T F

   Let

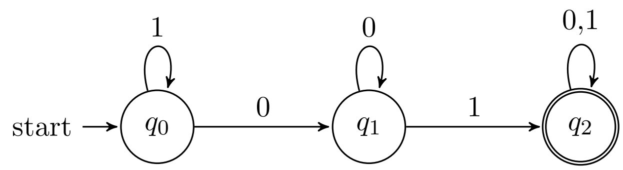
   $$D = \{w \mid w \text{ contains an equal number of occurrences of 01 and 10 as substrings}\}$$

   (i) Which of the following strings are in D?

   (a) 1001 (b) 101010 (c) 0111000 (d) 0101010 (e) None of the preceeding

   (j) D is regular. T F

2. Consider the following DFA $M = (Q, \Sigma, \delta, q_0, F)$:



   (a) (5 Marks) Give a regular expression for the set of strings that go from state q_0 to state q_2 passing only through state q_1:

   (b) (3 Marks) Draw the diagram of a machine for computing $L(M)^*$ which is obtained by adding *exactly* one state and three ϵ-transitions to M

(c) (2 Marks) Draw the diagram of a *deterministic* finite automaton that recognizes the language defined by the regular expression $(\mathtt{1}^*\mathtt{0}\mathtt{0}^*\mathtt{1}(\mathtt{0} \cup \mathtt{1})^*)^*$

3. (5 MARKS) For each T/F question, circle the correct answer.

   (a) If G is ambiguous, then some $w \in L(G)$ has more than one derivation. T F

   (b) If some $w \in L(G)$ has more than one derivation, then G is ambiguous. T F

   (c) For any ambiguous grammar G, there is an unambiguous grammar G' such that $L(G) = L(G')$. T F

   (d) If L is recognized by a DFA with n states then it is recognized by a PDA with n states. T F

   (e) Every CNF grammar is unambiguous T F

   (f) If L_1 is context-free and L_2 is regular, then $L_1 \cap L_2$ is regular. T F

   (g) If L_1 is context-free and L_2 is regular then $L_1 \cup L_2$ is regular T F

   (h) The CYK algorithm can decide whether or not $w \in L(G)$ in $O(n^3)$ steps, where n is the number of productions in G. T F

   (i) Which ONE of the following grammars generates $L((a(a \cup b)^* \cup b)^*)$ (circle just one answer)

      i.
      $$S \to SaU \mid Sb \mid \epsilon$$
      $$U \to a \mid b \mid Ua \mid Ub$$

      ii.
      $$S \to SX \mid \epsilon$$
      $$X \to aY \mid b$$
      $$Y \to UY \mid \epsilon$$
      $$U \to a \mid b$$

      iii.
      $$S \to SX$$
      $$X \to aY \mid b$$
      $$Y \to UY$$
      $$U \to a \mid b$$

      iv.
      $$S \to X | \epsilon$$
      $$X \to aY \mid b$$
      $$Y \to UY \mid \epsilon$$
      $$U \to a \mid b$$

      v. None of the above

   (j) Which of the following is an unambiguous grammar for the language of balanced parentheses (circle all the correct answers)

      i. $S \to (S) \mid (SS) \mid \epsilon$
      ii. $S \to (S)S \mid \epsilon$
      iii. $S \to (S) \mid SS \mid \epsilon$
      iv. $S \to (S) \mid SS \mid ()$
      v. None of the above

4. (a) (5 Marks) Convert the following grammar to CNF, using the algorithm presented in class: $S \to (S) \mid SS \mid \epsilon$. Show the grammar that results after each phase of the algorithm.

(b) (3 Marks) Is the CNF grammar produced in part (a) ambiguous? Why or why not?

(c) (2 Marks) Suppose we have any CNF grammar for the language $\{0^n1^n \mid n \geq 0\}$. How many steps will there be in the deriviation of 000111? Explain your answer.

5. (5 Marks) Circle the correct answer.

