# Performance Table Report

#### QQN Optimizer Benchmark

August 2, 2025

#### $1 \quad Problem: \ Sphere\_2D$

Optimize	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-		60.0	2.05e-1	2.68e-5	13.8	0.000
Adaptivel	Momentum					
QQN-Bise	ection-1	100.0	0.00e0	0.00e0	15.0	0.000
Adam-Ro		0.0	5.86e-2	2.27e-2	2502.0	0.055
QQN-		100.0	3.22e-14	2.46e-14	46.0	0.000
GoldenSee	ction					
Trust	Region-	10.0	1.25e3	7.55e-4	27.1	0.000
Standard	Ü					
Trust	Region-	30.0	7.95e3	2.26e-5	18.6	0.000
Aggressive						
Trust	Region-	35.0	6.68e2	1.21e-5	41.1	0.000
Adaptive	Ü					
GD-Neste	erov	100.0	2.51e-3	2.83e-4	16.4	0.000
L-BFGS-		100.0	4.28e-3	2.80e-3	156.2	0.004
Conservat	ive					
L-BFGS-		100.0	0.00e0	0.00e0	10.0	0.000
MoreThue	ente					
L-BFGS-I	Limited	100.0	1.24e-3	0.00e0	23.1	0.000
GD		100.0	4.93e-3	4.81e-3	152.6	0.004
Adam-		100.0	4.98e-3	4.94e-3	1113.1	0.023
WeightDe	ecay					
Trust	Region-	55.0	6.58e-1	5.18e-6	115.3	0.001
Precise						
Adam-Fas	$\operatorname{st}$	30.0	8.78e-2	2.82e-4	33.5	0.001
GD-Mome	$_{ m entum}$	100.0	4.85e-4	1.93e-4	16.9	0.000
GD-Weigh	htDecay	100.0	4.67e-3	4.31e-3	51.6	0.002
QQN-		100.0	0.00e0	0.00e0	12.0	0.000
CubicQua	draticInter	polation				
L-BFGS-		100.0	0.00e0	0.00e0	10.0	0.000
Aggressive	e					
Trust	Region-	100.0	1.69e-3	2.11e-4	602.0	0.004
Conservat	tive					
Adam-AN	<b>ISGrad</b>	0.0	7.20e-2	2.82e-2	2502.0	0.054
Adam		40.0	7.60e-3	4.96e-3	2432.4	0.048
QQN-Bise	ection-2	100.0	0.00e0	0.00e0	13.0	0.000
QQN-Stro	ongWolfe	100.0	0.00e0	0.00e0	11.0	0.000
L-BFGS		100.0	8.79e-5	0.00e0	13.5	0.000

