Table 1: Comprehensive Performance Comparison of Optimization Algorithms

| Problem | Optimizer | Mean Final Std Dev Value | Best Value | Worst Value | Mean Eva | I . | Success Rate (%) | Mean Time (s) | |
|--------------|----------------------|-------------------------------|----------------|----------------|--------------------|----------------|---------------------|------------------|-------|
| Sphere_2D | Adam-Rol | oust | 5.866 | e-2 | 2.41e-2 | 2.27e | -2 1.25e-1 | 2502.0 | 0.0 |
| | Adam-AMS | Grad | 7.20 | | 3.21e-2 | 2.82e | | | 0.0 |
| | Adam | Lib | 7.60 | | 2.83e-3 | 4.96e | | | 40.0 |
| | Adam-Weig GD | htDecay | 4.986 | | 2.03e-5 5.43e-5 | 4.94e 4.81e | | | 100.0 |
| | | n-Conservative | 1.696 | | 1.41e-3 | 2.11e | | | 100.0 |
| | L-BFGS-Co | | 4.286 | | 6.13e-4 | 2.80e | | | 100.0 |
| | GD-Weightl | | 4.676 | | 1.71e-4 | 4.31e | | | 100.0 |
| | Trust Regio | n-Precise | 6.586 | | 7.91e-1 | 5.18e | | | 55.0 |
| | Adam-Fast | | 8.78 | | 5.67e-2 | 2.82e | | | 30.0 |
| | GD-Nestero | v | 2.516 | e-3 | 2.02e-3 | 2.83e | -4 4.93e-3 | 3 16.4 | 100.0 |
| | GD-Momen | | 4.85 | | 1.02e-3 | 1.93e | | | 100.0 |
| | | veMomentum | 2.05 | | 2.50e-1 | 2.68e | | | 60.0 |
| | L-BFGS-Lir | | 1.24 | | 1.70e-3 | 0.00€ | | | 100.0 |
| | Trust Regio | | 6.68 | | 6.05e2 | 1.21e | | | 35.0 |
| | QQN-Golde | | 3.22e | | 4.92e-15 | 2.46e- | | | 100.0 |
| | | QuadraticInterpolation | 0.00 | | 0.00e0 | 0.00€ | | | 100.0 |
| | Trust Regio | n-Standard | 1.25 | | 2.01e3 | 7.55e | | | 10.0 |
| | L-BFGS QQN-Bisect | ion 2 | 8.796 | | 3.32e-4 0.00e0 | 0.006 | | | 100.0 |
| | QQN-Bisect | | 0.00 | | 0.00e0 0.00e0 | 0.006 | | | 100.0 |
| | | n-Aggressive | 7.95 | | 5.21e3 | 2.26e | | | 30.0 |
| | QQN-Strong | | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| | L-BFGS-Mo | oreThuente | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| | L-BFGS-Ag | | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| phere_10D | Adam-Rol | | 1.086 | | 3.40e-2 | 4.84e | | | 0.0 |
| • | Adam-AMS | | 2.64 | e-1 | 4.59e-2 | 1.95e | -1 3.63e-1 | 2502.0 | 0.0 |
| | Adam | | 4.986 | e-3 | 5.65e-5 | 4.94e | -3 5.21e-3 | 3 2391.8 | 95.0 |
| | Adam-Weig | | 4.92 | | 3.21e-5 | 4.88e | | | 100.0 |
| | _ | n-Conservative | 3.41 | | 3.63e-1 | 1.07e | | | 35.0 |
| | GD | | 4.91 | | 5.40e-5 | 4.82e | | | 100.0 |
| | L-BFGS-Co | | 4.51 | | 2.33e-4 | 4.20e | | | 100.0 |
| | Trust Regio | | 5.276 | | 7.84e-1 | 1.01e | | | 65.0 |
| | GD-Weightl | | 4.616 | | 1.67e-4 | 4.33e | | | 100.0 |
| | Trust Regio | n-Adaptive | 7.30 | | 5.96e2 | 2.03e | | | 30.0 |
| | Adam-Fast | :4 - 1 | 6.12 | | 2.40e-2 | 5.73e | | | 0.0 |
| | L-BFGS-Lir | veMomentum | 1.08e 5.51e | | 1.10e-3 6.74e-1 | 5.57e 2.42e | | | 100.0 |
| | GD-Adaptiv | | 1.596 | | 2.14e-1 | 1.53e | | | 65.0 |
| | GD-Nestero | | 1.70 | | 2.14e-1 2.92e-1 | 2.45e | | | 75.0 |
| | Trust Regio | | 6.75 | | 1.25e3 | 2.45e | | | 15.0 |
| | | n-Aggressive | 1.09 | | 1.85e3 | 5.346 | | | 0.0 |
| | QQN-Golde | | 1.60e | | 1.03e-14 | 1.45e- | | | 100.0 |
| | L-BFGS | | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| | QQN-Bisect | ion-2 | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| | QQN-Bisect | ion-1 | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| | | QuadraticInterpolation | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| | QQN-Strong | gWolfe | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| | L-BFGS-Mo | | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| | L-BFGS-Ag | | 0.00 | | 0.00e0 | 0.006 | | | 100.0 |
| osenbrock_2D | QQN-Gold | | 1.26 | | 5.76e-2 | 8.08e | | | 10.0 |
| | | QuadraticInterpolation | 3.73 | | 3.12e-2 | 5.16e | | | 40.0 |
| | QQN-Strong | | 5.676 | | 7.24e-2 | 4.58e | | | 35.0 |
| | QQN-Bisect Adam | JOH-1 | 4.65e | | 8.86e-1 3.51e-1 | 6.39e 4.86e | | | 5.0 |
| | L-BFGS-Ag | gressive | 3.18 | | 2.86e1 | 4.86e 4.34e | | | 0.0 |
| | L-BFGS-Ag | | 3.93 | | 5.18e0 | 3.26e | | 2251.6 | 0.0 |
| | GD | | 1.23 | | 1.46e0 | 7.46e | | | 0.0 |
| | I | n-Conservative | 2.84 | | 2.98e1 | 1.90e | | | 0.0 |
| | L-BFGS-Co | | 6.53 | | 2.16e-3 | 1.09e | | | 100.0 |
| | Adam-AMS | | 3.83 | | 1.33e0 | 4.66e | | | 0.0 |
| | L-BFGS-Mc | | 1.91 | | 6.42e-2 | 1.40e | | | 95.0 |
| | Adam-Robu | | 4.04 | | 8.59e-1 | 1.90€ | | 419.2 | 0.0 |
| | QQN-Bisect | | 6.07 | | 6.89e-2 | 4.36e | | | 30.0 |
| | Trust Regio | | 7.26 | | 8.07e0 | 3.766 | | | 0.0 |
| | | | | | | | | | |

| | | Tabl | e 1 - continue | ued from | previo | us page | | | | | _ |
|-----------------------|-------------------------------|---------------------|----------------|----------------|----------------|--------------------|----------------|-----|---------------------|------------------|---------------|
| Problem | Optimizer M | Iean Final Value | Std Dev | Best Value | Worst Value | Mean l Eva | | | e (%) | Mean Time (s) | |
| | Adam-Fast | | | 2.136 | e0 | 2.74e0 | 8.12e | -3 | 8.39e0 | 171.8 | 60.0 |
| | Trust Region-A | daptive | | 4.126 | e0 | 2.07e-1 | 3.836 | e0 | 4.42e0 | 494.4 | 0.0 |
| | GD-WeightDec | ay | | 3.65 | | 3.07e0 | 3.75e | | 1.07e1 | 58.7 | 0.0 |
| | L-BFGS | | | 1.366 | | 1.58e2 | 8.12e | | 5.03e2 | 121.5 | 0.0 |
| | GD-AdaptiveM | Iomentum | | 6.54e | | 1.23e0 | 3.23e | | 4.36e0 | 47.7 | 25.0 |
| | GD-Nesterov | _ | | 1.496 | | 1.88e0 | 5.05e | | 5.63e0 | 46.1 | 0.0 |
| | GD-Momentum | | | 5.61e 4.18e | | 3.44e0 1.83e-1 | 4.79e | | 1.33e1 4.51e0 | 23.8 89.6 | 0.0 |
| | Trust Region-S Trust Region-A | ggrossivo | | 4.186 | | 3.99e-1 | 4.016 | | 5.49e0 | 27.6 | 0.0 |
| Rosenbrock_5D | QQN-Golden | | | 6.13e | | 3.74e-1 | 2.60e | | 1.61e0 | 3314.1 | 55.0 |
| | Adam-Robust | | | 1.466 | | 6.99e0 | 6.12 | | 2.99e1 | 2502.0 | 0.0 |
| | Adam-AMSGra | ad | | 4.40 | e0 | 3.25e-1 | 3.256 | e0 | 4.82e0 | 2442.0 | 0.0 |
| | Adam | | | 3.926 | e0 | 4.66e-1 | 2.836 | e0 | 4.65e0 | 2471.6 | 0.0 |
| | QQN-CubicQu | | olation | 4.25e | | 1.40e-1 | 2.38€ | | 7.25e-1 | 1199.2 | 70.0 |
| | L-BFGS-Limite | | | 4.21e | | 3.55e-2 | 3.92€ | | 5.47e-1 | 3855.4 | 45.0 |
| | QQN-Bisection | | | 4.48e | | 1.63e-1 | 2.15e | | 9.11e-1 | 1588.3 | 55.0 |
| | L-BFGS-Conse | | | 2.026 | | 6.75e1 | 3.89€ | | 3.11e2 | 3106.7 | 20.0 |
| | QQN-Bisection | | | 6.94e | | 1.01e0 | 2.50e | | 4.64e0 | 1147.7 | 85.0 |
| | L-BFGS-Aggre | | | 8.07 | | 4.06e2 | 1.726 | | 1.19e3 | 3851.6 | 0.0 |
| | Adam-WeightD QQN-StrongWe | | | 2.07e 3.45e | | 2.05e0 4.37e-2 | 3.93e 2.58e | | 4.66e0 3.95e-1 | 1128.9 792.6 | 60.0 100.0 |
| | Trust Region-C | | | 1.026 | | 1.63e2 | 7.14 | | 1.31e3 | 3002.0 | 0.0 |
| | Trust Region-P | | | 1.016 | | 1.27e2 | 8.086 | | 1.35e3 | 3002.0 | 0.0 |
| | L-BFGS-MoreT | | | 9.01e | | 1.03e0 | 2.376 | | 3.50e0 | 1090.7 | 70.0 |
| | Trust Region-A | | | 8.416 | | 1.37e2 | 5.056 | | 1.11e3 | 3002.0 | 0.0 |
| | Trust Region-S | | | 6.236 | e1 | 7.73e1 | 4.666 | e0 | 2.53e2 | 2827.2 | 0.0 |
| | GD-Nesterov | | | 4.24 | | 5.00e0 | 3.90e | -1 | 1.31e1 | 335.4 | 10.0 |
| | Trust Region-A | | | 5.006 | | 4.17e-1 | 4.666 | | 5.93e0 | 776.1 | 0.0 |
| | GD-WeightDec | ay | | 7.30e | | 1.08e0 | 3.59€ | | 5.40e0 | 72.1 | 60.0 |
| | L-BFGS | | | 1.50 | | 2.28e2 | 1.986 | | 7.52e2 | 135.3 | 0.0 |
| | Adam-Fast | | | 1.446 | | 3.86e0 | 3.48e | | 1.86e1 | 44.4 | 5.0 |
| | GD Adamtica M | T # | | 5.096 | | 1.48e-1 | 4.756 | | 5.31e0 | 32.5 20.6 | 0.0 |
| | GD-AdaptiveM GD-Momentum | | | 4.60e | | 6.15e0 8.91e0 | 3.366 | | 5.66e1 4.95e1 | 20.6 | 0.0 |
| Rosenbrock_10D | Adam-Robus | | | 3.496 | | 9.88e0 | 1.616 | | 5.15e1 | 2502.0 | 0.0 |
| TOSONSTOCK=TOD | Adam-AMSGra | | | 9.696 | | 8.57e-3 | 9.666 | | 9.70e0 | 2353.0 | 100.0 |
| | L-BFGS-Aggre | | | 1.646 | | 3.20e2 | 5.826 | | 1.56e3 | 3850.3 | 0.0 |
| | Adam | | | 9.716 | e0 | 6.65e-2 | 9.686 | e0 | 9.93e0 | 2293.1 | 90.0 |
| | Trust Region-A | daptive | | 2.026 | e3 | 2.22e2 | 1.64 | е3 | 2.59e3 | 3002.0 | 0.0 |
| | Trust Region-P | | | 2.10 | | 1.92e2 | 1.66 | | 2.46e3 | 3002.0 | 0.0 |
| | Trust Region-S | | | 1.06 | | 2.06e2 | 6.70 | | 1.41e3 | 3002.0 | 0.0 |
| | Trust Region-C | | | 2.16 | | 2.33e2 | 1.76 | _ | 2.55e3 | 3002.0 | 0.0 |
| | Adam-WeightD | v | | 9.676 | | 2.09e-2 | 9.61 | | 9.70e0 | 789.4 | 100.0 |
| | Trust Region-A | | | 1.006 | | 3.85e-1 | 9.596 | | 1.09e1 | 1615.8 | 30.0 |
| | L-BFGS-Conse L-BFGS | rvative | | 9.686 | | 1.80e-2 1.68e2 | 9.636 | | 9.70e0 7.37e2 | 205.3 338.9 | 100.0 |
| | QQN-GoldenSe | ection | | 9.456 | | 1.08e2 1.57e-1 | 9.036 | | 9.66e0 | 277.2 | 100.0 |
| | QQN-Goldense QQN-CubicQu | | olation | 9.436 | | 5.83e-2 | 9.036 | | 9.70e0 | 109.6 | 100.0 |
| | L-BFGS-More | | | 9.20 | | 1.98e0 | 7.836 | | 1.75e1 | 212.8 | 95.0 |
| | QQN-Bisection | | | 8.476 | | 8.02e-1 | 7.176 | | 9.68e0 | 120.3 | 100.0 |
| | QQN-StrongWe | | | 8.696 | | 5.85e-1 | 7.76 | | 9.57e0 | 90.3 | 100.0 |
| | QQN-Bisection | | | 9.376 | e0 | 3.57e-1 | 8.02 | e0 | 9.67e0 | 83.2 | 100.0 |
| | Adam-Fast | | | 1.94 | | 1.08e1 | 9.18 | | 3.68e1 | 61.6 | 45.0 |
| | GD-WeightDec | ay | | 9.586 | | 5.37e-2 | 9.45 | | 9.67e0 | 35.7 | 100.0 |
| | GD | | | 9.75 | | 2.14e-1 | 9.59 | | 1.02e1 | 41.5 | 75.0 |
| | L-BFGS-Limite | | | 9.596 | | 4.89e-2 | 9.516 | | 9.68e0 | 62.3 | 100.0 |
| | GD-Momentum | 1 | | 3.536 | | 2.87e1 | 9.326 | | 7.27e1 | 35.0 | 55.0 |
| | GD-Nesterov GD-AdaptiveM | [ama as-t | | 9.316 | | 1.92e-1 9.83e0 | 8.976 | | 9.62e0 1.16e2 | 28.6 | 100.0 |
| Michalewicz_2D_m10 | QQN-Bisection | | | -1.996 | | 9.83e0 8.69e-2 | -3.996 | | 1.16e2 -1.17e-1; | 23.1 3 1278.7 | 0.0 |
| witchatewitz_2D_III1U | Adam | J11- 1 | | -4.986 | | 4.98e-1 | -9.976 | | -2.05e-12 | | 50.0 |
| | QQN-StrongWo | olfe | | -4.936 | | 1.75e-1 | -8.016 | | -8.45e-1 | | 0.0 |
| | QQN-Bisection | | | -4.016 | | 1.75e-1 1.75e-1 | -8.016 | | -4.27e-1 | | 0.0 |
| | QQN-GoldenSe | | | -7.526 | | 2.11e-6 | -9.60 | | -8.52e-13 | | 0.0 |
| | Adam-Fast | | | -4.716 | | 4.78e-1 | -1.00 | | -4.63e-13 | | 40.0 |
| | L-BFGS-Aggre | ssive | | -8.97 | | 2.69e-1 | -8.97 | | -2.07e-1 | | 0.0 |
| | L-BFGS-Conse | | | -7.20 | | 2.35e-6 | -1.076 | | -2.95e-1 | 1 838.7 | 0.0 |
| | OD WILLIAM | | | + | | | | | 0 =0 4 | 100.0 | |
| | GD-WeightDec | ay | | -3.246 | e-6 | 7.04e-6 | -2.45 | ÷-5 | -3.78e-13 | 3 423.0 | 0.0 |

Table 1 – continued from previous page

| | Table | $\geq 1 - contin$ | ued from | ı previo | us page | | | | | |
|---------------------|--|-------------------|------------------|----------|--------------------|--------------------|------------|----------------|-------------|---|
| Problem | Optimizer Mean Final | Std Dev | Best | Worst | Mean I | Func | Success | Mean Time | | |
| | Value | | Value | Value | Eval | I . | Rate (%) | (s) | | |
| | L-BFGS | | -9.996 | - 0 | 3.00e-1 | -1.00e0 | 1 1 22 2 1 | | 10.0 | |
| | L-BFGS-MoreThuente | | -9.996 -2.50e | | 4.32e-1 | -1.00eC | | | 25.0 | |
| | GD-Nesterov | | -9.986 | | 2.99e-1 | -9.99e-1 | | | 10.0 | |
| | GD-Nesterov GD | | -1.16 | | 2.04e-6 | -7.38e-6 | | | 0.0 | |
| | L-BFGS-Limited | | -2.50 | | 4.32e-1 | -1.00e0 | | | 25.0 | - |
| | Adam-AMSGrad | | -2.996 | | 4.57e-1 | -9.96e-1 | | | 30.0 | |
| | GD-Momentum | | -3.256 | | 1.42e-1 | -6.49e- | | | 0.0 | |
| | GD-AdaptiveMomentum | | -5.006 | | 2.18e-1 | -9.99e-1 | | | 5.0 | |
| | QQN-CubicQuadraticInterpo | olation | -5.136 | e-7 | 2.19e-6 | -1.00e-5 | 5 -4.20e-1 | 4 19.1 | 0.0 | |
| | Adam-Robust | | -3.656 | e-7 | 9.47e-7 | -3.26e-6 | 6 -9.07e-1 | 4 13.0 | 0.0 | |
| | Adam-WeightDecay | | -3.496 | e-7 | 6.17e-7 | -2.33e-6 | 6 -7.24e-1 | 4 13.0 | 0.0 | |
| | Trust Region-Precise | | -7.80 | e-7 | 1.28e-6 | -4.71e-6 | 6 -4.65e-1 | 4.5 | 0.0 | |
| | Trust Region-Conservative | | -8.816 | | 3.16e-6 | -1.46e- | | | 0.0 | |
| | Trust Region-Adaptive | | -4.40 | | 7.97e-7 | -2.59e-6 | | | 0.0 | |
| | Trust Region-Aggressive | | -5.336 | | 1.94e-6 | -8.89e-6 | | | 0.0 | |
| 16.1.1.50 | Trust Region-Standard | | -6.09 | | 2.15e-6 | -9.91e-6 | 1 | | 0.0 | |
| Michalewicz_5D_m10 | QQN-CubicQuadraticInt | erpolation | -8.496 | | 4.86e-1 | -1.88eC | | | 0.0 | |
| | QQN-StrongWolfe | | -1.76 | | 7.40e-1 | -2.79e0 | | | 25.0 | |
| | QQN-Bisection-1 | | -1.78 | | 6.73e-1 | -2.74e0 | | | 20.0 | |
| | QQN-GoldenSection QQN-Bisection-2 | | -1.36 -1.78 | | 7.15e-1 8.34e-1 | -2.69e0 -2.85e0 | | | 35.0 | |
| | L-BFGS-Aggressive | | -1.78 | | 3.88e-1 | -2.85eC | | | 0.0 | |
| | L-BFGS-Aggressive L-BFGS-Conservative | | -4.986 | | 7.13e-1 | -1.25eC | | | 30.0 | |
| | L-BFGS-Conservative L-BFGS-Limited | | -8.446 | | 6.65e-1 | -2.09e0 | | | 0.0 | |
| | GD | | -2.00 | | 7.11e-1 | -2.69e0 | | | 45.0 | |
| | Adam-AMSGrad | | -2.17 | | 6.28e-1 | -2.71e0 | | | 55.0 | |
| | Adam | | -2.46 | | 5.04e-1 | -2.71e0 | | | 80.0 | |
| | L-BFGS-MoreThuente | | -1.39 | | 5.83e-1 | -2.73e0 | | | 5.0 | |
| | GD-Nesterov | | -1.24 | | 6.70e-1 | -2.69e0 | | | 5.0 | |
| | Adam-WeightDecay | | -1.75 | | 5.79e-1 | -2.69e0 | | | 10.0 | |
| | GD-WeightDecay | | -1.24 | e0 | 5.21e-1 | -2.69e0 | -7.45e- | 1 86.0 | 5.0 | |
| | Adam-Robust | | -1.65 | e0 | 7.13e-1 | -2.69e0 | -8.20e- | 1 83.1 | 10.0 | |
| | Trust Region-Conservative | | -9.74 | e-1 | 4.08e-1 | -1.86e0 | -1.78e- | 3 263.6 | 0.0 | |
| | GD-Momentum | | -1.11 | | 8.01e-1 | -2.69e0 | | | 5.0 | |
| | Adam-Fast | | -2.62 | | 4.21e-1 | -3.29e0 | | | 65.0 | |
| | L-BFGS | | -2.16 | | 2.92e-1 | -8.96e-1 | | | 0.0 | |
| | GD-AdaptiveMomentum | | -1.09 | | 5.78e-1 | -2.69e0 | | | 5.0 | |
| | Trust Region-Precise | | -9.856 | | 6.29e-1 | -1.74e0 | | | 0.0 | |
| | Trust Region-Adaptive | | -7.586 | | 6.00e-1 | -1.73e0 | | | 0.0 | |
| | Trust Region-Standard | | -3.296 | | 4.87e-1 | -1.53e0 | | | 0.0 | |
| Michelowicz 10D m10 | Trust Region-Aggressive QQN-CubicQuadraticInte | onnolation | -1.81e | I . | 2.12e-1 1.51e0 | -7.69e-1 | | | 0.0 | |
| Michalewicz_10D_m10 | QQN-StrongWolfe | erpolation | -3.34 | | 1.16e0 | -6.27e0 | | | 5.0 | |
| | QQN-Bisection-1 | | -4.24 | | 1.09e0 | -5.36e0 | | | 0.0 | |
| | QQN-Bisection-2 | | -4.37 | | 1.24e0 | -6.26e0 | | | 5.0 | |
| | QQN-GoldenSection | | -4.14 | | 1.08e0 | -6.30e0 | | | 5.0 | |
| | L-BFGS-Limited | | -3.52 | | 1.38e0 | -5.59e0 | | | 0.0 | _ |
| | L-BFGS-Conservative | | -4.54 | | 9.46e-1 | -6.26e0 | | | 0.0 | |
| | L-BFGS-Aggressive | | -1.30 | e0 | 9.34e-1 | -3.26e0 | 3.39e-1 | 3050.9 | 0.0 | |
| | L-BFGS-MoreThuente | | -4.11 | | 7.64e-1 | -5.98e0 | | | 0.0 | |
| | Adam | | -5.84 | | 5.72e-1 | -6.27e0 | | | 20.0 | |
| | Adam-AMSGrad | | -5.74 | | 6.30e-1 | -6.28e0 | | | 30.0 | |
| | GD-WeightDecay | | -4.56 | | 7.54e-1 | -6.26e0 | | | 0.0 | |
| | Trust Region-Conservative | | -3.71 | | 9.15e-1 | -5.42e0 | | | 0.0 | |
| | Adam-WeightDecay | | -5.31 | | 5.80e-1 | -6.25e0 | | | 0.0 | |
| | Adam-Robust | | -5.03 | | 7.58e-1 | -6.26e0 | | | 0.0 | |
| | Adam-Fast GD-Momentum | | -6.12 | | 4.94e-1 | -6.77e0 | | | 35.0 0.0 | |
| | GD-Momentum GD-AdaptiveMomentum | | -3.79 -4.52 | | 1.08e0 7.73e-1 | -5.24e0 -6.23e0 | | | 0.0 | |
| | L-BFGS | | -3.66 | | 3.76e-1 | -6.23eC | | | 0.0 | |
| | Trust Region-Precise | | -3.69 | | 9.58e-1 | -1.14eC | | | 0.0 | |
| | GD-Nesterov | | -3.74 | | 1.19e0 | -6.19e0 | | | 0.0 | |
| | GD-Nestelov GD | | -3.74 | | 7.26e-1 | -5.04e0 | | | 0.0 | |
| | Trust Region-Adaptive | | -3.26 | | 9.04e-1 | -5.27e0 | | | 0.0 | |
| | Trust Region-Standard | | -2.49 | | 1.15e0 | -4.98e0 | | | 0.0 | |
| | Trust Region-Aggressive | | -1.68 | | 6.53e-1 | -3.16eC | | | 0.0 | |
| Rastrigin_2D | L-BFGS-Aggressive | | ' I | 2.81e1 | | 88e0 | 1.91e1 | 3.73e1 38 | 851.8 0.0 | 0 |
| | Trust Region-Conserv | | | 1.85e1 | 4.6 | 35e0 | 9.36e0 | 2.65e1 30 | 002.0 | 0 |
| | | | | _ | | | Continue | d on nort nore | • | |

