Chapter 1

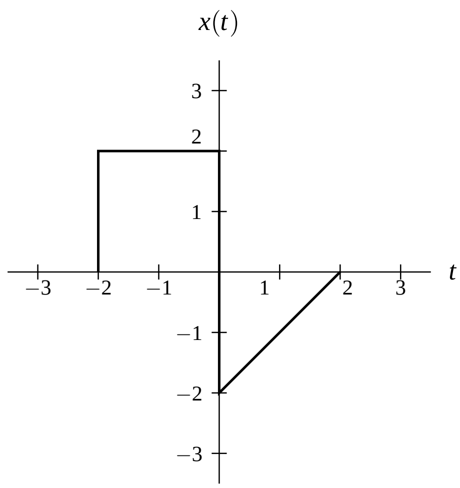
Continuous-Time Signals and Systems (Chapter 2)

2.1 Identify the time and/or amplitude transformations that must be applied to the signal $x(t)$ in order to obtain each of the signals specified below. Choose the transformations such that time shifting precedes time scaling and amplitude scaling precedes amplitude shifting. Be sure to clearly indicate the order in which the transformations are to be applied.
(a) $y(t) = x(2t-1)$;
(b) $y(t) = x(\frac{1}{2}t+1)$;
(c) $y(t) = 2x(-\frac{1}{2}t+1)+3$;
(d) $y(t) = -\frac{1}{2}x(-t+1)-1$; and
(e) $y(t) = -3x(2[t-1])-1$.

Solution.

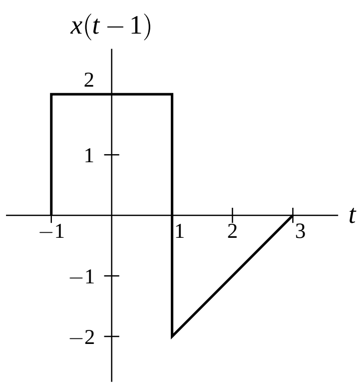
(e) To obtain $y(t)$ from $x(t)$, we apply the following transformations: 1) time shift by 2 (i.e., shift to the right by 2), 2) time scale by 2 (i.e., compress horizontally by 2), 3) amplitude scale by -3 (i.e., expand vertically by 3 and invert), and 4) amplitude shift by -1 (i.e., shift down by 1).

2.3 Given the signal $x(t)$ shown in the figure below, plot and label each of the following signals:
(a) $x(t-1)$;
(b) $x(2t)$;
(c) $x(-t)$;
(d) $x(2t+1)$; and
(e) $\frac{1}{4}x(-\frac{1}{2}t+1)-\frac{1}{2}$.

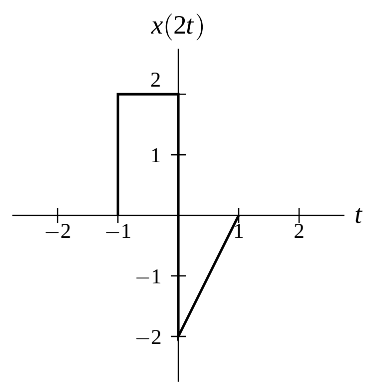


Solution.

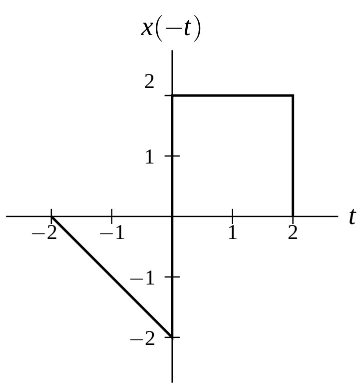
(a)



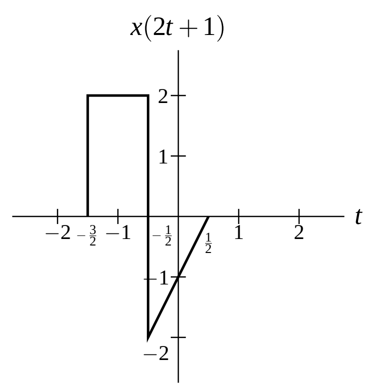
(b)



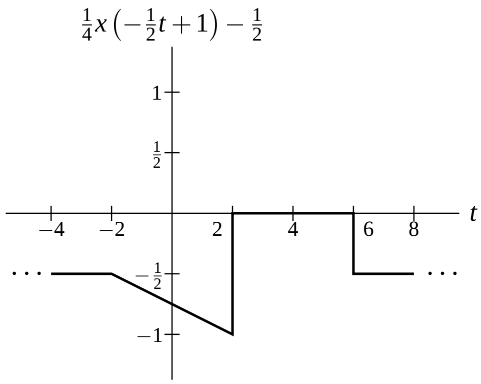
(c)



(d)



(e)



2.4 Determine whether each of the following functions is even, odd, or neither even nor odd:
(a) $x(t) = t^3$;
(b) $x(t) = t^3|t|$;
(c) $x(t) = \left|t^3\right|$;
(d) $x(t) = (\cos 2\pi t)(\sin 2\pi t)$;
(e) $x(t) = e^{j2\pi t}$; and
(f) $x(t) = \frac{1}{2}[e^t + e^{-t}]$.