#### 2 Problem: Sphere\_10D

Optimizer Success Rate (%) Mean Final Value	Best Value Mean Func Evals Mean Tim	ne (s)
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Trust         Region-         30.0         7.30e2         2.03e-4         139.3         0.001           Adamtyre         Adam-Fast         0.0         6.12e-1         5.73e-1         37.4         0.001           L-BFGS-Limited         100.0         1.08e-3         5.57e-7         40.3         0.001           Adam-AMSGrad         0.0         2.64e-1         1.95e-1         2502.0         0.060           L-BFGS-         100.0         4.51e-3         4.20e-3         197.5         0.005           Conservative         0.00         4.51e-3         4.20e-3         197.5         0.005           Conservative         0.00         1.70e-1         2.45e-5         19.4         0.001           Trust         Region-         65.0         5.27e-1         1.01e-4         504.0         0.004           Precise         0.00         0.00e0         0.00e0         11.0         0.000           QN-StrongWolfe         100.0         0.00e0         0.00e0         11.0         0.000           QN-StrongWolfe         100.0         0.00e0         0.00e0         15.0         0.000           QN-Bisection-2         100.0         0.00e0         0.00e0         15.0         0.00     <	GD-WeightDecay	100.0	4.61e-3	4.33e-3	62.5	0.002
Adam-Fast         0.0         6.12e-1         5.73e-1         37.4         0.001           L-BFGS-Limited         100.0         1.08e-3         5.57e-7         40.3         0.001           Adam-AMSGrad         0.0         2.64e-1         1.95e-1         2502.0         0.060           L-BFGS-         100.0         4.51e-3         4.20e-3         197.5         0.005           Conservative         GD-Momentum         75.0         1.70e-1         2.45e-5         19.4         0.001           GD-Momentum         75.0         1.70e-1         2.45e-5         19.4         0.001           Trust         Region-         65.0         5.27e-1         1.01e-4         504.0         0.004           Precise         V         0.00e0         0.00e0         11.0         0.004         0.004           QN-StrongWolfe         100.0         0.00e0         0.00e0         11.0         0.000<		30.0	7.30e2	2.03e-4	139.3	0.001
Adam-Fast         0.0         6.12e-1         5.73e-1         37.4         0.001           L-BFGS-Limited         100.0         1.08e-3         5.57e-7         40.3         0.001           Adam-AMSGrad         0.0         2.64e-1         1.95e-1         2502.0         0.060           L-BFGS-         100.0         4.51e-3         4.20e-3         197.5         0.005           Conservative         GD-Momentum         75.0         1.70e-1         2.45e-5         19.4         0.001           GD-Momentum         75.0         1.70e-1         2.45e-5         19.4         0.001           Trust         Region-         65.0         5.27e-1         1.01e-4         504.0         0.004           Precise         V         0.00e0         0.00e0         11.0         0.004         0.004           QN-StrongWolfe         100.0         0.00e0         0.00e0         11.0         0.000<	Adaptive					
Adam-AMSGrad         0.0         2.64e-1         1.95e-1         2502.0         0.060           L-BFGS-         100.0         4.51e-3         4.20e-3         197.5         0.005           Conservative           GD-Momentum         75.0         1.70e-1         2.45e-5         19.4         0.001           Trust Region-         65.0         5.27e-1         1.01e-4         504.0         0.004           Precise           QQN-StrongWolfe         100.0         0.00e0         0.00e0         11.0         0.000           GD-Nesterov         65.0         1.59e-1         1.53e-5         19.9         0.001           L-BFGS         100.0         0.00e0         0.00e0         15.0         0.000           Trust Region-         15.0         6.75e2         2.86e-5         51.8         0.000           Standard         L-BFGS-         100.0         0.00e0         0.00e0         10.0         0.00           MoreThuente         GD         100.0         4.91e-3         4.82e-3         192.4         0.005           QQN-         100.0         0.00e0         0.00e0         12.0         0.000           CubicQuadraticInterp		0.0	6.12e-1	5.73e-1	37.4	0.001
L-BFGS-   100.0   4.51e-3   4.20e-3   197.5   0.005	L-BFGS-Limited	100.0	1.08e-3	5.57e-7	40.3	0.001
Conservative	Adam-AMSGrad	0.0	2.64e-1	1.95e-1	2502.0	0.060
Conservative	L-BFGS-	100.0	4.51e-3	4.20e-3	197.5	0.005
Trust         Region- Precise         65.0         5.27e-1         1.01e-4         504.0         0.004           Precise         Frecise         7         65.0         0.00e0         0.00e0         11.0         0.000           GD-Nesterov         65.0         1.59e-1         1.53e-5         19.9         0.001           L-BFGS         100.0         0.00e0         0.00e0         15.0         0.000           QQN-Bisection-2         100.0         0.00e0         0.00e0         13.0         0.000           Trust         Region-         15.0         6.75e2         2.86e-5         51.8         0.000           Standard         Trust         Region-         0.00e0         0.00e0         10.0         0.000           MoreThuente         Trust         0.00e0         0.00e0         10.0         0.000           MoreThuente         Trust         Region-         100.0         0.00e0         0.00e0         10.0         0.000           QQN-         100.0         0.00e0         0.00e0         12.0         0.000         0.000           CubicQuadraticInterpolation         Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.052	Conservative					
Precise	GD-Momentum	75.0	1.70e-1	2.45e-5	19.4	0.001
QQN-StrongWolfe         100.0         0.00e0         0.00e0         11.0         0.000           GD-Nesterov         65.0         1.59e-1         1.53e-5         19.9         0.001           L-BFGS         100.0         0.00e0         0.00e0         15.0         0.000           QQN-Bisection-2         100.0         0.00e0         0.00e0         13.0         0.000           Trust         Region-         15.0         6.75e2         2.86e-5         51.8         0.000           Standard         L-BFGS-         100.0         0.00e0         0.00e0         10.0         0.000           MoreThuente         GD         100.0         4.91e-3         4.82e-3         192.4         0.005           QQN-         100.0         0.00e0         0.00e0         12.0         0.000           CubicQuadraticInterpolation         Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.000           Aggressive         Adam         95.0         4.98e-3         4.94e-3         2391.8         0.052           Adam-Robust         0.0         1.08e-1         4.84e-2         2502.0         0.061           QQN-Bisection-1         100.0         0.00e0	Trust Region-	65.0	5.27e-1	1.01e-4	504.0	0.004