| | | Table 1 | - continued | fron | n previou | s page | | | | | |
|---------------|----------------------|---|-------------|------------|------------------|----------|---------------------|---------------------|------------------|------------------|--------------|
| Problem | Optimizer Me | ean Final S Value | | est due | Worst Value | Mean Eva | | Success Rate (%) | Mean T | lime | |
| | Adam-AN | MSGrad | | | 1.02e1 | | 48e0 | 7.96e0 | 1.29e1 | 770.2 | 55.0 |
| | Adam | Conservative | | | 1.02e1 9.82e0 | | $\frac{48e0}{01e0}$ | 7.96e0 7.96e0 | 1.29e1 3.02e1 | 780.5 455.9 | 55.0 75.0 |
| | | gion-Precise | | | 9.82e0 9.71e0 | - | 37e0 | 7.96e0 7.96e0 | 1.29e1 | 915.9 | 10.0 |
| | Adam-We | eightDecay | | | 9.71e0 | 2. | 38e0 | 7.96e0 | 1.30e1 | 254.7 | 50.0 |
| | | MoreThuente | | | 8.56e0 | | 94e0 | 1.83e0 | 3.12e1 | 170.8 | 55.0 |
| | Adam-Ro QQN-Bise | | | | 9.22e0 8.59e0 | | 16e0 47e0 | 7.96e0 1.03e0 | 1.30e1 1.69e1 | 111.5 125.3 | 15.0 65.0 |
| | | denSection | | | 8.19e0 | | 01e0 | 1.05e0 1.29e-1 | 1.79e1 | 156.0 | 55.0 |
| | | oicQuadraticIn | terpolation | | 7.75e0 | | 18e0 | 2.17e0 | 1.59e1 | 64.2 | 80.0 |
| | | gion-Adaptive | | | 9.68e0 | | 46e0 | 7.96e0 | 2.21e1 | 241.0 | 15.0 |
| | QQN-Stro L-BFGS-I | | | | 8.32e0 9.13e0 | | 20e0 25e0 | 1.36e0 5.86e0 | 1.69e1 1.29e1 | 71.4 106.5 | 65.0 70.0 |
| | L-BFGS | Billited | | | 5.70e1 | | 05e1 | 2.49e0 | 1.31e2 | 88.4 | 5.0 |
| | Adam-Fas | | | | 9.46e0 | | 04e0 | 8.02e0 | 1.43e1 | 36.3 | 0.0 |
| | GD-Weigl | htDecay tiveMomentun | | | 1.01e1 8.98e0 | | 27e0 78e0 | 7.96e0 7.97e0 | 1.34e1 1.34e1 | 24.3 21.5 | 5.0 |
| | GD-Adap GD-Mom | | 1 | | 1.04e1 | | 44e0 | 7.97e0 7.98e0 | 1.54e1 1.57e1 | 21.5 | 0.0 |
| | GD-Neste | | | | 1.00e1 | | 35e0 | 7.96e0 | 1.37e1 | 20.1 | 5.0 |
| | QQN-Bise | | | | 8.57e0 | | 87e0 | 2.07e0 | 1.69e1 | 23.8 | 30.0 |
| | | gion-Standard | | | 9.02e0 | | 98e0 | 7.96e0 | 1.30e1 1.40e1 | 63.8 | 0.0 |
| | GD Trust Reg | gion-Aggressive | <u> </u> | _ | 1.04e1 1.29e1 | | 51e0 89e0 | 7.96e0 7.97e0 | 1.40e1 3.74e1 | 13.3 20.1 | 5.0 |
| Rastrigin_5D | | -Aggressive | , | | 7.97e1 | | 04e0 | 5.68e1 | 9.50e1 | 3852.0 | 0.0 |
| | | gion-Conservat | ive | | 6.91e1 | | 96e0 | 5.03e1 | 8.29e1 | 3002.0 | 0.0 |
| | Adam-AN | MSGrad gion-Precise | | | 2.38e1 2.39e1 | | 35e0 48e0 | 2.04e1 2.03e1 | 2.99e1 2.99e1 | 719.2 2558.8 | 40.0 35.0 |
| | Adam | gion-Frecise | | | 2.39e1 2.42e1 | | 48e0 17e0 | 2.03e1 2.03e1 | 2.99e1 2.99e1 | 745.3 | 30.0 |
| | | Conservative | | | 2.34e1 | | 92e0 | 1.99e1 | 3.61e1 | 881.9 | 40.0 |
| | | Adam-WeightDecay Trust Region-Adaptive | | | | | 13e0 | 2.02e1 | 2.99e1 | 246.9 | 55.0 |
| | | Trust Region-Adaptive QQN-GoldenSection | | | | | 02e0 | 2.02e1 | 4.05e1 | 646.1 | 60.0 |
| | | QQN-GoldenSection QQN-Bisection-1 | | | | | 30e0 08e0 | 7.97e0 1.80e1 | 4.97e1 3.38e1 | 266.0 186.6 | 35.0 25.0 |
| | | oicQuadraticIn | terpolation | | 2.48e1 2.30e1 | | 41e0 | 4.91e0 | 4.28e1 | 106.2 | 50.0 |
| | | MoreThuente | | | 2.77e1 | | 23e1 | 7.71e0 | 9.40e1 | 165.1 | 50.0 |
| | QQN-Stro Adam-Ro | | | | 2.36e1 2.24e1 | | 65e0 | 1.16e1 | 4.08e1 | 100.2 | 40.0 |
| | | otiveMomentun | 1 | | 2.24e1 2.14e1 | | 09e0 74e0 | 1.99e1 1.83e1 | 2.99e1 2.99e1 | 103.5 52.4 | 55.0 65.0 |
| | L-BFGS-I | | - | | 2.84e1 | | 49e1 | 1.70e1 | 6.88e1 | 115.2 | 35.0 |
| | L-BFGS | | | | 1.02e2 | | 33e1 | 5.89e1 | 1.42e2 | 98.2 | 0.0 |
| | GD-Neste | | | | 2.37e1 | | 00e0 | 1.99e1 | 3.05e1 | 41.2 | 45.0 |
| | GD-Weigl | gion-Standard | | | 2.25e1 2.31e1 | | 90e0 22e0 | 1.89e1 1.95e1 | 2.99e1 2.99e1 | 37.8 168.5 | 50.0 45.0 |
| | Adam-Fas | | | | 2.33e1 | | 56e0 | 1.92e1 | 3.17e1 | 48.9 | 40.0 |
| | GD-Mom | | | | 3.03e1 | | 57e0 | 2.04e1 | 4.94e1 | 28.9 | 5.0 |
| | QQN-Bise GD | ection-2 | | | 3.08e1 | | 62e1 | 1.10e1 | 6.07e1 | 36.6 15.7 | 15.0 30.0 |
| | _ | gion-Aggressive | <u> </u> | | 2.44e1 2.33e1 | | 48e0 41e0 | 1.99e1 1.82e1 | 3.05e1 4.25e1 | 45.7 | 50.0 |
| Rastrigin_10D | L-BFGS | -Aggressive | | | 1.63e2 | 1. | 31e1 | 1.40e2 | 1.87e2 | 3852.0 | 0.0 |
| | | gion-Conservat | ive | | 1.57e2 | | 05e1 | 1.28e2 | 1.71e2 | 3002.0 | 0.0 |
| | | gion-Precise Conservative | | | 9.70e1 4.68e1 | | 03e1 45e0 | 6.33e1 4.06e1 | 1.13e2 7.77e1 | 3002.0 2037.2 | 20.0 |
| | Adam-AN | | | | 4.48e1 | | 03e0 | 4.00e1 4.17e1 | 5.47e1 | 718.0 | 55.0 |
| | Adam | | | | 4.65e1 | | 42e0 | 4.16e1 | 5.97e1 | 731.5 | 30.0 |
| | | gion-Adaptive | | | 4.39e1 | | 92e0 | 4.16e1 | 4.98e1 | 1397.8 | 40.0 |
| | GD-Neste | erov eightDecay | | | 4.50e1 4.52e1 | | 13e0 97e0 | 4.02e1 4.15e1 | 5.49e1 5.48e1 | 249.0 253.8 | 25.0 35.0 |
| | | | | | 4.32e1 4.11e1 | | 33e0 | 2.56e1 | 6.07e1 | 291.6 | 65.0 |
| | QQN-Bise | L-BFGS-MoreThuente QQN-Bisection-1 | | | 5.11e1 | 1. | 61e1 | 3.88e1 | 8.86e1 | 216.4 | 45.0 |
| | | QQN-GoldenSection | | | 4.53e1 | | 18e1 | 2.11e1 | 7.26e1 | 275.4 | 50.0 |
| | | QQN-CubicQuadraticInterpolation | | | 4.52e1 5.16e1 | | 84e0 59e1 | 3.21e1 3.24e1 | 5.87e1 9.55e1 | 102.5 118.5 | 60.0 40.0 |
| | | QQN-StrongWolfe Adam-Robust | | | 4.44e1 | | 07e0 | 3.24e1 4.13e1 | 5.48e1 | 127.9 | 30.0 |
| | Trust Reg | Trust Region-Standard | | | 4.59e1 | | 59e0 | 4.10e1 | 7.11e1 | 346.6 | 25.0 |
| | | GD-WeightDecay | | | 4.39e1 | | 41e0 | 3.60e1 | 4.98e1 | 61.3 | 45.0 |
| | | Adam-Fast L-BFGS-Limited | | | 4.51e1 | | 26e0 | 3.98e1 3.39e1 | 5.44e1 5.47e1 | 71.9 122.0 | 30.0 50.0 |
| | L-BFGS-I | ышкеа | | - | 4.36e1 1.85e2 | | 97e0 27e1 | 3.39e1 1.42e2 | 2.59e2 | 102.0 | 0.0 |
| | 1 11 00 | | | | 1.0002 | 5. | _, | | ued on next | | 0.0 |

| | | Table | e 1 - continu | ued fro | m previou | s page | | | | |
|--------------------------|-----------|--------------------------------------|---------------|---------------|------------------|---------------------|--------------------|------------------|-----------------|--------------|
| Problem | Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | Mean Func Evals | Success Rate (% | | ime | |
| | QQN | N-Bisection-2 | | | 5.38e1 | 1.81e1 | 2.52e1 | 8.86e1 | 50.6 | 15.0 |
| | GD- | Momentum | | | 6.15e1 | 1.25e1 | 4.06e1 | 8.46e1 | 30.6 | 10.0 |
| | | AdaptiveMoment | | | 4.85e1 | 8.54e0 | 3.80e1 | 6.99e1 | 23.2 | 40.0 |
| | | t Region-Aggress | ive | | 4.36e1 | 3.69e0 | 3.87e1 | 5.26e1 | 93.5 | 45.0 |
| Asklass 2D a 20 kg 2 acc | GD I D | ECC Ammagain | | | 4.47e1 | 4.34e0 | 3.89e1 | 5.49e1 | 19.4 | 35.0 |
| Ackley_2D_a20_b0.2_c6.2 | | FGS-Aggressiv m-AMSGrad | e | | 3.74e0 3.57e0 | 3.90e-1 2.23e-5 | 2.80e0 3.57e0 | 4.38e0 3.57e0 | 3083.9 753.0 | 20.0 |
| | | V-CubicQuadration | Interpolation | | 3.44e0 | 4.86e-1 | 1.95e0 | 4.24e0 | 496.8 | 40.0 |
| | Adai | | . | | 3.57e0 | 5.40e-5 | 3.57e0 | 3.57e0 | 301.6 | 0.0 |
| | | FGS-Conservative | 9 | | 3.57e0 | 1.87e-8 | 3.57e0 | 3.57e0 | 183.6 | 0.0 |
| | L-BI | | | | 3.20e0 | 4.86e-1 | 1.89e0 | 3.57e0 | 202.7 | 50.0 |
| | | m-WeightDecay | | | 3.58e0 | 8.83e-4 | 3.57e0 | 3.58e0 | 119.4 | 0.0 |
| | | FGS-Limited N-GoldenSection | | | 3.37e0 3.18e0 | 7.29e-1 6.29e-1 | 2.64e-1 1.58e0 | 3.57e0 3.57e0 | 176.7 137.5 | 15.0 35.0 |
| | | WeightDecay | | | 3.59e0 | 2.07e-2 | 3.57e0 | 3.66e0 | 49.2 | 0.0 |
| | | Vergnt Decay V-StrongWolfe | | | 3.49e0 | 1.92e-1 | 3.00e0 | 3.57e0 | 62.8 | 15.0 |
| | | t Region-Conserv | ative | | 3.58e0 | 1.90e-2 | 3.57e0 | 3.64e0 | 171.2 | 0.0 |
| | Adaı | m-Fast | | | 3.62e0 | 6.41e-2 | 3.57e0 | 3.79e0 | 52.9 | 0.0 |
| | | N-Bisection-1 | | | 2.85e0 | 1.05e0 | 1.12e-2 | 3.57e0 | 53.6 | 60.0 |
| | | m-Robust | | | 3.58e0 | 3.34e-3 | 3.57e0 | 3.59e0 | 42.9 | 0.0 |
| | | Nesterov | | | 3.57e0 | 2.11e-3 | 3.57e0 | 3.58e0 | 30.6 | 0.0 |
| | | V-Bisection-2 FGS-MoreThuent | Δ | | 2.75e0 3.34e0 | 1.16e0 4.09e-1 | 3.79e-1 1.83e0 | 3.57e0 3.57e0 | 36.5 42.2 | 40.0 45.0 |
| | GD | G5-More i nuent | е | | 3.57e0 | 1.91e-11 | | 3.57e0 3.57e0 | 20.2 | 0.0 |
| | _ | AdaptiveMoment | ıım | | 3.57e0 | 6.21e-11 | | 3.57e0 | 15.2 | 0.0 |
| | | Momentum | | | 3.70e0 | 6.22e-2 | 3.57e0 | 3.79e0 | 14.7 | 0.0 |
| | Trus | t Region-Precise | | | 3.58e0 | 1.39e-2 | 3.58e0 | 3.64e0 | 39.5 | 0.0 |
| | | t Region-Adaptiv | | | 3.66e0 | 2.13e-1 | 3.32e0 | 4.52e0 | 14.8 | 5.0 |
| | | t Region-Aggress | | | 4.78e0 | 2.77e0 | 3.62e0 | 1.67e1 | 5.3 | 0.0 |
| Ackley_5D_a20_b0.2_c6.2 | | t Region-Standar .m-AMSGrad | rd. | | 5.19e0 3.57e0 | 3.91e0 2.74e-9 | 3.58e0 3.57e0 | 1.70e1 3.57e0 | 8.6 1172.5 | 0.0 |
| Ackiey_5D_a20_b0.2_c0.2 | | GS-Aggressive | | | 3.76e0 | 2.74e-3 2.74e-1 | 3.01e0 | 4.09e0 | 3085.2 | 20.0 |
| | Adai | | | | 3.57e0 | 1.27e-6 | 3.57e0 | 3.57e0 | 548.5 | 0.0 |
| | | V-CubicQuadratio | * | | 3.56e0 | 9.71e-2 | 3.14e0 | 3.63e0 | 206.9 | 5.0 |
| | | FGS-Conservative | e | | 3.57e0 | 5.12e-8 | 3.57e0 | 3.57e0 | 373.8 | 0.0 |
| | | FGS-Limited | | | 3.57e0 | 1.20e-8 | 3.57e0 | 3.57e0 | 310.5 | 0.0 |
| | | m-WeightDecay V-GoldenSection | | | 3.57e0 3.51e0 | 4.57e-4 2.78e-1 | 3.57e0 2.30e0 | 3.58e0 3.57e0 | 183.0 193.7 | 5.0 |
| | L-BI | | | | 3.46e0 | 4.50e-1 | 1.87e0 | 4.43e0 | 121.3 | 25.0 |
| | | m-Fast | | | 3.60e0 | 4.00e-2 | 3.57e0 | 3.73e0 | 83.3 | 0.0 |
| | | V-Bisection-1 | | | 3.38e0 | 3.60e-1 | 2.37e0 | 3.57e0 | 65.2 | 30.0 |
| | | t Region-Conserv | ative | | 3.58e0 | 8.66e-4 | 3.57e0 | 3.58e0 | 212.3 | 0.0 |
| | | N-StrongWolfe | | | 3.50e0 | 3.36e-1 | 2.03e0 | 3.57e0 | 53.0 | 5.0 |
| | | m-Robust | | | 3.58e0 | 3.67e-3 | 3.57e0 | 3.59e0 | 59.0 | 0.0 |
| | GD | N-Bisection-2 | | | 3.57e0 3.34e0 | 2.52e-11 3.86e-1 | 3.57e0 2.48e0 | 3.57e0 3.57e0 | 51.8 50.8 | 35.0 |
| | | AdaptiveMoment | um | + | 3.58e0 | 3.63e-3 | | 3.59e0 | 34.5 | 0.0 |
| | L-BI | FGS-MoreThuent | | + | 3.50e0 | 2.48e-1 | 2.54e0 | 3.57e0 | 51.0 | 10.0 |
| | GD- | Nesterov | | | 3.60e0 | 4.12e-2 | 3.57e0 | 3.76e0 | 17.0 | 0.0 |
| | | WeightDecay | | | 3.61e0 | 2.38e-2 | 3.58e0 | 3.69e0 | 15.3 | 0.0 |
| | | t Region-Precise | | | 3.62e0 | 1.68e-1 | 3.57e0 | 4.35e0 | 45.5 | 0.0 |
| | | Momentum t Region-Adaptiv | *** | | 3.72e0 | 1.33e-1 | 3.59e0 | 4.11e0 | 13.0 | 0.0 |
| | | t Region-Adaptiv t Region-Standar | | + | 3.70e0 4.49e0 | 1.47e-1 1.26e0 | 3.58e0 3.60e0 | 4.11e0 6.86e0 | 17.3 9.1 | 0.0 |
| | | t Region-Aggress | | | 4.45e0 4.15e0 | 2.82e-1 | 3.71e0 | 4.90e0 | 5.0 | 0.0 |
| Ackley_10D_a20_b0.2_c6 | | m-AMSGrad | - | | 3.57e0 | 2.45e-9 | 3.57e0 | 3.57e0 | 1241.4 | 0.0 |
| | Adaı | m | | | 3.57e0 | 7.49e-9 | 3.57e0 | 3.57e0 | 745.8 | 0.0 |
| | | V-CubicQuadratio | Interpolation | | 3.59e0 | 9.76e-2 | 3.33e0 | 3.84e0 | 471.4 | 5.0 |
| | | m-WeightDecay | | | 3.57e0 | 1.60e-5 | 3.57e0 | 3.57e0 | 281.3 | 0.0 |
| | | FGS-Conservative m-Robust | 2 | | 3.57e0 3.58e0 | 5.16e-9 3.01e-3 | 3.57e0 3.57e0 | 3.57e0 3.59e0 | 170.7 126.8 | 0.0 |
| | | N-GoldenSection | | + | 3.57e0 | 3.01e-3 1.01e-8 | 3.57e0 3.57e0 | 3.57e0 | 186.9 | 0.0 |
| | GD | . Jorgan Jacob Colon | | + | 3.57e0 | 1.76e-10 | | 3.57e0 | 99.3 | 0.0 |
| | L-BI | FGS-Aggressive | | - | 3.54e0 | 1.35e-1 | 2.95e0 | 3.57e0 | 185.8 | 5.0 |
| | L-BI | FGS-Limited | | | 3.57e0 | 4.05e-10 | | 3.57e0 | 142.2 | 0.0 |
| | | N-StrongWolfe | | | 3.57e0 | 5.84e-15 | | 3.57e0 | 79.2 | 0.0 |
| | | m-Fast | | | 3.60e0 | 4.10e-2 | 3.57e0 | 3.70e0 | 106.6 | 0.0 |
| | L-BI | . GD | | | 3.50e0 | 2.17e-1 | 2.72e0 | 3.58e0 | 133.3 | 15.0 |
| | | | | | | | Contin | nued on next | page | |

| | Table $1-$ continued in | rom previous | page | | | | |
|--------------------|---|--------------------|--------------------|---------------------|--------------------|------------------|--------------|
| Problem | Optimizer Mean Final Std Dev Best Value Value | | Mean Func Evals | Success Rate (%) | Mean T (s) | lime | |
| | QQN-Bisection-1 | 3.54e0 | 1.23e-1 | 3.08e0 | 3.57e0 | 96.2 | 10.0 |
| | Trust Region-Conservative | 3.58e0 | 2.44e-3 | 3.57e0 | 3.58e0 | 205.8 | 0.0 |
| | QQN-Bisection-2 | 3.54e0 | 7.29e-2 | 3.30e0 | 3.57e0 | 65.9 | 20.0 |
| | L-BFGS-MoreThuente | 3.57e0 | 1.12e-2 | 3.52e0 | 3.57e0 | 55.9 | 5.0 |
| | GD-WeightDecay | 3.59e0 | 6.14e-3 | 3.58e0 | 3.60e0 | 19.4 | 0.0 |
| | GD-AdaptiveMomentum GD-Nesterov | 3.60e0 3.66e0 | 1.35e-2 2.68e-2 | 3.57e0 3.62e0 | 3.63e0 3.72e0 | 16.1 14.0 | 0.0 |
| | GD-Nesterov GD-Momentum | 3.81e0 | 6.88e-2 | 3.64e0 | 3.72e0 3.91e0 | 14.5 | 0.0 |
| | Trust Region-Precise | 3.61e0 | 7.74e-2 | 3.58e0 | 3.92e0 | 42.5 | 0.0 |
| | Trust Region-Adaptive | 4.41e0 | 1.08e0 | 3.59e0 | 6.23e0 | 18.2 | 0.0 |
| | Trust Region-Standard | 4.36e0 | 8.00e-1 | 3.64e0 | 5.79e0 | 9.1 | 0.0 |
| | Trust Region-Aggressive | 4.22e0 | 2.38e-1 | 3.81e0 | 4.95e0 | 5.2 | 0.0 |
| StyblinskiTang_2D | Adam-Robust | -2.07e1 | 2.26e0 | -2.44e1 | -1.49e1 | 2502.0 | 0.0 |
| | Adam-AMSGrad | -3.03e1 | 1.58e0 | -3.29e1 | -2.78e1 | 2502.0 | 0.0 |
| | Adam Adam-WeightDecay | -2.94e1 -7.55e1 | 2.68e0 5.64e0 | -3.47e1 -7.83e1 | -2.46e1 -6.42e1 | 2502.0 1893.5 | 0.0 80.0 |
| | L-BFGS-Aggressive | -7.55e1 -5.13e1 | 1.89e1 | -7.77e1 | -0.42e1 -4.63e0 | 3848.1 | 0.0 |
| | Trust Region-Precise | -6.26e1 | 4.27e-1 | -6.35e1 | -6.22e1 | 3002.0 | 0.0 |
| | Trust Region-Conservative | -1.02e1 | 4.38e-1 | -1.15e1 | -9.64e0 | 3002.0 | 0.0 |
| | QQN-StrongWolfe | -7.34e1 | 6.74e0 | -7.83e1 | -6.42e1 | 284.4 | 65.0 |
| | L-BFGS-Conservative | -7.69e1 | 4.23e0 | -7.83e1 | -6.42e1 | 300.1 | 90.0 |
| | QQN-Bisection-1 | -7.40e1 | 7.84e0 | -7.83e1 | -5.01e1 | 385.6 | 70.0 |
| | L-BFGS-Limited | -7.41e1 | 6.48e0 | -7.83e1 | -6.42e1 | 500.0 | 70.0 |
| | Trust Region-Adaptive | -7.37e1 -7.69e1 | 1.73e1 | -7.83e1 | 3.85e-1 -6.42e1 | 885.1 | 70.0 |
| | QQN-GoldenSection QQN-CubicQuadraticInterpolation | -7.34e1 | 4.24e0 8.08e0 | -7.83e1 -7.83e1 | -5.01e1 | 159.8 74.5 | 90.0 |
| | QQN-Bisection-2 | -7.48e1 | 6.12e0 | -7.83e1 | -6.42e1 | 99.0 | 75.0 |
| | L-BFGS | -7.32e1 | 7.01e0 | -7.83e1 | -5.78e1 | 132.3 | 50.0 |
| | GD | -7.34e1 | 6.73e0 | -7.83e1 | -6.42e1 | 66.5 | 65.0 |
| | Adam-Fast | -7.48e1 | 4.67e0 | -7.83e1 | -6.16e1 | 67.3 | 35.0 |
| | Trust Region-Standard | 6.01e1 | 5.78e2 | -7.83e1 | 2.58e3 | 212.3 | 40.0 |
| | GD-WeightDecay GD-Momentum | -7.46e1 -5.68e1 | 6.18e0 1.59e1 | -7.83e1 -7.83e1 | -6.37e1 -2.48e1 | 39.9 35.9 | 35.0 15.0 |
| | GD-Momentum GD-Nesterov | -6.56e1 | 1.39e1 1.28e1 | -7.83e1 | -2.48e1 -3.59e1 | 29.1 | 10.0 |
| | L-BFGS-MoreThuente | -7.41e1 | 6.47e0 | -7.83e1 | -6.42e1 | 54.1 | 70.0 |
| | GD-AdaptiveMomentum | -3.87e1 | 1.37e1 | -7.83e1 | -1.65e1 | 24.4 | 5.0 |
| | Trust Region-Aggressive | -6.43e1 | 2.56e1 | -7.83e1 | 3.64e-1 | 49.3 | 20.0 |
| StyblinskiTang_5D | Adam-Robust | -5.32e1 | 3.79e0 | -6.12e1 | -4.49e1 | 2502.0 | 0.0 |
| | Adam-AMSGrad | -7.50e1 | 4.47e0 | -8.10e1 | -6.56e1 | 2502.0 | 0.0 |
| | Adam Adam-WeightDecay | -7.38e1 -1.90e2 | 4.48e0 6.36e0 | -8.30e1 -1.95e2 | -6.64e1 -1.82e2 | 2502.0 1865.5 | 0.0 65.0 |
| | L-BFGS-Aggressive | -1.90e2 -1.04e2 | 4.24e1 | -1.93e2 -1.83e2 | -1.02e2 -1.25e1 | 3847.9 | 0.0 |
| | Trust Region-Precise | -6.28e1 | 7.02e-1 | -6.40e1 | -6.17e1 | 3002.0 | 0.0 |
| | Trust Region-Conservative | -1.05e1 | 7.12e-1 | -1.22e1 | -9.39e0 | 3002.0 | 0.0 |
| | Trust Region-Adaptive | -1.88e2 | 1.11e1 | -1.95e2 | -1.67e2 | 2249.5 | 65.0 |
| | L-BFGS-Conservative | -1.89e2 | 1.03e1 | -1.95e2 | -1.59e2 | 567.8 | 65.0 |
| | L-BFGS-Limited | -1.89e2 | 6.97e0 | -1.96e2 | -1.82e2 | 839.9 | 50.0 |
| | QQN-StrongWolfe | -1.85e2 | 1.08e1 | -1.96e2 | -1.53e2 | 261.1 | 40.0 |
| | QQN-Bisection-1 QQN-GoldenSection | -1.89e2 -1.90e2 | 8.22e0 9.30e0 | -1.96e2 -1.96e2 | -1.68e2 -1.68e2 | 413.4 330.5 | 60.0 70.0 |
| | QQN-Bisection-2 | -1.89e2 | 9.39e0 | -1.96e2 -1.96e2 | -1.68e2 | 177.8 | 60.0 |
| | QQN-CubicQuadraticInterpolation | -1.82e2 | 1.48e1 | -1.96e2 | -1.53e2 | 129.7 | 45.0 |
| | L-BFGS-MoreThuente | -1.89e2 | 6.99e0 | -1.96e2 | -1.82e2 | 209.2 | 50.0 |
| | Trust Region-Standard | -1.88e2 | 8.33e0 | -1.96e2 | -1.67e2 | 568.8 | 55.0 |
| | GD | -1.86e2 | 1.17e1 | -1.95e2 | -1.53e2 | 95.7 | 50.0 |
| | L-BFGS | -1.86e2 | 7.77e0 | -1.96e2 | -1.74e2 | 153.3 | 30.0 |
| | GD-Momentum | -1.64e2 | 3.72e1 1.04e1 | -1.95e2 | -7.54e1 | 57.8 53.1 | 15.0 |
| | GD-WeightDecay GD-Nesterov | -1.88e2 -1.82e2 | 1.04e1 1.85e1 | -1.96e2 -1.96e2 | -1.53e2 -1.16e2 | 53.1 44.9 | 60.0 30.0 |
| | Adam-Fast | -1.86e2 | 1.14e1 | -1.96e2 | -1.62e2 | 65.0 | 55.0 |
| | GD-AdaptiveMomentum | -1.35e2 | 3.62e1 | -1.95e2 | -7.45e1 | 34.4 | 10.0 |
| | Trust Region-Aggressive | -1.90e2 | 1.11e1 | -1.96e2 | -1.53e2 | 144.5 | 45.0 |
| StyblinskiTang_10D | Adam-Robust | -1.05e2 | 5.69e0 | -1.13e2 | -9.23e1 | 2502.0 | 0.0 |
| | Adam-AMSGrad | -1.51e2 | 6.41e0 | -1.63e2 | -1.38e2 | 2502.0 | 0.0 |
| | Adam Weight Descri | -1.49e2 | 8.21e0 | -1.65e2 | -1.36e2 | 2502.0 | 0.0 |
| | Adam-WeightDecay L-BFGS-Aggressive | -3.73e2 -1.97e2 | 6.63e0 5.94e1 | -3.78e2 -3.07e2 | -3.63e2 -3.33e1 | 1837.7 3848.4 | 35.0 |
| | Trust Region-Precise | -6.35e1 | 9.97e-1 | -6.56e1 | -6.19e1 | 3002.0 | 0.0 |
| | 1 | 0.0001 | 3.0.01 | | ed on next | | |

