

Question 1 [10 marks]

Give a mathematical proof of the identity

$$\sum_{i=1}^n 2i = n^2 + n$$

for all integers $n \geq 1$.

Basis

When $n = 1$, $\sum_{i=1}^1 2i = 2 = 1^2 + 1$, so the identity holds.

Induction Hypothesis

Suppose the identity holds for some $n \geq 1$.

Induction Step

Consider $n + 1$.

$$\begin{aligned}\sum_{i=1}^{n+1} 2i &= 2 \cdot 1 + 2 \cdot 2 + \dots + 2 \cdot n + 2 \cdot (n + 1) \\ &= \left[\sum_{i=1}^n 2i \right] + 2(n + 1) \\ &= n^2 + n + 2(n + 1) \quad (\text{By the induction hypothesis}) \\ &= n^2 + n + 2n + 2 \\ &= (n^2 + 2n + 1) + (n + 1) \\ &= (n + 1)^2 + (n + 1)\end{aligned}$$

Therefore, the identity holds for $n + 1$, and, by induction, for all $n \geq 1$.

Question 2 [10 marks]

Arrange the following 5 functions into ascending order by their asymptotic complexity (with asymptotically smaller functions first):

$$n \log_2(n), \quad 2^{50}, \quad 2^{100}n, \quad \sqrt{5n}, \quad \log_2(n^5)$$

Write your answer in the spaces below.

$$2^{50} \quad \log_2(n^5) \quad \sqrt{5n} \quad 2^{100}n \quad n \log_2(n)$$