ANALYSIS OF ERROR CONTROL PROTOCOLS

Types of Error Control Protocols Considered

- We discuss three types of error control protocols all belonging to the family of Auto Repeat Request (ARQ):
  1. Stop-and-wait ARQ (SW ARQ)
  2. Go-back-N ARQ (GBN ARQ)
  3. Selective-Repeat ARQ (SR ARQ)

Automatic-Repeat-Request (ARQ)

- All ARQ techniques employ some kind of error detection coding of the transmitted data so that the receiver has the ability to detect the presence of errors.
- When an error is detected, the receiver requests a retransmission of the faulty data. ARQ techniques are simple to implement in hardware and they are especially effective when there is a reliable feedback channel connecting the receiver to the transmitter.

Stop-and-Wait ARQ (SW ARQ)

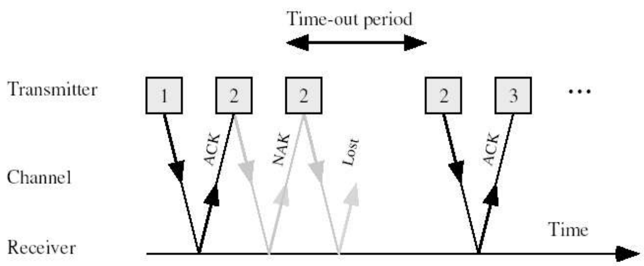
Stop-and-Wait ARQ (SW ARQ)

- Stop-and-wait (SW ARQ) protocol is a simple protocol, which is used when the round-trip time for a pair of packet/acknowledgment is close to the packet transmission time.
- The basic idea is to ensure that each packet has been received correctly before initiating transmission of the next packet.
- The required receiver buffer size is one packet only.

Stop-and-Wait ARQ (SW ARQ)

- When the sender transmits a packet on the forward channel, the receiver checks it for errors.
- If there are no errors, the receiver acknowledges the correct transmission by sending an acknowledge (ACK) signal through the feedback channel.
- In that case the transmitter proceeds to send the next packet.
- If there were errors in the received packet, the receiver sends a negative acknowledgment signal (NAK) and the sender sends the same packet again.
- If the sender does not receive ACK or NAK signals due to some problem in the feedback channel the sender waits for a certain timeout period and sends the packet again.

Stop-and-Wait ARQ (SW ARQ)

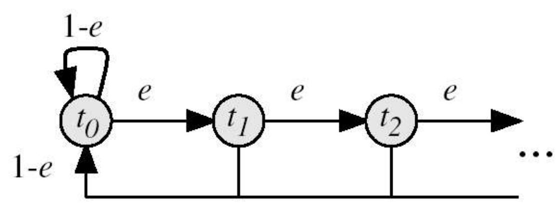


Modeling Stop-and-Wait ARQ

- We make the following assumptions for our analysis of the stop-and-wait ARQ (SW ARQ)

1. The average length of a packet is *n* bits.
2. The forward channel has random noise and the probability that a bit will be received in error is □. Another name for □ is bit error rate (BER).
3. The feedback channel is assumed noise free so that acknowledgment signals from the receiving station will always be transmitted to the sending station.
4. The sender will keep sending a packet until it is correctly received.

Modeling Stop-and-Wait ARQ



State t_i indicates i retransmission attempts

Modeling Stop-and-Wait ARQ

Probability that the packet contains one or more errors is given by

$$e = 1 - (1 - \epsilon)^n$$

$$e \approx \epsilon\, n \qquad \text{when} \qquad \epsilon\, n \ll 1$$

Modeling Stop-and-Wait ARQ

$$\mathbf{P} = \begin{bmatrix} 1-e & 1-e & 1-e & 1-e & \cdots \\ e & 0 & 0 & 0 & \cdots \\ 0 & e & 0 & 0 & \cdots \\ 0 & 0 & e & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Modeling Stop-and-Wait ARQ

At equilibrium the distribution vector is given by

$$\mathbf{s} = \begin{bmatrix} 1-e \\ (1-e)e \\ (1-e)e^2 \\ \vdots \end{bmatrix}$$

Stop-and-Wait ARQ Performance

The average number of retransmissions for a packet is given by

$$\begin{aligned} N_t &= s_1 + 2s_2 + 3s_3 + \cdots \\ &= \sum_{i=1}^{\infty} i\,(1-e)\,e^i \\ &= \frac{e}{1-e} \qquad \text{transmissions/packet} \end{aligned}$$

