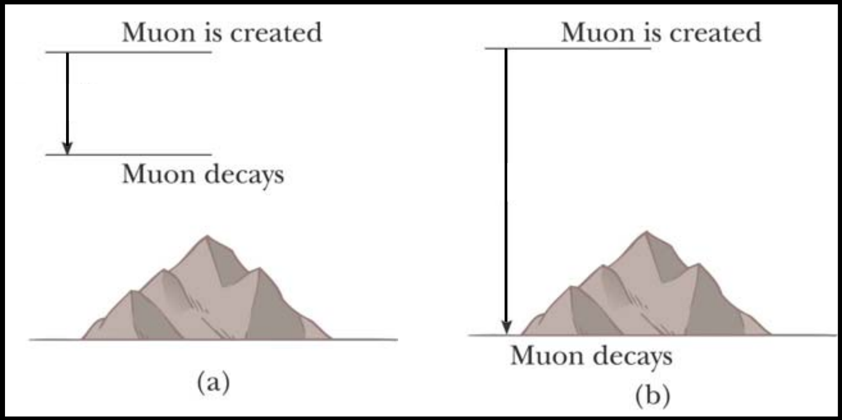
- **Experimental evidence :** Lifetime of a moving elementary particle.



(a) Without relativistic considerations, muons created in the atmosphere and traveling downward with a speed of 0.99c travel only about 6.6×10^2 m before decaying with an average lifetime of 2.2 μs. Thus, very few muons reach the surface of the Earth.

(b) With relativistic considerations, the muon's lifetime is dilated according to an observer on Earth. As a result, according to this observer, the muon can travel about 4.8×10^3 m before decaying. This results in many of them arriving at the surface.

- **Why we don't usually notice time dilation ?**

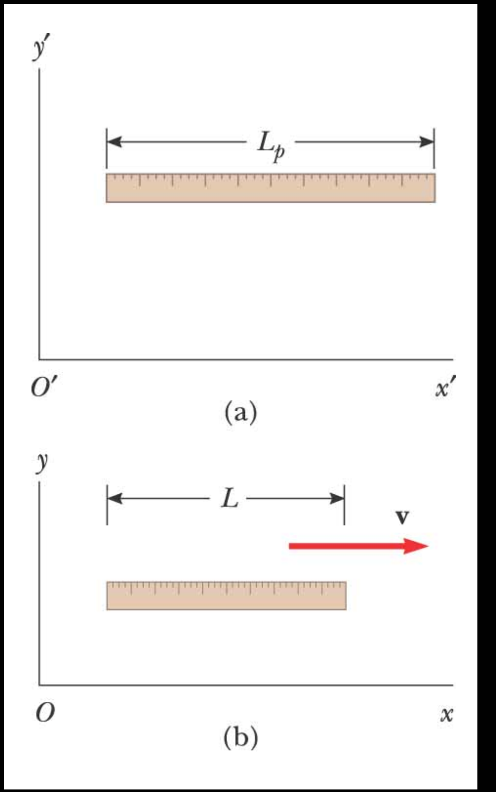
Example : (Time dilation for everyday speed.)

A car traveling 100 km/h covers a certain distance in 10 s according to the driver's watch. What does an observer at rest measure for the time interval ?

39.5.3. Length Contraction :

→ *Moving objects are shorter in the direction of motion* :

The length of an object is measured to be shorter when it is moving relative to the observer than when it is at rest.



(a) A meter stick measured by an observer in a frame attached to the stick has its proper length L_p.

(b) The stick measured by an observer in a frame in which the stick has a velocity v relative to the frame is measured to be shorter than its proper length L_p by a factor $(1- v^2/c^2)^{1/2}$.

$$\boxed{L = L_p / \gamma = L_p \left(1 - v^2/c^2 \right)^{1/2}}$$ (Length-contraction formula)

with proper length L_p (the length of the object – or distance between two points whose positions are measured at the same time – as determined by observers at rest with respect to it).