| | Table 1 | continued fro | m previou | s page | | | | |
|----------|--|-----------------------------------|--------------------|--------------------|---------------------|-------------------------|------------------|---------------|
| Problem | Optimizer Mean Final St Value | td Dev Best Value | Worst Value | Mean Func Evals | Success Rate (%) | Mean Ti (s) | me | |
| | Trust Region-Conservativ | ve | -1.09e1 | 7.10e-1 | -1.25e1 -2.54e2 | -9.28e0 | 3002.0 | 0.0 |
| | Trust Region-Adaptive L-BFGS-Conservative | | -2.51e2 -3.74e2 | 1.09e0 8.27e0 | -2.54e2 -3.82e2 | -2.49e2 -3.49e2 | 3002.0 543.3 | 30.0 |
| | QQN-StrongWolfe | | -3.60e2 | 1.80e1 | -3.86e2 | -3.21e2 | 362.2 | 15.0 |
| | QQN-Bisection-1 | | -3.74e2 | 9.58e0 | -3.89e2 | -3.49e2 | 412.5 | 30.0 |
| | QQN-GoldenSection | | -3.76e2 | 1.20e1 | -3.91e2 | -3.49e2 | 452.4 | 45.0 |
| | L-BFGS-Limited | | -3.76e2 | 7.70e0 | -3.86e2 | -3.63e2 | 550.6 | 35.0 |
| | Trust Region-Standard QQN-CubicQuadraticInte | ernolation | -3.68e2 -3.66e2 | 1.96e1 1.97e1 | -3.79e2 -3.91e2 | -3.00e2 -3.21e2 | 1120.5 172.1 | 40.0 15.0 |
| | QQN-Bisection-2 | erpolation | -3.72e2 | 1.22e1 | -3.90e2 | -3.49e2 | 234.2 | 35.0 |
| | GD | | -3.71e2 | 9.86e0 | -3.80e2 | -3.49e2 | 137.3 | 25.0 |
| | GD-Momentum | | -3.59e2 | 3.83e1 | -3.82e2 | -1.98e2 | 86.2 | 15.0 |
| | GD-WeightDecay L-BFGS-MoreThuente | | -3.75e2 -3.76e2 | 9.86e0 1.18e1 | -3.83e2 -3.91e2 | -3.49e2 -3.49e2 | 74.7 125.9 | 45.0 35.0 |
| | Trust Region-Aggressive | | -3.70e2 -3.71e2 | 1.18e1 1.92e1 | -3.91e2 -3.81e2 | -3.49e2 -3.13e2 | 284.1 | 50.0 |
| | GD-Nesterov | | -3.75e2 | 7.32e0 | -3.83e2 | -3.60e2 | 65.9 | 30.0 |
| | L-BFGS | | -3.76e2 | 1.21e1 | -3.90e2 | -3.46e2 | 132.2 | 45.0 |
| | GD-AdaptiveMomentum | | -3.21e2 | 6.46e1 | -3.82e2 | -1.87e2 | 53.4 | 20.0 |
| Poolo 2D | Adam-Fast | | -3.65e2 | 1.28e1 | -3.85e2 | -3.50e2 | 67.5 | 30.0 |
| Beale_2D | Adam-AMSGrad Adam-Robust | | 2.17e-1 4.58e-1 | 7.60e-2 2.48e-1 | 8.70e-2 1.68e-1 | 3.95e-1 1.07e0 | 2502.0 2502.0 | 0.0 |
| | Adam | | 7.19e-2 | 3.00e-2 | 3.04e-2 | 1.57e-1 | 2502.0 | 0.0 |
| | Adam-WeightDecay | | 1.50e-2 | 2.32e-5 | 1.49e-2 | 1.50e-2 | 1539.9 | 100.0 |
| | QQN-StrongWolfe | | 4.59e-1 | 1.93e0 | 3.96e-3 | 8.88e0 | 786.0 | 90.0 |
| | L-BFGS-Aggressive Trust Region-Conservativ | | 1.39e1 | 3.00e0 | 8.86e0 | 2.02e1 | 3851.9 2999.7 | 0.0 5.0 |
| | QQN-CubicQuadraticInte | | 4.33e0 4.97e-1 | 3.19e0 2.13e0 | 9.92e-3 2.46e-3 | 1.29e1 9.79e0 | 2999.7 | 95.0 |
| | QQN-Bisection-1 | orpolation | 4.13e-1 | 1.77e0 | 4.49e-3 | 8.13e0 | 277.4 | 95.0 |
| | GD | | 1.49e-2 | 4.62e-5 | 1.49e-2 | 1.50e-2 | 229.2 | 100.0 |
| | QQN-GoldenSection | | 6.90e-3 | 2.42e-3 | 2.42e-3 | 9.87e-3 | 347.2 | 100.0 |
| | L-BFGS-Conservative Trust Region-Precise | | 1.17e-2 2.17e-2 | 2.95e-3 1.24e-2 | 5.27e-3 3.20e-3 | 1.49e-2 4.17e-2 | 200.7 743.8 | 100.0 45.0 |
| | GD-WeightDecay | | 1.48e-2 | 1.25e-4 | 1.46e-2 | 1.49e-2 | 88.0 | 100.0 |
| | QQN-Bisection-2 | | 2.61e0 | 3.98e0 | 3.70e-3 | 9.20e0 | 83.8 | 70.0 |
| | L-BFGS | | 4.55e-1 | 8.11e-1 | 1.99e-3 | 3.15e0 | 122.0 | 65.0 |
| | L-BFGS-Limited L-BFGS-MoreThuente | | 1.00e-2 8.71e-3 | 4.71e-3 4.80e-3 | 3.68e-4 3.68e-4 | 1.49e-2 1.50e-2 | 110.5 78.1 | 100.0 |
| | Trust Region-Adaptive | | 1.27e-1 | 1.01e-1 | 3.79e-2 | 5.20e-1 | 179.6 | 0.0 |
| | GD-AdaptiveMomentum | | 3.09e-1 | 7.01e-1 | 6.42e-4 | 2.11e0 | 25.5 | 80.0 |
| | GD-Nesterov | | 9.18e-3 | 2.78e-3 | 4.69e-3 | 1.44e-2 | 27.7 | 100.0 |
| | Adam-Fast | | 1.59e0 | 1.58e-1 | 1.39e0 | 1.86e0 | 37.6 | 0.0 |
| | GD-Momentum Trust Region-Standard | | 1.32e0 2.71e2 | 9.54e-1 1.18e3 | 1.20e-2 1.24e-1 | 2.68e0 5.41e3 | 24.4 50.2 | 5.0 |
| | Trust Region-Aggressive | | 1.23e0 | 4.13e-1 | 1.33e-3 | 1.91e0 | 17.8 | 5.0 |
| Levi_2D | L-BFGS-Aggressive | | 2.87e0 | 8.03e-1 | 1.00e0 | 4.17e0 | 3851.3 | 0.0 |
| | QQN-StrongWolfe | | 1.44e0 | 9.74e-1 | 1.40e-1 | 3.07e0 | 450.4 | 25.0 |
| | L-BFGS-Limited QQN-CubicQuadraticInte | | 9.89e-1 1.04e0 | 6.68e-1 8.94e-1 | 1.13e-1 1.50e-1 | 2.50e0 2.75e0 | 828.4 189.6 | 25.0 40.0 |
| | Adam-AMSGrad | erpolation | 1.04e0 1.91e0 | 1.49e-1 | 1.30e-1 1.43e0 | 1.98e0 | 282.9 | 0.0 |
| | Adam | | 1.93e0 | 1.42e-1 | 1.43e0 | 1.98e0 | 242.4 | 0.0 |
| | QQN-GoldenSection | | 9.83e-1 | 8.11e-1 | 1.87e-2 | 1.98e0 | 262.0 | 45.0 |
| | L-BFGS-Conservative | | 1.94e0 | 1.32e-1 | 1.43e0 | 1.98e0 | 209.4 | 0.0 |
| | QQN-Bisection-1 Trust Region-Conservative | 70 | 1.30e0 1.95e0 | 7.24e-1 3.47e-1 | 1.54e-1 1.07e0 | 2.44e0 2.75e0 | 200.7 490.0 | 15.0 |
| | Adam-WeightDecay | ,,, | 1.95e0 | 2.71e-1 | 1.44e0 | 2.76e0 | 99.6 | 0.0 |
| | L-BFGS | | 4.40e0 | 4.35e0 | 1.84e-1 | 1.60e1 | 107.2 | 15.0 |
| | QQN-Bisection-2 | | 1.08e0 | 8.03e-1 | 4.47e-2 | 1.98e0 | 77.5 | 20.0 |
| | L-BFGS-MoreThuente Adam-Robust | | 1.48e0 1.88e0 | 6.68e-1 2.51e-1 | 1.36e-1 1.08e0 | 1.98e0 2.01e0 | 86.2 42.6 | 15.0 0.0 |
| | Adam-Robust Adam-Fast | | 2.09e0 | 2.68e-1 | 1.08e0 1.47e0 | 2.01e0 2.78e0 | 47.2 | 0.0 |
| | GD-WeightDecay | | 2.00e0 | 4.00e-1 | 1.19e0 | 2.77e0 | 28.1 | 0.0 |
| | GD-AdaptiveMomentum | | 2.06e0 | 3.04e-1 | 1.28e0 | 3.01e0 | 18.7 | 0.0 |
| | Trust Region-Precise GD | | 2.04e0 2.04e0 | 3.89e-1 4.26e-1 | 1.43e0 1.43e0 | 3.38e0 3.11e0 | 76.8 21.9 | 0.0 |
| | GD-Momentum | | 2.04e0 2.02e0 | 5.27e-1 | 7.01e-1 | 2.71e0 | 19.0 | 0.0 |
| | GD-Nesterov | | 1.93e0 | 3.46e-1 | 7.77e-1 | 2.32e0 | 15.2 | 0.0 |
| | Trust Region-Adaptive | | 2.09e0 | 5.10e-1 | 1.46e0 | 4.04e0 | 29.6 | 0.0 |
| | Trust Region-Standard | | 2.18e0 | 4.89e-1 | 1.43e0 | 3.40e0 led on next p | 10.7 | 0.0 |

Table 1- continued from previous page

| Nation Value Value Value Evels Rate (%) Colorate | Probl | lom | Optimizer | Mean Final | $\frac{1 - \text{contin}}{\text{Std Dev}}$ | Best | worst | | ean Func | Success | Mean T | lime | |
|--|-------------------|-----|-----------|-----------------|--|------|---------|-----|----------|---------|---------|--------|-------|
| Conditional Prince 21 | 11001 | lem | Optimizer | | Std Dev | | I | IVI | I | | | | |
| LBFKS Aggreeive | | | | | ive | | 1.33e2 | | 5.68e2 | 1.02e0 | 2.61e3 | 6.3 | 0.0 |
| Trust Region Conservative | GoldsteinPrice_2I | D | | | | | | | | | | | |
| Trust Region-Apartive 22063 1.0963 8.4062 4.2063 29438 0.0 | | | | | ative | | | | | 1 | | | |
| QQN StrongWille S.84e1 2.81e1 7.11e1 8.40e2 3.05 0.01 | | | | | weive | | | | | | | | |
| Adam Alfane S. 4062 3.176-2 8.4062 8.002 8.002 8.007 8 | | | | | е | | | | | | | | |
| Adam | | | | | | | | | | | | | |
| LeBFGS-MoreThurnte | | | | | | | | | | | | | I |
| Trust Region-Standard | | | | | | ı | | | | | | | |
| QQN-Bisection | | | | | | | | | | | | | |
| QNN-GoldenSection | | | | | u | | | | | 1 | | | |
| Adam. Weight Decay | | | QQN | -GoldenSection | | | | | 3.02e-11 | 8.40e1 | 8.40e1 | 383.2 | I |
| Trust Region-Aggressive | | | | | | | | | | | | | |
| LBFGS | | | | | ive | | | | | | | | |
| QQN Silection 2 1.58e2 1.99e2 8.40e1 8.40e2 21.2 5.0 GD-Nesterov 8.57e2 1.82e1 8.40e2 9.22e2 24.6 0.0 Adam-Fast 8.55e2 1.88e1 8.40e2 9.15e2 35.3 0.0 GD-AdaptiveMomentum 8.45e2 4.11e0 8.41e2 8.57e2 1.99 0.0 GD-Momentum 8.85e2 2.94e2 9.88e1 1.93e3 17.9 0.0 GD-Momentum 8.85e2 2.94e2 9.88e1 1.93e3 17.9 0.0 GD-Momentum 8.85e2 2.94e2 9.88e1 1.93e3 17.9 0.0 Adam-AMSGrad 2.50e-2 1.14e-5 2.50e-2 2.50e-2 6.29e 1.00.0 Adam-AMSGrad 2.50e-2 1.14e-5 2.50e-2 2.50e-2 6.29e 1.00.0 Adam-AMSGrad 2.50e-2 1.46e-5 2.50e-2 2.50e-2 6.242 1.00.0 GD-WoightDecay 2.50e-2 1.46e-5 2.50e-2 2.50e-2 6.242 1.00.0 GD-WoightDecay 2.50e-2 1.46e-5 2.50e-2 2.50e-2 2.23.7 1.00.0 Adam-MeightDecay 2.49e-2 5.67e-5 2.48e-2 2.50e-2 2.70e-2 1.00.0 GD-Momentum 2.49e-2 5.67e-5 2.48e-2 2.50e-2 7.10 1.00.0 GD-Momentum 2.49e-2 5.67e-5 2.48e-2 2.50e-2 7.10 1.00.0 GD-Momentum 2.49e-2 5.67e-5 2.48e-2 2.50e-2 7.10 1.00.0 GD-Momentum 2.50e-2 1.11e-4 2.6e-2 2.50e-2 7.10 1.00.0 GD-Momentum 2.50e-2 1.69e-3 2.46e-2 2.50e-2 7.10 1.00.0 GD-Momentum 2.50e-2 6.76e-3 1.12e-3 3.66e-2 2.17-4 95.0 GD-Momentum 2.50e-2 6.7 | | | | | | | | | | | | | |
| GD. Nesterow | | | | | | | | | | | | | |
| Adam-Fast S.55e2 1.88et 8.40e2 9.18e2 35.3 0.0 | | | | | | | | | | | | | |
| GD-AdaptiveMomentum | | | | | | | | | | | | | |
| GD-WeightDeay | | | GD-A | | um | | 8.45e2 | | 4.79e0 | 8.41e2 | 8.57e2 | 19.9 | 0.0 |
| GD-Momentum | | | _ | W-:L4D | | | | | | | | | |
| Matyas 2D | | | | | | | | | | | | | I |
| Adam | Matyas_2D | | | violiteitutii | | | | | | 1 | | | I |
| GD-WeightDecay | | | | | | | | | | | | | |
| Adam-WeightDecay | | | | | | | | | | | | | |
| GD-Momentum | | | | | | | | | | | | | |
| LBFGS-Aggressive | | | | | | | | | | | | | |
| Adam-Robust | | | | | | | | | | | | | |
| QQN-GoldenSection | | | | | | | | | | | | | |
| QQN-Bisection-2 | | | QQN | -GoldenSection | | | 1.55e-2 | | 6.76e-3 | 1.12e-3 | 2.47e-2 | 138.9 | 100.0 |
| QQN-Cubic Quadratic Interpolation 1.07c-2 9.48c-3 2.70c-30 2.47c-2 34.0 100.0 | | | | | um | | | | | | | | |
| QQN-Bisection-1 | | | -0 -0 | | Interpolation | 2 | | | | | | | |
| QQN-StrongWolfe 2.73e-29 3.90e-29 1.77e-32 1.69e-28 24.0 100.0 L-BFGS-Limited 1.73e-2 5.34e-3 7.16e-3 2.46e-2 24.6 100.0 L-BFGS 1.47e-2 9.70e-3 5.88e-4 2.50e-2 20.1 100.0 Adam-Fast 2.30e-2 1.45e-3 1.98e-2 2.49e-2 12.9 100.0 L-BFGS-MoreThuente 1.72e-2 8.74e-3 2.59e-5 2.46e-2 20.8 100.0 Trust Region-Standard 2.41e-1 1.91e-1 3.14e-2 3.36e-1 7.2 0.0 Trust Region-Adaptive 6.88e-2 1.85e-2 2.87e-2 5.0 0.0 Trust Region-Adaptive 6.88e-2 1.85e-2 2.87e-2 7.0 0.0 Trust Region-Aggressive 6.67e-1 6.95e-1 2.86e-2 1.50e0 6.3 0.0 Himmelblau.2D Adam-AMSGrad 8.17e1 5.99e0 7.19e1 9.17e1 2502.0 0.0 Adam-Augish Decay 2.41e-1 4.0e-3 | | | | | Interpolation | 1 | | | | | | | |
| L-BFGS-Limited 1.73e-2 5.34e-3 7.16e-3 2.46e-2 24.6 100.0 L-BFGS 1.47e-2 9.70e-3 5.88e-4 2.50e-2 20.1 100.0 Adam-Fast 2.30e-2 1.45e-3 1.98e-2 2.49e-2 12.9 100.0 L-BFGS-MoreThuente 1.72e-2 8.74e-3 2.59e-5 2.46e-2 20.8 100.0 Trust Region-Standard 2.41e-1 1.19e-1 3.14e-2 3.36e-1 7.2 0.0 Trust Region-Precise 4.71e-2 8.16e-3 3.00e-2 5.69e-2 5.0 0.0 Trust Region-Adaptive 6.88e-2 1.85e-2 2.83e-2 8.71e-2 7.0 0.0 Trust Region-Conservative 3.93e-2 6.90e-3 2.66e-2 5.75e-2 7.0 0.0 Trust Region-Conservative 6.67e-1 6.95e-1 2.86e-2 1.50e0 6.3 0.0 Adam-Robust 1.05e2 5.73e0 9.10e1 1.15e2 2502.0 0.0 Adam-MSGrad 8.17e1 5.99e0 7.19e1 9.17e1 2502.0 0.0 Adam-WeightDecay 2.41e-1 4.02e-3 2.34e-1 2.48e-1 1744.1 100.0 L-BFGS-Aggressive 3.44e1 1.81e1 4.95e0 7.59e1 3850.9 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Standard 1.50e-1 5.93e-2 7.13e-2 2.46e-1 2048.1 100.0 L-BFGS-Conservative 2.03e-1 5.93e-2 7.13e-2 2.46e-1 2048.1 100.0 Trust Region-Standard 1.50e-1 8.67e-2 3.76e-3 3.03e1 516.7 80.0 QQN-Bisection-1 1.09e-1 4.94e-2 2.06e-2 2.13e-1 107.9 100.0 QQN-StrongWolfe 9.18e-2 6.79e-2 9.60e-3 2.15e-1 68.0 100.0 QQN-StrongWolfe 9.18e-2 6.79e-2 9.60e-3 2.15e-1 61.5 100.0 QQN-GlodenSection 7.91e-2 6.51e-2 1.17e-4 2.36e-1 102.8 100.0 QQN-GlodenSection 7.91e-2 6.51e-2 1.17e-4 2.36e-1 102.8 100.0 QQN-GlodenSection 7.91e-2 6.51e-2 1.17e-4 2.36e-1 55.1 | | | | | | | | | | | | | 100.0 |
| L-BFGS | | | | | | | | | | | | | |
| Adam-Fast | | | | | | | | | | | | | |
| Trust Region-Standard 2.41e-1 1.19e-1 3.14e-2 3.36e-1 7.2 0.0 Trust Region-Precise 4.71e-2 8.16e-3 3.00e-2 5.69e-2 5.0 0.0 Trust Region-Adaptive 6.88e-2 1.85e-2 2.83e-2 8.71e-2 7.0 0.0 Trust Region-Conservative 3.93e-2 6.90e-3 2.66e-2 5.75e-2 7.0 0.0 Mader-Robust 1.05e2 5.73e0 9.10e1 1.15e2 2502.0 0.0 Adam-Robust 1.05e2 5.73e0 9.10e1 1.15e2 2502.0 0.0 Adam-AMSGrad 8.17e1 5.99e0 7.19e1 9.17e1 2502.0 0.0 Adam-WeightDecay 2.41e-1 4.02e-3 2.34e-1 2.48e-1 1744.1 100.0 Trust Region-Precise 3.44e1 1.81e1 4.95e0 7.59e1 3850.9 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Adaptive 1.32e-1 | | | Adan | n-Fast | | | 2.30e-2 | | 1.45e-3 | 1.98e-2 | 2.49e-2 | 12.9 | 100.0 |
| Trust Region-Precise 4.71e-2 8.16e-3 3.00e-2 5.69e-2 5.0 0.0 Trust Region-Adaptive 6.88e-2 1.85e-2 2.83e-2 8.71e-2 7.0 0.0 Trust Region-Conservative 3.93e-2 6.69e-3 2.66e-2 5.75e-2 7.0 0.0 Trust Region-Aggressive 6.67e-1 6.95e-1 2.86e-2 1.50e0 6.3 0.0 Adam-Robust 1.05e2 5.73e0 9.10e1 1.15e2 2502.0 0.0 Adam-AMSGrad 8.17e1 5.99e0 7.19e1 9.17e1 2502.0 0.0 Adam-WeightDecay 2.41e-1 4.02e-3 2.34e-1 2.48e-1 1744.1 100.0 L-BFGS-Aggressive 3.44e1 1.81e1 4.95e0 7.59e1 3850.9 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Adaptive 1.30e-1 5.99e-2 7.13e-2 2.46e-1 2048.1 100.0 L-BFGS-Conservative 2.03e | | | | | | | | | | | | | |
| Trust Region-Adaptive 6.88e-2 1.85e-2 2.83e-2 8.71e-2 7.0 0.0 Trust Region-Conservative 3.93e-2 6.90e-3 2.66e-2 5.75e-2 7.0 0.0 Trust Region-Aggressive 6.67e-1 6.95e-1 2.86e-2 1.50e0 6.3 0.0 Adam-Robust 1.05e2 5.73e0 9.10e1 1.15e2 2502.0 0.0 Adam-AMSGrad 8.17e1 5.99e0 7.19e1 9.17e1 2502.0 0.0 Adam 8.20e1 5.99e0 7.20e1 9.40e1 2502.0 0.0 Adam-WeightDecay 2.41e-1 4.02e-3 2.34e-1 2.48e-1 1744.1 100.0 L-BFGS-Aggressive 3.44e1 4.81e1 4.95e0 7.59e1 3850.9 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Adaptive 1.32e-1 5.93e-2 7.13e-2 2.46e-1 2048.1 100.0 L-BFGS-Conservative 2.03e-1 < | | | | | u | | | | | | | | |
| Trust Region-Aggressive 6.67e-1 6.95e-1 2.86e-2 1.50e0 6.3 0.0 Adam-Robust 1.05e2 5.73e0 9.10e1 1.15e2 2502.0 0.0 Adam-AMSGrad 8.17e1 5.99e0 7.19e1 9.17e1 2502.0 0.0 Adam-WeightDecay 2.41e-1 4.02e-3 2.34e-1 2.48e-1 1744.1 100.0 L-BFGS-Aggressive 3.44e1 1.81e1 4.95e0 7.59e1 3850.9 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Conservative 1.60e2 3.17e0 1.52e2 1.65e2 3002.0 0.0 Trust Region-Adaptive 1.32e-1 5.93e-2 7.13e-2 2.46e-1 2048.1 100.0 L-BFGS-Conservative 2.03e-1 2.91e-2 1.42e-1 2.47e-1 264.2 100.0 Trust Region-Standard 1.50e-1 8.67e-2 3.76e-3 3.03e-1 516.7 80.0 QQN-Bisection-1 1.09e-1 4.94e-2 2.06e-2 2.13e-1 107.9 100.0 QQN-CubicQuadraticInterpolation 9.46e-2 7.76e-2 5.19e-3 2.45e-1 68.0 100.0 Adam-Fast 5.29e0 2.08e-1 4.98e0 5.82e0 69.3 0.0 QQN-StrongWolfe 9.18e-2 6.79e-2 9.60e-3 2.15e-1 61.5 100.0 L-BFGS 6.22e0 5.64e0 1.56e-2 1.80e1 93.7 35.0 QQN-GoldenSection 7.91e-2 6.51e-2 1.17e-4 2.36e-1 102.8 100.0 QQN-Bisection-2 8.17e-2 6.08e-2 2.27e-3 2.05e-1 55.1 100.0 | | | Trust | Region-Adaptiv | | | 6.88e-2 | | 1.85e-2 | 2.83e-2 | 8.71e-2 | 7.0 | |
| Himmelblau_2D | | _ | | | | | | | | | | | I |
| Adam-AMSGrad 8.17e1 5.99e0 7.19e1 9.17e1 2502.0 0.0 Adam 8.20e1 5.99e0 7.20e1 9.40e1 2502.0 0.0 Adam-WeightDecay 2.41e-1 4.02e-3 2.34e-1 2.48e-1 1744.1 100.0 L-BFGS-Aggressive 3.44e1 1.81e1 4.95e0 7.59e1 3850.9 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Conservative 1.60e2 3.17e0 1.52e2 1.65e2 3002.0 0.0 Trust Region-Adaptive 1.32e-1 5.93e-2 7.13e-2 2.46e-1 2048.1 100.0 L-BFGS-Conservative 2.03e-1 2.91e-2 1.42e-1 2.47e-1 264.2 100.0 Trust Region-Standard 1.50e-1 8.67e-2 3.76e-3 3.03e-1 516.7 80.0 QQN-Bisection-1 1.09e-1 4.94e-2 2.06e-2 2.13e-1 107.9 100.0 Adam-Fast 5.29e0 2.08e-1 4.98e0 5.82e0 69.3 0.0 QQN-Str | Himmelblau 2D | | | | ive | | | | | | | | |
| Adam-WeightDecay 2.41e-1 4.02e-3 2.34e-1 2.48e-1 1744.1 100.0 L-BFGS-Aggressive 3.44e1 1.81e1 4.95e0 7.59e1 3850.9 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Conservative 1.60e2 3.17e0 1.52e2 1.65e2 3002.0 0.0 Trust Region-Adaptive 1.32e-1 5.93e-2 7.13e-2 2.46e-1 2048.1 100.0 L-BFGS-Conservative 2.03e-1 2.91e-2 1.42e-1 2.47e-1 264.2 100.0 Trust Region-Standard 1.50e-1 8.67e-2 3.76e-3 3.03e-1 516.7 80.0 QQN-Bisection-1 1.09e-1 4.94e-2 2.06e-2 2.13e-1 107.9 100.0 QQN-CubicQuadraticInterpolation 9.46e-2 7.76e-2 5.19e-3 2.45e-1 68.0 100.0 Adam-Fast 5.29e0 2.08e-1 4.98e0 5.82e0 69.3 0.0 QQN-StrongWolfe 9.18e-2 6.79e-2 9.60e-3 2.15e-1 61.5 100.0 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>8.17e1</td> <td></td> <td>5.99e0</td> <td>7.19e1</td> <td></td> <td>2502.0</td> <td></td> | | | | | | | 8.17e1 | | 5.99e0 | 7.19e1 | | 2502.0 | |
| L-BFGS-Aggressive 3.44e1 1.81e1 4.95e0 7.59e1 3850.9 0.0 Trust Region-Precise 1.07e2 3.03e0 1.02e2 1.12e2 3002.0 0.0 Trust Region-Conservative 1.60e2 3.17e0 1.52e2 1.65e2 3002.0 0.0 Trust Region-Adaptive 1.32e-1 5.93e-2 7.13e-2 2.46e-1 2048.1 100.0 L-BFGS-Conservative 2.03e-1 2.91e-2 1.42e-1 2.47e-1 264.2 100.0 Trust Region-Standard 1.50e-1 8.67e-2 3.76e-3 3.03e-1 516.7 80.0 QQN-Bisection-1 1.09e-1 4.94e-2 2.06e-2 2.13e-1 107.9 100.0 QQN-CubicQuadraticInterpolation 9.46e-2 7.76e-2 5.19e-3 2.45e-1 68.0 100.0 Adam-Fast 5.29e0 2.08e-1 4.98e0 5.82e0 69.3 0.0 QQN-StrongWolfe 9.18e-2 6.79e-2 9.60e-3 2.15e-1 61.5 100.0 L-BFGS 6.22e0 5.64e0 1.56e-2 1.80e1 93.7 35.0 | | | | | | | | | | | | | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | | | | | | | | | | | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | | | | | | | | | | | |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | Trust | Region-Conserv | | | 1.60e2 | | 3.17e0 | 1.52e2 | 1.65e2 | 3002.0 | 0.0 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | | | | | | | | | | | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | | | | | | | | | | | |
| QQN-CubicQuadraticInterpolation 9.46e-2 7.76e-2 5.19e-3 2.45e-1 68.0 100.0 Adam-Fast 5.29e0 2.08e-1 4.98e0 5.82e0 69.3 0.0 QQN-StrongWolfe 9.18e-2 6.79e-2 9.60e-3 2.15e-1 61.5 100.0 L-BFGS 6.22e0 5.64e0 1.56e-2 1.80e1 93.7 35.0 QQN-GoldenSection 7.91e-2 6.51e-2 1.17e-4 2.36e-1 102.8 100.0 QQN-Bisection-2 8.17e-2 6.08e-2 2.27e-3 2.05e-1 55.1 100.0 | | | | | | | | | | | | | |
| QQN-StrongWolfe 9.18e-2 6.79e-2 9.60e-3 2.15e-1 61.5 100.0 L-BFGS 6.22e0 5.64e0 1.56e-2 1.80e1 93.7 35.0 QQN-GoldenSection 7.91e-2 6.51e-2 1.17e-4 2.36e-1 102.8 100.0 QQN-Bisection-2 8.17e-2 6.08e-2 2.27e-3 2.05e-1 55.1 100.0 | | | QQN | -CubicQuadratic | Interpolation | ı | | | | | | | |
| L-BFGS 6.22e0 5.64e0 1.56e-2 1.80e1 93.7 35.0 QQN-GoldenSection 7.91e-2 6.51e-2 1.17e-4 2.36e-1 102.8 100.0 QQN-Bisection-2 8.17e-2 6.08e-2 2.27e-3 2.05e-1 55.1 100.0 | | | | | | | | | | | | | |
| QQN-GoldenSection 7.91e-2 6.51e-2 1.17e-4 2.36e-1 102.8 100.0 QQN-Bisection-2 8.17e-2 6.08e-2 2.27e-3 2.05e-1 55.1 100.0 | | | | | | | | | | 1 | | | I |
| | | | QQN | -GoldenSection | | | 7.91e-2 | | 6.51e-2 | 1.17e-4 | 2.36e-1 | 102.8 | 100.0 |
| | | | QQN | -Bisection-2 | | | 8.17e-2 | | 6.08e-2 | | | | 100.0 |

