## CSC 225 SPRING 2012 ALGORITHMS AND DATA STRUCTURES I MIDTERM EXAMINATION UNIVERSITY OF VICTORIA

1.	Student ID:	
2.	Name:	

- 3. DATE: 10 February 2012 DURATION: 50 MINUTES INSTRUCTOR: V. SRINIVASAN
- 4. THIS QUESTION PAPER HAS EIGHT PAGES (INCLUDING THE COVER PAGE).
- 5. THIS QUESTION PAPER HAS FOUR QUESTIONS.
- 6. ALL ANSWERS TO BE WRITTEN ON THIS EXAMINATION PAPER.
- 7. THIS IS A CLOSED BOOK EXAM. NO AIDS ARE ALLOWED.
- 8. KEEP YOUR ANSWERS SHORT AND PRECISE.

Q1(5)	
Q2(5)	
Q3(5)	
Q4(5)	
TOTAL(20) =	

- 1. The three parts of Question 1 test the basics of asymptotic analysis.
  - (a) State the definition of Big Oh. [1 Mark]

(b) Order the following functions by order of growth starting with the slowest. [2 Marks]

 $5n, (\log n)^5, n^5, 5, 5^n.$ 

(c) Consider the following sum:  $S(n) = \sum_{i=1}^{n} \log i$ . Give a simple function f(n) so that the sum S(n) is in  $\Theta(f(n))$ . Briefly explain why. [2 marks]

- 2. The two parts of Question 2 test our knowledge about solving **recurrence equations**.
  - (a) Solve the following recurrence equation to get a closed-formula for T(n). Assume the n is a power of two. [2.5 Marks]

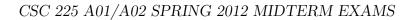
$$T(n) = 5 \text{ if } n = 1$$
$$= 2T(\frac{n}{2}) + 2n \text{ if } n \ge 2$$

(b) Using induction, prove that your closed formula from part (a) is correct. [2.5 Marks]

- 3. The three parts of Question 3 check if we understand **basic data structures** such as Priority Queues and Dictionaries.
  - (a) The priority queue ADT has many possible implementations, each with its own running time for the two operations, insertItem and removeMin. Complete the following table [1.5 Marks].

Priority Queue Implementation	Time for insertItem	Time for removeMin
Unsorted Array		
Sorted Array		
Heaps		

(b) Suppose you are given an input sequence  $S = [16\ 4\ 22\ 18\ 6\ 32]$ . Show how a heapsort algorithm runs on input S is built by constructing the heap at the end of each step of the algorithm.  $[2\ \text{Marks}]$ .



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(c) Define the load factor  $\alpha$  of a hash table with n keys and N slots. How is this related to the running time of an unsuccessful search when we use hashing with separate chaining [1.5 Marks]? Explain briefly.

- 4. Question 4 is based on **sorting algorithms**.
  - (a) The following table compares the time and space requirements of sorting algorithms on an input of size N. [2 Marks]

Algorithm	Worst-Case Running Time	Minimum Extra Space Needed
Selection Sort		
Quick Sort		
Merge Sort		
Heap Sort		

(b) Suppose that we are given an array A with n keys and k inversions. Here, an inversion is defined as a pair of entries that are out of order in the array. What is the running time of INSERTION SORT when it is used to sort A in Big Oh notation? Why? [1.5 Marks]

(b) Show how merge-sort algorithm works on the following input sequence using the merge-sort tree.  $[1.5~{\rm Marks}]$ 

$$S = [12\ 86\ 34\ 62\ 6\ 55\ 92\ 47]$$