Service Chain Composition in NFV Networks: Failures in A Game Perspective

Simeng Bian, Xi Huang, Ziyu Shao, Yang Yang
School of Information Science and Technology, ShanghaiTech University
Email: {biansm, huangxi, shaozy, yangyang}@shanghaitech.edu.cn

***Abstract*—Network function virtualization (NFV) initiates a revolution of network service (NS) delivery by forming different NSs as chains of virtualized network functions on commodity servers. Despite various benefits of NFV, deciding the chains that induce short latencies and less congestion remains a key challenge, *a.k.a.* service chain composition problem. Existing works mainly resort to centralized solutions that require full knowledge of network state with poor scalability to reach a social optimum, overlooking privacy issues and non-cooperative behaviors of users. Failures due to user/resource unavailability make it even more challenging. In this paper, we model service chain composition as a *potential game*; then we formulate it as a problem of searching one Nash Equilibrium (NE) of the game, which is essentially $\mathcal{NP}$-hard to solve. By exploiting the unique problem structure, we leverage Markov approximation framework and then propose DISCCA, an efficient distributed algorithm that leads the game towards a NE through user interactions. By extending the model with user and resource unavailability, we prove that the new game is still potential, while proposing a variant of DISCCA to deal with failures. Through extensive simulations, our results show that DISCCA efficiently induces a significant improvement in latency and congestion experienced by users, even in face of failures.**

I. Introduction

Propelled by the ever-growing variety and quality of network services (NSs), traditional NS provisioning based on dedicated hardwares has come to its end because of its poor flexibility and scalability in network management. Instead, Network function virtualization (NFV) revolutionizes the way of NS delivery by virtualizing and deploying network functions as applications on commodity servers, facilitating better flexibility in NS provisioning, manageability, reliability, and performance guarantee.

In a typical NFV system, each virtualized network function (VNF) has multiple instances distributed on different servers with fix locations. A NS is then constructed by chaining instances of different VNFs successively, *a.k.a.* service chain composition. Once deployed, users that subscribe the NS generate traffic that flow through the instances and acquire treatments in a pipeline fashion. One key challenge in NFV is to decide the service chains that induce short latencies and less congestion [1]. On the one hand, if the chained instances in a NS are distributed on servers with poor network conditions, users will experience long latency, leading to SLA violation and reduction in operator revenues. On the other hand, NSs may share the same instance on some server, inducing unfavorable resource contention and affecting service quality.

To address the challenge, existing works mainly resort to developing heuristics or approximation algorithms. Bari *et al.* [2] leverage ILP and propose a heuristic solution based on Viterbi algorithm, which provides solutions that are within 1.3 times of the optimal solution. Beck *et al.* [3] propose a heuristic method that coordinate the composition of VNF chains and their embedding into the substrate network, scaling well with only hundreds of nodes. Zhang *et al.* [4] apply the open Jackson queue network model and proposed heuristic algorithms for joint service chaining and request scheduling problems. All the above view the problem as the one that focus on coordinating NSs to reach a social optimum with fully knowledge of system state. Despite the intrinsic complexity of the problem [5], it may be impractical due to privacy issues and non-cooperative interaction among users in practice. Instead, game theory provides a framework that greatly captures the strategic interaction among players. Inspired by this, D'Oro *et al.* [6] formulate the problem as a congestion game and propose a distributed and privacy-preserving algorithm with provable convergence performance to a NE of the game in polynomial time. However, they do not consider the failures of users and resources which are inevitable in practice and may have a significant impact on the performance of the game as stated in [7]–[11].

In this paper, we address the service chain composition problem by modeling it as a non-cooperative game while considering the failures of users and resources. We prove that the game is a potential game with a given weighted potential function. Searching a Nash equilibrium (NE) of the game is essentially $\mathcal{NP}$-hard. By leveraging Markov approximation framework [12] and exploiting unique problem structure, we propose DISCCA, an low-complexity, distributed algorithm that achieves a NE with polynomial complexity. We show that the game is still potential in the more general setting and propose a variant of DISCCA. We summarize our main contributions as follows:

- ◇ **Modeling and Formulation:** We develop an exquisite model that accurately captures the system dynamics and user interactions. Based on the model, we are the first to formulate the service chaining composition as a non-cooperative game while taking both user and resource failures into consideration. We show the game is a potential game and formulate the problem as to find a NE of the game that induce short latency and low congestion.

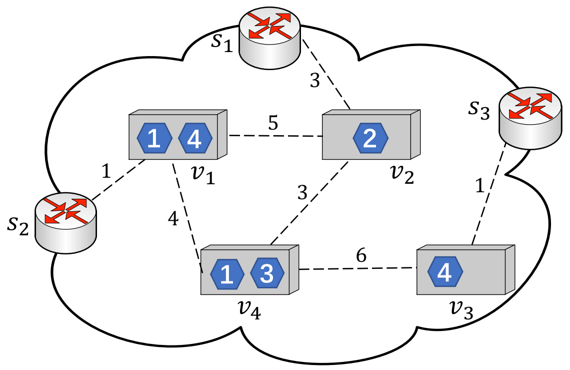


Fig. 1. A sample NFV system.

- ◇ **Algorithm Design** We leverage the unique problem structure and apply Markov approximation technique, and propose DISCCA, a distributed and low-complexity algorithm to solve the problem while handling failures.
- ◇ **Experimental Verification and Analysis** Simulation results show that DISCCA achieves NE within polynomial iteration steps. Specifically, the convergence time increases almost linearly with the size of the problem.

The rest of the paper is organized as follows. Section II first introduce some basic notations and the service chain composition problem. Section III reformulates it as a non-cooperative game while proving its potential property. In this part, we propose DISCCA, an efficient distributed algorithm to chain instances with near-optimal latency and congestion. Then Section IV extends the previous model with user and resource failures, proves that the new game is still potential, and proposes a variant of DISCCA. Section V presents our results from comprehensive simulations and the corresponding analysis, while Section VI concludes this paper.

II. NFV System Model

A. Model Notations

We consider a NFV system owned by a Telco Operator that provides network services to its users [6]. Figure 1 shows such a sample NFV system. Inside the system there is a set of servers $\mathcal{V}$, with a size of $V \triangleq |\mathcal{V}|$. Servers are able to run virtual network function (VNF) instances on virtual machines (VMs). Each server $v \in \mathcal{V}$ is equipped with a fixed set of VNF, denoted by $\mathcal{F}_v$. Each VNF has one instance on the server, while each instance is deployed in a standalone VM. Our model can be directly extended to the case with more than one instance in a VM or on a server. To distinguish instances of the same VNF on different servers, we use (v, f) to denote a specific VNF instance, where v denotes the server where the instance resides, and f denotes the VNF that the instance belongs to. Following this notation, we use $\mathcal{R}$ to denote the set of all VNF instances (VMs)[1] in this system. For each $r = (v, f) \in \mathcal{R}$, we denote v by r_1 and f by r_2.

[1]In this paper, we use the terms VNF instance and VM interchangeably.