| | | | Tabl | e 1 - contin | nued fro | m previou | ıs pa | age | | | | |
|-----------|---------|----------------------------|----------------------------------|----------------|---------------|--------------------|-------|---------------------|---------------------|--------------------|------------------|--------------|
| | Problem | Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | M | ean Func Evals | Success Rate (%) | Mean T | Time | |
| _ | | L-BF | GS-Limited | | | 9.58e-2 | | 8.02e-2 | 7.81e-3 | 2.40e-1 | 60.9 | 100.0 |
| | | GD | | | | 1.53e-1 | | 6.88e-2 | 2.80e-2 | 2.41e-1 | 42.9 | 100.0 |
| | | | VeightDecay | | | 3.93e-1 | | 4.16e-1 | 8.90e-2 | 1.20e0 | 29.6 | 75.0 |
| | | | Region-Aggress Nesterov | sive | | 6.71e-1 9.49e0 | | 4.77e-1 7.94e0 | 1.80e-2 3.27e-2 | 1.52e0 2.11e1 | 133.1 26.7 | 25.0 40.0 |
| | | I | AdaptiveMoment | um | | 6.94e1 | | 3.44e0 | 5.87e1 | 7.33e1 | 22.8 | 0.0 |
| | | | Momentum | | | 4.37e1 | | 1.60e1 | 3.43e0 | 7.63e1 | 24.3 | 0.0 |
| | | I | GS-MoreThuent | e | | 1.02e-1 | | 7.28e-2 | 8.92e-3 | 2.39e-1 | 36.3 | 100.0 |
| Booth_2D | 1 | | m-AMSGrad | | | 1.19e1 | | 2.01e0 | 8.69e0 | 1.53e1 | 2502.0 | 0.0 |
| | | Adam | n-Robust | | | 2.03e1 1.20e1 | | 2.55e0 2.17e0 | 1.43e1 8.76e0 | 2.49e1 1.67e1 | 2502.0 2502.0 | 0.0 |
| | | I | n-WeightDecay | | | 1.19e-1 | | 3.80e-4 | 1.19e-1 | 1.20e-1 | 1886.3 | 100.0 |
| | | | GS-Aggressive | | | 7.43e1 | | 5.77e0 | 6.24e1 | 8.60e1 | 3852.0 | 0.0 |
| | | | Region-Precise | | | 1.05e1 | | 5.90e0 | 2.82e0 | 2.22e1 | 3002.0 | 0.0 |
| | | Trust | Region-Conserv | vative | | 6.49e1 | | 6.14e0 | 5.04e1 | 7.80e1 | 3002.0 | 0.0 |
| | | | Region-Adaptiv | | | 6.12e-2 | | 3.24e-2 | 2.38e-2 | 1.24e-1 | 919.9 | 90.0 |
| | | I | GS-Conservative -Bisection-1 | 9 | | 1.10e-1 1.87e-2 | | 5.23e-3 1.86e-2 | 1.03e-1 1.21e-2 | 1.20e-1 9.94e-2 | 208.0 162.4 | 100.0 |
| | | GD | -Disection-1 | | | 1.17e-1 | | 1.31e-3 | 1.21e-2 1.15e-1 | 1.20e-1 | 86.0 | 100.0 |
| | | | -Bisection-2 | | | 1.49e-2 | | 2.92e-3 | 1.15e-2 | 2.20e-2 | 100.0 | 100.0 |
| | | | Region-Standar | | | 5.27e-1 | | 1.54e-1 | 3.46e-1 | 7.93e-1 | 235.1 | 0.0 |
| | | | -CubicQuadratio | cInterpolation | ı | 2.56e-6 | | 2.13e-6 | 2.81e-7 | 7.40e-6 | 56.0 | 100.0 |
| | | I | n-Fast GS-Limited | | | 3.09e0 6.14e-2 | | 2.17e-1 3.83e-2 | 2.74e0 2.04e-3 | 3.45e0 1.13e-1 | 56.3 56.1 | 0.0 |
| | | I | -GoldenSection | | | 3.39e-2 | | 3.83e-2 1.16e-2 | 2.04e-3 1.60e-2 | 5.86e-2 | 92.0 | 100.0 |
| | | | VeightDecay | | | 1.29e0 | | 5.10e-2 | 1.00e-2 1.15e-1 | 1.73e0 | 27.0 | 15.0 |
| | | | AdaptiveMoment | um | | 9.51e0 | | 8.38e-1 | 8.56e0 | 1.11e1 | 21.0 | 0.0 |
| | | I | Vesterov | | | 2.56e0 | | 2.92e-1 | 2.10e0 | 2.95e0 | 20.6 | 0.0 |
| | | L-BF | | | | 4.05e-2 | | 1.53e-2 | 1.17e-2 | 6.97e-2 | 42.4 | 100.0 |
| | | | Momentum | | | 7.27e0 | | 3.58e-1 | 6.20e0 4.00e-1 | 8.01e0 2.38e0 | 21.2 63.5 | 0.0 |
| | | | Region-Aggress GS-MoreThuent | | | 1.49e0 3.86e-2 | | 4.64e-1 3.42e-2 | 5.15e-4 | 2.38e0 1.13e-1 | 29.7 | 100.0 |
| | | | -StrongWolfe | | | 1.13e-2 | | 7.50e-4 | 9.93e-3 | 1.19e-1 1.29e-2 | 26.0 | 100.0 |
| Griewank. | _2D | | m-AMSGrad | | | 5.24e0 | | 4.70e-2 | 5.16e0 | 5.30e0 | 2502.0 | 0.0 |
| | | I | n-Robust | | | 5.36e0 | | 6.83e-2 | 5.26e0 | 5.48e0 | 2502.0 | 0.0 |
| | | | GS-Aggressive | | | 4.91e0 | | 7.72e-3 | 4.91e0 | 4.95e0 | 2427.6 | 0.0 |
| | | Adam | n-WeightDecay | | | 5.24e0 4.92e0 | | 4.43e-2 2.42e-6 | 5.16e0 4.92e0 | 5.30e0 4.92e0 | 2502.0 2231.2 | 0.0 |
| | | GD | i- weight Decay | | | 4.92e0 4.91e0 | | 3.24e-8 | 4.92e0 4.91e0 | 4.91e0 4.91e0 | 1668.0 | 0.0 |
| | | | GS-Limited | | | 4.91e0 | | 4.49e-8 | 4.91e0 | 4.91e0 | 927.3 | 0.0 |
| | | QQN | -StrongWolfe | | | 4.74e0 | | 3.50e-1 | 3.76e0 | 4.91e0 | 347.6 | 0.0 |
| | | | VeightDecay | | | 4.91e0 | | 1.03e-7 | 4.91e0 | 4.91e0 | 406.6 | 0.0 |
| | | L-BF | GS-Conservative | е | | 4.91e0 4.91e0 | | 3.09e-9 3.75e-8 | 4.91e0 4.91e0 | 4.91e0 4.91e0 | 504.6 295.6 | 0.0 |
| | | I | -GoldenSection | | | 4.91e0 4.91e0 | | 1.03e-13 | 4.91e0 4.91e0 | 4.91e0 4.91e0 | 258.3 | 0.0 |
| | | | Vesterov | | | 4.91e0 | | 1.90e-4 | 4.91e0 | 4.91e0 | 125.0 | 0.0 |
| | | | Momentum | | | 4.91e0 | | 4.49e-4 | 4.91e0 | 4.91e0 | 100.2 | 0.0 |
| | | | Region-Conserv | | | 4.92e0 | | 2.48e-3 | 4.91e0 | 4.92e0 | 336.5 | 0.0 |
| | | | -CubicQuadratio | cInterpolation | 1 | 4.80e0 | | 4.73e-1 | 2.74e0 | 4.91e0 | 77.3 | 0.0 |
| | | QQN | -Bisection-2 -Bisection-1 | | | 4.66e0 4.91e0 | | 2.21e-1 1.27e-13 | 4.25e0 4.91e0 | 4.91e0 4.91e0 | 94.6 76.3 | 0.0 |
| | | | n-Fast | | | 4.93e0 | | 6.09e-3 | 4.92e0 | 4.94e0 | 67.0 | 0.0 |
| | | | GS-MoreThuent | e | | 4.91e0 | | 1.88e-7 | 4.91e0 | 4.91e0 | 73.0 | 0.0 |
| | | | AdaptiveMoment | um | | 4.99e0 | | 5.58e-3 | 4.97e0 | 5.00e0 | 32.0 | 0.0 |
| | | | Region-Precise | | | 4.95e0 | | 1.38e-2 | 4.93e0 | 4.97e0 | 58.7 | 0.0 |
| | | | Region-Adaptiv | | | 5.11e0 | | 1.18e-1 | 5.00e0 | 5.54e0 | 16.4 | 0.0 |
| | | | Region-Aggress Region-Standar | | | 6.00e0 5.99e0 | | 6.58e-2 7.24e-2 | 5.90e0 5.90e0 | 6.12e0 6.16e0 | 5.0 | 0.0 |
| Griewank. | _5D | | N-StrongWolfe | | | 1.09e1 | | 2.09e0 | 5.74e0 | 1.33e1 | 1535.0 | 0.0 |
| | | Adam | n-Robust | | | 1.29e1 | | 6.91e-2 | 1.28e1 | 1.30e1 | 2502.0 | 0.0 |
| | | | n-AMSGrad | | | 1.25e1 | | 2.91e-2 | 1.25e1 | 1.26e1 | 2502.0 | 0.0 |
| | | Adan | า | | | 1.25e1 | | 2.35e-2 | 1.25e1 | 1.25e1 | 2502.0 | 0.0 |
| | | - AF | | | | | | 1.06e-3 | 1.22e1 | 1.22e1 | 1668.0 | 0.0 |
| | | GD | | | | 1.22e1 | | | | | | |
| | | Adan | n-WeightDecay | | | 1.23e1 | | 6.65e-3 | 1.23e1 | 1.23e1 | 1651.7 | 0.0 |
| | | Adam GD-V | | | | | | | | | | |
| | | Adam GD-V QQN QQN | n-WeightDecay WeightDecay | | | 1.23e1 1.22e1 | | 6.65e-3 1.86e-5 | 1.23e1 1.22e1 | 1.23e1 1.22e1 | 1651.7 915.3 | 0.0 |

| _ | | | Tabl | e 1 - contin | nued fro | m previou | us page | 3 | | | | |
|--------------|---------|-----------|--------------------------------------|--------------|---------------|------------------|---------|---------------------|---------------------|------------------|------------------|------|
| | Problem | Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | 1 | n Func vals | Success Rate (%) | | Cime | |
| | | L-BF | GS-Conservative | e | | 1.22e1 | | 5.00e-9 | 1.22e1 | 1.22e1 | 705.6 | 0.0 |
| | | | GS-Limited | | | 1.22e1 | | 6.36e-8 | 1.22e1 | 1.22e1 | 632.5 | 0.0 |
| | | | GS-Aggressive | | | 1.09e1 | | 2.56e0 | 5.15e0 | 1.22e1 | 1097.7 | 0.0 |
| | | | Nesterov Momentum | | | 1.22e1 1.22e1 | | 3.21e-9 2.51e-9 | 1.22e1 1.22e1 | 1.22e1 1.22e1 | 394.2 387.2 | 0.0 |
| | | L-BF | | | | 1.22e1 1.22e1 | | 4.05e-3 | 1.22e1 1.22e1 | 1.22e1 1.23e1 | 259.7 | 0.0 |
| | | | AdaptiveMoment | tum | | 1.23e1 | | 3.47e-2 | 1.22e1 | 1.23e1 | 133.1 | 0.0 |
| | | | -CubicQuadrati | | n | 1.28e1 | | 2.58e-1 | 1.22e1 | 1.32e1 | 115.1 | 0.0 |
| | | | t Region-Conser | | | 1.23e1 | | 2.20e-3 | 1.23e1 | 1.23e1 | 380.4 | 0.0 |
| | | | GS-MoreThuent | e | | 1.22e1 | | 2.35e-7 | 1.22e1 | 1.22e1 | 118.5 | 0.0 |
| | | | n-Fast | | | 1.23e1 | | 1.71e-2 3.58e-1 | 1.22e1 | 1.23e1 | 67.1 11.4 | 0.0 |
| | | Trust | t Region-Precise t Region-Adapti | vre | | 1.34e1 1.35e1 | | 3.00e-2 | 1.23e1 1.34e1 | 1.36e1 1.35e1 | 5.0 | 0.0 |
| | | | t Region-Standa | | | 1.35e1 | | 2.54e-2 | 1.34e1 | 1.36e1 | 5.0 | 0.0 |
| | | | t Region-Aggress | | | 1.35e1 | | 2.50e-2 | 1.35e1 | 1.35e1 | 5.0 | 0.0 |
| Griewank_ | 10D | QQI | N-StrongWolfe | | | 6.06e0 | | 1.00e1 | 9.86e-3 | 2.60e1 | 2281.2 | 0.0 |
| | | | I-GoldenSection | | | 2.35e1 | | 6.71e0 | 2.19e0 | 2.58e1 | 4343.6 | 0.0 |
| | | • • | I-Bisection-2 | | | 2.33e1 | | 7.17e0 | 1.67e0 | 2.58e1 | 3260.0 | 0.0 |
| | | | f-Bisection-1 n-AMSGrad | | | 2.19e1 2.54e1 | | 9.08e0 2.09e-2 | 5.93e-10 2.53e1 | 2.58e1 2.54e1 | 2381.1 2502.0 | 5.0 |
| | | | n-Robust | | | 2.54e1 2.55e1 | | 2.09e-2 2.00e-2 | 2.55e1 2.55e1 | 2.54e1 2.55e1 | 2502.0 | 0.0 |
| | | | n-WeightDecay | | | 2.42e1 | | 1.44e-2 | 2.41e1 | 2.42e1 | 2502.0 | 0.0 |
| | | L-BF | GS-Limited | | | 1.95e1 | | 3.46e0 | 8.48e0 | 2.38e1 | 2256.7 | 0.0 |
| | | | WeightDecay | | | 2.48e1 | | 3.83e-2 | 2.48e1 | 2.49e1 | 1668.0 | 0.0 |
| | | | Nesterov | | | 2.21e1 | | 2.17e-1 | 2.15e1 | 2.23e1 | 1668.0 | 0.0 |
| | | Adar | | | | 2.54e1 | | 2.20e-2 | 2.53e1 | 2.54e1 | 2502.0 | 0.0 |
| | | | GS-Aggressive Momentum | | | 6.09e0 2.20e1 | | 5.66e0 3.14e-1 | 1.01e0 2.16e1 | 2.16e1 2.24e1 | 3817.8 1668.0 | 0.0 |
| | | | GS-Conservative | 9 | | 2.33e1 | | 1.54e0 | 2.01e1 | 2.49e1 | 1668.3 | 0.0 |
| | | GD | | | | 2.56e1 | | 2.73e-2 | 2.55e1 | 2.57e1 | 1668.0 | 0.0 |
| | | | AdaptiveMoment | tum | | 1.90e1 | | 3.00e0 | 1.39e1 | 2.12e1 | 920.4 | 0.0 |
| | | | n-Fast | | | 1.17e1 | | 3.56e0 | 2.89e0 | 1.50e1 | 950.2 | 0.0 |
| | | Trust | Region-Conser | vative | | 2.08e1 | | 2.32e0 3.27e0 | 1.60e1 | 2.43e1 | 1557.3 | 0.0 |
| | | | GS-MoreThuent | | | 1.93e1 1.42e1 | | 5.29e0 | 1.20e1 1.93e0 | 2.43e1 2.12e1 | 483.9 499.3 | 0.0 |
| | | | -CubicQuadrati | | 1 | 2.45e1 | | 5.40e0 | 9.67e-1 | 2.12e1 2.58e1 | 108.6 | 0.0 |
| | | | t Region-Precise | 1 1 | | 2.60e1 | | 1.72e-2 | 2.60e1 | 2.60e1 | 5.0 | 0.0 |
| | | | t Region-Aggress | | | 2.60e1 | | 1.61e-2 | 2.60e1 | 2.60e1 | 5.0 | 0.0 |
| | | | t Region-Adapti | | | 2.60e1 | | 1.74e-2 | 2.60e1 | 2.60e1 | 5.0 | 0.0 |
| Schwefel_2 | D | | t Region-Standa: m-Robust | rd | | 2.60e1 9.37e2 | | 2.15e-2 6.80e-1 | 2.60e1 9.36e2 | 2.60e1 9.39e2 | 5.0 2502.0 | 0.0 |
| Schweier_2 | D | | n-AMSGrad | | | 9.35e2 | | 7.05e-1 | 9.34e2 | 9.36e2 9.36e2 | 2502.0 | 0.0 |
| | | | GS-Conservative | е | | 7.11e2 | | 3.94e-8 | 7.11e2 | 7.11e2 | 2095.3 | 0.0 |
| | | Adar | n-WeightDecay | | | 9.10e2 | | 8.43e-1 | 9.09e2 | 9.12e2 | 2502.0 | 0.0 |
| | | Adar | n | | | 9.35e2 | | 7.54e-1 | 9.33e2 | 9.36e2 | 2502.0 | 0.0 |
| | | GD | | | | 7.11e2 | | 9.48e-4 | 7.11e2 | 7.11e2 | 1668.0 | 0.0 |
| | | | n-Fast WeightDecay | | | 7.11e2 7.11e2 | | 9.21e-8 1.75e-8 | 7.11e2 7.11e2 | 7.11e2 7.11e2 | 1972.4 1115.0 | 0.0 |
| | | | t Region-Conser | vative | | 9.37e2 | | 8.10e-1 | 9.35e2 | 9.39e2 | 3002.0 | 0.0 |
| | | | t Region-Precise | | | 8.84e2 | | 7.64e-1 | 8.83e2 | 8.86e2 | 3002.0 | 0.0 |
| | | | t Region-Adapti | | | 7.11e2 | | 6.05e-2 | 7.11e2 | 7.11e2 | 2834.8 | 0.0 |
| | | | -StrongWolfe | | | -7.26e2 | | 2.56e3 | -1.07e4 | 5.92e2 | 379.3 | 50.0 |
| | | | GS-Limited | | | 7.11e2 | | 7.58e-7 | 7.11e2 | 7.11e2 | 513.3 | 0.0 |
| | | GD-I | Nesterov | | | 7.11e2 | | 1.78e-7 | 7.11e2 | 7.11e2 | 292.6 | 0.0 |
| | | | Momentum | | + | 7.11e2 7.11e2 | | 5.60e-4 1.13e-7 | 7.11e2 7.11e2 | 7.11e2 7.11e2 | 292.1 277.4 | 0.0 |
| | | | t Region-Standa | rd | | 7.11e2 7.12e2 | | 2.41e-1 | 7.11e2 7.12e2 | 7.11e2 7.12e2 | 711.8 | 0.0 |
| | | | GS-MoreThuent | | | 7.11e2 | | 1.08e-4 | 7.11e2 | 7.11e2 | 147.6 | 0.0 |
| | | GD-A | AdaptiveMoment | | | 7.12e2 | ; | 3.92e-2 | 7.12e2 | 7.12e2 | 89.0 | 0.0 |
| | | | I-GoldenSection | | | 7.11e2 | | 1.80e-4 | 7.11e2 | 7.11e2 | 145.1 | 0.0 |
| | | | GS-Aggressive | T | | 7.11e2 | | 2.18e-2 | 7.11e2 | 7.11e2 | 101.0 | 0.0 |
| | | | f-CubicQuadratic t Region-Aggress | | n | 7.11e2 7.17e2 | | 0.00e0 6.24e-1 | 7.11e2 7.16e2 | 7.11e2 7.18e2 | 57.9 179.0 | 0.0 |
| | | | t Region-Aggress I-Bisection-1 | oive. | | 7.17e2 7.11e2 | | 6.24e-1 3.40e-13 | 7.16e2 7.11e2 | 7.18e2 7.11e2 | 47.6 | 0.0 |
| | | | I-Bisection-2 | | | 7.11e2 7.11e2 | | 0.00e0 | 7.11e2 7.11e2 | 7.11e2 7.11e2 | 50.9 | 0.0 |
| Schwefel_51 | D | | FGS-Conserva | tive | | 1.78e3 | | 1.53e-7 | 1.78e3 | 1.78e3 | 2163.4 | 0.0 |
| DCIIWCICI_01 | | | | | | | | | | | | |
| | | | n-Robust n-AMSGrad | | | 2.34e3 | | 1.19e0 1.27e0 | 2.34e3 2.34e3 | 2.34e3 2.34e3 | 2502.0 2502.0 | 0.0 |

| | | Tabl | e 1 - continu | ed fro | m previou | ıs pa | age | | | | |
|--------------|-----------|--|----------------|---------------|--------------------|-------|----------------------|---------------------|--------------------|------------------|--------------|
| Problem | Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | M | ean Func Evals | Success Rate (%) | Mean 7 (s) | lime | |
| | | m-WeightDecay | | | 2.28e3 | | 1.09e0 | 2.27e3 | 2.28e3 | 2502.0 | 0.0 |
| | Ada | m | | | 2.34e3 | | 1.25e0 | 2.33e3 | 2.34e3 | 2502.0 | 0.0 |
| | GD Ada | m-Fast | | | 1.78e3 1.78e3 | | 1.99e-3 1.99e-7 | 1.78e3 1.78e3 | 1.78e3 1.78e3 | 1668.0 1907.4 | 0.0 |
| | | -WeightDecay | | | 1.78e3 | | 4.13e-8 | 1.78e3 | 1.78e3 | 1124.1 | 0.0 |
| | Trus | st Region-Conserv | vative | | 2.36e3 | | 1.17e0 | 2.35e3 | 2.36e3 | 3002.0 | 0.0 |
| | | st Region-Precise | | | 2.30e3 | | 1.15e0 | 2.30e3 | 2.31e3 | 3002.0 | 0.0 |
| | | st Region-Adaptiv | ve | | 2.12e3 | | 1.37e0 | 2.11e3 | 2.12e3 | 3002.0 | 0.0 |
| | | FGS-Limited N-StrongWolfe | | | 1.79e3 -1.74e3 | | 5.71e1 7.34e3 | 1.78e3 -3.30e4 | 2.04e3 1.48e3 | 661.0 349.1 | 50.0 |
| | | st Region-Standar | rd | | 1.78e3 | | 2.49e-1 | 1.78e3 | 1.78e3 | 1774.8 | 0.0 |
| | | -Nesterov | | | 1.78e3 | | 2.86e-7 | 1.78e3 | 1.78e3 | 339.1 | 0.0 |
| | | -Momentum | | | 1.78e3 | | 2.50e-7 | 1.78e3 | 1.78e3 | 323.9 | 0.0 |
| | L-Bi | | | | 1.78e3 | | 5.48e-4 | 1.78e3 | 1.78e3 | 325.7 | 0.0 |
| | | -AdaptiveMoment FGS-MoreThuent | | | 1.78e3 1.78e3 | | 1.03e-1 9.47e-6 | 1.78e3 1.78e3 | 1.78e3 1.78e3 | 124.3 158.2 | 0.0 |
| | | st Region-Aggress | | | 1.78e3 | | 9.41e-0 9.61e-1 | 1.78e3 | 1.78e3 | 446.0 | 0.0 |
| | | N-GoldenSection | | | 1.78e3 | | 4.96e-13 | 1.78e3 | 1.78e3 | 149.4 | 0.0 |
| | | FGS-Aggressive | | | 1.78e3 | | 4.59e-2 | 1.78e3 | 1.78e3 | 101.0 | 0.0 |
| | | N-CubicQuadratio | cInterpolation | | 1.78e3 | | 4.55e-13 | 1.78e3 | 1.78e3 | 57.3 | 0.0 |
| | | N-Bisection-1 N-Bisection-2 | | | 1.78e3 1.78e3 | | 4.55e-13 4.69e-13 | 1.78e3 1.78e3 | 1.78e3 1.78e3 | 52.0 52.0 | 0.0 |
| Schwefel_10D | | n-Bisection-2 am-Robust | | | 1.78e3 4.69e3 | | 4.69e-13 1.88e0 | 1.78e3 4.68e3 | 1.78e3 4.69e3 | 2502.0 | 0.0 |
| | | FGS-Conservative | e | | 3.55e3 | | 2.96e-7 | 3.55e3 | 3.55e3 | 2188.9 | 0.0 |
| | | m-AMSGrad | | | 4.67e3 | | 1.85e0 | 4.67e3 | 4.68e3 | 2502.0 | 0.0 |
| | | m-WeightDecay | | | 4.55e3 | | 1.50e0 | 4.55e3 | 4.56e3 | 2502.0 | 0.0 |
| | Ada GD | <u>m</u> | | | 4.67e3 3.55e3 | | 2.21e0 8.86e-3 | 4.67e3 3.55e3 | 4.68e3 3.55e3 | 2502.0 1668.0 | 0.0 |
| | | WeightDecay | | | 3.55e3 | | 6.30e-8 | 3.55e3 | 3.55e3 | 1194.6 | 0.0 |
| | | m-Fast | | | 3.55e3 | | 1.08e-7 | 3.55e3 | 3.55e3 | 1628.7 | 0.0 |
| | | st Region-Adaptiv | | | 4.48e3 | | 1.82e0 | 4.48e3 | 4.49e3 | 3002.0 | 0.0 |
| | | st Region-Conserv st Region-Precise | | | 4.72e3 4.67e3 | | 1.20e0 1.71e0 | 4.72e3 4.67e3 | 4.73e3 4.67e3 | 3002.0 3002.0 | 0.0 |
| | | st Region-Standar | | | 3.73e3 | | 2.12e0 | 3.73e3 | 3.74e3 | 3002.0 | 0.0 |
| | | FGS-Limited | | | 3.55e3 | | 3.01e1 | 3.42e3 | 3.55e3 | 577.7 | 0.0 |
| | | N-StrongWolfe | | | 8.35e2 | | 1.64e3 | -2.28e3 | 2.96e3 | 395.5 | 50.0 |
| | | Nesterov | | | 3.55e3 | | 8.72e-7 | 3.55e3 | 3.55e3 | 392.1 | 0.0 |
| | L-Bi | -Momentum | | | 3.55e3 3.55e3 | | 6.50e-7 1.15e-2 | 3.55e3 3.55e3 | 3.55e3 3.55e3 | 380.3 300.9 | 0.0 |
| | | st Region-Aggress | sive | | 3.56e3 | | 1.17e0 | 3.56e3 | 3.56e3 | 888.8 | 0.0 |
| | | -AdaptiveMoment | | | 3.55e3 | | 5.29e-2 | 3.55e3 | 3.55e3 | 164.1 | 0.0 |
| | | FGS-MoreThuent | ie e | | 3.55e3 | | 1.12e-3 | 3.55e3 | 3.55e3 | 159.3 | 0.0 |
| | | N-GoldenSection | | | 3.55e3 | | 9.37e-13 | 3.55e3 | 3.55e3 | 147.8 | 0.0 |
| | | N-Bisection-2 FGS-Aggressive | | | 3.55e3 3.55e3 | | 9.09e-13 4.65e-2 | 3.55e3 3.55e3 | 3.55e3 3.55e3 | 52.0 101.0 | 0.0 |
| | | N-CubicQuadratic | cInterpolation | | 3.55e3 | | 1.43e-3 | 3.55e3 | 3.55e3 | 56.9 | 0.0 |
| | QQI | N-Bisection-1 | • | | 3.55e3 | | 9.09e-13 | 3.55e3 | 3.55e3 | 52.0 | 0.0 |
| Levy_2D | | am-Robust | | | 4.34e-2 | | 2.28e-2 | 1.83e-2 | 9.34e-2 | 2502.0 | 0.0 |
| | Ada Ada | m-AMSGrad | | | 2.61e-2 5.69e-3 | | 1.26e-2 3.31e-3 | 1.24e-2 1.60e-3 | 5.59e-2 1.48e-2 | 2502.0 2502.0 | 0.0 |
| | | .m-WeightDecay | | | 9.83e-7 | | 7.75e-9 | 9.67e-7 | 9.98e-7 | 2229.0 | 100.0 |
| | | -Weight Decay | | | 9.96e-7 | | 1.94e-9 | 9.93e-7 | 1.00e-6 | 1453.5 | 100.0 |
| | GD | | | | 2.39e-4 | | 2.63e-5 | 1.85e-4 | 2.75e-4 | 1668.0 | 0.0 |
| | | FGS-Aggressive | | | 6.19e-4 | | 9.61e-4 | 6.80e-8 | 2.64e-3 | 1303.1 | 70.0 |
| | | FGS-Conservative FGS-Limited | e | | 9.25e-7 1.05e-6 | | 1.12e-6 8.37e-7 | 8.80e-8 1.49e-7 | 5.33e-6 4.54e-6 | 626.6 583.7 | 85.0 95.0 |
| | L-Bi | | | | 7.38e-3 | | 2.81e-2 | 1.43e-7 1.57e-7 | 1.29e-1 | 285.0 | 80.0 |
| | | FGS-MoreThuent | e | | 5.43e-6 | | 2.10e-5 | 7.84e-9 | 9.69e-5 | 286.5 | 95.0 |
| | | N-GoldenSection | | | 1.81e-7 | | 2.48e-7 | 1.48e-10 | 8.51e-7 | 300.9 | 100.0 |
| | | N-Bisection-1 | T . 1 .: | | 1.15e-7 | | 1.44e-7 | 2.81e-9 | 5.10e-7 | 102.2 | 100.0 |
| | | N-CubicQuadrationst Region-Conserv | | | 8.29e-8 1.34e-2 | | 1.84e-7 2.06e-3 | 5.12e-10 9.88e-3 | 8.65e-7 1.65e-2 | 87.6 385.2 | 100.0 |
| | | N-StrongWolfe | vaulve | | 2.98e-7 | | 3.89e-7 | 9.88e-3 1.89e-10 | 9.85e-7 | 79.0 | 100.0 |
| | QQl | N-Bisection-2 | | | 1.80e-7 | | 2.24e-7 | 2.53e-9 | 8.17e-7 | 93.0 | 100.0 |
| | Ada | m-Fast | | | 7.87e-2 | | 4.29e-3 | 7.26e-2 | 8.98e-2 | 36.5 | 0.0 |
| | | Nesterov | | | 1.31e-1 | | 2.65e-2 | 9.69e-2 | 1.81e-1 | 22.4 | 0.0 |
| | | -Momentum st Region-Precise | | | 1.57e-1 6.16e-2 | | 2.47e-2 3.23e-2 | 1.15e-1 3.16e-2 | 2.20e-1 1.88e-1 | 22.5 70.6 | 0.0 |
| | iius | ,, 10081011-1 TECISE | | | 0.100-2 | | 5.25C-Z | | ued on next | | 0.0 |