Solution. (a) From the definition of $x(t)$, we have

$$\begin{aligned} x(-t) &= (-t)^3 \\ &= -t^3 \\ &= -x(t). \end{aligned}$$

Since $x(t) = -x(-t)$ for all t, $x(t)$ is odd.

(e) From the definition of $x(t)$, we have

$$\begin{aligned} x(-t) &= e^{-j2\pi t} \\ &= \cos(-2\pi t) + j\sin(-2\pi t) \\ &= \cos 2\pi t - j\sin 2\pi t. \end{aligned}$$

Thus, we can clearly see that $x(-t) \neq x(t)$ and $x(-t) \neq -x(t)$. Therefore, $x(t)$ is neither even nor odd.

(f) From the definition of $x(t)$, we have

$$\begin{aligned} x(-t) &= \tfrac{1}{2}[e^{-t} + e^{-(-t)}] \\ &= \tfrac{1}{2}[e^t + e^{-t}] \\ &= x(t). \end{aligned}$$

Thus, we can clearly see that $x(-t) = x(t)$ for all t. Therefore, $x(t)$ is even.

2.5 Prove each of the following assertions:
(a) The sum of two even signals is even.
(b) The sum of two odd signals is odd.
(c) The sum of an even signal and an odd signal is neither even nor odd.
(d) The product of two even signals is even.
(e) The product of two odd signals is even.
(f) The product of an even signal and an odd signal is odd.

Solution.

(a) Let $y(t) = x_1(t) + x_2(t)$ where $x_1(t)$ and $x_2(t)$ are even. From the definition of $y(t)$, we have

$$y(-t) = x_1(-t) + x_2(-t).$$

Since $x_1(t)$ and $x_2(t)$ are even, we have that $x_1(-t) = x_1(t)$ and $x_2(-t) = x_2(t)$. So, we can simplify the above expression for $y(t)$ as

$$\begin{aligned} y(-t) &= x_1(t) + x_2(t) \\ &= y(t). \end{aligned}$$

Thus, $y(t) = y(-t)$. Therefore, $y(t)$ is even.

(c) Let $y(t) = x_1(t) + x_2(t)$, where $x_1(t)$ and $x_2(t)$ are even and odd signals, respectively. From the definition of $y(t)$, we have

$$y(-t) = x_1(-t) + x_2(-t).$$

Since $x_1(t)$ is even, $x_1(-t) = x_1(t)$. Similarly, since $x_2(t)$ is odd, $x_2(-t) = -x_2(t)$. Using these observations, we can simplify the above expression for $y(t)$ as

$$\begin{aligned} y(-t) &= x_1(t) + [-x_2(t)] \\ &= x_1(t) - x_2(t). \end{aligned}$$

Clearly, $y(-t) \neq y(t)$ and $y(-t) \neq -y(t)$. Thus, $y(t)$ is neither even nor odd.

(f) Let $y(t) = x_1(t)x_2(t)$, where $x_1(t)$ and $x_2(t)$ are even and odd signals, respectively. From the definition of $y(t)$, we have

$$y(-t) = x_1(-t)x_2(-t).$$

Since $x_1(t)$ is even, $x_1(-t) = x_1(t)$. Similarly, since $x_2(t)$ is odd, $x_2(-t) = -x_2(t)$. Using these observations, we can simplify the above expression for $y(t)$ as

$$\begin{aligned} y(-t) &= x_1(t)[-x_2(t)] \\ &= -x_1(t)x_2(t) \\ &= -y(t). \end{aligned}$$

Thus, we have that $y(t) = -y(-t)$. Therefore, $y(t)$ is odd.

2.8 Suppose $h(t)$ is a causal signal and has the even part $h_e(t)$ given by

$$h_e(t) = t[u(t) - u(t-1)] + u(t-1) \quad \text{for } t > 0.$$

Find $h(t)$ for all t.

Solution.