GD-Nesterov   65.0   1.59e-1   1.53e-5   19.9   0.001     L-BFGS   100.0   0.00e0   0.00e0   15.0   0.000     QQN-Bisection-2   100.0   0.00e0   0.00e0   13.0   0.000     Trust   Region   15.0   6.75e2   2.86e-5   51.8   0.000     Standard	Precise					
L-BFGS         100.0         0.00e0         0.00e0         15.0         0.000           QQN-Bisection-2         100.0         0.00e0         0.00e0         13.0         0.000           Trust         Region-         15.0         6.75e2         2.86e-5         51.8         0.000           Standard           L-BFGS-         100.0         0.00e0         0.00e0         10.0         0.000           MoreThuente           GD         100.0         4.91e-3         4.82e-3         192.4         0.005           QQN-         100.0         0.00e0         0.00e0         12.0         0.000           CubicQuadraticInterpolation           Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.000           CubicQuadraticInterpolation           Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.000           Agressive           Adam-Robust         0.0         4.98e-3         4.94e-3         2391.8         0.052           Adam-Robust         0.0         0.00e0         0.00e0         15.0         0.00e      <	QQN-StrongWolfe	100.0	0.00e0	0.00e0	11.0	0.000
QQN-Bisection-2         100.0         0.00e0         0.00e0         13.0         0.000           Trust         Region-         15.0         6.75e2         2.86e-5         51.8         0.000           Standard           L-BFGS-         100.0         0.00e0         0.00e0         10.0         0.000           MoreThuente           GD         100.0         4.91e-3         4.82e-3         192.4         0.005           QQN-         100.0         0.00e0         0.00e0         12.0         0.000           Cubic Quadratic Interpolation           Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.000           Cubic Quadratic Interpolation           Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.000           Adam Region-         0.0         1.09e4         5.34e3         30.3         0.052           Adam-Robust         0.0         1.08e-1         4.84e-2         2502.0         0.061           QQN- Bisection-1         100.0         0.00e0         0.00e0         15.0         0.00e           L-BFGS- 100.0 <td>GD-Nesterov</td> <td>65.0</td> <td>1.59e-1</td> <td>1.53e-5</td> <td>19.9</td> <td>0.001</td>	GD-Nesterov	65.0	1.59e-1	1.53e-5	19.9	0.001
Trust         Region-         15.0         6.75e2         2.86e-5         51.8         0.000           Standard           L-BFGS-         100.0         0.00e0         0.00e0         10.0         0.000           More Thuente           GD         100.0         4.91e-3         4.82e-3         192.4         0.005           QQN-         100.0         0.00e0         0.00e0         12.0         0.000           Cubic Quadratic Interpolation           Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.000           Aggressive           Adam Robust         0.0         1.08e-1         4.84e-3         2391.8         0.052           Adam-Robust         0.0         1.08e-1         4.84e-2         2502.0         0.061           QQN-Bisection-1         100.0         0.00e0         0.00e0         15.0         0.000           GoldenSection         L-BFGS-         100.0         0.00e0         0.00e0         10.0         0.00e           Adam-         100.0         4.92e-3         4.88e-3         968.0         0.022	L-BFGS	100.0	0.00e0	0.00e0	15.0	0.000
Standard         L-BFGS-       100.0       0.00e0       10.0       0.00e0         MoreThuente       C       0.00e0       10.0       0.005         GD       100.0       0.00e0       0.00e0       12.0       0.000         CubicQuadraticInterpolation         Trust Region-       0.0       1.09e4       5.34e3       30.3       0.000         Aggressive         Adam Pabust       0.0       4.98e-3       4.94e-3       2391.8       0.052         Adam-Robust       0.0       1.08e-1       4.84e-2       2502.0       0.061         QQN-Bisection-1       100.0       0.00e0       0.00e0       15.0       0.000         QQN-       100.0       1.60e-13       1.45e-13       46.0       0.000         GoldenSection         L-BFGS-       100.0       0.00e0       0.00e0       10.0       0.000         Aggressive         Adam-       100.0       4.92e-3       4.88e-3       968.0       0.022         WeightDecay         Trust       Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020 <t< td=""><td>QQN-Bisection-2</td><td>100.0</td><td>0.00e0</td><td>0.00e0</td><td>13.0</td><td>0.000</td></t<>	QQN-Bisection-2	100.0	0.00e0	0.00e0	13.0	0.000
L-BFGS-	Trust Region-	15.0	6.75e2	2.86e-5	51.8	0.000
More Thuente           GD         100.0         4.91e-3         4.82e-3         192.4         0.005           QQN-         100.0         0.00e0         0.00e0         12.0         0.000           Cubic Quadratic Interpolation           Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.000           Aggressive	Standard					
GD         100.0         4.91e-3         4.82e-3         192.4         0.005           QQN-         100.0         0.00e0         0.00e0         12.0         0.000           CubicQuadraticInterpolation           Trust         Region-         0.0         1.09e4         5.34e3         30.3         0.000           Aggressive         Adam         95.0         4.98e-3         4.94e-3         2391.8         0.052           Adam-Robust         0.0         1.08e-1         4.84e-2         2502.0         0.061           QQN-Bisection-1         100.0         0.00e0         0.00e0         15.0         0.000           QQN-         100.0         1.60e-13         1.45e-13         46.0         0.000           GoldenSection         UBFGS-         100.0         0.00e0         10.0         0.000           Aggressive         Adam-         100.0         4.92e-3         4.88e-3         968.0         0.022           WeightDecay         Trust         Region-         35.0         3.41e-1         1.07e-3         2968.8         0.020           Conservative         GD-         60.0         5.51e-1         2.42e-4         18.4         0.001	L-BFGS-	100.0	0.00e0	0.00e0	10.0	0.000