| Value Value Value Seals Rate (%) (%) | | | | le 1 - contin | | | | | | | |
|--|-------------|--------------|-------------------|----------------|-------------------|---------|-----------|---------|-------------|-------------|--------|
| Ch. Magniew Ammerum 3.36c. 5.15c. 2.35c. 4.11c. 16.4 16.7 Prost Region-Standard 3.560 7.590 174c. 3.77c. 10.0 Trust Region-Standard 3.560 7.590 174c. 3.27c. 10.0 Trust Region-Standard 7.64c. 1.8c. 2.05c. 2.05c. 2.05c. 2.000 Trust Region-Standard 7.64c. 1.8c. 2.05c. 2.05c. 2.05c. 2.000 Admar-MMSCrad 7.64c. 1.8c. 2.05c. 2.05c. 2.05c. 2.000 0.00 Admar-Mobist 6.85c. 2.35c. 2.35c. 1.05c. 2.000 0.00 CD. Weight Drow 1.9c. 2.35c. 2.35c. 2.35c. 1.05c. 2.000 0.00 CD. Weight Drow 1.9c. 2.35c. 2.35c. 2.35c. 3.65c. 1.05c. 2.05c. 0.00 Admar-Mobist 2.35c. 2.35c. 2.35c. 3.65c. 3.65c. 2.35c. 0.00 Admar-Mobist 2.35c. 2.35c. 2.35c. 3.65c. 3.65c. 2.35c. 0.00 CD. Weight Drow 1.9c. 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.05c. 0.00 CD. Weight Drow 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.05c. 0.00 CD. Weight Drow 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.05c. 0.00 CD. Weight Drow 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.05c. 0.00 CD. Weight Drow 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.05c. 0.00 CD. Weight Drow 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.05c. 0.00 CD. Weight Drow 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.05c. 0.00 CD. Weight Drow 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.05c. 0.00 CD. Weight Drow 2.35c. 2.35c. 3.65c. 3.65c. 3.65c. 0.00 CD. Weight Drow 2.35c. 3.65c. 3.65c. | Proble | em Optimizer | | Std Dev | Best | Worst | Mean Func | Success | | lime | |
| Prest Region-Standard | | | | | varue | | | ` ' | 1, | | |
| Trust Region Aggressive | | | | | | | | | | | |
| Levy.5D | | | | | - | | | | | | |
| Lawy_5D | | | | | $\overline{}$ | | | | | | |
| Adam | Levy_5D | Ada | am-AMSGrad | | | | 1.84e-2 | 5.66e-2 | 1.23e-1 | 2502.0 | 0.0 |
| GD Weight Deary | | | | | | | | | | | |
| Adam-WeightDeay | | | | | | | | | | | |
| GD | | | | | | | | | | | |
| L-BFCS-Limited | | | | | \longrightarrow | | | | | | |
| LPBCS-Aggressive | | | | | | | | | | | |
| L.BRGS Conservative | | | | | | | | | | | 75.0 |
| Trust Region Conservative | | L-B | FGS-Conservative | e | | | | | | | 100.0 |
| LBFGS MoreThmente | | Trus | st Region-Conserv | | | 1.23e-2 | 2.42e-3 | 8.28e-3 | 1.73e-2 | 996.6 | |
| LBFGS | | | | | | | | | | | 100.0 |
| GDNesterov | | | | je | | | | | | | 100.0 |
| QQN-ChibicQuadraticInterpolation | | | | | | | | | | | |
| QQN Bisection-1 | | | | -Internalation | - | | | | | | |
| QQN-StrongWolfe | | | | Interpolation | + | | | | | | |
| QQN-StrongWolfe | | | | | -+ | | | | | | 100.0 |
| GD-Momentum | | | | - | - | | | | | | 100.0 |
| Trust Region-Precise | | GD- | -Momentum | | | 1.72e-1 | 6.34e-2 | 9.75e-7 | 2.44e-1 | 56.5 | 10.0 |
| GB-AdaptiveMomentum | | Trus | st Region-Precise | | | | | | 5.90e-2 | | |
| Trust Region-Adaptive | | | | | | | | | | | |
| Trust Region-Standard | | | | | \longrightarrow | | | | | | |
| Trust Region-Aggressive 4.37c-1 1.9c-1 1.82c-1 1.11c0 8.2 0.0 | | | | | \longrightarrow | | | | | | |
| Levy_10D | | | | | | | | | | | |
| Adam-AMSGrad | Levv_10D | | | 3100 | | | | | | 1 | |
| Adam | <u> </u> | | | | | | | | | | |
| GD | | | | | | | 2.05e-3 | | 1.09e-2 | 2502.0 | 0.0 |
| Adam-WeightDecay | | | | | | | | | | | 100.0 |
| L-BFGS-Aggressive | | _ | | | | | | | | | |
| L-BFGS-Limited | | | | | | | | | | | |
| Trust Region-Conservative | | | | | | | | | | | |
| LBFGS-Conservative | | | | vative | \rightarrow | | | | | | |
| QQN-GoldenSection | | | | | | | | | | | 100.0 |
| L-BFGS | | | | | | | | | | | 100.0 |
| QQN-CubicQuadraticInterpolation 2.83e-7 2.18e-7 2.06e-9 7.21e-7 98.6 100.00 | | | | | | | | | | | 80.0 |
| QQN-Bisection-1 | | | | | | | | | | | 100.0 |
| QQN-Bisection-2 | | | | cInterpolation | 1 | | | | | | 100.0 |
| Trust Region-Precise 3.14e-2 1.13e-2 1.56e-2 5.18e-2 325.6 0.0 QQN-StrongWolfe 1.46e-7 2.27e-7 7.28e-10 8.42e-7 70.1 100.0 GD-Nesterov 1.74e-1 1.99e-2 1.40e-1 2.05e-1 28.4 0.0 GD-Momentum 2.58e-1 3.43e-2 1.93e-1 3.23e-1 28.1 0.0 Adam-Fast 3.12e-1 1.23e-2 2.89e-1 3.41e-1 36.9 0.0 GD-AdaptiveMomentum 6.50e-1 3.56e-2 5.54e-1 7.01e-1 21.9 0.0 GD-AdaptiveMomentum 6.50e-1 3.56e-2 1.96e-2 1.41e-1 84.5 0.0 Trust Region-Adaptive 7.36e-2 4.00e-2 1.96e-2 1.41e-1 84.5 0.0 Trust Region-Aggressive 6.17e-1 4.58e-1 2.22e-1 1.93e-0 11.8 0.0 Trust Region-Aggressive 6.17e-1 4.58e-1 2.22e-1 1.93e-0 11.8 0.0 Adam-AMSGrad 9.29e-2 2.94e-2 4.85e-2 1.34e-1 2502.0 Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569-1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658-2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS QQN-ColdenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | | | | | | | | |
| QQN-StrongWolfe 1.46e-7 2.27e-7 7.28e-10 8.42e-7 70.1 100.0 GD-Nesterov 1.74e-1 1.99e-2 1.40e-1 2.05e-1 28.4 0.0 GD-Momentum 2.58e-1 3.43e-2 1.93e-1 3.23e-1 28.1 0.0 Adam-Fast 3.12e-1 1.23e-2 2.89e-1 3.41e-1 36.9 0.0 GD-AdaptiveMomentum 6.50e-1 3.56e-2 5.54e-1 7.01e-1 21.9 0.0 Trust Region-Adaptive 7.36e-2 4.00e-2 1.96e-2 1.41e-1 84.5 0.0 Trust Region-Aggressive 6.17e-1 4.58e-1 2.22e-1 1.93e0 11.8 0.0 Zakharov_2D Adam-AMSGrad 9.29e-2 2.94e-2 4.85e-2 1.34e-1 2502.0 Adam Robust 3.83e-2 2.87e-2 6.91e-3 1.35e-1 2502.0 Adam WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 | | | | | | | | | | | |
| GD-Nesterov 1.74e-1 1.99e-2 1.40e-1 2.05e-1 28.4 0.0 | | | | | + | | | | | | |
| GD-Momentum 2.58e-1 3.43e-2 1.93e-1 3.23e-1 28.1 0.0 Adam-Fast 3.12e-1 1.23e-2 2.89e-1 3.41e-1 36.9 0.0 GD-AdaptiveMomentum 6.50e-1 3.56e-2 5.54e-1 7.01e-1 21.9 0.0 Trust Region-Adaptive 7.36e-2 4.00e-2 1.96e-2 1.41e-1 84.5 0.0 Trust Region-Standard 2.04e0 7.08e0 8.10e-2 3.28e1 29.6 0.0 Trust Region-Aggressive 6.17e-1 4.58e-1 2.22e-1 1.93e0 11.8 0.0 Zakharov_2D Adam-AMSGrad 9.29e-2 2.94e-2 4.85e-2 1.34e-1 2502.0 Adam-Mosta 3.83e-2 2.87e-2 6.91e-3 1.35e-1 2502.0 Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 6.44e-6 863.5 Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 | | | | | $\overline{}$ | | | | | | |
| GD-AdaptiveMomentum 6.50e-1 3.56e-2 5.54e-1 7.01e-1 21.9 0.0 Trust Region-Adaptive 7.36e-2 4.00e-2 1.96e-2 1.41e-1 84.5 0.0 Trust Region-Standard 2.04e0 7.08e0 8.10e-2 3.28e1 29.6 0.0 Trust Region-Aggressive 6.17e-1 4.58e-1 2.22e-1 1.93e0 11.8 0.0 Zakharov_2D Adam_AMSGrad 9.29e-2 2.94e-2 4.85e-2 1.34e-1 2502.0 Adam_Robust 3.83e-2 2.87e-2 6.91e-3 1.35e-1 2502.0 Adam_WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 6.44e-6 863.5 Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | GD. | -Momentum | | | | | 1.93e-1 | | | |
| Trust Region-Adaptive 7.36e-2 4.00e-2 1.96e-2 1.41e-1 84.5 0.0 Trust Region-Standard 2.04e0 7.08e0 8.10e-2 3.28e1 29.6 0.0 Zakharov_2D Adam-AMSGrad 9.29e-2 2.94e-2 4.85e-2 1.34e-1 2502.0 Adam-Robust 3.83e-2 2.87e-2 6.91e-3 1.35e-1 2502.0 Adam 1.77e-3 1.69e-3 5.24e-5 6.40e-3 2502.0 Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 6.44e-6 863.5 Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise | | | | | | | | | | | |
| Trust Region-Standard 2.04e0 7.08e0 8.10e-2 3.28e1 29.6 0.0 Zakharov_2D Adam-AMSGrad 9.29e-2 2.94e-2 4.85e-2 1.34e-1 2502.0 Adam-Robust 3.83e-2 2.87e-2 6.91e-3 1.35e-1 2502.0 Adam 1.77e-3 1.69e-3 5.24e-5 6.40e-3 2502.0 Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 6.44e-6 863.5 Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 | | | | | | | | | | | |
| Trust Region-Aggressive 6.17e-1 4.58e-1 2.22e-1 1.93e0 11.8 0.0 | | | | | | | | | | | |
| Zakharov_2D Adam-AMSGrad 9.29e-2 2.94e-2 4.85e-2 1.34e-1 2502.0 Adam-Robust 3.83e-2 2.87e-2 6.91e-3 1.35e-1 2502.0 Adam 1.77e-3 1.69e-3 5.24e-5 6.40e-3 2502.0 Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 6.44e-6 863.5 Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<> | | | | | | | | | | | |
| Adam-Robust 3.83e-2 2.87e-2 6.91e-3 1.35e-1 2502.0 Adam 1.77e-3 1.69e-3 5.24e-5 6.40e-3 2502.0 Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 6.44e-6 863.5 Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 <t< td=""><td>Zakharov 2D</td><td> 11 us</td><td></td><td></td><td>1</td><td>0.176-1</td><td></td><td></td><td></td><td></td><td></td></t<> | Zakharov 2D | 11 us | | | 1 | 0.176-1 | | | | | |
| Adam 1.77e-3 1.69e-3 5.24e-5 6.40e-3 2502.0 Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 6.44e-6 863.5 Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | Zamarov_22 | | | | | | | | | | |
| Adam-WeightDecay 9.88e-9 7.13e-11 9.74e-9 9.97e-9 2040.6 L-BFGS-Conservative 6.39e-7 1.89e-6 5.41e-10 6.44e-6 863.5 Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | - | - | | | | | | |
| Trust Region-Conservative 8.84e-1 1.50e0 1.05e-2 6.57e0 2569.1 L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | | | | 9.88e-9 | | 9.74e-9 | | 2040.6 |
| L-BFGS-Limited 6.46e-7 2.30e-6 1.04e-9 1.04e-5 658.2 GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | | | | | | | | |
| GD 9.76e-9 9.66e-11 9.61e-9 9.97e-9 417.4 L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | | tive | | | | | | |
| L-BFGS-Aggressive 3.75e0 5.80e0 4.91e-10 1.50e1 1213.0 Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | <u>_imited</u> | | | | | | | |
| Trust Region-Precise 6.91e-2 9.24e-2 8.62e-3 4.38e-1 503.2 QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | A sersecive | | + | | | | | |
| QQN-StrongWolfe 4.83e-9 2.70e-9 1.51e-9 9.93e-9 95.0 L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | | | + | | | | | |
| L-BFGS 2.88e-2 5.11e-2 5.69e-10 1.73e-1 158.7 QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | | | | | | | | |
| QQN-CubicQuadraticInterpolation 1.52e-9 3.10e-9 1.85e-11 9.62e-9 89.2 QQN-GoldenSection 7.14e-11 3.11e-10 5.21e-21 1.43e-9 180.9 | | | | 1110 | | | | | | | |
| | | | | | nterpolat | ion | 1.52e-9 | 3.10e-9 | 1.85e-11 | 9.62e-9 | 89.2 |
| Continued on next page | | | QQN-Gold | denSection | | | 7.14e-11 | | | | 180.9 |
| | | | | | | | | Continu | ued on next | page | |

| | Tabl | e 1 - continue | nued from | previo | us page | | | | |
|-------------------------------------|---------------------|--------------------------------|---------------|----------------|---------------------|----------------------|----------------------|----------------------|-----------------------|
| Problem Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | Mean Func Evals | Success Rate (%) | Mean (s) | - | |
| _ | QQN-Bis | | | | 7.75e-14 | 8.83e-14 | 9.76e-18 | 3.54e-13 | 114.6 |
| | QQN-Bis | | | | 9.02e-14 | 2.34e-13 | 4.37e-17 | 9.97e-13 | 84.0 |
| | GD-Weig | ntDecay MoreThuente | <u> </u> | | 8.53e-2 7.07e-9 | 5.34e-2 1.81e-9 | 8.63e-9 3.76e-9 | 1.56e-1 9.71e-9 | 47.9 67.3 |
| | | gion-Adaptive | | | 4.96e3 | 2.16e4 | 3.14e-2 | 9.91e4 | 121.0 |
| | Adam-Fa | st | | | 2.81e-1 | 1.57e-2 | 2.54e-1 | 3.10e-1 | 37.5 |
| | | tiveMomentu | ım | | 7.05e-1 | 1.56e-1 | 4.12e-1 | 1.03e0 | 16.6 |
| | GD-Neste | | | | 6.00e-1 | 7.84e-2 | 5.01e-1 | 7.53e-1 | 16.7 |
| | GD-Mom | entum gion-Standaro | 1 | | 1.08e0 2.38e0 | 1.28e-1 4.10e0 | 8.96e-1 1.22e-1 | 1.34e0 1.46e1 | 17.1 38.3 |
| | | gion-Aggressi | | | 5.16e4 | 2.24e5 | 5.33e-1 | 1.40e1 1.03e6 | 18.9 |
| Zakharov_5D | | - A ggressive | | | 4.22e1 | 8.64e0 | 2.82e1 | 6.13e1 | 3309.5 |
| | Adam-Al | | | | 2.23e-2 | 1.32e-2 | 4.59e-3 | 5.39e-2 | 2502.0 |
| | Adam-Ro | bust | | | 7.06e-2 | 7.45e-2 | 6.18e-3 | 2.91e-1 | 2502.0 |
| | Adam We | eightDecay | | | 2.82e-3 8.64e-3 | 2.38e-3 1.13e-2 | 2.65e-4 9.76e-9 | 7.47e-3 2.86e-2 | 2502.0 1586.3 |
| | | gion-Standard | 1 | | 2.28e3 | 8.27e2 | 6.92e2 | 3.84e3 | 3002.0 |
| | | gion-Precise | - | | 3.50e3 | 7.96e2 | 2.04e3 | 4.73e3 | 3002.0 |
| | Trust Reg | gion-Conserva | | | 3.28e3 | 6.15e2 | 2.05e3 | 4.46e3 | 3002.0 |
| | | gion-Adaptive | | | 3.10e3 | 8.27e2 | 1.77e3 | 4.98e3 | 3002.0 |
| | L-BFGS-1 | gion-Aggressi | ve | | 5.65e1 | 1.12e2 4.45e-8 | 6.02e-1 7.85e-9 | 4.25e2 2.13e-7 | 2415.8 1078.6 |
| | | Limitea MoreThuente | | | 1.94e-8 9.32e-9 | 3.60e-10 | 8.60e-9 | 2.13e-7 9.95e-9 | 634.6 |
| | GD | Wiore i nueme | <u>'</u> | | 9.78e-9 | 9.76e-11 | 9.63e-9 | 9.97e-9 | 477.1 |
| | L-BFGS-0 | Conservative | | | 7.32e-9 | 2.98e-9 | 2.42e-10 | 9.99e-9 | 492.9 |
| | | picQuadratic | Interpolatio | on | 1.52e-9 | 1.59e-9 | 3.50e-12 | 5.23e-9 | 205.0 |
| | QQN-Str | | | | 2.41e-9 | 3.17e-9 | 2.91e-14 | 9.91e-9 | 100.3 |
| | QQN-Bis | | | | 1.50e-9 1.12e-12 | 1.97e-9 2.99e-12 | 2.56e-15 5.59e-14 | 6.16e-9 1.40e-11 | 114.7 118.0 |
| | | denSection | | | 2.83e-12 | 2.99e-12 2.87e-12 | 2.45e-13 | 1.40e-11 1.21e-11 | 138.0 |
| | GD-Weig | | | | 7.62e-1 | 4.69e-1 | 9.24e-9 | 1.31e0 | 52.5 |
| | L-BFGS | | | | 2.69e1 | 2.46e1 | 2.58e-1 | 8.10e1 | 93.0 |
| | Adam-Fa | | | | 9.66e0 6.97e1 | 1.31e0 | 7.78e0 | 1.15e1 | 34.8 |
| | | tiveMomentu | ım | | 6.97e1 | 1.64e1 | 4.83e1 | 1.09e2 | 18.5 |
| | GD-Mom GD-Neste | | | | 5.45e1 5.05e0 | 1.33e1 2.73e0 | 3.48e1 1.42e0 | 9.04e1 1.10e1 | 18.9 18.1 |
| Zakharov_10D | Adam-R | | | | 3.12e0 | 6.81e0 | 2.92e-2 | 2.78e1 | 2502.0 |
| | L-BFGS | | | | 2.07e4 | 8.30e3 | 6.23e3 | 3.58e4 | 3286.7 |
| | | MoreThuente | , | | 1.67e-6 | 9.70e-7 | 4.87e-7 | 4.27e-6 | 2886.3 |
| | | Aggressive | | | 9.79e-1 | 8.37e-1 | 1.37e-2 | 3.03e0 | 3811.6 |
| | Adam-Al Adam | ASGrad | | | 1.09e-1 4.52e-2 | 3.02e-2 4.19e-2 | 3.67e-2 1.69e-3 | 1.63e-1 1.18e-1 | 2016.2 2253.9 |
| | L-BFGS-1 | Limited | | | 9.29e-2 | 2.40e-1 | 5.26e-9 | 9.86e-1 | 2588.7 |
| | | gion-Conserva | ative | | 5.85e5 | 6.69e4 | 4.55e5 | 7.39e5 | 3002.0 |
| | | gion-Precise | | | 5.73e5 | 9.27e4 | 4.07e5 | 7.69e5 | 3002.0 |
| | | gion-Aggressi | | | 5.60e5 | 8.22e4 | 4.02e5 | 7.12e5 | 3002.0 |
| | | gion-Adaptive gion-Standare | | | 5.56e5 5.72e5 | 9.19e4 1.14e5 | 4.08e5 3.74e5 | 7.62e5 8.01e5 | 3002.0 3002.0 |
| | GD | gion-Standard | 1 | | 3.37e-1 | 2.28e-1 | 9.65e-9 | 9.78e-1 | 686.0 |
| | _ | eightDecay | | | 1.79e-1 | 3.62e-2 | 1.29e-1 | 2.61e-1 | 683.1 |
| | QQN-Cul | oicQuadraticl | Interpolatio | on | 1.75e-9 | 9.69e-10 | 4.53e-10 | 4.49e-9 | 312.8 |
| | QQN-Stro | | | | 9.67e-4 | 4.21e-3 | 2.12e-14 | 1.93e-2 | 281.4 |
| | QQN-Bis | | | | 1.58e-9 1.50e-9 | 9.47e-10 | 1.13e-10 | 3.31e-9 8.28e-9 | 159.2 |
| | QQN-Gol | denSection | | | 1.50e-9 1.96e-9 | 2.57e-9 2.69e-9 | 3.14e-19 4.68e-16 | 8.28e-9 8.14e-9 | 233.1 161.5 |
| | GD-Weig | | | | 1.51e1 | 1.85e1 | 8.77e-9 | 5.75e1 | 67.5 |
| | L-BFGS-0 | Conservative | | | 7.82e23 | 1.19e24 | 1.68e5 | 4.59e24 | 68.5 |
| | Adam-Fa | | | | 1.13e3 | 5.38e2 | 3.62e2 | 2.13e3 | 34.7 |
| | | tiveMomentu | ım | | 6.98e3 | 2.36e3 | 4.52e3 | 1.36e4 | 20.9 |
| | GD-Neste GD-Mom | | | | 8.62e2 6.41e3 | 6.04e2 3.22e3 | 4.52e1 1.95e3 | 1.76e3 1.10e4 | 20.2 21.1 |
| IllConditionedRosenbrock_2D_alpha10 | | entum oldenSection | n | | 6.41e3 1.25e-1 | 5.78e-2 | 1.95e3 6.99e-5 | 1.10e4 1.93e-1 | $\frac{21.1}{4459.4}$ |
| , | QQN-Str | | | | 5.46e-2 | 7.38e-2 | 9.29e-9 | 3.11e-1 | 2343.8 |
| | QQN-Cul | oicQuadraticl | Interpolatio | on | 3.48e-2 | 3.38e-2 | 4.34e-10 | 9.89e-2 | 1722.9 |
| | QQN-Bis | ection-1 | | | 4.65e-1 | 8.86e-1 | 1.58e-8 | 3.18e0 | 2369.2 |
| | Adam | O | | | 1.22e0 | 3.51e-1 | 4.86e-1 | 1.76e0 | 2502.0 |
| | | Conservative Aggressive | | | 2.69e-4 3.18e1 | 1.04e-3 2.86e1 | 2.05e-9 4.34e0 | 4.76e-3 1.12e2 | 1800.6 3852.0 |
| | n-pr. go | -281 CDD1 V.C | | | 0.1001 | | ed on next | | 3002.0 |