TABLE I
Notations used in this paper.

Notation	Description
$\mathcal{V}$	Set of commodity servers
V	Number of servers, $V \triangleq |\mathcal{V}|$
$\mathcal{F}_v$	Set of VNFs provided by server v
$\mathcal{R}$	Set of VMs (VNF instances) in the system
$\mathcal{S}$	Set of gateway routers
l_{u_1,u_2}	Latency between node u_1 and u_2
$\mathcal{N}$	Set of users
N	Number of users, $N \triangleq |\mathcal{N}|$
λ_i	Traffic rate of user i
s_i, t_i	Ingress and egress gateway routers of user i
$\mathcal{F}_i$	Network service request of user i
$f_{i,j}$	The *jth* VNF in user i's NS request
$\mathbf{w}_i$	Service chaining decision of user i
$w_{i,j}$	The VM chosen for the *jth* VNF of user i's request
$\mathbf{w}_{-i}$	Service chaining decisions of other users excluding i
$\mathbf{w}$	Combination of all users' decisions

Besides servers, the system also contains a set of gateway routers $\mathcal{S}$ that connect the servers and manage the communication with outer networks. For any two nodes $u_1, u_2 \in \mathcal{S} \cup \mathcal{R}$, l_{u_1,u_2} denotes the latency between them.[2]

Meanwhile, there is a set of network users $\mathcal{N}$, with a size of $N \triangleq |\mathcal{N}|$. Each user $i \in \mathcal{N}$ generates a traffic flow with rate λ_i. The traffic flows into the system through some gateway router $s_i \in \mathcal{S}$, and leaves the system through another router $t_i \in \mathcal{S}$. The user traffic requests a specific network service (NS) $\mathcal{F}_i$, denoted by an ordered chain of VNFs $\{f_{i,1}, f_{i,2}, \cdots, f_{i,|\mathcal{F}_i|}\}$. All required VNFs are assumed available in the system, *i.e.*, $\bigcup_{i\in\mathcal{N}} \mathcal{F}_i \subseteq \bigcup_{v\in\mathcal{V}} \mathcal{F}_v$.

For each user's request $\mathcal{F}_i$, we use $\mathbf{w}_i$ to denote the service chaining decision. $w_{i,j} = r$ means that user i's *jth* VNF $f_{i,j}$ will be assigned onto VM r to execute. Note that a VNF f can only be assigned to those VMs that run VNF f, *i.e.*, $w_{i,j} \in \{r \in \mathcal{R} \mid r_2 = f_{i,j}\}$. For simplicity, we use $\mathbf{w}_{-i}$ to denote the service chaining decisions of all other users excluding i, *i.e.*, $\mathbf{w}_{-i} \triangleq \{\mathbf{w}_k \mid k \in \mathcal{N}/\{i\}\}$, and denote $\mathbf{w} \triangleq (\mathbf{w}_i, \mathbf{w}_{-i})$. Table I summarizes the main notations.

B. Cost Functions

1) Latency: Response time often makes one of the most concerned requirements in user SLAs. However, under different service chaining decisions, users' traffic may experience different levels of latency. Given decision $\mathbf{w}_i$, we define the total latency experienced by user i as

$$c_i^{(\mathrm{L})}(\mathbf{w}_i) = l_{s_i,w_{i,1}} + \sum_{j=2}^{|\mathcal{F}_i|} l_{w_{i,j-1},w_{i,j}} + l_{w_{i,|\mathcal{F}_i|},t_i}. \quad (1)$$

2) Congestion: Since traffic flows of different users may adopt the same instance on same server, the congestion due to their resource contention is inevitable. Given the set of users that select instance r, denoted by

$$\mathcal{M}_r(\mathbf{w}) \triangleq \{k \in \mathcal{N} | r \in \mathbf{w}_k\}, \quad (2)$$

[2]The latency can be set as the number of hops, round-trip time, *etc*.

we define the workload on instance r as $\delta_r(\mathbf{w})$

$$\delta_r(\mathbf{w}) \triangleq \sum_{i \in \mathcal{M}_r(\mathbf{w})} \lambda_i. \tag{3}$$

Then the congestion cost of user i is defined as

$$c_i^{(\mathrm{C})}(\mathbf{w}) \triangleq \sum_{r \in \mathbf{w}_i} \delta_r(\mathbf{w}). \tag{4}$$

Combining the cost of latency and congestion together, we define total cost for user i as

$$c_i(\mathbf{w}) \triangleq \alpha c_i^{(\mathrm{L})}(\mathbf{w}_i) + c_i^{(\mathrm{C})}(\mathbf{w}). \tag{5}$$

where α is a positive weighting parameter.

III. Problem Formulation and Algorithm Design

Given the model in the previous section, one can formulate the service chain composition problem as an optimization problem that coordinates users' behavior towards a social optimum such as minimizing the overall congestion cost and average latency. However, that may require the full information of network state and users' cooperation, which may be impractical due to privacy concerns and non-cooperative interaction of competing for rather than sharing the resources among users. Therefore, we shift our mindset by viewing previous service chaining problem from a game perspective.

A. Preliminary to Game Theory

Typically, a game consists of a set of players. Each player has her own action space and a function that maps all players' actions to a resultant cost or payoff. In a non-cooperative game, each rational player strategically chooses action in response to others' actions so as to minimize her cost (maximize the payoff). A *Nash Equilibrium* (NE) refers to such a state that no player has the incentive to change her current action unilaterally. Among different types of game, potential games are often favorable due to its nice property that guarantee the existence of NE. A game is said to be *potential* if any player's incentive to change her own action can be equivalently expressed by a global potential function.

B. Reformulating as A Potential Game

We consider a game among a set of players $\mathcal{N}$. Player i's weight is her own traffic rate λ_i. The VNF instances $\mathcal{R}$ are actually the resources competed by players. Each player i picks an action, *i.e.* a chaining decision $\mathbf{w}_i$, from a finite action space $\mathcal{W}_i$, including all possible service chaining decisions of player i. A special case is when all player requests have a length of F and each server is equipped with all F VNF instances, then the size of each player's action space is V^F. In general, we use $\mathcal{W} \triangleq \bigcup_{i \in \mathcal{N}} \mathcal{W}_i$ to denote the combination of all players' action spaces, also referred as state space. The cost function of player i is defined as in (5). Accordingly, we define the game $\mathcal{G}$ by the following quadruple

$$\mathcal{G} \triangleq (\mathcal{N}, (\lambda_i)_{i \in \mathcal{N}}, \mathcal{W}, (c_i)_{i \in \mathcal{N}}). \tag{6}$$

By investigating the game's structure, we have the following theorem.