39.5.4. Four-Dimensional Space-Time

→ Four-dimensional space-time coordinate :

Space takes up three-dimensions and time is a ***fourth dimension***.

→ Space and time are intimately connected, and can intermix. Time-dilation and length contraction balance each other.

39.4 The Lorentz transformation equations

→ Correct transformation equation for $0 < v < c$.

1) Space–time transformation for S':

$$x' = \gamma\,(x - vt\,)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\,[\,t - (v/c^2)\,x\,]$$

Velocity transformation :

$$u_x' = (\,u_x - v\,)\,/\,[\,1 - (u_x v/c^2)\,]$$

with relativistic gamma factor :

$$\gamma = \gamma(v) = (\,1 - v^2/c^2\,)^{-1/2}$$

39.6 Relativistic linear momentum and energy

1) Relativistic equation for linear momentum $\boldsymbol{p}$:

$$\boxed{\boldsymbol{p} \equiv \gamma\, m\, \boldsymbol{u}}$$

when $u \ll c$, $\gamma \to 1$, and
$\boldsymbol{p} \to m\,\boldsymbol{u}$ (classical linear momentum)

Relativistic force $\boldsymbol{F}$: $\boldsymbol{F} \equiv d\boldsymbol{p}/dt \quad \Rightarrow \quad a \propto (1 - u^2 / c^2)^{3/2}$

when $u \to c$, $a \to 0$.

It is impossible to accelerate a particle to a speed *c*.

3) Relativistic kinetic energy K :

$$\boxed{K = \gamma\, mc^2 - mc^2}$$

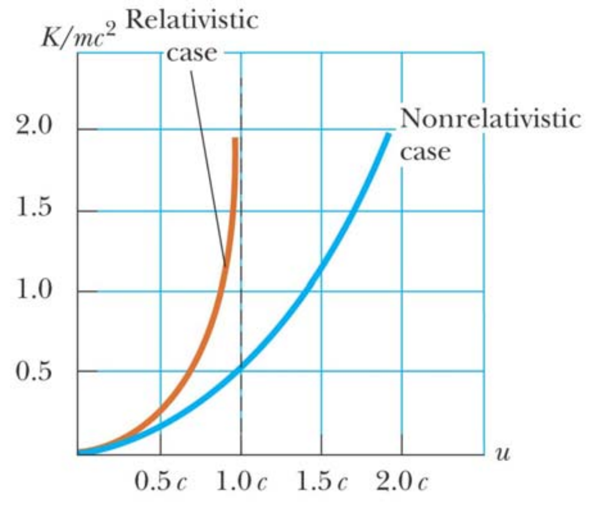
when $u \ll c$, $K \rightarrow mu^2/2$ (classical kinetic energy)

Total energy $E = K + E_R$, with rest energy $E_R = mc^2$

$$= (\gamma - 1)\, mc^2 + mc^2$$

When a particle is at rest ($K = 0$) → $E = mc^2 = E_R$

⇒ Concept of ***mass-energy equivalence***.
A concept fundamental to nuclear energy.



4) Relationship between E and p :

$$\boxed{E^2 = p^2c^2 + (mc^2)^2}$$

When a particle is at rest ($p = 0$), $E = E_R = mc^2$

For photons (zero mass) : $\boxed{E = pc}$

39.5.5. Relativistic Doppler Effect

The shift in frequency found for light emitted by the source in motion as opposed to light emitted by the source at rest.

- If a light source and an observer recede each other with a relative speed v, the frequency f_{Obs} measured by the observer is :

$$\boxed{f_{\mathrm{Obs}} = f_{\mathrm{Source}} \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}} \qquad \text{(receding)}$$

→ Expanding universe ($v \approx 0.15\,c \sim 0.91c$).

Example :

It is found that light from a distant galaxy is shifted down in frequency (red-shifted) by a factor of 3; that is, $f_{obs}/f_{source} = 1/3$. Is the galaxy approaching us or receding? And what is its speed?