We have that

$$\text{for } t > 0: \quad \begin{aligned} h_e(t) &= t[u(t) - u(t-1)] + u(t-1) \\ &= tu(t) + (-t+1)u(t-1). \end{aligned}$$

Since $h_e(t)$ is even, we can deduce that

$$\text{for } t < 0: \quad \begin{aligned} h_e(t) &= h_e(-t) \\ &= (-t)u(-t) + (t+1)u(-t-1). \end{aligned}$$

Let $h_o(t)$ denote the odd part of $h(t)$. Since $h(t)$ is causal and $h(t) = h_e(t) + h_o(t)$, we know that

$$\text{for } t < 0: \quad h_o(t) = -h_e(t).$$

Using this observation, we can deduce that

$$\begin{aligned} \text{for } t < 0: \quad h_o(t) &= -h_e(t) \\ &= -[(-t)u(-t) + (t+1)u(-t-1)] \\ &= (t)u(-t) + (-t-1)u(-t-1). \end{aligned}$$

Since $h_o(t)$ is odd, we have that

$$\begin{aligned} \text{for } t > 0: \quad h_o(t) &= -h_o(-t) \\ &= -[(-t)u(t) + (t-1)u(t-1)] \\ &= tu(t) + (-t+1)u(t-1). \end{aligned}$$

Therefore, we can conclude that

$$\begin{aligned} h(t) &= h_e(t) + h_o(t) \\ &= [tu(t) + (-t+1)u(t-1)] + [tu(t) + (-t+1)u(t-1)] \\ &= (2t)u(t) + (2-2t)u(t-1). \end{aligned}$$

2.9 Determine whether each of the signals given below is periodic. If the signal is periodic, find its fundamental period.
(a) $x(t) = \cos 2\pi t + \sin 5t$;
(b) $x(t) = [\cos(4t - \frac{\pi}{3})]^2$;
(c) $x(t) = e^{j2\pi t} + e^{j3\pi t}$; and
(d) $x(t) = 1 + \cos 2t + e^{j5t}$.

Solution.

(a) Let T_1 and T_2 denote the periods of $\cos 2\pi t$ and $\sin 5t$, respectively. We have

$$T_1 = \tfrac{2\pi}{2\pi} = 1 \quad \text{and} \quad T_2 = \tfrac{2\pi}{5}.$$

Since T_1/T_2 is irrational, $x(t)$ is not periodic.

(b) Since $\cos\left(4t - \frac{\pi}{3}\right)$ is periodic and the square of a periodic function is periodic, $x(t)$ is periodic. From the definition of $x(t)$, we can write

$$\begin{aligned} x(t) &= \cos^2\left(4t - \tfrac{\pi}{3}\right) \\ &= \left[\tfrac{1}{2}\left(e^{j(4t-\pi/3)} + e^{-j(4t-\pi/3)}\right)\right]^2 \\ &= \tfrac{1}{4}\left[e^{j(4t-\pi/3)(2)} + 2 + e^{-j(4t-\pi/3)(2)}\right] \\ &= \tfrac{1}{4}\left[e^{j(8t-2\pi/3)} + e^{-j(8t-2\pi/3)} + 2\right] \\ &= \tfrac{1}{4}[2\cos(8t - 2\pi/3) + 2] \\ &= \tfrac{1}{2}\cos(8t - 2\pi/3) + \tfrac{1}{2}. \end{aligned}$$

Thus, the period T of $x(t)$ is $T = \frac{2\pi}{8} = \frac{\pi}{4}$.

(c) Let T_1 and T_2 denote the periods of $e^{j2\pi t}$ and $e^{j3\pi t}$, respectively. We have

$$T_1 = \tfrac{2\pi}{2\pi} = 1, \quad T_2 = \tfrac{2\pi}{3\pi} = \tfrac{2}{3}, \quad \text{and} \quad \frac{T_1}{T_2} = \frac{1}{2/3} = \tfrac{3}{2}.$$

Since T_1/T_2 is rational, $x(t)$ is periodic. The period T of $x(t)$ is $T = 2T_1 = 2$.

2.10 Evaluate the following integrals:
(a) $\int_{-\infty}^{\infty} \sin\left(2t+\frac{\pi}{4}\right)\delta(t)dt$;
(b) $\int_{-\infty}^{t} [\cos\tau]\delta(\tau+\pi)d\tau$;
(c) $\int_{-\infty}^{\infty} x(t)\delta(at-b)dt$ where a and b are real constants and $a>0$;
(d) $\int_{0}^{2} e^{j2t}\delta(t-1)dt$; and
(e) $\int_{-\infty}^{t} \delta(\tau)d\tau$.

Solution.