Theorem 1: The game $\mathcal{G}$ is a weighted potential game with potential function

$$\Phi(\mathbf{w}) = 2\alpha \sum_{k \in \mathcal{N}} \lambda_k c_k^{(\mathrm{L})}(\mathbf{w}_k) + \sum_{r \in \mathcal{R}} [\delta_r(\mathbf{w})]^2 \tag{7}$$

such that for any state pair $\mathbf{w} = (\mathbf{w}_i, \mathbf{w}_{-i})$ and $\mathbf{w}' = (\mathbf{w}'_i, \mathbf{w}_{-i})$ that only player i changes her action,

$$\Phi(\mathbf{w}) - \Phi(\mathbf{w}') = 2\lambda_i \left[c_i(\mathbf{w}) - c_i(\mathbf{w}')\right]. \tag{8}$$

Proof 1: $\Phi(\mathbf{w})$ is a potential function if and only if we can deduce (8) from (7).

For any two states $\mathbf{w} = (\mathbf{w}_i, \mathbf{w}_{-i})$ and $\mathbf{w}' = (\mathbf{w}'_i, \mathbf{w}_{-i})$,

$$\begin{aligned} &\Phi(\mathbf{w}) - \Phi(\mathbf{w}') \\ &= 2\lambda_i \left[\alpha c_i^{(\mathrm{L})}(\mathbf{w}_i) - \alpha c_i^{(\mathrm{L})}(\mathbf{w}'_i)\right] + \sum_{r \in \mathcal{R}} \left[\delta_r^2(\mathbf{w}) - \delta_r^2(\mathbf{w}')\right]. \end{aligned} \tag{9}$$

Next, we focus on the second summation term on the right-hand side of (9). Let's divide the VMs in $\mathcal{R}$ into three subsets.

- ◇ $\mathcal{R}_1$: VMs adopted by player i in $\mathbf{w}_i$ but not in $\mathbf{w}'_i$;
- ◇ $\mathcal{R}_2$: VMs adopted in decision $\mathbf{w}'_i$ but not in $\mathbf{w}_i$;
- ◇ $\mathcal{R}_3$: VMs that are both adopted in $\mathbf{w}_i$ and $\mathbf{w}'_i$ or neither.

Note that $|\mathcal{R}_1| = |\mathcal{R}_2|$ because for any VM r such that $r \in \mathbf{w}_i, r \notin \mathbf{w}'_i$, there must exist another VM r' such that $r' \in \mathbf{w}'_i, r' \notin \mathbf{w}_i$. Then,

$$\begin{aligned} &\sum_{r \in \mathcal{R}} \left[\delta_r^2(\mathbf{w}) - \delta_r^2(\mathbf{w}')\right] \\ &= \sum_{r \in \mathcal{R}_1} \left\{\delta_r^2(\mathbf{w}) - [\delta_r(\mathbf{w}) - \lambda_i]^2\right\} \\ &\quad + \sum_{r \in \mathcal{R}_2} \left\{[\delta_r(\mathbf{w}') - \lambda_i]^2 - \delta_r^2(\mathbf{w}')\right\} \\ &= \sum_{r \in \mathcal{R}_1} \left[2\lambda_i \delta_r(\mathbf{w}) - \lambda_i^2\right] + \sum_{r \in \mathcal{R}_2} \left[-2\lambda_i \delta_r(\mathbf{w}') + \lambda_i^2\right] \\ &= 2\lambda_i \left\{\sum_{r \in \mathcal{R}_1} \delta_r(\mathbf{w}) - \sum_{r \in \mathcal{R}_2} \delta_r(\mathbf{w}')\right\} - |\mathcal{R}_1|\lambda_i^2 + |\mathcal{R}_2|\lambda_i^2 \\ &= 2\lambda_i \left[c_i^{(\mathrm{C})}(\mathbf{w}) - c_i^{(\mathrm{C})}(\mathbf{w}')\right]. \end{aligned} \tag{10}$$

Substituting (10) into (9), we obtain

$$\begin{aligned} &\Phi(\mathbf{w}) - \Phi(\mathbf{w}') \\ &= 2\lambda_i \left[\alpha c_i^{(\mathrm{L})}(\mathbf{w}_i) - \alpha c_i^{(\mathrm{L})}(\mathbf{w}'_i)\right] + 2\lambda_i \left[c_i^{(\mathrm{C})}(\mathbf{w}) - c_i^{(\mathrm{C})}(\mathbf{w}')\right] \\ &= 2\lambda_i \left[c_i(\mathbf{w}) - c_i(\mathbf{w}')\right]. \end{aligned} \tag{11}$$

So far we have deduced (8) from (7). Proof of *Theorem 1* is done.■

In fact, we have another way to represent the potential function, *i.e.*, by defining function $B_i : \mathcal{W}_{-i} \to \mathbb{R}$ as

$$B_i(\mathbf{w}_{-i}) \triangleq 2\alpha \sum_{k\in\mathcal{N}/\{i\}} \lambda_k c_k^{(\mathrm{L})}(\mathbf{w}_k) - |\mathcal{F}_i|\lambda_i^2 + \sum_{r\in\mathcal{R}} \left(\sum_{k\in\mathcal{M}_r(\mathbf{w})/\{i\}} \lambda_k \right)^2 . \tag{12}$$

Thus we can rewrite the potential function as

$$\Phi(\mathbf{w}) = 2\lambda_i c_i(\mathbf{w}) + B_i(\mathbf{w}_{-i}), \tag{13}$$

where $B_i(\mathbf{w}_{-i})$ is unrelated to $\mathbf{w}_i$, the action of player i.

C. Problem Formulation

Though it has been proved that any finite potential game admits at least one pure NE [13], [14], it still remains open and challenging how to find the solution in a low-complexity way. To address the challenge, we leverage Markov approximation framework [12] by taking its advantage of distributed algorithm design that achieves the optimum asymptotically through random sampling techniques.

Corollary 1: If we find a state $\mathbf{w}^*$ such that

$$\Phi(\mathbf{w}^*) = \min_{\mathbf{w}} \Phi(\mathbf{w}), \tag{14}$$

then $\mathbf{w}^*$ must be a NE.

Proof 2: Assume $\mathbf{w}^*$ is current state and a player i prepares to change her action from $\mathbf{w}_i^*$ to $\mathbf{w}_i \in \mathcal{W}_i$. From (14) we know that

$$\Phi(\mathbf{w}_i^*, \mathbf{w}_{-i}^*) - \Phi(\mathbf{w}_i, \mathbf{w}_{-i}^*) \le 0. \tag{15}$$

According to Theorem 1, we have

$$c_i(\mathbf{w}_i^*, \mathbf{w}_{-i}^*) - c_i(\mathbf{w}_i, \mathbf{w}_{-i}^*) \le 0, \tag{16}$$

which indicates that any action change from $\mathbf{w}_i^*$ won't brings a lower cost to player i.