QQN-       100.0       0.00e0       0.00e0       12.0       0.000         CubicQuadraticInterpolation         Trust       Region-       0.0       1.09e4       5.34e3       30.3       0.000         Aggressive       ***         Adam       95.0       4.98e-3       4.94e-3       2391.8       0.052         Adam-Robust       0.0       1.08e-1       4.84e-2       2502.0       0.061         QQN-Bisection-1       100.0       0.00e0       0.00e0       15.0       0.000         QQN-       100.0       1.60e-13       1.45e-13       46.0       0.000         GoldenSection       ***         L-BFGS-       100.0       0.00e0       0.00e0       10.0       0.000         Aggressive       **         Adam-       100.0       4.92e-3       4.88e-3       968.0       0.022         WeightDecay       **         Trust       Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020         Conservative       **         GD-       60.0       5.51e-1       2.42e-4       18.4       0.001	MoreThuente					
CubicQuadraticInterpolation         Trust       Region-       0.0       1.09e4       5.34e3       30.3       0.000         Aggressive       Adam       95.0       4.98e-3       4.94e-3       2391.8       0.052         Adam-Robust       0.0       1.08e-1       4.84e-2       2502.0       0.061         QQN-Bisection-1       100.0       0.00e0       0.00e0       15.0       0.000         QQN-       100.0       1.60e-13       1.45e-13       46.0       0.000         GoldenSection       U-BFGS-       100.0       0.00e0       10.0       0.000         Aggressive       Adam-       100.0       4.92e-3       4.88e-3       968.0       0.022         WeightDecay       Trust Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020         Conservative       GD-       60.0       5.51e-1       2.42e-4       18.4       0.001		100.0	4.91e-3	4.82e-3	192.4	0.005
Trust     Region-     0.0     1.09e4     5.34e3     30.3     0.000       Aggressive       Adam     95.0     4.98e-3     4.94e-3     2391.8     0.052       Adam-Robust     0.0     1.08e-1     4.84e-2     2502.0     0.061       QQN-Bisection-1     100.0     0.00e0     0.00e0     15.0     0.000       QQN-     100.0     1.60e-13     1.45e-13     46.0     0.000       Golden Section     1.08e-1     0.00e0     10.0     10.0     0.000       Aggressive       Adam-     100.0     4.92e-3     4.88e-3     968.0     0.022       Weight Decay       Trust     Region-     35.0     3.41e-1     1.07e-3     2968.8     0.020       Conservative       GD-     60.0     5.51e-1     2.42e-4     18.4     0.001	QQN-	100.0	0.00e0	0.00e0	12.0	0.000
Aggressive       Adam       95.0       4.98e-3       4.94e-3       2391.8       0.052         Adam-Robust       0.0       1.08e-1       4.84e-2       2502.0       0.061         QQN-Bisection-1       100.0       0.00e0       0.00e0       15.0       0.000         QQN-       100.0       1.60e-13       1.45e-13       46.0       0.000         GoldenSection       IL-BFGS-       100.0       0.00e0       10.0e0       10.0       0.000         Aggressive         Adam-       100.0       4.92e-3       4.88e-3       968.0       0.022         WeightDecay         Trust Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020         Conservative       GD-       60.0       5.51e-1       2.42e-4       18.4       0.001	CubicQuadraticInterpola	ation				
Adam       95.0       4.98e-3       4.94e-3       2391.8       0.052         Adam-Robust       0.0       1.08e-1       4.84e-2       2502.0       0.061         QQN-Bisection-1       100.0       0.00e0       0.00e0       15.0       0.000         QQN-       100.0       1.60e-13       1.45e-13       46.0       0.000         GoldenSection       I-BFGS-       100.0       0.00e0       0.00e0       10.0       0.000         Aggressive         Adam-       100.0       4.92e-3       4.88e-3       968.0       0.022         WeightDecay         Trust Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020         Conservative       GD-       60.0       5.51e-1       2.42e-4       18.4       0.001		0.0	1.09e4	5.34e3	30.3	0.000
Adam-Robust       0.0       1.08e-1       4.84e-2       2502.0       0.061         QQN-Bisection-1       100.0       0.00e0       0.00e0       15.0       0.000         QQN-       100.0       1.60e-13       1.45e-13       46.0       0.000         GoldenSection       L-BFGS-       100.0       0.00e0       0.00e0       10.0       0.000         Aggressive       Adam-       Union of the property of the pro	Aggressive					
QQN-Bisection-1       100.0       0.00e0       0.00e0       15.0       0.000         QQN-       100.0       1.60e-13       1.45e-13       46.0       0.000         GoldenSection       L-BFGS-       100.0       0.00e0       0.00e0       10.0       0.000         Aggressive       Adam-       100.0       4.92e-3       4.88e-3       968.0       0.022         WeightDecay       Trust Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020         Conservative       GD-       60.0       5.51e-1       2.42e-4       18.4       0.001	Adam	95.0	4.98e-3	4.94e-3	2391.8	0.052
QQN-       100.0       1.60e-13       1.45e-13       46.0       0.000         GoldenSection       L-BFGS-       100.0       0.00e0       0.00e0       10.0       0.000         Aggressive       Adam-       100.0       4.92e-3       4.88e-3       968.0       0.022         WeightDecay       Trust       Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020         Conservative       GD-       60.0       5.51e-1       2.42e-4       18.4       0.001	Adam-Robust	0.0				
GoldenSection L-BFGS- 100.0 0.00e0 0.00e0 10.0 0.0000 Aggressive Adam- 100.0 4.92e-3 4.88e-3 968.0 0.022 WeightDecay Trust Region- 35.0 3.41e-1 1.07e-3 2968.8 0.020 Conservative GD- 60.0 5.51e-1 2.42e-4 18.4 0.001		100.0	0.00e0	0.00e0	15.0	0.000
L-BFGS- 100.0 0.00e0 0.00e0 10.0 0.000 Aggressive Adam- 100.0 4.92e-3 4.88e-3 968.0 0.022 WeightDecay Trust Region- 35.0 3.41e-1 1.07e-3 2968.8 0.020 Conservative GD- 60.0 5.51e-1 2.42e-4 18.4 0.001		100.0	1.60e-13	1.45e-13	46.0	0.000
Aggressive         Adam-       100.0       4.92e-3       4.88e-3       968.0       0.022         WeightDecay       Trust Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020         Conservative         GD-       60.0       5.51e-1       2.42e-4       18.4       0.001						
Adam-     100.0     4.92e-3     4.88e-3     968.0     0.022       WeightDecay       Trust Region-     35.0     3.41e-1     1.07e-3     2968.8     0.020       Conservative       GD-     60.0     5.51e-1     2.42e-4     18.4     0.001	L-BFGS-	100.0	0.00e0	0.00e0	10.0	0.000
WeightDecay         Trust       Region-       35.0       3.41e-1       1.07e-3       2968.8       0.020         Conservative       GD-       60.0       5.51e-1       2.42e-4       18.4       0.001						
Trust     Region-     35.0     3.41e-1     1.07e-3     2968.8     0.020       Conservative       GD-     60.0     5.51e-1     2.42e-4     18.4     0.001		100.0	4.92e-3	4.88e-3	968.0	0.022
Conservative GD- 60.0 5.51e-1 2.42e-4 18.4 0.001						
GD- 5.51e-1 2.42e-4 18.4 0.001	Trust Region-	35.0	3.41e-1	1.07e-3	2968.8	0.020
		60.0	5.51e-1	2.42e-4	18.4	0.001
AdaptiveMomentum	${\bf Adaptive Momentum}$					

