UNIVERSITY OF VICTORIA

THIS IS A PRACTICE EXAM

MATHEMATICS 211 MATRIX ALGEBRA CRN 22613

Name:________________________________ Student ID:______________

TO BE ANSWERED ON THE PAPER AND ON THE BUBBLE SHEET Duration: 3 hours

Instructor: M. Laca

Question	Value	Marks
1-8	32	
9	5	
10	5	
11	5	
12	5	
13	5	
14	5	
15	5	
Total	67	

THIS EXAM HAS 4 PAGES PLUS THIS COVER SHEET.

YOU MUST COUNT THE NUMBER OF PAGES IN THIS PAPER BEFORE BEGINNING TO WRITE AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

INSTRUCTIONS:

1. **Enter your** $< lastname\ firstname >$ **and student number (omit the V00 at the beginning) and FILL IN THE CORRESPONDING BUBBLES on the green bubble sheet NOW.**
2. Your **name** and **student number** must also be recorded on the **front and back** of this examination and you must sign the coloured back page.
3. The exam consists of two parts: **Part I** has 8 multiple choice questions worth 4 marks each; the answers to these multiple-choice questions should be filled out on the bubble sheet supplied. **Part II** has 7 long-answer questions worth 5 marks each. All solutions should appear clearly explained on the exam paper and your final answer to each long-answer question should be clearly stated and boxed, circled or underlined. Use the back of the page if necessary for your calculations.
4. You may only use a Sharp EL-510R calculator or its newer version EL-510RNB. **No headphones and no cell phones** are allowed during the final examination.
5. This is a practice test to help you get used to the format and, to a limited extent, to represent the content of the actual final exam. Please do not take things too literally. The final exam will be different from the practice exam, and even if I had tried, I would not have been to able make them entirely similar. A list of the sections covered in the course is posted in Moodle. Make sure you review the HWs and both Midterms.

1. Let $M = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ and $K = \begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$. Then $(KM)^T =$

A. $\begin{bmatrix} 1 & -1 \\ -5 & 5 \end{bmatrix}$ B. $\begin{bmatrix} 6 & -6 \\ 3 & -3 \end{bmatrix}$ C. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ F. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ G. $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$ H. Not defined.

I. None of the above

2. Let M be a 4×4 matrix with $\det M = 5$. Then $\det(-2M) =$

A. -160 B. -80 C. -40 D. -20 E. -10
F. 10 G. 20 H. 40 I. 80 J. None of the above

3. Let $u = \begin{bmatrix} 1 \\ 7 \\ 0 \\ 7 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix}$, $w = \begin{bmatrix} 3 \\ 0 \\ 1 \\ 3 \end{bmatrix}$. Compute $(\boldsymbol{u} + \boldsymbol{w}) \cdot (3\boldsymbol{v} - \boldsymbol{w})$.

A. -20 B. -19 C. -10 D. 0 E. 60
F. 100 G. 120 H. 140 I. 160 J. None of the above

4. Calculate the projection of the vector $\vec{u} = \begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}$ onto the vector $\vec{v} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$.

A. $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ B. $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ C. $\begin{bmatrix} 1/3 \\ -1/3 \\ -1/6 \end{bmatrix}$ D. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ E. $\begin{bmatrix} 1/3 \\ -1/3 \\ 1/3 \end{bmatrix}$

F. $\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$ G. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ H. $\begin{bmatrix} 4/3 \\ -4/3 \\ -2/3 \end{bmatrix}$ I. $\begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$ J. None of the above

5. Let W be the subspace of $\mathbb{R}^3$ spanned by vectors $w_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, $w_2 = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$.

   Find a basis for $W^\perp$

   A. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ C. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ D. $\begin{bmatrix} 4 \\ 1 \\ 4 \end{bmatrix}$ E. $\begin{bmatrix} 1 \\ -10 \\ -4 \end{bmatrix}$

   F. $\begin{bmatrix} 0 \\ 1 \\ 1/\sqrt{2} \end{bmatrix}$ G. $\begin{bmatrix} -1/3 \\ 1/3 \\ 1 \end{bmatrix}$ H. $\begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$ I. $\begin{bmatrix} 1/3 \\ 1 \\ -1/3 \end{bmatrix}$ J. None of the above

6. If $\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$, for which values of k is the vector $w = \begin{bmatrix} k \\ 2 \\ 1 \end{bmatrix}$ in the linear span of $\{\vec{u}_1, \vec{u}_2\}$?

   A. -3 B. -2 C. -1 D. $-1/3$ E. 0
   F. $1/3$ G. 1 H. 2 I. 3 J. None of the above

7. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - 2y + z \\ 5x + 3z \\ 4x + y + 2z \end{bmatrix}$

   What is the second **column** (written as a row vector to save space) of the standard matrix of the linear transformation T?

   A. $(1, -2, 1)$ B. $(-2, 0, 1)$ C. $(1, 3, 2)$ D. $(1, 5, 4)$ E. $(5, 0, 3)$

   F. $(4, 1, 2)$ G. $(x, -2y, z)$ H. $(x, 5x, 4x)$ I. $(x, 0, x)$ J. None of the above

8. If $\mathcal{B} = \left\{ \vec{v}_1 = \begin{bmatrix} 2/\sqrt{6} \\ 1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \right\}$ is an orthonormal basis of $\mathbb{R}^3$, find $[\vec{w}]_B$, the coordinate vector of $\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ with respect to B.

A. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ B. $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ C. $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ D. $\begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$ E. $\begin{bmatrix} 1/\sqrt{6} \\ 5/\sqrt{2} \\ -2/\sqrt{3} \end{bmatrix}$

F. $\begin{bmatrix} 1/\sqrt{6} \\ 5/\sqrt{2} \\ 2/\sqrt{3} \end{bmatrix}$ G. $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ H. $\begin{bmatrix} \sqrt{6} \\ \sqrt{2} \\ \sqrt{3} \end{bmatrix}$ I. $\mathcal{B}$ is not an orthonormal basis.

J. None of the above

Long-answer questions:

9. Find the standard matrix of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that reflects the points with respect to the line $y = -x$.

10. Use Cramer's rule to find z in the solution of the system $2x + y + 4z + 2t = 1$; $2y + z + 2t = 1$; $2y + z + t = 0$; $2x + 2z = 0$.

11. The vectors $x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$ form a basis for a subspace W of $\mathbb{R}^3$.

Apply the Gram-Schmidt Process to obtain an orthogonal basis for W.

12. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & -1 \\ 0 & 2 & -2 \end{bmatrix}$, $b = \begin{bmatrix} -5 \\ 1 \\ 4 \end{bmatrix}$, and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. With this information, find:

(a) the solution to the system of linear equations $Ax = b$, using row reduction;

(b) a basis for the row space of A;

(c) a basis for the nullspace of A;

(d) a basis for the column space of A;

(e) the rank of the matrix A.

13. Give the factors L and U in the LU decomposition of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 4 \end{bmatrix}$, and use it to solve $A\vec{x} = \vec{b}$ for $\vec{b} = \begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix}$ (by solving first $L\vec{c} = \vec{b}$ and then $U\vec{x} = \vec{c}$).

14. The matrix $A = \begin{bmatrix} 5 & -1 \\ 2 & 2 \end{bmatrix}$ has eigenvectors $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

    (a) Is A similar to the diagonal matrix $D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$?

    (b) Find A^{10}.

15. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

    (a) Find the eigenvalues of A.

    (b) For each of the eigenvalues from part (a) find a basis for a corresponding eigenspace.

    (c) Is A diagonalizable?

[END]