Therefore, $\mathbf{w}^*$ is a NE because no player has the incentive to change her action unilaterally.■

According to corollary 1, we could find the NE of $\mathcal{G}$ by finding the optimal solution $\mathbf{w}$ that minimizes Φ, *i.e.*,

$$\underset{\mathbf{w}\in\mathcal{W}}{\text{Minimize}} \quad \Phi(\mathbf{w}). \tag{17}$$

Problem (17) is essentially a combinatorial problem which is in general $\mathcal{NP}$-hard to solve. By applying *log-sum-exponential* approximation [12], we transform (17) into the following form:

$$\begin{aligned} \underset{\mathbf{p}\succeq 0}{\text{Minimize}} \quad & \sum_{\mathbf{w}\in\mathcal{W}} p_{\mathbf{w}}\Phi(\mathbf{w}) + \frac{1}{\beta}\sum_{\mathbf{w}\in\mathcal{W}} p_{\mathbf{w}} \log p_{\mathbf{w}} \\ \text{Subject to} \quad & \sum_{\mathbf{w}\in\mathcal{W}} p_{\mathbf{w}} = 1 \end{aligned} \tag{18}$$

with an approximation gap bounded by $\frac{1}{\beta}\log|\mathcal{W}|$, where β is a tunable positive parameter. Problem (18) can also be viewed as the problem of deciding the time proportion $p_{\mathbf{w}}$ that the system spends on decision state $\mathbf{w}$, in a bid to minimize the time-average overall cost $\mathbf{\Phi}$ over a period of time with an entropy term. By solving the KKT conditions of problem (18), the optimal probability distribution turns out to be

$$p_{\mathbf{w}}^* = \frac{\exp[-\beta\Phi(\mathbf{w})]}{\sum_{\mathbf{w}'\in\mathcal{W}} \exp[-\beta\Phi(\mathbf{w}')]}, \ \forall\ \mathbf{w} \in \mathcal{W}. \tag{19}$$

D. Constructing a Markov Chain

It is impractical to solve the optimal solution (19) directly since that requires the full knowledge of states in the enormous search space $\mathcal{W}$. Instead, we resort to designing a discrete-time Markov chain while adopting random sampling techniques to compute $p_{\mathbf{w}}^*$ in an asymptotic manner. In general, we have two degrees of freedom in designing the Markov chain.

1) Topology design: We construct a Markov chain with its state space as $\mathcal{W}$. Thus each state in the chain corresponds to one possible combination of all players' actions, *i.e.*, $\mathbf{w} \in \mathcal{W}$. For simplicity, we define $H_{\mathbf{w},\mathbf{w}'}$ as the Hamming distance between the two states $\mathbf{w}$ and $\mathbf{w}'$.

$$H_{\mathbf{w},\mathbf{w}'} \triangleq \sum_{i\in\mathcal{N}} 1_{\mathbf{w}_i \ne \mathbf{w}_i'}, \tag{20}$$

in which 1_A is an indicator that equals 1 when A occurs and 0 otherwise. In the topology, two states are connected if their Hamming distance $H_{\mathbf{w},\mathbf{w}'} \le 1$, *i.e.*, at most one player changes her action.

To design a time-reversible Markov chain, we should first prove the connectivity of the topology.

Proof 3: Actually, the Hamming distance $H_{\mathbf{w},\mathbf{w}'}$ represents the number of steps needed to reach state $\mathbf{w}'$ from state $\mathbf{w}$. From the definition of Hamming distance in (20), we can conclude that

$$H_{\mathbf{w},\mathbf{w}'} \le N, \tag{21}$$

because there are at most N differences. Therefore any state pairs in the topology are reachable within finite steps, *i.e.*, the topology is connected. ■

2) Transition probability design: We define the self transition probability of state $\mathbf{w}$ as follows:

$$q_{\mathbf{w},\mathbf{w}} \triangleq \frac{1}{N}\sum_{i\in\mathcal{N}} \frac{\exp[-\beta\Phi(\mathbf{w})]}{\sum_{\bar{\mathbf{w}}\in\mathcal{A}_i} \exp[-\beta\Phi(\bar{\mathbf{w}})]}, \tag{22}$$

where $\mathcal{A}_i$ is a subset of the state space that only player i's action is changed while others' remain unchanged, *i.e.*, $\mathcal{A}_i = \{(\bar{\mathbf{w}}_i, \mathbf{w}_{-i}) \mid \bar{\mathbf{w}}_i \in \mathcal{W}_i\}$.

Next, we consider two states $\mathbf{w}$ and $\mathbf{w}'$ with $H_{\mathbf{w},\mathbf{w}'} = 1$. For simplicity, we use i to denote the only player that changes her action. Then, the transition probability from $\mathbf{w}$ to $\mathbf{w}'$ is defined as

$$q_{\mathbf{w},\mathbf{w}'} \triangleq \frac{1}{N}\frac{\exp[-\beta\Phi(\mathbf{w}')]}{\sum_{\bar{\mathbf{w}}\in\mathcal{A}_i} \exp[-\beta\Phi(\bar{\mathbf{w}})]}. \tag{23}$$

The transition probability for all other state pairs are all zeros.

We should verify the following two things to assure the designed Markov chain is correct and time-reversible.

- ◇ Transition probabilities from one state should sum to 1. For any state $\mathbf{w} \in \mathcal{W}$,

$$\begin{aligned}
&\sum_{\mathbf{w}' \in \mathcal{W}} q_{\mathbf{w},\mathbf{w}'} = \sum_{\substack{\mathbf{w}' \in \mathcal{W} \\ H_{\mathbf{w},\mathbf{w}'}=0}} q_{\mathbf{w},\mathbf{w}'} + \sum_{\substack{\mathbf{w}' \in \mathcal{W} \\ H_{\mathbf{w},\mathbf{w}'}=1}} q_{\mathbf{w},\mathbf{w}'} \\
=& q_{\mathbf{w},\mathbf{w}} + \sum_{i \in \mathcal{N}} \sum_{\substack{\hat{\mathbf{w}} \in \mathcal{A}_i \\ \hat{\mathbf{w}} \neq \mathbf{w}}} q_{\mathbf{w},\hat{\mathbf{w}}} \\
=& \frac{1}{N} \sum_{i \in \mathcal{N}} \frac{\exp[-\beta\Phi(\mathbf{w})]}{\sum_{\bar{\mathbf{w}} \in \mathcal{A}_i} \exp[-\beta\Phi(\bar{\mathbf{w}})]} \qquad (24) \\
&+ \sum_{i \in \mathcal{N}} \sum_{\substack{\hat{\mathbf{w}} \in \mathcal{A}_i \\ \hat{\mathbf{w}} \neq \mathbf{w}}} \frac{1}{N} \frac{\exp[-\beta\Phi(\hat{\mathbf{w}})]}{\sum_{\bar{\mathbf{w}} \in \mathcal{A}_i} \exp[-\beta\Phi(\bar{\mathbf{w}})]} \\
=& \frac{1}{N} \sum_{i \in \mathcal{N}} \frac{\sum_{\hat{\mathbf{w}} \in \mathcal{A}_i} \exp[-\beta\Phi(\hat{\mathbf{w}})]}{\sum_{\bar{\mathbf{w}} \in \mathcal{A}_i} \exp[-\beta\Phi(\bar{\mathbf{w}})]} = 1.
\end{aligned}$$