#### 3 Problem: Rosenbrock\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-Momentum	0.0	5.61e0	4.79e-1	23.8	0.001
L-BFGS-Limited	0.0	3.93e0	3.26e-2	2251.6	0.025
GD-WeightDecay	0.0	3.65e0	3.75e-2	58.7	0.002
Trust Region-	0.0	4.18e0	3.95e0	89.6	0.001
Standard					
L-BFGS	0.0	1.36e2	8.12e-1	121.5	0.002
Trust Region-	0.0	7.26e0	3.76e0	946.2	0.006
Precise					
L-BFGS-	95.0	1.91e-2	1.40e-4	651.0	0.011
MoreThuente					
QQN-	10.0	1.26e-1	8.08e-5	4248.6	0.083
GoldenSection					
Adam-AMSGrad	0.0	3.83e0	4.66e-1	678.1	0.015
Trust Region-	0.0	4.12e0	3.83e0	494.4	0.003
Adaptive					
Trust Region-	0.0	2.84e1	1.90e-1	2770.7	0.017
Conservative					

Trust	Region-	0.0	4.66e0	4.01e0	27.6	0.000
Aggressive						
QQN-		40.0	3.73e-2	5.16e-3	1619.8	0.065
CubicQuadr	aticInterpo	olation				
Adam		0.0	1.22e0	4.86e-1	2502.0	0.049
Adam-Fast		60.0	2.13e0	8.12e-3	171.8	0.003
QQN-Strong	gWolfe	35.0	5.67e-2	4.58e-3	2004.3	0.058
L-BFGS-		100.0	6.53e-3	1.09e-3	985.0	0.016
Conservative	e					
L-BFGS-		0.0	3.18e1	4.34e0	3852.0	0.027
Aggressive						
GD-Nestero	V	0.0	1.49e0	5.05e-2	46.1	0.001
Adam-		0.0	4.11e0	1.55e-2	231.9	0.005
WeightDeca	У					
Adam-Robu	st	0.0	4.04e0	1.90e0	419.2	0.009
QQN-Bisect	ion-2	30.0	6.07e-2	4.36e-3	304.7	0.007
GD		0.0	1.23e0	7.46e-1	854.0	0.021
GD-		25.0	6.54 e-1	3.23e-3	47.7	0.002
AdaptiveMo	mentum					
QQN-Bisect	ion-1	5.0	4.65e-1	6.39e-3	2363.3	0.052

#### 4 Problem: Rosenbrock\_5D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust Region-	0.0	5.00e0	4.66e0	776.1	0.005
Aggressive					
GD-	0.0	4.60 e1	3.36e1	20.6	0.001
AdaptiveMomentum					
L-BFGS-	70.0	9.01e-1	2.37e-1	1090.7	0.019
MoreThuente					
QQN-	70.0	4.25e-1	2.38e-1	1199.2	0.049
CubicQuadraticInterp	polation				
GD-Momentum	0.0	3.55e1	1.96e1	20.8	0.001
Adam-	60.0	2.07e0	3.93e-1	1128.9	0.024
WeightDecay					
Trust Region-	0.0	6.23e1	4.66e0	2827.2	0.018
Standard					
Adam-Robust	0.0	1.46e1	6.12e0	2502.0	0.058
Adam	0.0	3.92e0	2.83e0	2471.6	0.050
Adam-AMSGrad	0.0	4.40e0	3.25e0	2442.0	0.056
L-BFGS-	0.0	8.07e2	1.72e1	3851.6	0.029
Aggressive					
Trust Region-	0.0	1.01e3	8.08e2	3002.0	0.019
Precise					
QQN-Bisection-2	55.0	4.48e-1	2.15e-1	1588.3	0.039
QQN-Bisection-1	85.0	6.94e-1	2.50e-1	1147.7	0.029
${\bf QQN\text{-}StrongWolfe}$	100.0	3.45e-1	2.58e-1	792.6	0.024
Trust Region-	0.0	1.02e3	7.14e2	3002.0	0.021
Conservative					
GD-WeightDecay	60.0	7.30e-1	3.59e-1	72.1	0.002
QQN-	55.0	6.13e-1	2.60e-1	3314.1	0.061
GoldenSection					
Adam-Fast	5.0	1.44e1	3.48e-1	44.4	0.001
GD-Nesterov	10.0	4.24e0	3.90e-1	335.4	0.011
L-BFGS-Limited	45.0	4.21e-1	3.92e-1	3855.4	0.045
L-BFGS-	20.0	2.02e1	3.89e-1	3106.7	0.032
Conservative					