(a) From the sifting property of the unit-impulse function, we have

$$\begin{aligned}\int_{-\infty}^{\infty} \sin\left(2t+\tfrac{\pi}{4}\right)\delta(t)dt &= \left[\sin\left(2t+\tfrac{\pi}{4}\right)\right]\Big|_{t=0} \\ &= \sin\left(\tfrac{\pi}{4}\right) \\ &= \tfrac{1}{\sqrt{2}}.\end{aligned}$$

(b) From the sifting property of the unit-impulse function, we have

$$\begin{aligned}\int_{-\infty}^{t} [\cos\tau]\delta(\tau+\pi)d\tau &= \begin{cases} [\cos\tau]|_{\tau=-\pi} & \text{for } t>-\pi \\ 0 & \text{for } t<-\pi \end{cases} \\ &= \begin{cases} \cos(-\pi) & \text{for } t>-\pi \\ 0 & \text{for } t<-\pi \end{cases} \\ &= \begin{cases} -1 & \text{for } t>-\pi \\ 0 & \text{for } t<-\pi \end{cases} \\ &= -u(t+\pi).\end{aligned}$$

(c) We use a change of variable. Let $\lambda = at$ so that $t=\lambda/a$ and $d\lambda = adt$. Performing the change of variable and simplifying yields

$$\begin{aligned}\int_{-\infty}^{\infty} x(t)\delta(at-b)dt &= \begin{cases} \int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)\left(\frac{1}{a}\right)d\lambda & \text{for } a>0 \\ \int_{\infty}^{-\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)\left(\frac{1}{a}\right)d\lambda & \text{for } a<0 \end{cases} \\ &= \begin{cases} \frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)d\lambda & \text{for } a>0 \\ -\frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)d\lambda & \text{for } a<0 \end{cases} \\ &= \tfrac{1}{|a|}\int_{-\infty}^{\infty} x\left(\tfrac{\lambda}{a}\right)\delta(\lambda-b)d\lambda \\ &= \tfrac{1}{|a|}\left[x\left(\tfrac{\lambda}{a}\right)\right]\Big|_{\lambda=b} \\ &= \tfrac{1}{|a|}x\left(\tfrac{b}{a}\right).\end{aligned}$$

(d) Since the nonzero part of $\delta(t-1)$ is contained on the interval $[0,2]$, we can deduce from the sifting property that

$$\begin{aligned}\int_{0}^{2} e^{j2t}\delta(t-1)dt &= \left[e^{j2t}\right]\Big|_{t=1} \\ &= e^{j2}.\end{aligned}$$

(e) We have

$$\begin{aligned}\int_{-\infty}^{t} \delta(\tau) d\tau &= \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases} \\ &= u(t).\end{aligned}$$

Chapter 12

MATLAB (Appendix E)

E.101 Indicate whether each of the following is a valid MATLAB identifier (i.e., variable/function name):
(a) `4ever`
(b) `$rich$`
(c) `foobar`
(d) `foo_bar`
(e) `_foobar`

Solution.

(a) A MATLAB identifier cannot begin with a numeric character. Thus, `4ever` is not a valid identifier.

(b) A MATLAB identifier cannot contain the `$` character. Thus, `$rich$` is not a valid identifier.

(c) The name `foobar` is a valid identifier.

(d) The name `foo_bar` is a valid identifier.

(e) A MATLAB identifier cannot begin with an underscore character. Thus, `_foobar` is not a valid identifier.

E.102 Let T_C, T_F, and T_K denote the temperature measured in units of Celsius, Fahrenheit, and Kelvin, respectively. Then, these quantities are related by

$$T_F = \tfrac{9}{5}T_C + 32 \quad \text{and}$$
$$T_K = T_C + 273.18.$$

Write a program that generates a temperature conversion table. The first column of the table should contain the temperature in Celsius. The second and third columns should contain the corresponding temperatures in units of Fahrenheit and Kelvin, respectively. The table should have entries for temperatures in Celsius from -50 to 50 in steps of 10.

Solution.

The temperature conversion table can be produced with the following code:

```
display(sprintf('%-8s %-8s %-8s', 'Celsius', 'Fahrenheit', 'Kelvin'));
for celsius = -50 : 10 : 50
    fahrenheit = 9 / 5 * celsius + 32;
    kelvin = celsius + 273.18;
    display(sprintf('%8.2f %8.2f %8.2f', celsius, fahrenheit, kelvin));
end
```

The code produces the following output:

```
Celsius  Fahrenheit Kelvin
  -50.00   -58.00   223.18
  -40.00   -40.00   233.18
  -30.00   -22.00   243.18
  -20.00    -4.00   253.18
  -10.00    14.00   263.18
    0.00    32.00   273.18
   10.00    50.00   283.18
   20.00    68.00   293.18
   30.00    86.00   303.18
   40.00   104.00   313.18
   50.00   122.00   323.18
```

E.106 Suppose that the vector `v` is defined by the following line of code:

```
v = [0 1 2 3 4 5]
```

Write an expression in terms of `v` that yields a new vector of the same dimensions as `v`, where each element t of the original vector `v` has been replaced by the given quantity below. In each case, the expression should be as short as possible.
(a) $2t-3$;
(b) $1/(t+1)$;
(c) t^5-3; and
(d) $|t|+t^4$.

Solution.

(a) `2 * v - 3`
(b) `1 ./ (v + 1)`
(c) `v .^ 5 - 3`
(d) `abs(v) + v .^ 4`

