3.2.2. S(A, B, C, D)

A → B, B → C, B → D

b) A is never in right hand side, hence it must be part of every candidate key.

	closure.
A B C D	A B C D
A C D	A B C D
A B D	A B C D
A D	A B C D
A B C	A B C D
A C	A B C D
A B	A B C D
A	A B C D

all superkeys.

A is the minimal

⇒ A is only candidate key.

c) Any superset of A (except {A}):

$$\left\{ \begin{array}{l} ABCD \\ ACD \\ ABD \\ AD \\ ABC \\ AC \\ AB \end{array} \right\}$$

T(A, B, C, D)

AB → C, BC → D, CD → A, AD → B.

b)

	A	B	C	D	Closure.
✓	A	B	C	D	A B C D
✓		B	C	D	B C D A
✓	A		C	D	A C D B.
✓			C	D	C D A B
✓	A	B		D	A B D C
		B		D	B D
✓	A			D	A D B C
				D	D
✓	A	B	C		A B C D
✓		B	C		B C D A
	A		C		A C
			C		C
✓	A	B			A B C D
		B			B
	A				A

Superkeys marked with ✓

Minimal are:

$\boxed{\underline{AD, CD, AB}, BC}$

Candidate Keys

c) Superkeys not candidate keys

A	B	C	D
A	B	C	D
	B	C	D
A		C	D
A	B		D
A	B	C	

U(A, B, C, D)

A → B, B → C, C → D, D → A

This one is easier. We can do it as the previous one (compute the closure of all combinations of attributes)

But in this case, all single attrs are CK:

$\{A\}^+ = \{A B C D\}$

$\{B\}^+ = \{A B C, D\}$

$\{C\}^+ = \{A B C D\}$

$\{D\}^+ = \{A B C D\}$

A, B, C, D are all candidate keys.

c) Any combination of attributes is a superkey. Hence superkeys not CKs:

ABCD
ABC
AB D
AB
A CD
A C
A D

BCD
BC
B D

CD

3.24.

a) Instance R(A,B,C) $A \to B$ does not imply $B \to A$

R

A	B	C
1	5	0
2	3	2
4	3	3
1	5	2

same A(1) implies same B(5)

$1 \to 5$
$2 \to 3$
$4 \to 3$

Value of 3 in B does not imply same value of A

$3 \to 2$ and $3 \to 3$.

You could also prove it formally

b) $\left\{ \begin{array}{l} AB \to C \\ A \to C \end{array} \right\}$ does not imply $B \to C$

R (A B C D)

A	B	C	D
1	2	3	4
1	2	3	$\cancel{0}$
2	3	5	4
2	2	5	9

$AB \to C$ $1,2 \to 3$

$A \to C \left\{ \begin{array}{l} 1 \to 3 \\ 2 \to 5 \end{array} \right.$

$AB \to C \left\{ \begin{array}{l} 2,3 \to 5 \\ 2,2 \to 5 \end{array} \right.$

$\}\ 2 \to 3$

$2 \to 5$

So $B \to C$ does not hold.!!

You could have prove it

$$\{B\}^+ = \{B\}^+ \Rightarrow B \not\to C$$

3.2.10

a) $AB \to D$ $\quad R(ABCDE)$

$C \to E$

$D \to C$

$E \to A$

Compute $S = \pi_{ABC} R$

ABC	$\cancel{ABC}\,\cancel{D}\,\cancel{E}$
AB	$\cancel{AB}\,\cancel{D}\,C\,\cancel{E}$
$A\ C$	$\cancel{AC}\,\cancel{E}$
A	$\cancel{A}$
BC	$\cancel{BC}\,\cancel{E}\,A\,\cancel{D}$
B	$\cancel{B}$
C	$\cancel{C}\,\cancel{E}\,A$

$\Rightarrow$ $AB \to C$

$BC \to A$

$C \to A$

Not minimal:

$BC \to A$ redundant (we can generate it using $C \to A$)

$$\Rightarrow \left\{ \begin{array}{l} AB \to C \\ C \to A \end{array} \right\}$$

This is a minimal basis.

b) $R(ABCDE)$

$A \to D$

$BD \to E$

$AC \to E$

$DE \to B$.

ABC	~~ABC~~ D E
AB	~~AB~~ D E
AC	~~A~~ B ~~C~~ D E
A	~~A~~ D
BC	~~BC~~
B	~~B~~
C	~~C~~

$AC \to B$

This is a minimal basis

---

c) $R(ABCDE)$

$AB \to D$

$AC \to E$

$BC \to D$

$D \to A$

$E \to B$

ABC	~~ABC~~ D E
AB	~~AB~~ D
AC	~~A~~ B ~~C~~ D E
A	~~A~~
BC	A ~~BC~~ D E
B	~~B~~
C	~~C~~

$AC \to B$

$BC \to A$

Minimal basis

3.3.1

a) $R(A,B,C,D)$

$AB \to C$, $C \to D$, $D \to A$

Not SK.

$\{AB\}^+ = \{ABCD\}$

$\{C\}^+ = \{CDA\}$

$\{D\}^+ = \{DA\}$ ⇒ Violations.

Decompose ABCD

$C \to D$

$\{C\}^+ = ACD$

ACD | CB ← BCNF.

Compute FDs.

~~ACD~~	~~ACD~~
AC	~~AC~~D
~~A D~~	~~AD~~
~~A~~	~~A~~
CD	~~CD~~A
C	~~C~~DA
D	~~D~~A

$AC \to D$ ✓

$CD \to A$ ✓

$C \to DA$ ✓

$D \to A$

Violation.

$\{D\}^+ = \{DA\}$

ACD

$D \to A$

AD | CD

BCNF

Decomposition:

AD, CD, CB.

b) $R(A, B, C, D)$

$B \to C$, $B \to D$

$\{B\}^+ = \{BCD\}$ B NOT a SK

$\Rightarrow$ Not BCNF

ABCD

$B \to C$

BCD $\{B\}^+ = BCD$

AB BCNF.

FDs.

~~BCD~~	~~BCD~~
BC	~~BC~~D
B D	~~BCD~~
B	~~B~~CD
~~CD~~	~~CD~~
~~C~~	~~C~~
~~D~~	~~D~~

$$\left.\begin{array}{l} BC \to D \\ BD \to C \\ B \to C \\ B \to D. \end{array}\right\} \text{BCNF.}$$

Decomposition: BCD, AB.

c) $R(A, B, C, D)$

$AB \to C$, $BC \to D$, $CD \to A$, $AD \to B$.

$\{AB\}^+ = \{ABCD\}$

$\{BC\}^+ = \{BCDA\}$

$\{CD\}^+ = \{CDAB\}$

$\{AD\}^+ = \{ADBC\}$

All are SKs.

$\Rightarrow$ Table already in BCNF