GD Trust Adaptive	Region-	0.0 0.0	5.09e0 8.41e2	4.75e0 5.05e2	32.5 3002.0	0.001 0.019
L-BFGS		0.0	1.50e2	1.98e1	135.3	0.002

#### 5 Problem: Rosenbrock\_10D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust Region-	0.0	2.02e3	1.64e3	3002.0	0.022
Adaptive					
Trust Region-	0.0	1.06e3	6.70e2	3002.0	0.020
Standard					
QQN-	100.0	9.45e0	9.03e0	277.2	0.005
GoldenSection					
Adam-Fast	45.0	1.94e1	9.18e0	61.6	0.001
L-BFGS-	95.0	9.20e0	7.83e0	212.8	0.003
MoreThuente					
GD	75.0	9.75e0	9.59e0	41.5	0.001
GD-Nesterov	100.0	9.31e0	8.97e0	28.6	0.001
L-BFGS-	100.0	9.68e0	9.63e0	205.3	0.005
Conservative					
Adam-	100.0	9.67e0	9.61e0	789.4	0.018
WeightDecay					
QQN-Bisection-2	100.0	9.37e0	8.02e0	83.2	0.002
GD-WeightDecay	100.0	9.58e0	9.45e0	35.7	0.001
Trust Region-	30.0	1.00e1	9.59e0	1615.8	0.012
Aggressive					
GD-Momentum	55.0	3.53e1	9.32e0	35.0	0.001
Adam	90.0	9.71e0	9.68e0	2293.1	0.050
L-BFGS	0.0	1.19e2	2.51e1	338.9	0.005
QQN-	100.0	9.60 e0	9.48e0	109.6	0.004
CubicQuadraticInter	polation				
GD-	0.0	1.02e2	8.45e1	23.1	0.001
AdaptiveMomentum					
Trust Region-	0.0	2.16e3	1.76e3	3002.0	0.020
Conservative					
QQN-Bisection-1	100.0	8.47e0	7.17e0	120.3	0.003
QQN-StrongWolfe	100.0	8.69e0	7.76e0	90.3	0.003
L-BFGS-Limited	100.0	9.59e0	9.51e0	62.3	0.001
L-BFGS-	0.0	1.64e2	5.82e1	3850.3	0.050
Aggressive					
Adam-AMSGrad	100.0	9.69 e0	9.66e0	2353.0	0.058
Adam-Robust	0.0	3.49e1	1.61e1	2502.0	0.062
Trust Region-	0.0	2.10e3	1.66e3	3002.0	0.021
Precise					

#### 6 Problem: Michalewicz\_2D\_m10

Optimize	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	-6.09e-7	-9.91e-6	4.2	0.000
Standard						
QQN-		0.0	-7.52e-7	-9.60e-6	1573.8	0.026
GoldenSe	ction					
QQN-Bise	ection-1	0.0	-1.99e-2	-3.99e-1	1278.7	0.036
GD-Mome	entum	0.0	-3.25e-2	-6.49e-1	42.9	0.001

Trust	Region-	0.0	-5.33e-7	-8.89e-6	4.2	0.000
Aggressive						
QQN-Strong	gWolfe	0.0	-4.01e-2	-8.01e-1	1192.1	0.031
L-BFGS		10.0	-9.99e-2	-1.00e0	802.0	0.013
GD-		5.0	-5.00e-2	-9.99e-1	29.5	0.001
AdaptiveMo	omentum					
Adam-Fast		40.0	-4.71e-1	-1.00e0	1058.0	0.021
QQN-Bisect	tion-2	0.0	-4.01e-2	-8.01e-1	1074.5	0.027
Adam-Robu	ıst	0.0	-3.65e-7	-3.26e-6	13.0	0.000
Adam		50.0	-4.98e-1	-9.97e-1	1642.0	0.032
Trust	Region-	0.0	-8.81e-7	-1.46e-5	4.3	0.000
Conservative	e					
L-BFGS-		25.0	-2.50e-1	-1.00e0	752.4	0.010
MoreThuent	te					
L-BFGS-		0.0	-8.97e-2	-8.97e-1	1145.5	0.015
Aggressive						
QQN-		0.0	-5.13e-7	-1.00e-5	19.1	0.001
CubicQuadr	raticInterpola	tion				
L-BFGS-Lir	mited	25.0	-2.50e-1	-1.00e0	466.0	0.006
Trust	Region-	0.0	-4.40e-7	-2.59e-6	4.3	0.000
Adaptive						
GD-Nestero	v	10.0	-9.98e-2	-9.99e-1	272.0	0.008
L-BFGS-		0.0	-7.20e-7	-1.07e-5	838.7	0.014
Conservative	e					
GD-Weightl	Decay	0.0	-3.24e-6	-2.45e-5	423.0	0.013
Adam-AMS	Grad	30.0	-2.99e-1	-9.96e-1	192.4	0.004
Adam-		0.0	-3.49e-7	-2.33e-6	13.0	0.000
WeightDeca	ıy					
Trust	Region-	0.0	-7.80e-7	-4.71e-6	4.5	0.000
Precise	-					
$\operatorname{GD}$		0.0	-1.16e-6	-7.38e-6	252.8	0.006