   (a) Define $f(w) = 1w$ (1 concatenated with w). Then f is a bijection between $\{0,1\}^*$ and the natural numbers. T F

   (b) There are uncountably many undecidable languages. T F

   (c) There is a languge L such that neither L nor $\overline{L}$ is recognizable. T F

   (d) If L_1 is decidable, and $L_2 \subseteq L_1$, then L_2 is decidable. T F

   (e) If L_1 is recognizable, and $L_2 \subseteq L_1$, then L_2 is recognizable. T F

   (f) If A is undecidable and $\overline{A}$ is not recognizable, then A must be recognizable. T F

   (g) If A is recognizable and $\overline{A}$ is recognizable, then A must be decidable T F

   (h) If A is recognizable and A is not decidable, then $\overline{A}$ is not recognizable. T F

   (i) A is decidable iff $A \leq_m \{\epsilon\}$ T F

   (j) If $A \leq_m \emptyset$, then A is decidable. T F

6. (a) (5 Marks) Suppose there is a TM E which enumerates the elements of the language L. Explain how we can use E to build a TM M which, on input w, will reach an accepting state iff $w \in L$

(b) (3 Marks) Let $2HALT_{TM}$ be the language defined as follows:

$$2HALT_{TM} = \{\langle M \rangle \mid M \text{ halts on exactly 2 inputs}\}$$

Use the undecidability of the $HALT_{TM}$ to prove that $2HALT_{TM}$ is undecidable.

i. Fill in the blanks to define the reduction f:
$f(\langle M, w \rangle) =$

On input x:

if ____________ **then** __________________________

else if ____________ **then** __________________________

else

ii. Prove that f is a reduction from $HALT_{TM}$ to $2HALT_{TM}$

(c) (2 Marks) Let $10CELL_{TM}$ be the language defined as follows:

$$10CELL_{TM} = \{\langle M, w\rangle \mid M \text{ writes on no more than 10 tape cells on input } w\}$$

Give a reduction which proves that $10CELL_{TM}$ is undecidable, or give a high-level description of a decision procedure for this language.

7. (5 MARKS) Suppose that L_1 and L_2 are languages over $\{0, 1\}$, and that L_1 is reducible to L_2, and that the reduction can be done by a TM with running time $O(n^2)$.

   (a) If L_2 is RE then L_1 is RE. T F

   (b) If L_1 is RE then L_2 is RE. T F

   (c) If L_2 is undecidable then L_1 is undecidable. T F

   (d) If L_2 is in P then L_1 is recursive. T F

   (e) If L_1 is NP-complete and L_2 is in NP, then L_2 is NP-complete. T F

   (f) If L_2 is in NP then L_1 is in P. T F

   (g) If L_1 is not RE, then L_2 is not in P. T F

   (h) If $\overline{L_2}$ is decidable, then L_1 is decidable. T F

   (i) If $\overline{L_2}$ is RE, then L_1 is RE. T F

   (j) Suppose that L_2 is decided by a TM with running time $O(n^3)$. Then L_1 is decided by a TM with running time $O(n^6)$. T F

8. (a) (5 Marks) Is CFL $\subseteq$ P? Give an argument supporting your answer.

   (b) (3 Marks) Is every language in NP decidable? Give an argument supporting your answer

(c) (2 MARKS) Suppose that ϕ is a 3-CNF expression which contains n variables. A truth assignment T for ϕ is *heavy* if it sets at least $\lfloor \frac{n}{2} \rfloor$ of the variables to `1`. For example, if ϕ is

$$(\overline{x}_1 \vee \overline{x}_2 \vee \overline{x}_3) \wedge (x_1 \vee x_2 \vee \overline{x}_4) \wedge (x_1 \vee \overline{x}_2 \vee x_3),$$

then E contains 4 variables: x_1, x_2, x_3, x_4. The truth assignment T which sets x_1, x_2 and x_4 to `1` and x_3 `0` is heavy, since it sets $\frac{3}{4}$ of the variables to `1`.

PROBLEM: Satisfiability by a heavy truth assignment (HEAVYSAT)

INPUT: A 3-CNF expression ϕ

OUTPUT: "Yes" if and only if ϕ is satisfied by a heavy truth assignment.

Prove that HEAVYSAT is NP-complete. (HINT: use a reduction from 3SAT which adds new variables and clauses. This isn't hard!)

END