Definition of an algorithm

Hilbert's tenth problem: (1900) Is there an algorithm to determine if any multivariate polynomial (with integer co-efficients) has an integer root? But no clear definition of algorithm until Turing machines and similar systems.

No, in 1970. The language $D = \{p \mid p \text{ is a polynomial with an integral root}\}$ is undecidable.

But it is *recognizable*. Try every possible integer. If you knew bounds for the possible values then it would be decidable.

Coding Structured Inputs to TMs

- TM Input $=$ string
- We would like to consider TMs that take other objects, such as numbers, graphs, automata, TMs (and combinations thereof) as inputs
- We can *represent* these objects as strings
- For an object O we write $\langle O \rangle$ to denote its representation
- For a sequnce $O_1, \ldots, O_k$ we write $\langle O_1, \ldots, O_k \rangle$ to denote its representation
- When a TM expecting a certain type of object gets a string w is input, it must first check whether the string has the right format (i.e. $w = \langle O \rangle$ for an object O) and then must *decode* it into some internal representation

Describing a TM to test for graph connectivity

- A TM which decides the language $A = \{\langle G \rangle \mid G \text{ is a connected undirected graph}\}$.
- The *encoding* of a graph $\langle G \rangle$ is a list of nodes followed by a list of edges, e.g., $\langle G \rangle = (1, 2, 3, 4), ((1, 2), (2, 3), (1, 3), (1, 4))$
- Easiest might be to assume a three tape machine, with the input on the first tape.
- M checks that the input has the correct format
- IDEA: Tape 1 stores the graph. Tape 2 stores the queue of nodes to explore in a BFS. Tape 3 is a work tape.
- M copies the first node's name on Tape 2. While Tape 2 is not blank:
  1. M scans each unmarked edge on tape 1 and tries to match an endpoint of the edge to the front of the queue (leftmost node name on the second tape).
  2. If it matches, the edge is marked and M puts the other endpoint at the right end of Tape 2's list.
  3. When M completes this scan, it removes the node at the start of the queue and marks it on Tape 1.
- M scans Tape 1 to see if every node is marked. If it is M accepts; else it rejects.