## 7 Problem: Michalewicz\_5D\_m10

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD	45.0	-2.00e0	-2.69e0	738.0	0.019
GD-Nesterov	5.0	-1.24e0	-2.69e0	159.9	0.005
QQN-Bisection-2	35.0	-1.78e0	-2.85e0	1315.8	0.032
L-BFGS-	0.0	-4.98e-1	-1.25e0	3524.2	0.028
Aggressive					
L-BFGS-	30.0	-1.78e0	-2.69e0	1302.2	0.027
Conservative					
L-BFGS-	5.0	-1.39e0	-2.73e0	519.5	0.010
MoreThuente					
Trust Region-	0.0	-9.74e-1	-1.86e0	263.6	0.002
Conservative					
Trust Region-	0.0	-9.85e-1	-1.74e0	41.0	0.000
Precise					
L-BFGS-Limited	0.0	-8.44e-1	-1.86e0	2003.6	0.025
QQN-Bisection-1	20.0	-1.78e0	-2.74e0	1432.2	0.036
GD-WeightDecay	5.0	-1.24e0	-2.69e0	86.0	0.003
Adam-	10.0	-1.75e0	-2.69e0	178.5	0.004
WeightDecay					
Trust Region-	0.0	-3.29e-1	-1.53e0	5.5	0.000
Standard					
Adam-Fast	65.0	-2.62e0	-3.29e0	70.8	0.002
L-BFGS	0.0	-2.16e-1	-8.96e-1	90.4	0.001

QQN-Stroi	ngWolfe	25.0	-1.76e0	-2.79e0	1288.2	0.041
QQN-		5.0	-1.36e0	-2.69e0	1751.2	0.033
GoldenSec	tion					
QQN-		0.0	-8.49e-1	-1.88e0	1726.7	0.056
CubicQuae	draticInterpola	ation				
GD-		5.0	-1.09e0	-2.69e0	27.8	0.001
AdaptiveM	Iomentum					
Adam-AM	$\operatorname{SGrad}$	55.0	-2.17e0	-2.71e0	439.6	0.011
$\operatorname{Adam}$		80.0	-2.46e0	-2.71e0	474.6	0.010
Adam-Rob	oust	10.0	-1.65e0	-2.69e0	83.1	0.002
GD-Mome:	$_{ m ntum}$	5.0	-1.11e0	-2.69e0	51.9	0.002
Trust	Region-	0.0	-7.58e-1	-1.73e0	10.8	0.000
Adaptive						
Trust	Region-	0.0	-1.81e-1	-7.69e-1	5.0	0.000
Aggressive						

#### 8 Problem: Michalewicz\_10D\_m10

Optimize	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-I	Limited	0.0	-3.52e0	-5.59e0	3009.3	0.038
Adam-Fas	t	35.0	-6.12e0	-6.77e0	106.2	0.002
QQN-		0.0	-3.34e0	-6.26e0	1730.8	0.073
	draticInter	polation				
QQN-		5.0	-4.14e0	-6.30e0	2197.4	0.045
GoldenSec	ction					
QQN-Stro	ongWolfe	5.0	-4.71e0	-6.27e0	1916.5	0.063
$\operatorname{GD}$ -	Ü	0.0	-4.52e0	-6.23e0	27.6	0.001
Adaptive	Momentum					
Adam-Rol	bust	0.0	-5.03e0	-6.26e0	111.0	0.003
L-BFGS		0.0	-3.66e-1	-1.14e0	64.8	0.001
$\operatorname{Adam}$		20.0	-5.84e0	-6.27e0	508.9	0.012
QQN-Bise	ection-1	0.0	-4.24e0	-5.36e0	2030.0	0.055
L-BFGS-		0.0	-4.54e0	-6.26e0	2566.8	0.037
Conservat	ive					
GD-Neste	rov	0.0	-3.74e0	-6.19e0	26.9	0.001
QQN-Bise	ection-2	5.0	-4.37e0	-6.26e0	1808.5	0.047
Adam-AM	ISGrad	30.0	-5.74e0	-6.28e0	450.1	0.012
Adam-		0.0	-5.31e0	-6.25e0	179.3	0.004
WeightDe	cay					
Trust	Region-	0.0	-1.68e0	-3.16e0	5.3	0.000
Aggressive	-					
$\overrightarrow{\mathrm{GD}}$		0.0	-3.59e0	-5.04e0	17.2	0.000
Trust	Region-	0.0	-3.69e0	-5.30e0	114.2	0.001
Precise	Ü					
Trust	Region-	0.0	-3.26e0	-5.27e0	28.7	0.000
Adaptive						
Trust	Region-	0.0	-3.71e0	-5.42e0	644.0	0.005
Conservat	_					
GD-Mome	entum	0.0	-3.79e0	-5.24e0	37.3	0.001
Trust	Region-	0.0	-2.49e0	-4.98e0	9.8	0.000
Standard	Ü					
L-BFGS-		0.0	-4.11e0	-5.98e0	925.2	0.020
MoreThue	ente					
L-BFGS-		0.0	-1.30e0	-3.26e0	3050.9	0.021
Aggressive	9					
GD-Weigh		0.0	-4.56e0	-6.26e0	174.7	0.006
- GD-Weigi	1012 Cay	0.0	-1.0000	-0.2000	117.1	0.000