- ◇ Transition probabilities between any two states should satisfy the detailed balance equation.
  The detailed balance equation is naturally satisfied for state self transition. For any two different connected states $\mathbf{w}$ and $\mathbf{w}'$, recalling (19) and (23), we have

$$\begin{aligned}
&p^*_{\mathbf{w}} q_{\mathbf{w},\mathbf{w}'} \\
=& \frac{\exp[-\beta\Phi(\mathbf{w})]}{\sum_{\bar{\mathbf{w}} \in \mathcal{W}} \exp[-\beta\Phi(\bar{\mathbf{w}})]} \cdot \frac{1}{N} \frac{\exp[-\beta\Phi(\mathbf{w}')]}{\sum_{\bar{\mathbf{w}} \in \mathcal{A}_i} \exp[-\beta\Phi(\bar{\mathbf{w}})]} \\
=& \frac{\exp[-\beta\Phi(\mathbf{w}')]}{\sum_{\bar{\mathbf{w}} \in \mathcal{W}} \exp[-\beta\Phi(\bar{\mathbf{w}})]} \cdot \frac{1}{N} \frac{\exp[-\beta\Phi(\mathbf{w})]}{\sum_{\bar{\mathbf{w}} \in \mathcal{A}_i} \exp[-\beta\Phi(\bar{\mathbf{w}})]} \\
=& p^*_{\mathbf{w}'} q_{\mathbf{w}',\mathbf{w}}.
\end{aligned} \qquad (25)$$

From all above, we prove that the Markov chain we design in this paper is time-reversible.

E. Algorithm Design

In algorithm implementation, at each step a player is chosen uniformly randomly from N players to update her action. When player i is chosen, we define the in-step transition probability from state $\mathbf{w} = (\mathbf{w}_i, \mathbf{w}_{-i})$ to $\mathbf{w}' = (\mathbf{w}'_i, \mathbf{w}_{-i})$ as $q^{(i)}_{\mathbf{w},\mathbf{w}'}$. Note that $\mathbf{w}'_i$ may equals to $\mathbf{w}_i$ here, which indicates self transition.

$$q^{(i)}_{\mathbf{w},\mathbf{w}'} \triangleq \frac{\exp[-\beta\Phi(\mathbf{w}')]}{\sum_{\bar{\mathbf{w}} \in \mathcal{A}_i} \exp[-\beta\Phi(\bar{\mathbf{w}})]}. \qquad (26)$$

The transition probabilities for all other state pairs (*e.g.*, two states that another player j picks a new action) are all zeros.

However, such transition probabilities are unpractical in implementation because potential values are unhandy to obtain in real NFV networks. Recall that we have already established the relationship between potential and cost function in (13), we can reformulate the transition probability (26) as

$$q^{(i)}_{(\mathbf{w}_i,\mathbf{w}_{-i}),(\mathbf{w}'_i,\mathbf{w}_{-i})} = \frac{\exp[-2\beta\lambda_i c_i(\mathbf{w}'_i, \mathbf{w}_{-i})]}{\sum_{\bar{\mathbf{w}}_i \in \mathcal{W}_i} \exp[-2\beta\lambda_i c_i(\bar{\mathbf{w}}_i, \mathbf{w}_{-i})]}. \qquad (27)$$

Hence, the transition probability only depends on the cost, which can be obtained handily in real networks.

There are various ways to implement the uniform player selection during steps. One possible way is to have players change their actions in a Round-Robin fashion. In the long run, this is equivalent to select one of the N players uniformly randomly. In practice, we can actually implement Round-Robin in a distributed manner. Assume each player is given a index ranging from 0 to $N-1$ and all players maintain a variable t which indicates current step. Initially, t is set as 0. At current step t, the player $i = t\%N$ will get the chance to update her action. After the updating, the player will increase t by 1 and broadcast the new t to all other players. In such way, players will perform this procedure in turns.

Based on previous designs, we propose an algorithm named DIstribute Service Chain Composition Algorithm (DISCCA). DISCCA solves the problem (18) and hence solve the initial problem (17) in an approximate way. We present the pseudo-code of DISCCA in **Algorithm 1**. Initially, all players randomly pick a action from their action space, set her variable $t = 0$, and execute **Algorithm 1**.

Algorithm 1 DIstributed Service Chain Composition Algorithm (DISCCA) of player i

```
1: Repeat do
2: if receive a step increment signal then
3:     $t \leftarrow t + 1$
4: end if
5: if $t\%N == i$ then
6:     Perform action $\mathbf{w}'_i$ chosen from action space $\mathcal{W}_i$ with
       probability $q^{(i)}_{(\mathbf{w}_i,\mathbf{w}_{-i}),(\mathbf{w}'_i,\mathbf{w}_{-i})}$ in (27)
7:     Broadcast a step increment signal
8: end if
```

Additionally, for DISCCA, we have the following remarks:

- ◇ DISCCA requires only local information of each player; therefore, it can be executed in a distributed manner.
- ◇ The transition probability expression (27) is actually a weighted normalization. With a larger β, the system would spend a greater proportion of time on actions that incur lower costs. Thus intuitively, by increasing β, we can reduce the approximation gap of the converged cost; but that also leads to a longer time to converge to the optimal distribution [12]. In practice, the choice is often left to the system designers to make.

IV. Player and VM Failures

Sometimes, players may be offline due to network failures; likewise, VNF instances (VMs) may also break down due to various reasons. The availability of any player or instance may greatly influence the equilibrium state of the system [9]–[11]. Therefore, it is highly demanding to consider player and VM failures when making chaining decisions.

We consider failures that happen independently in our model. Nonetheless, our model can be a basis for system designers to consider more complex scenarios that involve correlated failures. We assume all players (and VMs) have identical survival probability, *i.e.*, Pr{player i survives} $=$

θ, $\forall i \in \mathcal{N}$ and $\Pr\{\text{VM } r \text{ survives}\} = \eta$, $\forall r \in \mathcal{R}$. Further, all service chains are assumed to have the same length of F. Our model can be extended to more general cases with various survival probability and chain lengths.

Next, we define the cost for player i given state $(\mathbf{w}_i, \mathbf{w}_{-i})$.

- ◇ If player i fails, it will receive an incompletion cost ω;
- ◇ If player i survives, but at least one of the selected VMs fails, it will also receive an incompletion cost ω;
- ◇ If both player i and its chosen VMs survive, its latency cost remains unchanged as defined in (1), but its congestion cost also depends on other players' survival status. We define the new congestion cost as

$$\hat{c}_i^{(\mathrm{C})}(\mathbf{w}_i, \mathbf{w}_{-i}) \triangleq \sum_{r\in\mathbf{w}_i} \left(\lambda_i + \sum_{k\in\mathcal{M}_r(\mathbf{w})/\{i\}} \theta\cdot\lambda_k\right) \tag{28}$$

Accordingly, the new cost function of player i is

$$\begin{aligned}\hat{c}_i(\mathbf{w}_i, \mathbf{w}_{-i}) &= (1-\theta)\cdot\omega + \theta\cdot\left(1-\eta^F\right)\cdot\omega \\ &\quad + \theta\cdot\eta^F\cdot\left(\alpha c_i^{(\mathrm{L})}(\mathbf{w}_i) + \hat{c}_i^{(\mathrm{C})}(\mathbf{w}_i, \mathbf{w}_{-i})\right)\end{aligned} \tag{29}$$

Though extended with player and VM failures, we show that the new game is still potential, as explicated by the following theorem.