For a typical channel $e \approx \epsilon\, n \ll 1$ and the average number of transmissions can be approximated as

$$N_t \approx \epsilon\, n$$

Stop-and-Wait ARQ Performance

The efficiency of the SW ARQ protocol is the inverse of the total number of transmissions which includes the first transmission plus the average number of retransmissions.

$$\eta = \frac{1}{1+N_t} = 1 - e$$

For a typical channel $e \approx \epsilon\, n \ll 1$ and the average number of transmissions can be approximated as

$$\eta \approx 1 - \epsilon\, n$$

Example

Assume an SW ARQ protocol in which the packet size is $n = 1000$ and the bit error rate is $\epsilon = 10^{-4}$. Find the performance of the SW ARQ protocol for this channel. Repeat the example when the bit error rate increases by a factor of 10.

Solution:

Average number of retransmissions for a window is

$$N_t = 0.1052$$

and the efficiency is

$$\eta = 90.48\%$$

Notice that because the bit error rate is low, we need just about one transmission to correctly receive a packet.

Now we increase the bit error rate to $\epsilon = 10^{-3}$ and get the following results.

$$N_t = 1.7196$$

and the efficiency is

$$\eta = 36.77\%$$

Notice that when the bit error rate is increased by one order of magnitude, the average number of packet transmission is 2.7.

More on performance study

- What is the throughput in terms of packet/sec?
  - (1-e) packet per round trip time (sec)
- Consider Forward Error Correction (FEC) coding, such that up-to k-bit errors in a packet of size n bits can be corrected:

$$e = 1 - \sum_{i=0}^{k} \binom{n}{i} \epsilon^i (1-\epsilon)^{n-i}$$

Other practical issues

- How to consider re-transmission limit?
- How to consider ACK error?
- How to consider idle probability?
- ……

Go-Back-N ARQ (GBN ARQ)

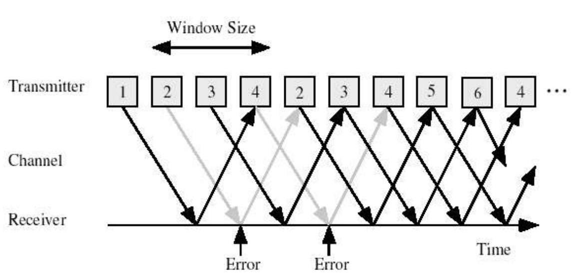
Go-Back-N (GBN ARQ) Protocol

- GBN Applications
  - Appears in various standard Data Link Control (DLC) Protocols, e.g., HDLC
  - The basis of error recovery procedure at the transport layer, e.g., in TCP protocol

Go-Back-N (GBN ARQ) Protocol

- Incoming packets to a transmitting DLC for a link from A to B are numbered sequentially.
- Sequence number (SN) is in the header of each frame.
- Several successive packets (within the transmission window) can be sent without waiting, and the transmitter keeps a copy in a buffer.
- Receiving DLC at B operates essentially the same as SW ARQ; it accepts packets in the correct order and sends acknowledgments back to A.
- The required receiver buffer size is one packet.
- Go-Back-N: N is a parameter. Not allow (i+N)-th packet being sent out before receiving an ACK of the i-th packet.

Go-Back-N (GBN ARQ) Protocol



Go-back-N ARQ protocol with buffer size $N = 3$. Solid arrows indicate ACK signals and grey arrows indicate NAK signals.

Go-Back-N (GBN ARQ) Protocol

We make the following assumptions for our analysis of the go-back-N protocol.

1. Each window contains N packets.
2. The average length of a packet is n bits.
3. The forward channel has random noise and the probability that a bit will be received in error is ϵ. Another name for ϵ is bit error rate (BER)
4. The feedback channel is assumed noise free so that acknowledgment signals from the receiving station will always be transmitted to the sending station.
5. The maximum number of retransmissions is k_m after which, the sender will declare the channel to be not functioning.

Modeling GBN ARQ Protocol

We can represent the state of the sender as a Markov chain having the following properties:

1. The states of the Markov chain are grouped into the sets $\mathcal{T}$, $\mathcal{R}_1$, $\mathcal{R}_2$, etc. These sets are explained below.
2. The number of states is infinite since no upper bound is placed on the number of retransmissions.
3. The time step is taken equal to the sum of transmission delay of one packet $T = \tau_t$. Thus a window that contains N packets will require N time steps to be transmitted.

