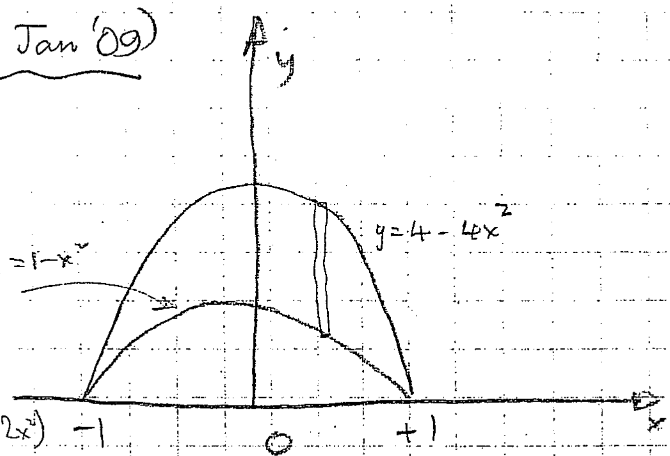
SOLNS; MATH 101 Assgt #1 (Jan '09)

1 a) Intersection Pts:

$4-4x^2 = 1-x^2$

$3 = 3x^2 \Rightarrow x = \pm 1$



$$V = 2\int_0^1 \pi\left[(4-4x^2)^2-(1-x^2)^2\right]dx$$

$16-32x^2+16x^4-(1+x^4-2x^2)$

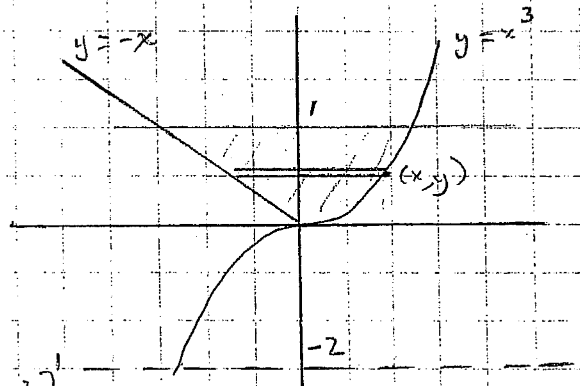
$$= 2\int_0^1 15\pi\left[1-2x^2+x^4\right]dx = \underline{\underline{16\pi}}$$

b) $$V = 2\int_0^1 \pi\left[(5-4x^2)^2-(2-x^2)^2\right]dx = 2\int_0^1 3\pi\left[7-12x^2+5x^4\right]dx$$

$$= 6\pi\left[7x-4x^3+x^5\right]_0^1 = \underline{\underline{24\pi}}$$

2.

$$\Delta V_i = 2\pi(y^{1/3}+y)(y+2)\,dy$$



$$V = \int_0^1 2\pi(y+2)(y^{1/3}+y)\,dy$$

$$= 2\pi\int_0^1 \left[y^{4/3}+2y^{1/3}+y^2+2y\right]dy$$

$$= 2\pi\left[\frac{3}{7}y^{7/3}+\frac{3}{2}y^{4/3}+\frac{y^3}{3}+y^2\right]_0^1 = \underline{\underline{137\pi/21}}$$

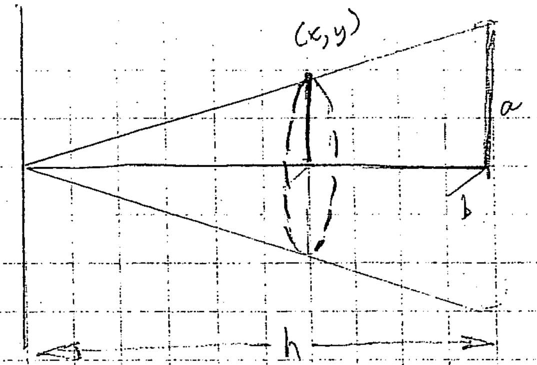
3. $$y = \frac{x^4+48}{24x} = \frac{x^3}{24}+\frac{2}{x} \quad \& \quad \left[1+(y')^2\right]^{1/2} = \left[1+\left(\frac{x^2}{8}-\frac{2}{x^2}\right)^2\right]^{1/2}$$

$$S = \int_2^3 \sqrt{1+(y')^2}\,dx \qquad = \sqrt{1+\frac{x^4}{64}-\frac{1}{2}+\frac{4}{x^4}} = \sqrt{\frac{1}{64x^4}(x^8+32x^4+256)}$$

so $$S = \int_2^3 \frac{1}{8x^2}(x^4+16)\,dx = \frac{1}{8}\int_2^3\left(x^2+\frac{16}{x^2}\right)dx = \frac{1}{8}\left[\frac{x^3}{3}-\frac{16}{x}\right]_2^3 =$$

4.

$$\Delta V_i = \pi \frac{a}{h} x \cdot \frac{b}{h} x \, dx$$

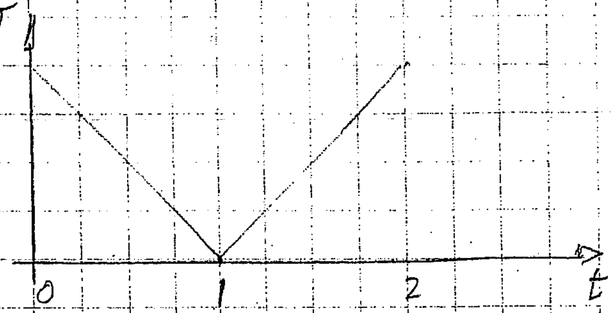


$$V = \int_0^h \pi \frac{ab}{h^2} x^2 dx$$

$$= \pi \frac{ab}{h^2} \cdot \left[\frac{x^3}{3}\right]_0^h = \frac{1}{3} \pi ab h$$

5. a) $\int_0^2 v(t) dt = \int_0^1 (-1)(t-1) dt + \int_1^2 (t-1) dt$

$= 1$



b) $\int_0^2 |v(t)| dt = \int_0^2 v(t) dt = 1.$