CSC 225 B02 - SUMMER 2014 - QUIZ 2

Name:
Student Number:

Question 1 [10 marks]

Solve the following recurrence equation to get a closed-formula for $T(n)$. Assume that n is a power of four. It is not necessary to prove your closed form.

$$\begin{aligned} T(n) &= 1 \text{ if } n = 1 \\ &= T\left(\frac{n}{4}\right) + 2 \text{ if } n \geq 4 \end{aligned}$$

Either of the two answers below is acceptable.

$$\begin{aligned} T(n) &= T\left(\frac{n}{4}\right) + 2 \\ &= \left[T\left(\frac{n}{16}\right) + 2\right] + 2 \\ &= \left[T\left(\frac{n}{64}\right) + 2\right] + 2 + 2 \\ &\vdots \\ &= T\left(\frac{n}{4^i}\right) + 2i \end{aligned}$$

Taking $i = \log_4(n)$ gives $T(n) = T\left(\frac{n}{4^{\log_4(n)}}\right) + 2\log_4(n) = T(1) + 2\log_4(n) = 1 + 2\log_4(n)$.

Using the substitution $n = 4^k$,

$$\begin{aligned} T(n) &= T\left(\frac{n}{4}\right) + 2 \\ T(4^k) &= T\left(\frac{4^k}{4}\right) + 2 \\ &= T\left(4^{k-1}\right) + 2 \\ &= \left[T\left(4^{k-2}\right) + 2\right] + 2 \\ &= \left[T\left(4^{k-3}\right) + 2\right] + 2 + 2 \\ &\vdots \\ &= T\left(4^{k-i}\right) + 2i \end{aligned}$$

Taking $i = k$ gives $T(4^k) = T\left(4^{k-k}\right) + 2k = T(1) + 2k$. Since $k = \log_4(n)$, this gives $T(n) = 1 + 2\log_4(n)$.

Question 2 [10 marks]

The pseudocode below gives a recursive algorithm for computing the maximum value in an array. Let $T(n)$ denote the running time of the algorithm on an array of size n, where n is a power of 3. Assume that the function MAX on line 14 runs in constant time.

```
1:  procedure RecursiveMax(A, n)
2:      if n = 1 then
3:          return A[0]
4:      end if
5:      Create arrays A1, A2, A3 of size n/3
6:      for i ← 0, ..., n/3 do
7:          A1[i] ← A[i]
8:          A2[i] ← A[(n/3) + i]
9:          A3[i] ← A[(2n/3) + i]
10:     end for
11:     m1 ← RecursiveMax(A1, n/3)
12:     m2 ← RecursiveMax(A2, n/3)
13:     m3 ← RecursiveMax(A3, n/3)
14:     return Max(m1, m2, m3)
15: end procedure
```

(a) Fill in the blanks below to complete the recurrence relation for $T(n)$:

$$\begin{aligned} T(n) &= 1 \text{ if } n = 1 \\ &= 3T\left(\frac{n}{3}\right) + 2 \text{ if } n \geq 3 \end{aligned}$$

(b) True or False?

(i) $T(n) \in \omega(1)$: **True**
(ii) $T(n) \in O(\log n)$: **False**
(iii) $T(n) \in \Omega(n)$: **True**
(iv) $T(n) \in o(3^n)$: **True**

(c) Give a non-recursive function $f(n)$ such that

$$T(n) \in \Theta(f(n))$$

$$f(n) = n \log n$$