Modeling GBN ARQ Protocol

The set $\mathcal{T}$ represents the states of the sender while it is transmitting a window for freshly arrived N packets.

$$\mathcal{T} = \left\{ \begin{array}{cccc} t_1 & t_2 & \cdots & t_N \end{array} \right\}$$

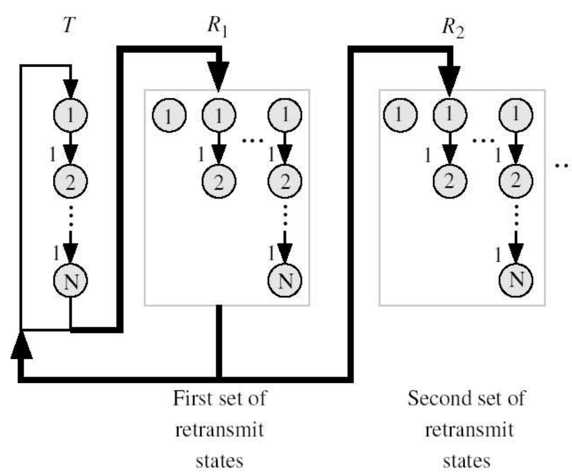
Modeling GBN ARQ Protocol

The set $\mathcal{R}_1$ represents the states of the sender while it is retransmitting packets for the first time. This set is the union of several subsets

$$\mathcal{R}_1 = R_{1,1} \cup R_{1,2} \cup \cdots \cup R_{1,N}$$

where subset $R_{1,i}$ is the subset of $\mathcal{R}_1$ that contain i states corresponding to retransmitting i packets for the first time.

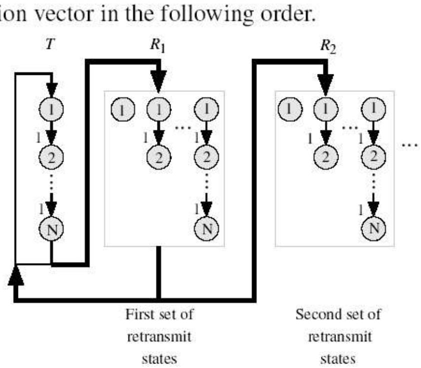
Modeling GBN ARQ Protocol



Modeling GBN ARQ Protocol

We organize the state distribution vector in the following order.

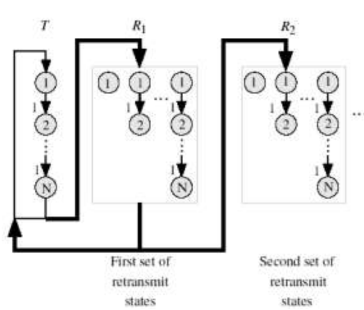
$$\mathbf{s} = \left[\begin{array}{cccc} \mathcal{T} & \mathcal{R}_1 & \mathcal{R}_2 & \cdots \end{array} \right]^t$$



Modeling GBN ARQ Protocol

For the case when $N = 2$ the distribution vector can be written as

$$\mathbf{s} = \left[\begin{array}{ccccccccccc} t & t & | & r_{1,1} & r_{1,2} & r_{1,2} & | & r_{2,1} & r_{2,2} & r_{2,2} & | & \cdots \end{array} \right]^t$$



Modeling GBN ARQ Protocol

The corresponding transition matrix of the sender for the case when $N = 2$ is given by

$$\mathbf{P} = \begin{bmatrix} 0 & p_{2,0} & p_{1,0} & 0 & p_{2,0} & p_{1,0} & 0 & p_{2,0} & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & p_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & p_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & p_{1,1} & 0 & p_{2,1} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & p_{2,2} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

GBN ARQ Protocol Performance

We can solve for $\mathbf{s}$ using iterations since the size of the matrix is large for any practical case.

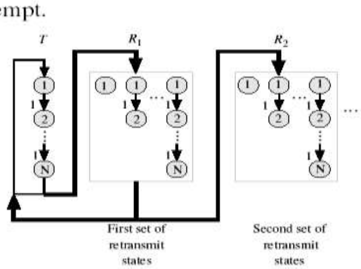
1. Assign to each transmit state some value, for example
$$t = 1 \qquad (1)$$
2. Estimate the retransmit states for $\mathcal{R}_1$ using the iterative expressions.
3. Estimate the values of the other retransmit states $\mathcal{R}_k$ using the iterative expressions.
4. Find the sum of all states
$$\mathcal{S} = N\, t + \sum_{k=1}^{K_m} \sum_{j=1}^{N} j\, r_{k,j}$$
5. Normalize the distribution vector.