Theorem 2: The new game $\hat{\mathcal{G}}$ with player and VM failures is still a weighted congestion game with potential function

$$\phi(\mathbf{w}) = 2\alpha\theta\sum_{k\in\mathcal{N}}\lambda_k\cdot c_k^{(\mathrm{L})}(\mathbf{w}_k) + \sum_{r\in\mathcal{R}}\left(\sum_{k\in\mathcal{M}_r(\mathbf{w})}\theta\cdot\lambda_k\right)^2, \tag{30}$$

such that for any state $\mathbf{w}'$ that only player i changes her action,

$$\phi(\mathbf{w}) - \phi(\mathbf{w}') = \frac{2\lambda_i}{\eta^F}\left[\hat{c}_i(\mathbf{w}) - \hat{c}_i(\mathbf{w}')\right]. \tag{31}$$

Proof 4: From the definition of ϕ in (30), we derive that for any two states $\mathbf{w}$ and $\mathbf{w}'$ that only differs in player i's action:

$$\begin{aligned}\phi(\mathbf{w}) - \phi(\mathbf{w}') &= 2\theta\lambda_i\left[\alpha c_i^{(\mathrm{L})}(\mathbf{w}_i) - \alpha c_i^{(\mathrm{L})}(\mathbf{w}_i')\right] \\ &\quad + \sum_{r\in\mathcal{R}}\left[\left(\sum_{k\in\mathcal{M}_r(\mathbf{w})}\theta\lambda_k\right)^2 - \left(\sum_{k\in\mathcal{M}_r(\mathbf{w}')}\theta\lambda_k\right)^2\right]\end{aligned} \tag{32}$$

Let's focus on the second summation term first. We divide the set of VMs $\mathcal{R}$ into three subsets $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$ similar as in *proof* 1.

$$\begin{aligned}&\sum_{r\in\mathcal{R}}\left[\left(\sum_{k\in\mathcal{M}_r(\mathbf{w})}\theta\lambda_k\right)^2 - \left(\sum_{k\in\mathcal{M}_r(\mathbf{w}')}\theta\lambda_k\right)^2\right] \\ &= \sum_{r\in\mathcal{R}_1}\left[\left(\sum_{k\in\mathcal{M}_r(\mathbf{w})}\theta\lambda_k\right)^2 - \left(\sum_{k\in\mathcal{M}_r(\mathbf{w})}\theta\lambda_k - \theta\lambda_i\right)^2\right] \\ &\quad + \sum_{r\in\mathcal{R}_2}\left[\left(\sum_{k\in\mathcal{M}_r(\mathbf{w}')}\theta\lambda_k - \theta\lambda_i\right)^2 - \left(\sum_{k\in\mathcal{M}_r(\mathbf{w}')}\theta\lambda_k\right)^2\right] \\ &= 2\theta\lambda_i\left[\sum_{r\in\mathcal{R}_1}\left(\sum_{k\in\mathcal{M}_r(\mathbf{w})}\theta\lambda_k\right) - \sum_{r\in\mathcal{R}_2}\left(\sum_{k\in\mathcal{M}_r(\mathbf{w}')}\theta\lambda_k\right)\right] \\ &= 2\theta\lambda_i\left(\hat{c}_i^{(\mathrm{C})}(\mathbf{w}) - \hat{c}_i^{(\mathrm{C})}(\mathbf{w}')\right)\end{aligned} \tag{33}$$

Recalling (29), we have

$$\begin{aligned}&\hat{c}_i(\mathbf{w}) - \hat{c}_i(\mathbf{w}') \\ &= \theta\eta^F\cdot\left[\alpha c_i^{(\mathrm{L})}(\mathbf{w}_i) - \alpha c_i^{(\mathrm{L})}(\mathbf{w}_i') + \hat{c}_i^{(\mathrm{C})}(\mathbf{w}) - \hat{c}_i^{(\mathrm{C})}(\mathbf{w}')\right]\end{aligned} \tag{34}$$

Combining (32), (33) and (34), we obtain

$$\phi(\mathbf{w}) - \phi(\mathbf{w}') = \frac{2\lambda_i}{\eta^F}\left(\hat{c}_i(\mathbf{w}) - \hat{c}_i(\mathbf{w}')\right) \tag{35}$$

The proof is done. ■

Similar as in previous non-failure case, we can reformulate the new potential function ϕ as

$$\phi(\mathbf{w}_i, \mathbf{w}_{-i}) = 2\theta\lambda_i\hat{c}_i(\mathbf{w}_i, \mathbf{w}_{-i}) + \hat{B}_i(\mathbf{w}_{-i}) \tag{36}$$

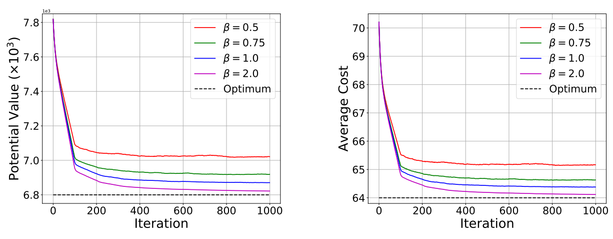
where

$$\begin{aligned}\hat{B}_i(\mathbf{w}_{-i}) &= 2\alpha\theta\sum_{k\in\mathcal{N}/\{i\}}\lambda_k c_k^{(\mathrm{L})}(\mathbf{w}_k) - |\mathcal{F}_i|\theta^2\lambda_i^2 \\ &\quad + \sum_{r\in\mathcal{R}}\left(\sum_{k\in\mathcal{M}_r(\mathbf{w})/\{i\}}\theta\cdot\lambda_k\right)^2.\end{aligned} \tag{37}$$

To find a NE of this game, we transform the problem into a combinatorial problem that finds the optimal state $\mathbf{w}$ to minimize the potential function ϕ. Applying Markov approximation techniques, we solve its respective approximation problem by designing another Markov chain similar as in section III.D, while the transition probability within an iteration that player i is chosen is

$$\hat{q}^{(i)}_{(\mathbf{w}_i,\mathbf{w}_{-i}),(\mathbf{w}_i',\mathbf{w}_{-i})} = \frac{\exp[-2\beta\theta\lambda_i\hat{c}_i(\mathbf{w}_i', \mathbf{w}_{-i})]}{\sum_{\bar{\mathbf{w}}_i\in\mathcal{W}_i}\exp[-2\beta\theta\lambda_i\hat{c}_i(\bar{\mathbf{w}}_i, \mathbf{w}_{-i})]}. \tag{38}$$

The algorithm is similar to **Algorithm 1**, while the transition probability is replaced by (38). Actually we can view previous model as a special case by setting both the surviving probabilities θ and η as 1.



(a) Potential value. (b) Per-player average cost.