| Develolore | 0 | | e 1 – contii | | | | | | T): | |
|--------------------------|----------------|----------------------|---------------------------|---------------|----------------|--------------------|---------------------|--------------------|-------------------|------------------|
| Problem | Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | Mean Func Evals | Success Rate (%) | Mean (s) | | |
| | | | MoreThuente | | | 1.52e-2 | 6.51e-2 | 3.79e-9 | 2.99e-1 | 1487.5 |
| | | L-BFGS-I | Limited | | | 3.93e0 | 5.18e0 | 3.26e-2 | 1.96e1 | 2251.6 |
| | | GD | ion-Conserva | tirro | | 1.23e0 2.84e1 | 1.46e0 2.98e1 | 7.46e-1 1.90e-1 | 6.37e0 1.23e2 | 854.0 2770.7 |
| | | Adam-AN | , | ilive | | 3.83e0 | 1.33e0 | 4.66e-1 | 4.75e0 | 678.1 |
| | | QQN-Bise | | | | 5.88e-2 | 7.05e-2 | 6.64e-9 | 2.52e-1 | 479.6 |
| | | Adam-Ro | | | | 4.04e0 | 8.59e-1 | 1.90e0 | 4.73e0 | 419.2 |
| | | Adam-Fas | | | | 2.13e0 | 2.75e0 | 2.59e-5 | 8.39e0 | 313.6 |
| | | | ion-Precise | | | 7.26e0 | 8.07e0 | 3.76e0 | 3.52e1 | 946.2 |
| | | | ightDecay | | | 4.11e0 | 9.66e-1 | 1.55e-2 | 4.69e0 | 231.9 |
| | | GD-Weigl | ion-Adaptive | 9 | | 4.12e0 | 2.07e-1 3.07e0 | 3.83e0 3.75e-2 | 4.42e0 1.07e1 | 494.4 58.7 |
| | | L-BFGS | ntDecay | | | 3.65e0 1.36e2 | 1.58e2 | 8.12e-1 | 5.03e2 | 121.5 |
| | | | tiveMomentu | ım | | 8.18e-1 | 1.24e0 | 4.17e-2 | 4.36e0 | 49.2 |
| | | GD-Neste | | | | 1.49e0 | 1.88e0 | 5.05e-2 | 5.63e0 | 46.1 |
| | | GD-Mom | entum | | | 5.61e0 | 3.44e0 | 4.79e-1 | 1.33e1 | 23.8 |
| | | Trust Reg | ion-Standard | l | | 4.18e0 | 1.83e-1 | 3.95e0 | 4.51e0 | 89.6 |
| | | | ion-Aggressi | | | 4.66e0 | 3.99e-1 | 4.01e0 | 5.49e0 | 27.6 |
| IllConditionedRosenbr | ock_5D_alpha10 | | oldenSection | | | 4.32e-1 | 5.14e-1 | 5.69e-8 | 1.61e0 | 3805.3 |
| | | QQN-Cut Adam-Ro | oicQuadraticl | nterpolati | on | 1.97e-1 | 2.84e-1 | 4.07e-9 6.12e0 | 7.25e-1 2.99e1 | 1403.3 2502.0 |
| | | Adam-Ro Adam-AN | | | | 1.46e1 4.40e0 | 6.99e0 3.25e-1 | 3.25e0 | 2.99e1 4.82e0 | 2502.0 |
| | | Adam Adam | isGrad | | | 3.92e0 | 4.66e-1 | 2.83e0 | 4.65e0 | 2471.6 |
| | | L-BFGS-I | Limited | | | 4.05e-1 | 5.13e-2 | 3.00e-1 | 5.47e-1 | 4035.2 |
| | | QQN-Bise | | | | 2.99e-1 | 2.82e-1 | 4.91e-8 | 9.11e-1 | 1647.8 |
| | | L-BFGS-I | MoreThuente | : | | 1.74e8 | 7.57e8 | 3.07e-7 | 3.47e9 | 2320.8 |
| | | QQN-Bise | | | | 4.19e-1 | 1.11e0 | 2.00e-9 | 4.64e0 | 1511.2 |
| | | | eightDecay | | | 1.84e0 | 2.24e0 | 1.84e-3 | 4.66e0 | 1797.3 |
| | | QQN-Stro | | | | 1.59e-7 | 1.68e-7 | 2.70e-9 | 5.77e-7 | 1191.3 |
| | | L-BFGS-A | Conservative | | | 2.02e1 8.07e2 | 6.75e1 4.06e2 | 2.46e-1 1.72e1 | 3.11e2 1.19e3 | 3386.6 3851.6 |
| | | | ion-Precise | | | 1.01e3 | 1.27e2 | 8.08e2 | 1.19e3 1.35e3 | 3002.0 |
| | | Trust Reg | gion-Conserva | ative | | 1.02e3 | 1.63e2 | 7.14e2 | 1.31e3 | 3002.0 |
| | | | ion-Adaptive | | | 8.41e2 | 1.37e2 | 5.05e2 | 1.11e3 | 3002.0 |
| | | Trust Reg | gion-Standard | l | | 6.23e1 | 7.73e1 | 4.66e0 | 2.53e2 | 2827.2 |
| | | GD-Weigl | | | | 7.32e-1 | 1.11e0 | 1.46e-1 | 5.40e0 | 459.3 |
| | | GD-Neste | | | | 4.24e0 | 5.00e0 | 3.82e-1 | 1.31e1 | 372.4 |
| | | | gion-Aggressi | ve | | 5.00e0 | 4.17e-1 | 4.66e0 | 5.93e0 | 776.1 |
| | | L-BFGS Adam-Fas | ·+ | | | 1.50e2 1.43e1 | 2.28e2 3.92e0 | 1.98e1 4.36e-2 | 7.52e2 1.86e1 | 135.3 49.9 |
| | | GD | 56 | | | 5.09e0 | 1.48e-1 | 4.30e-2 4.75e0 | 5.31e0 | 32.5 |
| | | _ | tiveMomentu | ım | | 4.60e1 | 6.15e0 | 3.36e1 | 5.66e1 | 20.6 |
| | | GD-Mom | | | | 3.55e1 | 8.91e0 | 1.96e1 | 4.95e1 | 20.8 |
| Ill Conditioned Rosenbra | ock_10D_alpha1 | | oldenSection | | | 4.56e0 | 1.82e0 | 4.63e-1 | 6.66e0 | 4477.5 |
| | | | oicQuadraticl | Interpolati | on | 6.00e-1 | 1.52e0 | 1.13e-7 | 4.99e0 | 1666.2 |
| | | Adam-Ro | | | | 3.49e1 | 9.88e0 | 1.61e1 | 5.15e1 | 2502.0 |
| | | Adam-AN QQN-Stro | | | | 9.31e0 5.85e-1 | 3.61e-1 1.24e0 | 8.08e0 9.35e-8 | 9.69e0 4.79e0 | 2490.9 1847.0 |
| | | Adam | nig wone | | | 9.11e0 | 4.86e-1 | 7.98e0 | 9.93e0 | 2475.8 |
| | | | MoreThuente | : | | 3.41e0 | 3.98e0 | 3.35e-5 | 1.75e1 | 2853.3 |
| | | QQN-Bise | | | | 1.22e0 | 2.80e0 | 1.19e-7 | 9.52e0 | 1844.9 |
| | | GD-Neste | | | | 1.17e0 | 1.66e-2 | 1.12e0 | 1.19e0 | 1514.9 |
| | | | Aggressive | | | 1.64e2 | 3.20e2 | 5.82e1 | 1.56e3 | 3850.3 |
| | | L-BFGS-I | | | | 3.86e0 | 5.50e-1 | 2.71e0 | 5.45e0 | 4036.8 |
| | | | eightDecay | | | 2.38e0 | 4.13e0 | 6.46e-6 | 9.61e0 | 2074.9 |
| | | QQN-Bise GD-Weigl | | | | 4.40e0 1.34e0 | 1.40e0 1.81e0 | 1.07e-7 1.70e-1 | 5.50e0 5.25e0 | 1646.8 1217.3 |
| | | | Conservative | | | 1.90e4 | 6.07e4 | 1.70e-1 1.03e0 | 2.56e5 | 3278.5 |
| | | | gion-Precise | | | 2.10e3 | 1.92e2 | 1.66e3 | 2.46e3 | 3002.0 |
| | | | gion-Standard | l | | 1.06e3 | 2.06e2 | 6.70e2 | 1.41e3 | 3002.0 |
| | | | Trust Region-Conservative | | | | 2.33e2 | 1.76e3 | 2.55e3 | 3002.0 |
| | | | gion-Adaptive | 9 | | 2.02e3 | 2.22e2 | 1.64e3 | 2.59e3 | 3002.0 |
| | | GD-Mom | | | | 3.04e1 | 3.31e1 | 5.05e-1 | 7.27e1 | 517.6 |
| | | | gion-Aggressi | ve | | 6.55e1 | 2.02e2 | 9.60e0 | 9.28e2 | 1618.0 |
| | | L-BFGS Adam-Fas | ** | | | 1.19e2 | 1.68e2 | 2.51e1 | 7.37e2 | 338.9 |
| | | GD Adam-Fas | 56 | | | 1.53e1 9.96e0 | 1.47e1 2.61e-1 | 1.91e-1 9.64e0 | 3.68e1 1.05e1 | 149.2 46.6 |
| | | | | | | 0.3000 | | っ・ロサビリ | | |
| | | | tiveMomentu | ım | | 1.02e2 | 9.83e0 | 8.45e1 | 1.16e2 | 23.1 |

| | | | Tabl | e 1 – contii | nued fror | n previo | us page | | | | |
|----------|------------|-----------|--|--|---------------|----------------|---|---|---|---|---|
| | Problem | Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | Mean Func Evals | Success Rate (%) | Mean (s) | | |
| Trigonom | netric_2D | | Adam-A | MSGrad | | | 1.08e-6 9.77e-7 | 3.54e-7 1.59e-8 | 9.47e-7 9.40e-7 | 2.57e-6 9.98e-7 | 1241.2 1269.5 |
| | | | Adam-Ro | bust | | | 3.10e-3 | 1.20e-2 | 5.08e-7 | 5.56e-2 | 442.4 |
| | | | GD | | | | 9.80e-7 | 1.07e-8 | 9.61e-7 | 9.99e-7 | 361.1 |
| | | | L-BFGS- | | | | 3.62e-4 | 1.35e-3 | 7.20e-8 | 6.20e-3 | 843.0 |
| | | | | eightDecay | | | 1.76e-5 | 4.01e-5 | 3.77e-7 | 1.63e-4 | 362.9 |
| | | | | denSection | | | 1.28e-3 | 5.56e-3 | 6.12e-10 | 2.55e-2 | 462.6 |
| | | | L-BFGS-I | | | | 7.02e-6 | 2.23e-5 | 2.15e-8 | 1.00e-4 | 433.9 |
| | | | QQN-Bise | | | | 2.06e-7 2.08e-7 | 2.73e-7 2.21e-7 | 7.96e-10 1.49e-9 | 8.24e-7 7.99e-7 | 207.1 220.8 |
| | | | QQN-Stro | | | | 4.90e-7 | 2.21e-7 2.12e-7 | 2.93e-9 | 9.96e-7 | 107.8 |
| | | | GD-Weig | | | | 2.11e-4 | 6.48e-4 | 8.75e-7 | 2.81e-3 | 102.4 |
| | | | | oicQuadraticl | Interpolati | on | 3.11e-7 | 3.76e-7 | 6.47e-10 | 9.84e-7 | 95.8 |
| | | | | Conservative | | | 5.22e-7 | 2.95e-7 | 3.92e-8 | 9.76e-7 | 129.4 |
| | | | L-BFGS | | | | 4.06e-3 | 1.77e-2 | 8.58e-10 | 8.12e-2 | 93.3 |
| | | | L-BFGS-1 | MoreThuente | ; | | 4.44e-7 | 3.28e-7 | 1.25e-8 | 9.94e-7 | 85.4 |
| | | | Adam-Fas | | | | 6.08e-3 | 1.20e-2 | 1.89e-8 | 5.66e-2 | 47.6 |
| | | | 1 . | tiveMomentu | ım | | 7.65e-3 | 4.04e-3 | 5.90e-5 | 1.24e-2 | 23.2 |
| | | | GD-Neste | | | | 2.31e-2 | 2.66e-2 | 5.79e-4 | 7.14e-2 | 24.2 |
| | | | GD-Mom | | | | 3.43e-2 | 2.95e-2 | 2.30e-4 | 8.79e-2 | 22.1 |
| | | | | ion-Conserva ion-Aggressi | | | 8.94e-3 2.50e1 | 1.37e-2 1.94e1 | 1.87e-3 3.36e-2 | 4.88e-2 5.71e1 | 43.2 18.3 |
| | | | | ion-Aggressi ion-Precise | ve | | 2.50e1 1.11e-1 | 2.96e-1 | 3.30e-2 1.34e-2 | 1.39e0 | 16.8 |
| | | | | gion-Precise gion-Standard | 1 | | 1.11e-1 1.22e1 | 2.96e-1 2.59e1 | 2.19e-1 | 9.57e1 | 13.5 |
| | | | | ion-Standard | | | 8.77e0 | 2.18e1 | 1.18e-2 | 8.47e1 | 12.8 |
| Trigonom | netric_5D | | | MSGrad | 5 | | 2.94e-2 | 3.49e-2 | 9.71e-7 | 8.56e-2 | 2192.1 |
| | | | QQN-Stro | ongWolfe | | | 7.54e-7 | 4.98e-7 | 1.51e-8 | 1.93e-6 | 1274.7 |
| | | | Adam | | | | 1.12e-2 | 2.74e-2 | 8.42e-7 | 8.54e-2 | 1728.1 |
| | | | L-BFGS- | | | | 1.22e0 | 5.81e-1 | 1.48e-1 | 2.56e0 | 3851.9 |
| | | | Adam-Ro | | | | 1.43e-2 | 2.36e-2 | 2.34e-7 | 6.11e-2 | 978.1 |
| | | | | MoreThuente | } | | 5.44e-4 | 2.01e-3 | 1.15e-7 | 9.19e-3 | 1087.8 |
| | | | | denSection | | | 2.34e-7 | 2.85e-7 | 3.03e-9 | 8.60e-7 | 1123.5 |
| | | | L-BFGS-I | eightDecay | | | 2.14e-3 1.95e-2 | 9.31e-3 3.08e-2 | 7.20e-7 3.70e-7 | 4.27e-2 8.53e-2 | 1331.0 784.6 |
| | | | | oicQuadraticl | Internolati | on | 4.16e-7 | 3.74e-7 | 1.09e-9 | 9.80e-7 | 389.1 |
| | | | QQN-Bise | | interpolati | .011 | 8.71e-4 | 3.70e-3 | 1.72e-10 | 1.70e-2 | 664.0 |
| | | | | Conservative | | | 2.73e-3 | 1.19e-2 | 1.66e-7 | 5.45e-2 | 887.4 |
| | | | GD | | | | 9.83e-7 | 7.74e-9 | 9.62e-7 | 9.94e-7 | 399.2 |
| | | | QQN-Bise | ection-2 | | | 3.22e-7 | 3.08e-7 | 6.17e-10 | 9.20e-7 | 305.6 |
| | | | GD-Weig | ntDecay | | | 1.36e-2 | 2.77e-2 | 8.18e-7 | 8.54e-2 | 105.4 |
| | | | L-BFGS | | | | 4.77e0 | 8.95e0 | 1.26e-2 | 4.30e1 | 233.9 |
| | | | | ion-Conserva | ative | | 1.91e-2 | 1.87e-2 | 8.29e-4 | 5.63e-2 | 425.6 |
| | | | Adam-Fas | | | | 6.17e-2 | 5.21e-2 | 7.00e-4 | 1.62e-1 | 54.8 |
| | | | GD-Neste | | | | 1.60e-2 | 2.60e-2 | 4.85e-4 | 7.53e-2 | 33.5 |
| | | | GD-Adap GD-Mom | tiveMomentu | ım | | 1.79e-2 1.57e-1 | 3.35e-2 1.93e-1 | 1.30e-4 4.87e-3 | 1.56e-1 8.47e-1 | 30.6 25.0 |
| | | | | gion-Precise | | | 5.59e-2 | 2.09e-2 | 1.47e-2 | 9.38e-2 | 84.8 |
| | | | | ion-Adaptive | | | 1.25e-1 | 7.39e-2 | 2.86e-2 | 3.04e-1 | 26.1 |
| | | | | ion-Standard | | | 4.09e-1 | 1.81e-1 | 1.21e-1 | 7.66e-1 | 9.8 |
| | | | | ion-Aggressi | | | 1.41e0 | 4.40e-1 | 6.56e-1 | 2.25e0 | 5.2 |
| Trigonom | netric_10D | | | \mathbf{MSGrad} | | | 3.00e-2 | 3.66e-2 | 7.20e-6 | 8.75e-2 | 2502.0 |
| | | | L-BFGS-1 | Limited | | | 7.31e-3 | 1.99e-2 | 9.82e-6 | 8.19e-2 | 3943.6 |
| | | | Adam | | | | 9.70e-2 | 2.10e-1 | 9.68e-7 | 7.20e-1 | 2080.5 |
| | | | Adam-Ro | | | | 7.13e-2 | 1.09e-1 | 9.71e-7 | 5.20e-1 | 1825.9 |
| | | | | MoreThuente | ; | | 1.34e-2 | 3.73e-2 | 4.78e-7 | 1.60e-1 | 1949.3 |
| | | | | Conservative denSection | | | 9.45e0 4.35e-4 | 4.12e1 1.06e-3 | 7.52e-7 | 1.89e2 | 3123.2 1834.2 |
| | | | GD GD | denogen011 | | | 4.39e-3 | 1.06e-3 1.89e-2 | 1.12e-7 9.77e-7 | 3.88e-3 8.69e-2 | 958.0 |
| | | | L-BFGS- | Aggressive | | | 1.00e1 | 4.32e0 | 2.42e0 | 1.75e1 | 3852.0 |
| | | | | | ative | | 1.22e0 | 1.60e0 | 3.82e-3 | 5.77e0 | 2651.3 |
| | | | Trust Res | ion-Conserva | | | | | | | |
| | | | Trust Reg QQN-Bise | gion-Conserva ection-1 | 20110 | | 7.53e-4 | 2.61e-3 | 2.91e-8 | 1.18e-2 | 858.0 |
| | | | QQN-Bise | | 20170 | | 7.53e-4 4.25e-2 | 2.61e-3 1.14e-1 | 2.91e-8 8.69e-7 | 1.18e-2 5.20e-1 | 858.0 803.8 |
| | | | QQN-Bise Adam-We QQN-Cul | ection-1 eightDecay picQuadraticl | | on | 4.25e-2 4.48e-7 | 1.14e-1 2.76e-7 | 8.69e-7 9.99e-9 | 5.20e-1 9.71e-7 | 803.8 329.2 |
| | | | QQN-Bise Adam-We QQN-Cul QQN-Stre | ection-1 eightDecay oicQuadraticl ongWolfe | | on | 4.25e-2 4.48e-7 2.92e-7 | 1.14e-1 2.76e-7 2.02e-7 | 8.69e-7 9.99e-9 9.27e-9 | 5.20e-1 9.71e-7 7.23e-7 | 803.8 329.2 347.3 |
| | | | QQN-Bise Adam-We QQN-Cul QQN-Stro QQN-Bise | ection-1 eightDecay oicQuadraticlongWolfe ection-2 | | on | 4.25e-2 4.48e-7 2.92e-7 3.56e-7 | 1.14e-1 2.76e-7 2.02e-7 2.58e-7 | 8.69e-7 9.99e-9 9.27e-9 8.26e-8 | 5.20e-1 9.71e-7 7.23e-7 9.59e-7 | 803.8 329.2 347.3 357.8 |
| | | | QQN-Biss Adam-We QQN-Cul QQN-Stro QQN-Bise GD-Weig | ection-1 eightDecay oicQuadraticl ongWolfe ection-2 ntDecay | | on | 4.25e-2 4.48e-7 2.92e-7 3.56e-7 8.48e-3 | 1.14e-1 2.76e-7 2.02e-7 2.58e-7 2.56e-2 | 8.69e-7 9.99e-9 9.27e-9 8.26e-8 8.63e-7 | 5.20e-1 9.71e-7 7.23e-7 9.59e-7 1.07e-1 | 803.8 329.2 347.3 357.8 134.4 |
| | | | QQN-Biss Adam-We QQN-Cul QQN-Stro QQN-Bise GD-Weig | ection-1 eightDecay bicQuadraticl ongWolfe ection-2 ntDecay cion-Precise | | on | 4.25e-2 4.48e-7 2.92e-7 3.56e-7 | 1.14e-1 2.76e-7 2.02e-7 2.58e-7 | 8.69e-7 9.99e-9 9.27e-9 8.26e-8 | 5.20e-1 9.71e-7 7.23e-7 9.59e-7 | 803.8 329.2 347.3 357.8 |

| Problem Optimize | | $rac{{ m e} 1 - { m contin}}{{ m l} { m Std} { m Dev}}$ | Best Value | Worst Value | Mean Func Evals | Success Rate (%) | Mean (s) | | |
|-----------------------|----------------------|---|---------------|-------------|--------------------|---------------------|--------------------|-------------------|------------------|
| | GD-Adap | tiveMomentu | ım | | 1.41e-1 | 7.60e-2 | 4.06e-2 | 3.86e-1 | 48.0 |
| | GD-Neste | erov | | | 1.85e-1 | 7.94e-2 | 7.60e-2 | 4.35e-1 | 45.1 |
| | L-BFGS | | | | 4.76e1 | 6.11e1 | 8.87e0 | 2.11e2 | 77.5 |
| | GD-Mome | entum gion-Adaptive | | | 1.17e0 1.86e-1 | 1.48e0 1.19e-1 | 2.40e-3 2.37e-2 | 4.14e0 4.38e-1 | 34.0 118.4 |
| | | gion-Standard | | | 6.02e-1 | 2.65e-1 | 1.37e-1 | 1.10e0 | 34.1 |
| | Trust Reg | gion-Aggressi | | | 1.90e0 | 5.88e-1 | 9.79e-1 | 3.67e0 | 12.2 |
| PenaltyI_2D_alpha1e6 | | rongWolfe | | | 1.14e0 | 1.65e-2 | 1.12e0 | 1.19e0 | 3162.1 |
| | | Aggressive denSection | | | 4.69e4 1.13e0 | 1.12e5 1.00e-4 | 1.17e0 1.12e0 | 3.51e5 1.13e0 | 2664.3 2567.9 |
| | | MoreThuente | | | 4.78e0 | 2.66e0 | 1.12e0 1.18e0 | 8.48e0 | 2878.8 |
| | QQN-Bise | | ' | | 1.12e0 | 8.85e-7 | 1.12e0 | 1.13e0 | 1032.0 |
| | L-BFGS-I | Limited | | | 1.18e0 | 5.23e-2 | 1.13e0 | 1.31e0 | 4341.4 |
| | | gion-Aggressi | ve | | 1.39e5 | 7.17e4 | 1.23e4 | 2.70e5 | 3002.0 |
| | | gion-Precise | | | 1.67e5 | 7.86e4 | 2.57e4 | 2.92e5 | 3002.0 |
| | | gion-Adaptive gion-Conserva | | | 1.32e5 1.39e5 | 8.14e4 8.98e4 | 1.57e4 8.35e3 | 3.15e5 4.03e5 | 3002.0 3002.0 |
| | | gion-Conserva | | | 1.43e5 | 8.01e4 | 3.60e4 | 3.21e5 | 3002.0 |
| | | Conservative | | | 1.13e0 | 2.13e-2 | 1.12e0 | 1.22e0 | 1183.8 |
| | Adam-AN | 4SGrad | | | 1.16e0 | 5.02e-2 | 1.13e0 | 1.35e0 | 647.9 |
| | Adam | | | | 1.17e0 | 1.42e-1 | 1.13e0 | 1.79e0 | 625.9 |
| | QQN-Bise | ection-2 eightDecay | | | 2.42e0 1.59e0 | 1.73e0 9.85e-1 | 1.17e0 1.14e0 | 6.97e0 4.57e0 | 299.3 226.2 |
| | Adam-Ro | | | | 2.11e0 | 2.17e0 | 1.14e0 1.13e0 | 1.02e1 | 107.1 |
| | L-BFGS | | | | 7.22e4 | 1.26e5 | 1.52e0 | 3.42e5 | 109.5 |
| | | oicQuadraticl | | on | 1.12e0 | 2.16e-16 | 1.12e0 | 1.12e0 | 38.0 |
| | | tiveMomentu | ım | | 7.74e2 | 1.18e3 | 1.15e0 | 3.84e3 | 17.6 |
| | Adam-Fas GD-Neste | | | | 1.01e2 1.46e2 | 2.80e2 3.37e2 | 1.26e0 1.38e0 | 1.09e3 1.23e3 | 22.0 13.9 |
| | GD-Weigh | | | | 5.08e0 | 1.33e1 | 1.34e0 | 6.31e1 | 13.9 |
| | GD | 102 000) | | | 1.37e1 | 3.13e1 | 1.28e0 | 1.22e2 | 14.9 |
| | GD-Mom | | | | 6.67e1 | 2.53e2 | 1.26e0 | 1.16e3 | 13.0 |
| PenaltyI_5D_alpha1e6 | | rongWolfe | | | 4.21e0 | 1.19e0 | 2.86e0 | 7.74e0 | 3121.1 |
| | QQN-Bise | denSection | | | 2.81e0 2.81e0 | 6.12e-4 7.61e-5 | 2.81e0 2.81e0 | 2.81e0 2.81e0 | 2178.7 3026.4 |
| | | MoreThuente | · | | 1.29e1 | 4.81e0 | 3.57e0 | 1.92e1 | 2860.6 |
| | L-BFGS-I | | | | 2.96e0 | 9.11e-2 | 2.84e0 | 3.13e0 | 4259.2 |
| | | Conservative | | | 2.85e0 | 5.01e-2 | 2.81e0 | 3.03e0 | 3087.5 |
| | | Aggressive | | | 2.82e5 | 2.24e5 | 3.49e0 | 6.03e5 | 1110.7 |
| | | gion-Adaptive gion-Standard | | | 3.71e5 3.74e5 | 1.47e5 1.32e5 | 9.78e4 1.16e5 | 6.40e5 7.18e5 | 3002.0 3002.0 |
| | | ion-Standard | | | 3.74e5 3.90e5 | 1.32e5 1.18e5 | 1.10e5 1.47e5 | 5.82e5 | 3002.0 |
| | | gion-Aggressi | | | 3.61e5 | 1.19e5 | 1.76e5 | 5.61e5 | 3002.0 |
| | | gion-Precise | | | 4.20e5 | 1.11e5 | 2.36e5 | 6.26e5 | 3002.0 |
| | Adam-AN | 4SGrad | | | 2.85e0 | 9.04e-2 | 2.82e0 | 3.24e0 | 747.3 |
| | Adam QQN-Bise | action 2 | | | 2.90e0 1.03e1 | 1.17e-1 5.36e0 | 2.82e0 3.91e0 | 3.24e0 2.42e1 | 733.5 469.8 |
| | | eightDecay | | | 7.65e0 | 7.96e0 | 2.86e0 | 3.08e1 | 260.1 |
| | Adam-Ro | | | | 2.16e1 | 2.85e1 | 2.82e0 | 1.26e2 | 120.4 |
| | L-BFGS | | | | 7.00e4 | 1.12e5 | 4.88e0 | 5.16e5 | 95.2 |
| | | oicQuadraticl | Interpolati | on | 2.81e0 | 7.09e-16 | 2.81e0 | 2.81e0 | 38.0 |
| | Adam-Fas | st tiveMomentu | 1772 | | 4.69e2 5.87e2 | 8.74e2 9.45e2 | 3.30e0 3.36e0 | 2.81e3 2.82e3 | 29.0 13.9 |
| | GD-Rdap GD-Weigl | | 1111 | | 3.50e1 | 9.45e2 9.28e1 | 3.50e0 3.50e0 | 4.19e2 | 14.5 |
| | GD | 102 000) | | | 8.04e1 | 1.22e2 | 3.17e0 | 4.27e2 | 17.1 |
| | GD-Neste | | | | 3.89e2 | 8.55e2 | 3.28e0 | 3.56e3 | 14.1 |
| | GD-Mom | | | | 9.24e1 | 2.66e2 | 3.99e0 | 1.22e3 | 14.4 |
| PenaltyI_10D_alpha1e6 | • • | rongWolfe | | | 1.10e1 | 3.01e0 | 8.12e0 | 2.06e1 | 3057.8 |
| | QQN-Gol QQN-Bise | denSection ection-1 | | | 5.63e0 5.63e0 | 1.88e-4 2.30e-3 | 5.62e0 5.63e0 | 5.63e0 5.63e0 | 4382.9 2161.3 |
| | L-BFGS-I | | | | 5.89e0 | 1.07e-1 | 5.78e0 | 6.13e0 | 4213.4 |
| | L-BFGS-0 | Conservative | | | 5.73e0 | 1.30e-1 | 5.63e0 | 6.12e0 | 2611.7 |
| | | MoreThuente | | | 2.89e1 | 6.69e0 | 1.06e1 | 3.74e1 | 2859.4 |
| | QQN-Bise | | 1 | | 1.86e1 | 5.50e0 | 9.12e0 | 2.63e1 | 1070.2 |
| | | gion-Standard gion-Adaptive | | | 7.40e5 7.20e5 | 2.39e5 1.83e5 | 2.96e5 3.49e5 | 1.19e6 1.14e6 | 3002.0 3002.0 |
| | 1 LIUST D.E9 | non-auadity | - | | | | | | |
| | | | | | 7.72e5 | 1.26e5 | 5.48e5 | 1.03e6 | 3002.0 |
| | Trust Reg | gion-Conserva gion-Precise | | | 7.72e5 7.75e5 | 1.26e5 1.73e5 | 5.48e5 4.16e5 | 1.03e6 9.96e5 | 3002.0 3002.0 |