GBN ARQ Protocol Performance

The probability that the source is in the k-th retransmission state is

$$\alpha_k = \sum_{j=1}^{N} j\, r_{k,j}$$

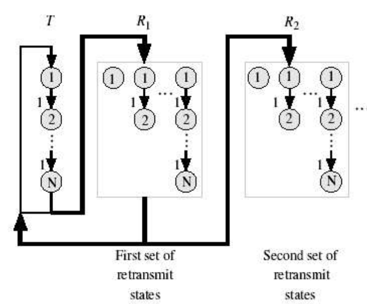
α_k also represents the average number of packets sent at the k-th retransmission attempt.



GBN ARQ Protocol Performance

The average number of retransmission is given by

$$R_k = \sum_{k=1}^{k_m} k \alpha_k$$



GBN ARQ Protocol Performance

The delay associated with the k-th retransmission is given by

$$t_k = \sum_{j=1}^{k} \alpha_j$$

The average delay for transmitting a given frame is given by

$$T_a = \sum_{k=1}^{k_m} t_k\, \alpha_k$$

GBN ARQ Protocol Performance

The average number of packets sent due to all retransmissions is given by

$$N_a = \sum_{k=1}^{k_m} n_k\, \alpha_k$$

where n_k is the average number of packets sent at the k-th retransmission attempt.

GBN ARQ Protocol Performance

The efficiency of the GBN ARQ protocol is the ratio of frame size to the total number of packets transmitted:

$$\eta = \frac{N}{N + N_a}$$

When there are no errors $N_a = 0$ and we get 100% efficiency.

Example

Assume a GBN ARQ protocol with the following parameters.

$$\begin{aligned} n &= 500 && \text{bits} \\ N &= 8 && \text{packets} \\ \epsilon &= 10^{-4} \\ K_m &= 16 \end{aligned}$$

Find the performance of the GBN ARQ protocol for this channel.

Solution:

The average number of retransmissions for a given frame is

$$R_a = 0.2195$$

he average delay for a given frame is

$$T_a = 0.0306$$

the average number of retransmitted packets for a given frame is

$$N_a = 0.026$$

and the efficiency is

$$\eta = 99.68\%$$

Alternative models

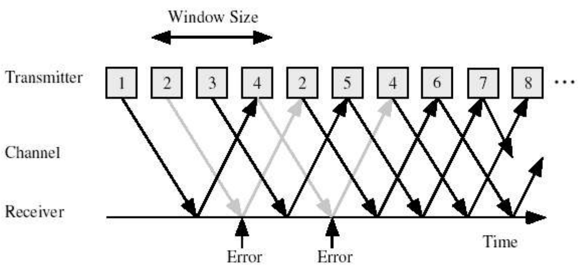
- Delayed-ACK: the receiver transmits a single ACK for the whole window of transmissions
- Immediate-ACK: the receiver transmits one ACK/NACK for each arrival
- Considering the above two cases, how to obtain the throughput performance (packet/sec)?

Selective Repeat ARQ (SR-ARQ)

Selective-Repeat (SR ARQ) Protocol

- Transmitter groups the packets into windows of N packets.
- When the sender sends packets within a window, the receiver stores the packets of the current window and checks for errors.
- After a complete window has been received, or after the proper timeout period, the receiver instructs the transmitter to resend only the packets that contained errors.
- That results in a more efficient protocol compared to GBN ARQ that resends packets in error as well as error-free packets.

SR-ARQ Protocol



Selective-repeat ARQ protocol with buffer size $N = 3$. Solid arrows indicate ACK signals and grey arrows indicate NAK signals.

Modeling SR-ARQ Protocol

We can use the model developed for GBN ARQ protocol.

The transition probability $p_{i,j}$ is the probability that j packets need to be retransmitted given that a frame of i packets was sent. For instance $p_{5,2}$ indicates the probability that two packets have to be retransmitted given that five packets were sent.

$p_{i,j}$ is given by the expression

$$p_{i,j} = \binom{i}{j} (1 - e)^{i-j} e^j$$

Hybrid ARQ

- In advanced wireless systems (LTE/WiMax/ UMTS/...), hybrid ARQ (HARQ) is used, using both Forward Error Correction (FEC) and ARQ
  - Some FEC codes can both detect and correct error, e.g., Reed-Solomon code
- Type I HARQ (simplest), using FEC to correct error, ED to detect error and then ARQ
- Type II HARQ, retransmit FEC+ED, and followed by msg+ED if needed
- HARQ with soft combining, i.e., combining multiple copies of the same bit received