Fig. 2. Performance of DISCCA under various values of β in terms of (a) potential value and (b) per-player average cost.

V. Numerical Results

In this part, we evaluate DISCCA under various simulation scenarios. We assume there are 100 players, each with a traffic rate of 1Mbps. Besides, there is a cluster of 5 servers, each equipped with three VNFs, *e.g.*, Firewall (FW), Load Balancer (LB), and Intrusion Detection System (IDS). For simplicity, the latency between any gateway router $s \in \mathcal{S}$ and VM $r \in \mathcal{R}$, and the latency between two VMs within the same server are set as 1. Other latency values are sampled uniformly randomly from the interval [2, 6]. We assume all players require the same network service, *i.e.*, FW $\rightarrow$ LB $\rightarrow$ IDS. Parameter α and β are set as 1 by default. All the results in the following are obtained by averaging over 100 simulation runs.

A. Performance

We first investigate how potential value and per-player average cost changes over iterations. We define the per-player average cost (hereinafter referred to as average cost) as

$$\bar{C} = \frac{1}{N} \sum_{i \in \mathcal{N}} c_i(\mathbf{w}_i, \mathbf{w}_{-i}). \tag{39}$$

Before presenting the results of DISCCA, we calculate the optimal value of potential value and average cost. In the case with 100 player and 5 servers, the optimal state is that each server holds 20 players and each player's request is processed within a single server. In such case, all players have the same cost and the optimal average cost is

$$\bar{C}_{opt} = (1 + 1 + 1 + 1) + (20 + 20 + 20) = 64. \tag{40}$$

Similarly, the optimal potential value is

$$\Phi_{opt} = 2 \times 100 \times 4 + 5 \times 3 \times 20 = 6800. \tag{41}$$

Figure 2 present the performance of DISCCA in terms of (a) potential value and (b) average cost with different values of parameter β, in which the black dotted lines present the corresponding optimal values (6800 and 64). We have the following observations:

1) Both potential value and average cost goes down gradually and levels off as the number of iterations increases, which verifies the convergence of DISCCA.
2) The values after convergence (at iteration 1000 as an example) of both potential value and average cost decrease as β increases from 0.5 to 2. This is consistent with our previous analysis: larger β makes the Markov chain stays at low-potential (low-cost) states with higher probabilities, hence incurs lower potential value (average cost).
3) With $\beta = 2$, the curve approximates the optimal value closely within small number of iterations. For example, in Figure (a) the potential value at iteration 400 is about 6842, which is only 0.62% higher than the optimal value 6800. Similarly in Figure (b), the average cost at iteration 400 is 64.22, which is only 0.34% higher than the optimal value 64.

B. Scalability

An algorithm should be scalable to be deployed in practical, large-scale systems. In this part, we investigate how the following factors affect the scalability of DISCCA:

◇ N: number of players
◇ V: number of servers
◇ F: length of service chain
◇ D: number of VNF replications

By default, D is set as the number of servers V, *i.e.*, each VNF type has a instance on all servers.

Figure 3(a) shows how factors N and V affect the convergence time of DISCCA. We say the process converges if the sum of n successive potential differences is less than a small value ϵ. In this part we set $n = 5$ and $\epsilon = 1$. From Figure 3(a), we have the following two observations:

1) The convergence time increase almost linearly with the number of players N.
2) Increasing the number of servers nearly has no effect on the convergence of DISCCA.

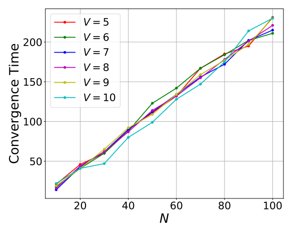
Figure 3(b) presents how factors N and V affect the average cost. We have the following observations:

1) The average cost increases as N increases from 10 to 100. This is because with larger N, each individual will competes with more players, thereby suffering a higher congestion cost.
2) The average cost decreases as V increases from 5 to 10. In fact, with more servers, players are able to be distributed more separately, therefore the congestion cost is reduced.

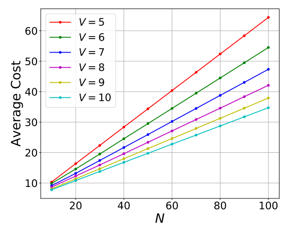
Figure 4(a) shows how F and D affect the convergence time. We observe that the convergence time increases as F almost linearly, while parameter D has little effect. Figure 4(b) presents the average cost under different values of F and D. The average cost goes up as the service chain goes longer. The reason is that with a longer SC, the player experiences larger latency cost as well as higher congestion level. We also observe that with the same F, increasing the number of VNF replications D helps reduce the average cost. The reason is similar as increasing V, both cases bring more resources and lower the congestion cost.

C. Flexibility

In practical NFV networks, users usually join and leave dynamically [15]. Moreover, a VM may be offline due some

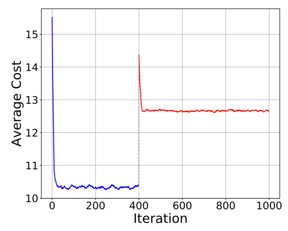


(a) Convergence time.

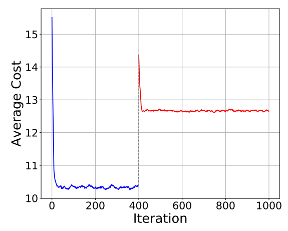


(b) Per-player average cost.

Fig. 3. Performance of DISCCA under different settings of N and V in terms of (a) convergence time and (b) per-player average cost.

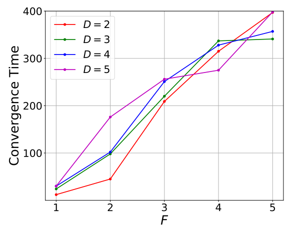


(a) Player dynamics.

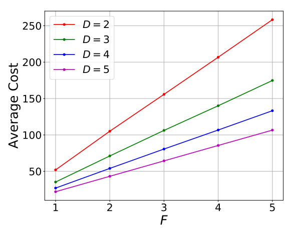


(b) VM dynamics.

Fig. 5. Average cost v.s. iteration with (a) player and (b) VM dynamics.



(a) Convergence time.



(b) Average cost.

Fig. 4. Performance of DISCCA under different settings of F and D in terms of (a) convergence time and (b) per-player average cost.

TABLE II
AVERAGE COST UNDER DIFFERENT $\bar{\theta}$ AND $\bar{\eta}$.

$\bar{\eta}$ / $\bar{\theta}$	0	0.1	0.2	0.3	0.4	0.5
0	43.2	73.1	100	123.4	143.8	161.1
0.1	55.6	83.1	107.7	129.5	148.3	164.1
0.2	68.7	93.7	116.1	135.9	153	167.4
0.3	82.6	105	125	142.7	157.9	170.8
0.4	97.2	116.8	134.4	149.8	163.2	174.4
0.5	112.6	129.3	144.2	157.3	168.6	178.2

inevitable factors and restart again after a while. Therefore, we should investigate the flexibility of DISCCA, *i.e.*, how quickly can DISCCA react to such network dynamics.