| Problem Optimizer Mean Final Std Dow Value Value Value State | | | | e 1 – contin | | | | 1 | 1 - | | |
|--|-----------------------|-----------|---------|---------------|------------|----|---------|----------|-------------|---------|--------|
| Adam AMSCrad | Problem | Optimizer | | Std Dev | | | | | | I . | |
| Adam | | | | | ve | | 7.20e5 | | | 1.04e6 | |
| Adam-Neight-Decay | | | | ISGrad | | | | | | | |
| Adam Faboust | | | | :-l-+D | | | | | | | |
| LBRGS | | | | | | | | | | | |
| Adam-Past QQN-Cubacquartsticlaterpolation 1.1763 2.053 6.260 8.053 1.281 | | | | bust | | | | | | | |
| LFFCS Aggressive | | | | t | | | | | | | |
| GD | | | QQN-Cub | icQuadraticI | nterpolati | on | 5.62e0 | 1.32e-15 | 5.62e0 | 5.62e0 | 38.0 |
| GD-AdaptiveMomentum | | | | Aggressive | | | | | | | |
| GD-Weight Decay | | | - | | | | | | | | |
| GD.Nesterov | | | | | ım | | | | | | |
| GD-Momentum | | | | | | | | | | | |
| Barrier_2D_mm0.1 | | | | | | | | | | | |
| Adam | Barrier_2D_mu0.1 | | Adam-R | obust | | | | | | l . | |
| Adam-WeightDecay | | | | ISGrad | | | 4.13e-1 | | | | |
| L-BFGS-Conservative | | | | | | | | | | | |
| GP | | | | | | | | | | | |
| Trust Region-Conservative | | | | Conservative | | | | | | | |
| L-BFGS-Limited | | | | ion-Conserva | tive | | | | | | |
| GD-WeightDecay | | | | | 66146 | | | | | | |
| Tust Region-Precise | | | | | | | | | | | |
| GD-Nesterov | | | | | | | 4.55e-1 | | | 8.14e-1 | |
| GD-Momentum | | | | | | | | | | | |
| Trust Region-Adaptive | | | | | | | | | | | |
| Trust Region-Standard | | | | | | | | | | | |
| GD-AdaptiveMomentum | | | | | | | | | | | |
| Trust Region-Aggressive 2,25e0 1,10e0 1,10e0 1,0fe0 2502,0 2,0e0 | | | | | | | | | | | |
| Adam-Robust | | | | | | | | | | | |
| Adam | Barrier_5D_mu 0.1 | | | | | | | | 1.01e0 | | |
| Adam-WeightDecay | | | | bust | | | | | | | |
| L.BEGS-Conservative | | | | . 1.15 | | | | | | | |
| Trust Region-Conservative | | | | | | | | | | | |
| GD | | | | | tive | | | | | | |
| L-BFGS-Limited 1.00e0 7.64e-3 9.99e-1 1.03e0 182.4 | | | | ion-consci va | 501 V C | | | | | | |
| GD-WeightDecay | | | | imited | | | | | | | |
| Adam-Fast | | | | | | | 1.02e0 | 1.49e-2 | 1.00e0 | | |
| GD-Nesterov | | | | | | | | | | | |
| GD-Momentum | | | | | | | | | | | |
| Trust Region-Adaptive | | | | | | | | | | | |
| Trust Region-Standard Trust Region-Aggressive Barrier_10D_mu0.1 Adam-Robust Adam-Robust 2.00e0 2.51e-3 2.00e0 2.51e-3 2.00e0 2.01e0 2.01e0 2.02e0 Adam-AMSGrad 2.04e0 1.32e-2 2.02e0 2.07e0 2.00e0 2.0 | | | | | <u> </u> | | | | | | |
| Barrier_10D_mu0.1 | | | | | | | | | | | |
| Barrier_10D_mu0.1 Adam-Robust 2.00e0 2.51e-3 2.00e0 2.01e0 2502.0 Adam 2.04e0 1.32e-2 2.02e0 2.07e0 2502.0 Adam 2.00e0 4.64e-7 2.00e0 2.00e0 2502.0 Adam-WeightDecay 2.00e0 5.45e-9 2.00e0 2.00e0 1144.5 Trust Region-Conservative 2.05e0 5.65e-2 2.00e0 2.18e0 2487.1 L-BFGS-Conservative 2.00e0 1.04e-8 2.00e0 2.00e0 555.1 GD 2.00e0 4.28e-11 2.00e0 2.00e0 305.0 L-BFGS-Limited 2.00e0 4.28e-11 2.00e0 2.00e0 195.4 Trust Region-Precise 2.04e0 8.08e-2 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.37e0 413.8 GD-Momentum 3.39e 2.4fee 2.4fee 2.7ie0 | | | | | | | | | | | |
| Adam 2.00e0 4.64e-7 2.00e0 2.00e0 2502.0 Adam-WeightDecay 2.00e0 5.45e-9 2.00e0 2.00e0 1144.5 Trust Region-Conservative 2.05e0 5.65e-2 2.00e0 2.18e0 2487.1 L-BFGS-Conservative 2.00e0 1.04e-8 2.00e0 2.00e0 555.1 GD 2.00e0 4.28e-11 2.00e0 2.00e0 305.0 L-BFGS-Limited 2.00e0 5.46e-9 2.00e0 2.00e0 195.4 Trust Region-Precise 2.04e0 8.08e-2 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.37e0 404.4 Trust Region-Adaptive 2.17e0 1.35e-1 2.00e0 2.37e0 106.3 Adam-Fast 2.58e0 8.67e-2 2.45e0 2.7te0 31.5 GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Aggressive 3.43e0 1.45e0 2.00e0 7.0 | $Barrier_10D_mu0.1$ | | Adam-R | obust | | | | 2.51e-3 | 2.00e0 | 2.01e0 | 2502.0 |
| Adam-WeightDecay 2.00e0 5.45e-9 2.00e0 2.00e0 1144.5 Trust Region-Conservative 2.05e0 5.65e-2 2.00e0 2.18e0 2487.1 L-BFGS-Conservative 2.00e0 1.04e-8 2.00e0 2.00e0 555.1 GD 2.00e0 4.28e-11 2.00e0 2.00e0 305.0 L-BFGS-Limited 2.00e0 5.46e-9 2.00e0 2.00e0 195.4 Trust Region-Precise 2.04e0 8.08e-2 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.37e0 406.3 Adam-Fast 2.58e0 8.67e-2 2.45e0 2.71e0 31.5 GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.32e0 30.9 Trust Region-Aggressive 3.43e0 1.45e0 2.00e0 7.03e0 10.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | ISGrad | <u> </u> | | | | | | |
| Trust Region-Conservative 2.05e0 5.65e-2 2.00e0 2.18e0 2487.1 L-BFGS-Conservative 2.00e0 1.04e-8 2.00e0 2.00e0 555.1 GD 2.00e0 4.28e-11 2.00e0 2.00e0 305.0 L-BFGS-Limited 2.00e0 5.46e-9 2.00e0 2.00e0 195.4 Trust Region-Precise 2.04e0 8.08e-2 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.37e0 413.8 Trust Region-Adaptive 2.17e0 1.35e-1 2.00e0 2.37e0 106.3 Adam-Fast 2.58e0 8.67e-2 2.45e0 2.71e0 31.5 GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Aggressive 3.43e0 4.45e-1 2.00e0 3.32e0 30.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e | | | | . 1.15 | | | | | | | |
| L-BFGS-Conservative 2.00e0 1.04e-8 2.00e0 2.00e0 555.1 GD | | | | | 4 | | | | | | _ |
| GD 2.00e0 4.28e-11 2.00e0 2.00e0 305.0 L-BFGS-Limited 2.00e0 5.46e-9 2.00e0 2.00e0 195.4 Trust Region-Precise 2.04e0 8.08e-2 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.00e0 49.4 Trust Region-Adaptive 2.17e0 1.35e-1 2.00e0 2.37e0 106.3 Adam-Fast 2.58e0 8.67e-2 2.45e0 2.71e0 31.5 GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.09e 30.9 NoisySphere_2D_sigma0.01 QN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2 | | | | | tive | | | | | | |
| L-BFGS-Limited 2.00e0 5.46e-9 2.00e0 2.00e0 195.4 Trust Region-Precise 2.04e0 8.08e-2 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.00e0 49.4 Trust Region-Adaptive 2.17e0 1.35e-1 2.00e0 2.37e0 106.3 Adam-Fast 2.58e0 8.67e-2 2.45e0 2.71e0 31.5 GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.32e0 30.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 | | | | Joinsol value | | | | | | | |
| Trust Region-Precise 2.04e0 8.08e-2 2.00e0 2.37e0 413.8 GD-WeightDecay 2.00e0 1.69e-5 2.00e0 2.00e0 49.4 Trust Region-Adaptive 2.17e0 1.35e-1 2.00e0 2.37e0 106.3 Adam-Fast 2.58e0 8.67e-2 2.45e0 2.71e0 31.5 GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.32e0 30.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 25.7 | | | | Limited | | | | | | | |
| Trust Region-Adaptive 2.17e0 1.35e-1 2.00e0 2.37e0 106.3 Adam-Fast 2.58e0 8.67e-2 2.45e0 2.71e0 31.5 GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.32e0 30.9 Trust Region-Aggressive 3.43e0 1.45e0 2.00e0 7.03e0 10.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | | | | | 8.08e-2 | 2.00e0 | | 413.8 |
| Adam-Fast 2.58e0 8.67e-2 2.45e0 2.71e0 31.5 GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.32e0 30.9 Trust Region-Aggressive 3.43e0 1.45e0 2.00e0 7.03e0 10.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | | | | | | | | |
| GD-Nesterov 2.50e0 4.65e-2 2.43e0 2.64e0 21.1 GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.32e0 30.9 Trust Region-Aggressive 3.43e0 1.45e0 2.00e0 7.03e0 10.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | |) | | | | | | |
| GD-Momentum 3.39e0 2.40e-1 2.91e0 3.84e0 21.2 Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.32e0 30.9 Trust Region-Aggressive 3.43e0 1.45e0 2.00e0 7.03e0 10.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | | | | | | | | |
| Trust Region-Standard 2.34e0 4.45e-1 2.00e0 3.32e0 30.9 Trust Region-Aggressive 3.43e0 1.45e0 2.00e0 7.03e0 10.9 NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | | | | | | | | |
| NoisySphere_2D_sigma0.01 Trust Region-Aggressive 3.43e0 1.45e0 2.00e0 7.03e0 10.9 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | | 1 | | | | | | |
| NoisySphere_2D_sigma0.01 QQN-Bisection-1 1.33e0 6.30e-1 2.16e-1 2.48e0 41.2 L-BFGS-Conservative 1.57e0 7.01e-2 1.46e0 1.66e0 97.0 Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | | | | | | | | |
| Adam-WeightDecay 2.18e0 2.38e-1 1.76e0 2.70e0 27.6 Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | NoisySphere_2D_sigma | 0.01 | QQN-Bis | section-1 | | | 1.33e0 | 6.30e-1 | 2.16e-1 | 2.48e0 | 41.2 |
| Adam-AMSGrad 2.10e0 2.73e-1 1.65e0 2.60e0 25.7 Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | | | | | 7.01e-2 | | | |
| Adam 2.13e0 2.38e-1 1.72e0 2.60e0 21.4 | | | | | | | | | | | |
| | | | | ISGrad | | | | | | | |
| Continued on next page | | | Adam | | | | 2.13e0 | | | | 21.4 |
| | - | | | | | | | Continu | iea on next | page | |

| Problem Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | us page Mean Fur Evals | nc Succes | | Time s) | |
|---------------------------|-----------------------|--|---------------|-----------------------|----------------------------------|--------------------|-------------------------|------------------|------------------|
| | QQN-Stro | ngWolfe | | | 9.23e-1 | 6.21e-1 | 2.83e-2 | 2.06e0 | 30.5 |
| | L-BFGS | | | | 1.02e1 | 1.19e1 | 7.26e-2 | 3.76e1 | |
| | Adam-Rol | $\frac{\text{oust}}{\text{MoreThuente}}$ | | | 2.10e0 1.84e0 | 2.43e-1 7.24e-1 | 1.76e0 1.12e-1 | 2.74e0 2.67e0 | |
| | QQN-Gold | | | | 2.04e0 | 2.79e-1 | 1.45e0 | 2.45e0 | |
| | L-BFGS-L | | | | 1.64e0 | 6.02e-1 | 7.63e-1 | 2.63e0 | 31.4 |
| | GD | | | | 2.20e0 | 6.02e-1 | 1.42e0 | 2.96e0 | |
| | Adam-Fas | | | | 2.26e0 | 7.44e-1 | 1.40e0 | 4.41e0 | - |
| | GD-Nester | icQuadraticI | nterpolati | on | 2.04e0 2.82e0 | 2.13e-1 9.93e-1 | 1.70e0 1.53e0 | 2.34e0 4.52e0 | |
| | GD-Weigh | | | | 2.30e0 | 5.19e-1 | 1.61e0 | 3.10e0 | |
| | | tiveMomentu | m | | 2.39e0 | 7.39e-1 | 1.45e0 | 3.76e0 | |
| | GD-Mome | | | | 2.45e0 | | 1.43e0 | 4.08e0 | |
| | L-BFGS-A | | | | 2.05e0 2.19e0 | 2.75e-1 | 1.69e0 | 2.57e0 2.54e0 | |
| | | ion-Precise ion-Conserva | tive | | 2.19e0 2.06e0 | 2.25e-1 2.96e-1 | 1.76e0 1.68e0 | 2.54e0 2.87e0 | |
| | | ion-Aggressiv | | | 2.09e0 | 2.18e-1 | | 2.50e0 | |
| | Trust Reg | ion-Adaptive | ; | | 2.03e0 | 2.74e-1 | 1.67e0 | 2.62e0 | 4.2 |
| | | ion-Standard | | | 2.10e0 | 2.61e-1 | 1.70e0 | 2.64e0 | |
| M. C.I. FD. : 0.01 | QQN-Bise | | | | 1.97e0 | 7.03e-3 | 1.96e0 | 1.97e0 | |
| NoisySphere_5D_sigma0.01 | QQN-Bis | Section-1 Conservative | | | 4.46e0 4.42e0 | 9.06e-1 1.19e-1 | 2.43e0 4.14e0 | 6.17e0 4.57e0 | |
| | Adam | onservative | | | 5.42e0 | 2.68e-1 | 4.14e0 4.73e0 | 5.93e0 | |
| | Adam-AM | ISGrad | | | 5.42e0 | 3.72e-1 | 4.94e0 | 6.37e0 | |
| | Adam-Rol | oust | | | 5.11e0 | 3.42e-1 | 4.60e0 | 5.73e0 | |
| | L-BFGS | | | | 1.44e1 | 8.91e0 | 2.91e0 | | |
| | Adam-We | ightDecay | | | 5.22e0 5.52e0 | 3.07e-1 8.70e-1 | 4.62e0 4.36e0 | 5.81e0 6.84e0 | |
| | GD-Mome | ntum | | | 5.52e0 5.71e0 | 9.79e-1 | 4.30e0 4.41e0 | 7.61e0 | |
| | L-BFGS-L | | | | 3.90e0 | 8.53e-1 | 2.63e0 | 5.48e0 | |
| | Adam-Fas | t | | | 5.99e0 | 1.16e0 | 4.17e0 | 8.14e0 | |
| | | tiveMomentu | m | | 5.68e0 | 8.70e-1 | 4.36e0 | 7.22e0 | |
| | | IoreThuente | | | 4.61e0 | 8.49e-1 | 3.27e0 | 5.99e0 | |
| | GD-Nester GD-Weigh | | | | 5.61e0 5.35e0 | 1.00e0 6.50e-1 | 4.26e0 4.47e0 | 6.77e0 6.45e0 | |
| | QQN-Stro | | | | 3.67e0 | 1.35e0 | 1.11e0 | 5.33e | |
| | QQN-Gold | denSection | | | 4.88e0 | 7.47e-1 | 3.49e0 | 6.27e0 | |
| | | icQuadraticI | nterpolati | on | 5.26e0 | 3.22e-1 | 4.75e0 | 5.60e0 | |
| | | ion-Precise | | | 5.28e0 | 4.50e-1 | 4.65e0 | 6.08e0 | |
| | | ion-Conserva ion-Standard | | | 5.20e0 5.20e0 | 3.98e-1 3.94e-1 | 4.59e0 4.73e0 | 5.78e0 6.28e0 | |
| | | ion-Adaptive | | | 5.32e0 | 3.69e-1 | 4.75e0 4.65e0 | 6.03e0 | |
| | L-BFGS-A | | <u> </u> | | 5.18e0 | 4.15e-1 | 4.66e0 | 5.74e0 | |
| | | ion-Aggressiv | ve | | 5.27e0 | 3.43e-1 | 4.65e0 | 5.82e0 | |
| NoisySphere_10D_sigma0.01 | QQN-Bis | | | | 9.08e0 | 1.54e0 | 4.69e0 | 1.10e1 | |
| | | Conservative dam-Weight | Decay | | 9.49e0 | 1.72e-1 1.04e1 | 9.16e0 4.40e-1 | 9.70e0 9.79e0 | 93.8 1.14e1 |
| | | dam-AMSG | | | | 1.04e1 | 6.28e-1 | 9.75e0 | 1.18e1 |
| | | dam | | | | 1.07e1 | 5.64e-1 | 9.87e0 | 1.20e1 |
| | | dam-Robust | ; | | | 1.05e1 | 6.12e-1 | 9.71e0 | 1.19e1 |
| | | HD A L | N.f. | | | 1.10e1 | 7.03e-1 | 9.54e0 | 1.19e1 |
| | | D-Adaptive -BFGS | Momentur | n | | 1.03e1 2.28e1 | 7.39e-1 1.18e1 | 9.45e0 7.46e0 | 1.20e1 4.18e1 |
| | | D-Nesterov | | | | 1.09e1 | 8.10e-1 | 9.63e0 | 1.25e1 |
| | L | -BFGS-More | | | | 9.77e0 | 1.48e0 | 6.61e0 | 1.14e1 |
| | | GD-Momentu | | | | 1.11e1 | 9.34e-1 | 9.66e0 | 1.26e1 |
| | | QN-Golden | | | | 1.03e1 | 5.55e-1 | 9.18e0 | 1.11e1 |
| | | GD-WeightDe QN-StrongV | | | | 1.10e1 9.41e0 | 7.21e-1 1.31e0 | 9.66e0 7.23e0 | 1.25e1 1.10e1 |
| | | dam-Fast | vone | | | 1.11e1 | 1.31e0 1.47e0 | 9.46e0 | 1.10e1 1.54e1 |
| | | -BFGS-Limi | ted | | | 9.06e0 | 1.17e0 | 6.79e0 | 1.08e1 |
| | | rust Region- | | ive | | 1.04e1 | 3.84e-1 | 9.82e0 | 1.13e1 |
| | | rust Region- | | | | 1.06e1 | 3.31e-1 | 9.93e0 | 1.12e1 |
| | | L-BFGS-Aggressive | | | | 1.06e1 | 5.02e-1 | 9.75e0 | 1.11e1 |
| | | Trust Region-Aggressive | | | | 1.04e1 1.06e1 | 5.45e-1 5.93e-1 | 9.77e0 9.80e0 | 1.14e1 1.16e1 |
| | | Trust Region-Standard QQN-CubicQuadraticInterpolatio | | | | 1.00e1 1.04e1 | 1.61e-1 | 1.03e1 | 1.07e1 |
| | (| QN-CubicQ | uaaraticir | Trust Region-Adaptive | | | | 1.0001 | 1.0761 |
| | | | | |)II | 1.04e1 | 6.06e-1 tinued on ne | 9.73e0 | 1.07e1 1.17e1 |

| Problem Optimizer | Mean Final Value | $oxed{ Std Dev }$ | Best Value | Worst Value | Mear | n Func vals | Success Rate (% | | an Time (s) | | |
|-------------------------------------|---------------------|------------------------------------|---------------|----------------|------|----------------|------------------------------------|------------------------------------|--------------------|--------------------|-------------|
| SparseRosenbrock_4D | | QQN-Golde : QQN-StrongV | | | | 3.33 | | 1.76e-1 6.04e-7 6.28e-1 2.24e-8 | | 6.84e-1 3.00e0 | 4 |
| | (| QQN-CubicQ | uadraticIn | terpolatio | n | | | 6.41e-2 | 6.10e-9 | 2.15e-1 | 1 |
| | | QQN-Bisectio | | | | | | 4.62e-1 | 9.12e-8 | 1.60e0 | 2 |
| | | Adam-AMSG: Adam | rad | | | | 9e0 6e0 | 1.45e0 4.83e-1 | 1.65e0 1.19e0 | 8.86e0 3.18e0 | 2 |
| | | L-BFGS-More | Thuente | | | | 5e0 | 2.72e0 | 2.55e-7 | 8.30e0 | 2 |
| | | L-BFGS-Limi | | | | 2.3 | | 4.08e0 | 7.38e-2 | 1.74e1 | 3 |
| | | L-BFGS-Cons | | | | | 7e2 | 5.46e2 | 3.78e-5 | 2.51e3 | 3 |
| | | L-BFGS-Aggr Trust Region- | | ive | | 6.3 | | 4.42e1 4.50e1 | 8.59e0 3.37e-1 | 1.72e2 1.58e2 | 3 |
| | | Adam-Weight | | ive | | | 7e0 | 3.97e0 | 1.75e-2 | 9.30e0 | - 2 |
| | | Adam-Robust | | | | | 3e0 | 1.96e0 | 3.60e0 | 8.99e0 | (|
| | | GD | | | | | 8e0 | 2.48e0 | 9.10e-1 | 9.50e0 | į |
| | | Trust Region- Trust Region- | | | | 2.1 8.3 | | 2.75e1 2.62e-1 | 7.75e0 7.78e0 | 1.10e2 8.82e0 | 1 |
| | | Adam-Fast | Adaptive | | | 1.9 | | 3.76e0 | 1.14e-3 | 1.01e1 | |
| | | QQN-Bisectio | n-2 | | | 1.4 | | 1.80e0 | 5.00e-7 | 3.95e0 | |
| | | GD-WeightDe | cay | | | | 4e0 | 4.49e0 | 4.73e-2 | 1.17e1 | 1 |
| | | L-BFGS GD-Nesterov | | | | 9.6 | | 1.12e2 | 7.18e0 | 4.84e2 | |
| | | GD-Nesterov GD-Adaptivel | Momentur | n | | 3.3 | 8e0 1e0 | 3.99e0 4.23e0 | 1.58e-1 2.41e-2 | 1.22e1 1.97e1 | + |
| | | Trust Region- | | - | | 8.2 | | 2.76e-1 | 7.68e0 | 8.68e0 | 1 |
| | | GD-Momentu | | | | 1.1 | 0e1 | 6.57e0 | 4.39e-2 | 3.10e1 | |
| Count Daniel 1 10D | | Trust Region- | | • | | | | 4.61e-1 | 7.92e0 | 9.65e0 | |
| SparseRosenbrock_10D | | QQN-Golde: QQN-StrongV | | | | 5.84 | | 5.22e-1 7.63e-1 | 5.69e-7 1.90e-9 | 2.38e0 3.16e0 | 2 |
| | | $\frac{QQN-Strong v}{QQN-CubicQv}$ | | terpolation | n | 1.45 | | 1.61e-1 | 1.89e-8 | 3.94e-1 | 1 |
| | | QQN-Bisectio | | F | | 6.70 | | 9.09e-1 | 1.26e-7 | 3.85e0 | 2 |
| | | Adam-AMSG | | | | | | 9.36e-1 | 4.14e0 | 7.62e0 | 2 |
| | | Adam-Robust | | | | 1.1 | | 2.30e0 | 9.15e0 | 2.07e1 | 2 |
| | | Adam L-BFGS-More | Thuente | | | | 1e0 8e0 | 1.16e0 8.03e0 | 3.14e0 2.91e-4 | 7.20e0 2.32e1 | 2 |
| | | L-BFGS-Limi | | | | 1.2 | | 3.17e0 | 3.17e-1 | 1.51e1 | 3 |
| | | Adam-Weight | | | | 3.2 | 7e0 | 7.74e0 | 1.06e-2 | 2.19e1 | 2 |
| | | L-BFGS-Cons | | | | 1.0 | | 4.58e7 | 2.96e-2 | 2.10e8 | 3 |
| | | L-BFGS-Aggr QQN-Bisectio | | | | | 1e2 1e0 | 7.47e1 5.76e0 | 8.09e1 3.42e-1 | 4.01e2 1.61e1 | 3 |
| | | Trust Region- | | | | 1.0 | | 6.99e1 | 1.18e0 6.91e1 | 2.43e2 | 3 |
| | | Trust Region- | | ive | | | | 4.85e1 | | 2.68e2 | 3 |
| | | Trust Region- | Adaptive | | | 2.9 | | 2.81e1 | 2.02e1 | 1.48e2 | 2 |
| | | GD F. (| | | | 3.2 | | 5.80e0 | 1.20e0 | 2.11e1 | = 3 |
| | | Adam-Fast GD-WeightDe | cav | | | | 0e0 4e0 | 5.64e0 9.20e0 | 5.27e-2 1.18e-1 | 2.63e1 2.47e1 | 1 2 |
| | | GD-Nesterov | cay | | | | | 5.92e-1 | 5.84e-2 | 2.48e0 | |
| | | Trust Region- | Standard | | | | | 3.97e-1 | 1.95e1 | 2.11e1 | |
| | | L-BFGS | | | | | 0e2 | 3.25e2 | 2.70e1 | 1.32e3 | |
| | | GD-Adaptivel Trust Region- | | | | 1.3 | 1e1 | 1.44e1 8.10e-1 | 5.08e-2 1.97e1 | 3.27e1 2.26e1 | + |
| | | GD-Momentu | | | | | 5e1 | 1.33e1 | 3.12e-1 | 4.42e1 | <u> </u> |
| $SparseQuadratic_5D_pattern[1,3]$ | | Adam-Robu | \mathbf{st} | | | 6.93 | 3e-2 | 2.46e-2 | 2.83e-2 | 1.11e-1 | 2 |
| | | Adam-AMSG | rad | | | | | 5.56e-2 | 1.19e-1 | 3.19e-1 | 2 |
| | | Adam Adam-Weight | Doggy | | | | | 1.54e-3 7.96e-9 | 5.91e-4 9.73e-7 | 5.76e-3 9.99e-7 | 2 |
| | | Trust Region- | | ive | | | 6e0 | 2.02e0 | 9.73e-7 8.95e-3 | 5.15e0 | 1 |
| | (| GD | | - | | | 7e-7 | 1.24e-8 | 9.59e-7 | 9.96e-7 | 1 |
| | | L-BFGS-Cons | | | | | | 3.62e-7 | 1.15e-10 | 9.31e-7 | 3 |
| | | GD-WeightDe L-BFGS-Limi | | | | | | 4.76e-8 | 8.42e-7 4.26e-9 | 9.96e-7 9.93e-7 | |
| | | Trust Region- | | | | | 4.57e-7 2.77e-7 4.92e-1 5.29e-1 | | 4.26e-9 3.14e-2 | 9.93e-7 1.49e0 | 1 |
| | | QQN-GoldenS | | | | | | 1.38e-7 | 2.49e-9 | 4.81e-7 | 1 |
| | (| QQN-CubicQuadraticInterpolation | | | | | 4.31e-7 3.85e-8 | | 3.60e-7 | 4.92e-7 | |
| | | QQN-Bisection-1 | | | | | | 2.00e-7 | 4.95e-9 | 7.40e-7 | |
| | | L-BFGS | n ? | | | | | 3.20e-7 2.16e-7 | 1.77e-8 1.83e-9 | 9.73e-7 7.77e-7 | 1 |
| | | QQN-Bisection-2 Adam-Fast | | | | | | 2.16e-7 1.39e-2 | 3.36e-1 | 3.82e-1 | +- |
| | | GD-Nesterov | | | | | | 3.40e-2 | 2.73e-1 | 3.98e-1 | + |
| | | GD-Adaptivel | Momentur | n | | 9.75 | | 6.24e-2 | 8.63e-1 | 1.09e0 | |
| | | | | | | | Conti | nued on | next page | | |