We run the simulation within 1000 iterations, and a new player with rate 5Mbps will join at iteration 400. Figure 5(a) present the performance of DISCCA in such case with $N = 10$. We could observe that when the new player joins, the per-player average cost increases immediately and sharply. This is because the arrival of the new player increases the congestion costs of some players. But in a short period (less than ten iterations) the cost decreases and converges again. Results from Figure 5(a) verify that DISCCA can adapt to player dynamics quickly.

We also consider VM dynamics in the simulation. If one VM is temporally closed, all the tasks will be migrated to another randomly selected VM of the same type. We assume at iteration 400, the VNF instance Firewall on the first server is closed. From Figure 5(b) we see that when the VM is closed, the cost rises up immediately because the random migration may cause imbalanced player distribution and incur high congestion cost. From the figure, we notice that after the sharp increment, the cost converges within ten iterations again. Figure 5(b) verifies that DISCCA can adapt to VM dynamics quickly.

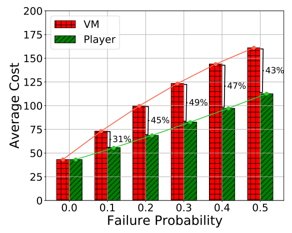
Combining Figure 5(a) and 5(b) together, we conclude that DISCCA algorithm can react to both player and VM dynamics in a flexible way.

D. DISCCA with Failure

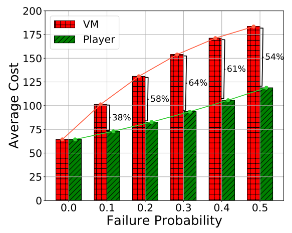
As mentioned before, players and VMs may fail with some positive probability. In this part we investigate how the failure probability affect the performance of DISCCA. For simplicity we introduce $\bar{\theta} = 1 - \theta$ as the failure probability of players, and $\bar{\eta} = 1-\eta$ as the failure probability of VMs. In Table II we show the average cost under different settings of $\bar{\theta}$ and $\bar{\eta}$ while the length of SC is set as 2. From this table, we observe that the average cost increases from left to right, top to bottom. In summary, larger failure probability (whether player failure or VM failure) incurs larger average cost.

We plotted Figure 6(a) to present the first row and first column of Table II. The red meshed bars present the case when $\bar{\eta}$ increases from 0 to 0.5 and $\bar{\theta}$ is default as 0 (the first row). Similarly the green stripped bars present the case when $\bar{\theta}$ ranges from 0 to 0.5 and $\bar{\eta} = 0$ (the first column). Figure 6(b) presents similar results but under the setting with $F = 3$. We have the following observations:

1) Higher failure probabilities lead to higher average costs for both players and VMs.
2) The red meshed bars are always higher than the green stripped bars when failure probability ranges from 0.1 to 0.5. In other words, with the same probability value, VM failure has a greater impact on the average cost than player failure. The reason is that a player uses multiple



(a) $F = 2$.



(b) $F = 3$.

Fig. 6. Average cost v.s. failure probability of player (green stripped bar) and VM (red meshed bar) under different settings of SC length F.

VMs in a service request, so the impact of VM failures will be magnified.

3) With fixed failure probability (0.2 as an example), the difference between the two bars in Figure 6(b) (58%) is larger than that in Figure 6(a) (45%). The reason is that the impact of VM failures expands further as the service chain grows.

In summary, DISCCA performs well at different levels of failures.

VI. Conclusion

In this paper, we investigate the service chain composition problem in NFV networks. We consider both agent and resource failure in our model, and then formulate the problem as a game. We prove the game is a potential game and present a weighted potential function. By leveraging Markov approximation framework, we propose a distributed and low-complexity algorithm DISCCA to achieve the NE of the game. Comprehensive simulation results shows that DISCCA achieves NE within polynomial iterations, approximates the optima closely, and adapts to network dynamics quickly, even in face of failures.

References

[1] P. Quinn and T. Nadeau, "Problem statement for service function chaining," Tech. Rep., 2015.

[2] M. F. Bari, S. R. Chowdhury, R. Ahmed, and R. Boutaba, "On orchestrating virtual network functions," in *International Conference on Network and Service Management (CNSM)*. IEEE, 2015, pp. 50–56.

[3] M. T. Beck and J. F. Botero, "Scalable and coordinated allocation of service function chains," *Computer Communications*, vol. 102, pp. 78–88, 2017.

[4] Q. Zhang, Y. Xiao, F. Liu, J. C. Lui, J. Guo, and T. Wang, "Joint optimization of chain placement and request scheduling for network function virtualization," in *International Conference on Distributed Computing Systems (ICDCS)*. IEEE, 2017, pp. 731–741.

[5] R. Cohen, L. Lewin-Eytan, J. S. Naor, and D. Raz, "Near optimal placement of virtual network functions," in *International Conference on Computer Communications (INFOCOM)*. IEEE, 2015, pp. 1346–1354.

[6] S. DOro, L. Galluccio, S. Palazzo, and G. Schembra, "Exploiting congestion games to achieve distributed service chaining in nfv networks," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 2, pp. 407–420, 2017.

[7] P. Gill, N. Jain, and N. Nagappan, "Understanding network failures in data centers: measurement, analysis, and implications," in *ACM SIGCOMM Computer Communication Review*, vol. 41, no. 4. ACM, 2011, pp. 350–361.

[8] D. Cotroneo, L. De Simone, A. K. Iannillo, A. Lanzaro, R. Natella, J. Fan, and W. Ping, "Network function virtualization: Challenges and directions for reliability assurance," in *International Symposium on Software Reliability Engineering Workshops (ISSREW)*. IEEE, 2014, pp. 37–42.

[9] M. Penn, M. Polukarov, and M. Tennenholtz, "Congestion games with failures," in *Proceedings of the 6th ACM Conference on Electronic Commerce*. ACM, 2005, pp. 259–268.

[10] R. Meir, M. Tennenholtz, Y. Bachrach, and P. Key, "Congestion games with agent failures." in *AAAI*, vol. 12, 2012, pp. 1401–1407.

[11] Y. Li, Y. Jia, H. Tan, R. Wang, Z. Han, and F. C. M. Lau, "Congestion game with agent and resource failures," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 3, pp. 764–778, 2017.

[12] M. Chen, S. C. Liew, Z. Shao, and C. Kai, "Markov approximation for combinatorial network optimization," *IEEE Transactions on Information Theory*, vol. 59, no. 10, pp. 6301–6327, 2013.

[13] R. W. Rosenthal, "A class of games possessing pure-strategy nash equilibria," *International Journal of Game Theory*, vol. 2, no. 1, pp. 65–67, 1973.

[14] D. Monderer and L. S. Shapley, "Potential games," *Games and economic behavior*, vol. 14, no. 1, pp. 124–143, 1996.

[15] D. Shah and J. Shin, "Dynamics in congestion games," *ACM SIGMETRICS Performance Evaluation Review*, vol. 38, no. 1, pp. 107–118, 2010.