| Problem Optimizer Mean F Value | inal St | d Dev | Best Value | Worst Value | Mea | n Func Evals | Success Rate (%) | | an Time (s) | | |
|---|---------------------------|-------------------------|------------------------|----------------|-----|-----------------|--|----------------------|-------------------------------|--------------------|---|
| | | t Region- Momentu | Adaptive | | | 1.2 | | 4.09e2 1.96e-2 | 5.86e-2 4.06e-1 | 1.37e3 6.07e-1 | |
| | | V-StrongV | | | | | | .26e-7 | 4.00e-8 | 4.52e-7 | |
| | | GS-Aggr | | | | | | 3.06e-8 | 2.36e-7 | 3.36e-7 | |
| | | FGS-More | | | | | | .36e-7 | 1.12e-8 | 6.56e-7 | |
| | | t Region- | Standard Aggressive | 2 | | | | 7.08e0 7.17e2 | 1.67e2 1.51e4 | 1.90e2 1.86e4 | |
| SparseQuadratic_10D_pattern[1, 3] | | m-Robu | | , | | | | 3.53e-2 | 4.23e-2 | 1.71e-1 | 2 |
| | | n-AMSG | rad | | | 2.33 | | 1.06e-2 | 1.70e-1 | 3.20e-1 | 2 |
| | Adar | | D | | | | | 8.81e-4 | 3.96e-4 | 3.78e-3 | 2 |
| | | n-Weight | Decay Conservat | ive | | 9.8 | | 7.84e-9 5.92e-1 | 9.70e-7 6.04e-1 | 9.98e-7 3.15e0 | 3 |
| | GD | t region- | Consci vac | 110 | | | | .20e-8 | 9.60e-7 | 1.00e-6 | |
| | | GS-Cons | | | | | | 2.96e-7 | 2.80e-9 | 9.97e-7 | |
| | | t Region- | | | | | | 3.28e-1 | 2.31e-3 | 1.54e0 | |
| | | WeightDe I-GoldenS | | | | | | 3.38e-8 3.27e-7 | 8.75e-7 7.74e-9 | 9.98e-7 9.91e-7 | |
| | | | | terpolatio | n | | | .47e-8 | 2.26e-9 | 4.52e-8 | |
| | L-BF | FGS | | 1 | | | | 3.35e-3 | 1.06e-8 | 3.83e-2 | - |
| | | GS-Limi | | | | | | 2.74e-7 | 5.73e-8 | 9.57e-7 | |
| | | -Bisectio | | | | | | 3.82e-7 | 2.29e-8 | 9.90e-7 | |
| | | t Region- I-Bisectio | Adaptive | | | 1.2 | | 4.25e2 3.20e-7 | 5.42e-2 2.03e-8 | 1.43e3 9.46e-7 | |
| | | V-Strong V | | | | | | .50e-7 | 8.82e-8 | 8.12e-7 | |
| | Adar | n-Fast | | | | 7.28 | Be-1 2 | 2.79e-2 | 6.71e-1 | 7.71e-1 | |
| | | | Momentur | n | | 1.5 | | 1.23e-2 | 1.41e0 | 1.64e0 | |
| | | Nesterov Momentu | ··· | | | 6.03 9.13 | | 2.70e-2 5.54e-2 | 5.68e-1 8.60e-1 | 7.02e-1 1.06e0 | |
| | 1 | GS-Aggr | | | | | | .67e-8 | 1.72e-7 | 2.44e-7 | |
| | | GS-More | | | | | | 2.42e-7 | 2.06e-8 | 8.82e-7 | |
| | | | Standard | | | | | 3.48e1 | 2.21e-1 | 1.29e2 | |
| I | | | Aggressive | е | | | | 4.68e3 | 8.89e-1 | 2.17e4 | - |
| LogisticRegression_100samples_5features_reg0.01 | | WeightI Nesterov | Jecay | | | 3.1 | | 3.54e-4 3.66e-7 | 3.26e-1 3.15e-1 | 3.27e-1 3.15e-1 | 1 |
| | GD | TCBUCTOV | | | | 3.7 | | 2.81e-3 | 3.72e-1 | 3.81e-1 | 1 |
| | 1 | Momentu | m | | | 3.15 | | 3.78e-7 | 3.15e-1 | 3.15e-1 | 1 |
| | | n-Robust | , | | | 4.35 | | .22e-2 | 4.13e-1 3.87e-1 | 4.60e-1 | 2 |
| | 1 | n-AMSG n-Weight | | | | 3.25 | | 0.76e-3 1.09e-3 | | 4.23e-1 3.24e-1 | 2 |
| | | GS-Limi | | | | 3.15 | | 1.97e-5 1 9.33e-3 | 3.20e-1 3.15e-1 3.92e-1 | 3.15e-1 | 3 |
| | Adar | | | | | 4.05 | | | | 4.21e-1 | 2 |
| | L-BF | FGS | | | | 3.10 | 6e-1 7 | 7.17e-4 | 3.15e-1 | 3.19e-1 | 2 |
| | | GS-Aggr | | | | | | .24e-4 | 3.15e-1 | 3.16e-1 | 1 |
| | | Adaptivel GS-Cons | Momentur | n | | 3.15 | | 1.41e-9 .83e-11 | 3.15e-1 3.15e-1 | 3.15e-1 3.15e-1 | |
| | | I-Goldens | | | | | | .54e-12 | 3.15e-1 3.15e-1 | 3.15e-1 3.15e-1 | |
| | QQN | V-Bisectio | n-1 | | | 3.15 | 5e-1 2 | .67e-12 | 3.15e-1 | 3.15e-1 | |
| | | | | terpolatio | n | 3.15 | | .75e-12 | 3.15e-1 | 3.15e-1 | |
| | | GS-More I-Bisectio | | | | 3.15 | | 2.25e-5 .84e-12 | 3.15e-1 3.15e-1 | 3.15e-1 3.15e-1 | : |
| | | n-Fast | 11-2 | | | | | .09e-4 | 3.16e-1 | 3.16e-1 | |
| | QQN | V-StrongV | Volfe | | | 3.15 | 5e-1 2 | .43e-12 | 3.15e-1 | 3.15e-1 | - |
| | | | Conservat | ive | | 4.24 | | 2.86e-3 | 4.20e-1 | 4.28e-1 | |
| | | t Region- | | | | | | 2.08e-2 | 6.48e-1 | 7.23e-1 | |
| | | | Aggressive Adaptive | = | | | | 2.53e-2 3.03e-2 | 6.53e-1 6.37e-1 | 7.51e-1 7.46e-1 | |
| | Trus | t Region- | Standard | | | | | 3.18e-2 | 6.55e-1 | 7.75e-1 | |
| LogisticRegression_200samples_10features_reg0.0 | 1 GD- | WeightI | | | | 3.39 | 9e-1 4 | 1.45e-4 | 3.38e-1 | 3.40e-1 | 1 |
| | | Nesterov | | | | | | 6.77e-7 | 3.23e-1 | 3.23e-1 | 1 |
| | GD-I | Momentu | m | | | | | 7.07e-7 3.28e-3 | 3.23e-1 3.93e-1 | 3.23e-1 4.05e-1 | 1 |
| | | GS-Limi | ted | | | | | 3.38e-5 | 3.23e-1 | 3.24e-1 | 3 |
| | Adar | n-Robust | | | | | 3.23e-1 8.38e-5 3.23e-1 4.12e-1 8.61e-3 3.96e-1 | | | 4.28e-1 | 2 |
| | | n-AMSG | | | | | | 5.85e-3 | 3.84e-1 | 4.09e-1 | 2 |
| | Adam-WeightDecay | | | | | | | 8.69e-4 | 3.25e-1 | 3.29e-1 | 2 |
| | Adam L-BFGS-Aggressive | | | | | | | 0.22e-3 7.06e-4 | 3.75e-1 3.23e-1 | 4.09e-1 3.26e-1 | 2 |
| | | | | | | | | 2.14e-5 | 3.23e-1 3.23e-1 | 3.23e-1 | 2 |
| | ' | | L-BFGS | | | | | ued on r | next page | | |

| Problem Optimizer Mean F Valu | inal | Std Dev | Best Value | Worst Value | Mea | n Func Evals | Succes Rate (| | ean Time (s) | |
|--|---------------------------------|--|---------------|----------------|-----|-----------------|--------------------|----------------------|--------------------|--------------------|
| - | | GD-Adaptivel L-BFGS-Cons | | n | | | 3e-1 3e-1 | 4.50e-9 9.68e-11 | 3.23e-1 3.23e-1 | 3.23e-1 3.23e-1 |
| | | QQN-GoldenS | | | | | 3e-1 | 1.79e-12 | | 3.23e-1 3.23e-1 |
| | | QQN-Bisectio | | | | | 3e-1 | 1.05e-12 | | 3.23e-1 |
| | | QQN-CubicQ | | terpolatio | n | | 3e-1 | 1.34e-12 | | 3.23e-1 |
| | | QQN-Bisectio | | | | | 3e-1 | 1.19e-12 | | 3.23e-1 |
| | | QQN-StrongV Adam-Fast | voiie | | | | 3e-1 4e-1 | 1.79e-12 2.14e-4 | 3.23e-1 3.24e-1 | 3.23e-1 3.25e-1 |
| | | L-BFGS-More | Thuente | | | | 3e-1 | 2.80e-4 | 3.23e-1 | 3.24e-1 |
| | | Trust Region- | | ive | | | 4e-1 | 2.76e-3 | 4.39e-1 | 4.49e-1 |
| | | Γrust Region- | | | | | 1e-1 | 2.67e-2 | 6.59e-1 | 7.56e-1 |
| | | Γrust Region- Γrust Region- | | 9 | | | 3e-1 1e-1 | 2.08e-2 2.90e-2 | 6.65e-1 6.45e-1 | 7.53e-1 7.53e-1 |
| | | Trust Region- | | | | | 6e-1 | 2.50e-2 2.50e-2 | 6.56e-1 | 7.48e-1 |
| LinearRegression_100samples_5features_reg0.01 | | Adam-AMS | | | | | 6e0 | 3.82e-1 | 2.47e0 | 3.69e0 |
| | | Adam-Robust | | | | | 9e0 | 5.55e-1 | 4.01e0 | 6.12e0 |
| | | Adam | D | | | | 8e0 | 4.16e-1 | 2.36e0 | 3.78e0 7.21e-2 |
| | | Adam-Weight Frust Region- | | ive | | | 8e-2 7e0 | 1.52e-4 9.98e-1 | 7.16e-2 6.26e0 | 9.77e0 |
| | | L-BFGS-Cons | | ive | | | 5e-2 | 3.25e-10 | | 7.15e-2 |
| | | GD | | | | 7.1 | 5e-2 | 5.94e-11 | 7.15e-2 | 7.15e-2 |
| | | L-BFGS-More | | | | | 6e-2 | 5.33e-5 | 7.15e-2 | 7.18e-2 |
| | Trust Region-Precise | | | | | | 8e-1 | 8.18e-2 | 7.38e-2 | 4.03e-1 |
| | | L-BFGS-Limi GD-WeightDe | | | | | 5e-2 5e-2 | 2.22e-10 2.05e-10 | | 7.15e-2 7.15e-2 |
| | | QQN-GoldenS | | | | | 5e-2 | 1.34e-9 | 7.15e-2 7.15e-2 | 7.15e-2 7.15e-2 |
| | | Trust Region- | | | | | 5e3 | 6.64e2 | 1.22e-1 | 1.96e3 |
| | | L-BFGS | | | | | 8e-1 | 2.48e-1 | 7.15e-2 | 1.21e0 |
| | | QQN-Bisectio | | . 1 | | | 5e-2 | 9.50e-15 | | 7.15e-2 |
| | | QQN-CubicQ | | terpolatio | n | | 5e-2 5e-2 | 1.77e-9 8.70e-14 | 7.15e-2 7.15e-2 | 7.15e-2 7.15e-2 |
| | QQN-StrongWolfe QQN-Bisection-2 | | | | | | 5e-2 | 3.89e-14 | 7.15e-2 7.15e-2 | 7.15e-2 7.15e-2 |
| | | L-BFGS-Aggr | | | | | 5e-2 | 3.23e-14 | 7.15e-2 | 7.15e-2 |
| | Adam-Fast | | | | | 0e-1 | 6.82e-2 | 7.25e-2 | 2.46e-1 | |
| | | Trust Region- | Standard | | | | 0e2 | 5.00e1 | 2.92e-1 | 1.65e2 |
| | | GD-Nesterov GD-Momentu | m | | | | 9e-1 6e0 | 2.74e-2 6.13e-2 | 8.36e-1 1.22e0 | 9.57e-1 1.50e0 |
| | | GD-Adaptivel | | n | | | 0e0 | 4.19e-2 | 1.90e0 | 2.04e0 |
| | 7 | Trust Region- | Aggressive | | | | 2e4 | 1.65e4 | 9.10e-1 | 5.27e4 |
| LinearRegression_200samples_10features_reg0.01 | | Adam-AMS | | | | | | 1.53e0 | 5.00e1 | 5.75e1 |
| | | Adam-Robust | | | | | 4e1 2e0 | 2.14e0 4.49e-1 | 5.92e1 1.66e0 | 6.79e1 3.54e0 |
| | | Adam-Weight Adam | Decay | | | | 4e1 | 2.38e0 | 4.86e1 | 5.85e1 |
| | | Trust Region- | Precise | | | | 9e1 | 3.12e0 | 4.85e1 | 5.96e1 |
| | 7 | Trust Region- | Conservat | ive | | 1.0 | 6e2 | 1.88e0 | 1.03e2 | 1.10e2 |
| | | Γrust Region- | Adaptive | | | | 8e3 | 8.92e2 | 5.40e-1 | 2.10e3 |
| | | GD L-BFGS-Cons | ervetivo | | | | 2e-1 2e-1 | 3.41e-6 2.33e-5 | 4.82e-1 4.82e-1 | 4.82e-1 4.82e-1 |
| | | Trust Region- | | | | | 6e3 | 4.67e3 | 7.13e-1 | 1.23e4 |
| | - A | Adam-Fast | | | | 5.1 | 4e-1 | 4.25e-2 | 4.82e-1 | 5.75e-1 |
| | | GD-WeightDe | | | | | 2e-1 | 9.82e-6 | 4.82e-1 | 4.82e-1 |
| | | L-BFGS-Aggr | | | | | 2e-1 | 2.65e-5 | 4.82e-1 | 4.82e-1 |
| | | L-BFGS-Limi QQN-Bisectio | | | | | 2e-1 2e-1 | 9.14e-5 1.93e-6 | 4.82e-1 4.82e-1 | 4.82e-1 4.82e-1 |
| | | QQN-GoldenS | | | | | 2e-1 2e-1 | 1.59e-5 | 4.82e-1 | 4.82e-1 |
| | 7 | Trust Region- | Aggressive | e | | 3.9 | 2e4 | 2.34e4 | 1.28e0 | 6.10e4 |
| | | QN-Bisectio | n-2 | | | | 2e-1 | 1.86e-6 | 4.82e-1 | 4.82e-1 |
| | | QQN-StrongV | Volfe | | | | 2e-1 | 1.42e-5 | 4.82e-1 | 4.82e-1 |
| | | L-BFGS L-BFGS-MoreThuente | | | | | 9e-1 2e-1 | 9.28e-2 8.59e-5 | 4.82e-1 4.82e-1 | 8.97e-1 4.82e-1 |
| | | L-BFGS-MoreThuente QQN-CubicQuadraticInterpolation | | | | | 4.82e-1 4.82e-1 | | 4.82e-1 | 4.82e-1 |
| | | GD-Nesterov | | | | | 0e0 | 2.65e-5 1.42e-2 | 1.78e0 | 1.83e0 |
| | GD-Momentum | | | | | 6e0 | 2.68e-2 | 2.41e0 | 2.51e0 | |
| NouvelNetwork 100commics leaves 5 10 2 | | GD-AdaptiveMomentum | | | | | 5e0 | 5.26e-2 | 4.95e0 | 5.12e0 |
| NeuralNetwork_100samples_layers_5_10_3 | | GD-WeightDecay GD | | | | | 4e-1 3e-1 | 3.79e-3 2.81e-3 | 1.80e-1 1.98e-1 | 1.96e-1 2.08e-1 |
| | | Adam-AMSG: | rad | | | | 4e-1 | 9.68e-3 | 1.40e-1 | 1.74e-1 |
| | | Adam-Robust | | | | | 7e-1 | 9.86e-3 | 1.48e-1 | 1.86e-1 |
| | | | _ | | | | Cont | tinued on | next page | |

| | Problem | Optimizer | Mean Fina Value | $\begin{array}{c c} \mathbf{bie} & 1 - \mathbf{contin} \\ \mathbf{l} & \mathbf{Std} & \mathbf{Dev} \end{array}$ | Best Value | Worst Value | Mear | n Func vals | Success Rate (%) | | an Time (s) | | |
|----------|--------------|-----------------|--------------------|---|------------------------|----------------|------|----------------|---------------------|--------------------|--------------------|--------------------|--------|
| | | | | Adam L-BFGS-More | Thuonto | | | 1.53 | | 5.87e-3 7.64e-3 | 1.40e-1 1.40e-1 | 1.64e-1 1.64e-1 | 2 |
| | | | | QQN-StrongV | | | | 1.4 | | .72e-3 | 1.38e-1 | 1.53e-1 | 1 |
| | | | | Adam-Weight | Decay | | | 1.4 | | 2.52e-3 | 1.40e-1 | 1.51e-1 | 1 |
| | | | | L-BFGS-Aggr | | | | 1.50 | | 5.38e-3 | 1.40e-1 | 1.62e-1 | 2 |
| | | | | L-BFGS-Limi L-BFGS-Cons | | | | 1.4 | | .08e-2 0.87e-3 | 1.40e-1 1.40e-1 | 1.76e-1 1.70e-1 | 2 1 |
| | | | | QQN-Golden | | | | 1.4 | | 3.06e-3 | 1.38e-1 | 1.51e-1 | 2 |
| | | | | L-BFGS | | | | 2.39 | | 1.10e-2 | 1.41e-1 | 4.48e-1 | 1 |
| | | | | QQN-CubicQ QQN-Bisection | | terpolatio: | n | 1.40 | | 67e-3 76e-4 | 1.37e-1 1.38e-1 | 1.58e-1 1.43e-1 | |
| | | | | QQN-Bisection | | | | 1.40 | | .16e-4 | 1.38e-1 1.39e-1 | 1.45e-1 1.45e-1 | |
| | | | | Adam-Fast | | | | 1.43 | I | .31e-3 | 1.39e-1 | 1.57e-1 | |
| | | | | Trust Region- | | ive | | 2.20 | | .45e-3 | 2.18e-1 | 2.23e-1 | |
| | | | | Trust Region- GD-Nesterov | Adaptive | | | 8.19 2.2' | I | 2.38e0 5.51e-3 | 1.95e-1 2.20e-1 | 1.12e1 2.35e-1 | |
| | | | | GD-Nesterov GD-Momentu | m | | | 2.3 | | 5.51e-3 5.59e-3 | 2.25e-1 2.25e-1 | 2.35e-1 2.46e-1 | |
| | | | | Trust Region- | | | | 2.23 | | .48e-2 | 1.97e-1 | 3.24e-1 | |
| | | | | Trust Region- | | | | 4.0 | | .17e-1 | 2.08e-1 | 1.77e0 | |
| | | | | GD-Adaptive Trust Region- | | | | 2.50 | | .34e-2 3.20e-1 | 2.32e-1 2.20e-1 | 2.89e-1 1.24e0 | |
| NeuralNe | etwork_100sa | mples_layers_10 | _20_5 | GD | Aggressive | - | | | | 20e-1 01e-3 | 2.20e-1 1.18e-1 | 1.24e0 1.34e-1 | 1 |
| | | | | GD-WeightDe | | | | 9.58 | Be-2 2 | .99e-3 | 9.05e-2 | 1.02e-1 | 1 |
| | | | | Adam-AMSG | | | | | | 5.01e-3 | 3.82e-2 | 5.96e-2 | 2 |
| | | | | Adam-Robust Adam | | | | | I | .49e-3 .45e-3 | 4.85e-2 3.82e-2 | 7.16e-2 5.22e-2 | 2 |
| | | | | L-BFGS-More | Thuente | | | | I | .08e-3 | 5.29e-2 | 6.95e-2 | 2 |
| | | | | L-BFGS-Limi | | | | | | 5.80e-3 | 4.16e-2 | 8.19e-2 | 3 |
| | | | | L-BFGS-Aggr | | | | | I | .60e-3 | 4.81e-2 | 6.46e-2 | 3 |
| | | | | L-BFGS-Cons QQN-Goldens | | | | | | .59e-3 .37e-4 | 3.82e-2 3.79e-2 | 5.53e-2 4.07e-2 | 2 |
| | | | | Adam-Weight | | | | | | .26e-5 | 3.79e-2 3.82e-2 | 3.82e-2 | 1 |
| | | | | L-BFGS | - | | | 1.30 | Ge-1 1 | .66e-1 | 4.42e-2 | 8.11e-1 | 1 |
| | | | | QQN-StrongV | | | | | I | .88e-3 | 3.75e-2 | 5.26e-2 | 1 |
| | | | | QQN-CubicQ QQN-Bisection | uadraticIn | terpolatio: | n | | I | .40e-4 | 3.77e-2 3.78e-2 | 3.82e-2 3.82e-2 | |
| | | | | GD-Nesterov | 11-1 | | | 1.23 | I | .62e-2 | 4.04e-2 | 1.69e-1 | |
| | | | | QQN-Bisectio | | | | 3.80 |)e-2 1 | .53e-4 | 3.77e-2 | 3.82e-2 | |
| | | | | GD-Momentu | m | | | 1.65 | | 2.68e-2 | 4.69e-2 | 1.76e-1 | |
| | | | | Adam-Fast Trust Region- | Concervat | ivo | | 1.48 | | .53e-3 .17e-3 | 3.71e-2 1.28e-1 | 5.58e-2 1.53e-1 | |
| | | | | GD-Adaptive | | | | 1.74 | | .09e-3 | 1.66e-1 | 1.83e-1 | |
| | | | | Trust Region- | Adaptive | | | | | .58e-2 | 1.36e-1 | 3.17e-1 | |
| | | | | Trust Region- | | | | 1.6 | | 2.68e-3 | 1.54e-1 | 1.66e-1 | |
| | | | | Trust Region- Trust Region- | Aggressive Standard | | | 7.5 | | .49e-1 .79e-1 | 3.26e-1 2.05e-1 | 1.01e0 1.05e0 | |
| SVM_100 | samples_5fea | tures_C1 | | Adam-AMS | | | | | | .44e-5 | 6.43e-1 | 6.43e-1 | 1 |
| | - | | | Adam | | | | 6.43 | | .18e-7 | 6.43e-1 | 6.43e-1 | 1 |
| | | | | Adam-Robust Adam-Weight | | | | 6.43 | | .65e-4 .36e-6 | 6.43e-1 6.43e-1 | 6.44e-1 6.43e-1 | 1 |
| | | | | GD Adam-weight | Decay | | | 6.43 | | .30e-6 | 6.43e-1 6.43e-1 | 6.43e-1 6.43e-1 | |
| | | | | L-BFGS-Limi | | | | 6.43 | Be-1 1 | .94e-5 | 6.43e-1 | 6.43e-1 | |
| | | | | GD-Weight De | | | | 6.43 | | 2.96e-6 | 6.43e-1 | 6.43e-1 | |
| | | | | QQN-Goldens L-BFGS-Cons | | | | 6.43 | | 87e-5 59e-5 | 6.43e-1 6.43e-1 | 6.43e-1 6.43e-1 | |
| | | | | QQN-Bisection | | | | 6.43 | | .39e-5 | 6.43e-1 | 6.43e-1 | |
| | | | | Trust Region- | Conservat | ive | | 2.8 | 2e0 : | 1.25e0 | 6.45e-1 | 3.96e0 | |
| | | | | QQN-Bisectio | | 4.0mm -1 -1 | | 6.43 | | .80e-5 | 6.43e-1 | 6.43e-1 | |
| | | | | QQN-CubicQ QQN-StrongV | | terpolatio: | n | 6.43 | | 7.64e-7 7.43e-5 | 6.43e-1 6.43e-1 | 6.43e-1 6.43e-1 | |
| | | | | GD-Nesterov | , 5110 | | | 6.64 | | .99e-3 | 6.50e-1 | 6.76e-1 | |
| | | | GD-Momentum | | | | | Be-1 9 | 0.01e-3 | 6.53e-1 6.43e-1 | 6.91e-1 | | |
| | | | | L-BFGS-MoreThuente | | | | | 6.43e-1 1.35e-5 | | | 6.43e-1 | |
| | | | | L-BFGS Adam-Fast | | | | | | .38e-12 2.44e-2 | 6.43e-1 6.44e-1 | 6.43e-1 7.29e-1 | |
| | | | | Adam-Fast L-BFGS-Aggressive | | | | | | 41e-10 | 6.43e-1 | 6.43e-1 | |
| | | | | Trust Region- | Precise | | | 9.38 | Be-1 2 | 2.76e-1 | 6.73e-1 | 1.58e0 | |
| | | | | GD-Adaptive | Momentur | n | | 7.53 | | 2.76e-2 | 7.12e-1 | 8.06e-1 | |
| | | | | Contin | ued on | next page | | | | | | | |

| Problem Optimizer Mean l | Final Std Dev | Best | Worst | Mean | Func | Success | Mea | an Time | | |
|------------------------------|-----------------|-------------------------|-------------|------|--------|---------|-------------------------------|---------|---------|----|
| Valu | ıe | Value | Value | Ev | als | Rate (% | (%) (s) | | | |
| | Trust Region | n-Adaptive | | | 4.6 | 8e0 | 6.93e0 | 7.07e-1 | 3.43e1 | |
| | Trust Region | n-Aggressiv | e | | 1.00e3 | | 4.36e3 | 6.57e-1 | 2.00e4 | |
| | Trust Region | | | | 3.4 | 7e2 | 9.80e2 | 7.26e-1 | 3.28e3 | |
| SVM_200samples_10features_C1 | Adam-AM | \mathbf{SGrad} | | | 6.86 | 6e-1 | 1.05e-5 | 6.86e-1 | 6.86e-1 | 1 |
| | Adam | | | | 6.86 | 6e-1 | 9.76e-6 | 6.86e-1 | 6.86e-1 | 1 |
| | QQN-Bisect | ion-1 | | | 6.86 | 6e-1 | 5.90e-6 | 6.86e-1 | 6.86e-1 | 1. |
| | QQN-Strong | | | | 6.86 | 6e-1 | 3.82e-6 | 6.86e-1 | 6.86e-1 | 1 |
| | | QQN-Bisection-2 | | | | | 1.16e-5 | 6.86e-1 | 6.86e-1 | 6 |
| | GD | | | | 6.86 | 6e-1 | 1.43e-6 | 6.86e-1 | 6.86e-1 | 4 |
| | | Adam-WeightDecay | | | | | 7.69e-6 | 6.86e-1 | 6.86e-1 | 6 |
| | Adam-Robu | Adam-Robust | | | | | 1.01e-5 | 6.86e-1 | 6.86e-1 | 5 |
| | QQN-Golder | QQN-GoldenSection | | | | | 6.68e-6 | 6.86e-1 | 6.86e-1 | 6 |
| | L-BFGS-Mo | reThuente | | | 5.1 | 6e3 | 2.25e4 | 6.86e-1 | 1.03e5 | 5 |
| | L-BFGS-Lin | nited | | | 6.86 | 6e-1 | 1.04e-4 | 6.86e-1 | 6.87e-1 | 7 |
| | L-BFGS-Cor | nservative | | | 6.86 | 6e-1 | 5.85e-6 5.68e-6 6.77e-7 | 6.86e-1 | 6.86e-1 | 6 |
| | QQN-Cubic | QuadraticIı | nterpolatio | n | 6.86 | 6e-1 | | 6.86e-1 | 6.86e-1 | 2 |
| | GD-WeightI | Decay | | | 6.86 | 6e-1 | | 6.86e-1 | 6.86e-1 | 1 |
| | L-BFGS | | | | 7.68 | Be-1 | 3.55e-1 | 6.86e-1 | 2.32e0 | 3 |
| | Trust Region | n-Conserva | tive | | 2.4 | 9e0 | 1.56e0 | 6.88e-1 | 4.05e0 | 1 |
| | L-BFGS-Ag | gressive | | | 6.86 | | 6.90e-6 | 6.86e-1 | 6.86e-1 | 1 |
| | Adam-Fast | | | | 7.22 | 2e-1 | 2.34e-2 | 6.87e-1 | 7.51e-1 | |
| | GD-Nestero | V | | | 7.03 | Be-1 | 4.03e-3 | 6.94e-1 | 7.12e-1 | |
| | GD-Moment | | | | 7.08 | | 5.01e-3 | 6.99e-1 | 7.17e-1 | |
| | Trust Region | | | | 9.09 | 9e-1 | 1.95e-1 | 7.00e-1 | 1.21e0 | |
| | GD-Adaptiv | GD-AdaptiveMomentum | | | | Be-1 | 1.76e-2 | 7.62e-1 | 8.35e-1 | |
| | | Trust Region-Adaptive | | | | | | 6.97e-1 | 3.46e1 | |
| | | Trust Region-Standard | | | | 6e-1 | 1.40e-1 | 6.90e-1 | 1.09e0 | |
| | Trust Region | Trust Region-Aggressive | | | | | 4.05e-1 | 7.56e-1 | 2.23e0 | |