# 9 Problem: Rastrigin\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Fast	0.0	9.46e0	8.02e0	36.3	0.001
Adam	55.0	1.02e1	7.96e0	780.5	0.016
GD-	0.0	8.98e0	7.97e0	21.5	0.001
AdaptiveMomentum	L				
Adam-AMSGrad	55.0	1.02e1	7.96e0	770.2	0.018
GD-Momentum	0.0	$1.04\mathrm{e}1$	7.98e0	22.7	0.001
GD-Nesterov	5.0	1.00e1	7.96e0	20.1	0.001
Adam-Robust	15.0	9.22e0	7.96e0	111.5	0.003
Trust Region-	0.0	1.29e1	7.97e0	20.1	0.000
Aggressive					
L-BFGS-Limited	70.0	9.13e0	5.86e0	106.5	0.001
L-BFGS-	0.0	2.81e1	1.91e1	3851.8	0.024
Aggressive					
QQN-Bisection-2	30.0	8.57e0	2.07e0	23.8	0.001
L-BFGS	5.0	5.70e1	2.49e0	88.4	0.001
GD	5.0	1.04e1	7.96e0	13.3	0.000
Trust Region-	0.0	1.85e1	9.36e0	3002.0	0.019
Conservative					
QQN-	80.0	7.75e0	2.17e0	64.2	0.002
CubicQuadraticInter					
QQN-	55.0	8.19e0	1.29e-1	156.0	0.002
GoldenSection					
Trust Region-	0.0	9.02e0	7.96e0	63.8	0.000
Standard	0.0	0.0200		00.0	0.000
GD-WeightDecay	5.0	1.01e1	7.96e0	24.3	0.001
QQN-Bisection-1	65.0	8.59e0	1.03e0	125.3	0.003
Trust Region-	15.0	9.68e0	7.96e0	241.0	0.002
Adaptive	10.0	0.0000		211.0	0.002
L-BFGS-	75.0	9.82e0	7.96e0	455.9	0.006
Conservative	10.0	0.0200	1.0000	100.0	0.000
Adam-	50.0	9.71e0	7.96e0	254.7	0.006
WeightDecay	00.0	0.1100	1.5000	201.1	0.000
L-BFGS-	55.0	8.56e0	1.83e0	170.8	0.003
MoreThuente	00.0	0.0000	1.0000	110.0	0.000
Trust Region-	10.0	9.71e0	7.96e0	915.9	0.006
Precise Region-	10.0	0.1100	1.5000	010.0	0.000
QQN-StrongWolfe	65.0	8.32e0	1.36e0	71.4	0.002
& & 1 - Durong Mone	00.0	0.9200	1.0000	11.1	0.002

## $10 \quad Problem: \ Rastrigin\_5D$

Optimiz	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	35.0	2.39e1	2.03e1	2558.8	0.017
Precise						
QQN-Bis	section-1	25.0	2.48e1	1.80e1	186.6	0.004
L-BFGS-		50.0	2.77e1	7.71e0	165.1	0.003
MoreThu	iente					
QQN-Bis	section-2	15.0	3.08e1	1.10e1	36.6	0.001
GD-Nest	erov	45.0	2.37e1	1.99e1	41.2	0.001
GD		30.0	2.44e1	1.99e1	15.7	0.000
GD-	GD-		2.14e1	1.83e1	52.4	0.002
AdaptiveMomentum						
Trust	Region-	60.0	2.34e1	2.02e1	646.1	0.004
Adaptive	:					

cay	55.0	2.27e1	2.02e1	246.9	0.006
ive					
***					
Region-	0.0	$6.91\mathrm{e}1$	5.03e1	3002.0	0.020
	0.0	1.02e2	5.89e1	98.2	0.001
-					
Region-	45.0	2.31e1	1.95e1	168.5	0.001
draticInterpola					
	50.0	2.30e1	4.91e0	106.2	0.003
ntum	5.0	3.03e1	2.04e1	28.9	0.001
oust	55.0	2.24e1	1.99e1	103.5	0.003
SGrad					0.017
			2.03e1		0.016
	35.0	2.84e1	1.70e1	115.2	0.002
ve	20.0	2.0101	1.0001	001.0	0.000
	40.0	2.34e1	1.99e1	881.9	0.009
	0.0	1.0101	0.0001	0002.0	0.020
01011	0.0	7 97e1	5.68e1	3852.0	0.029
tion	JJ.U	2.3161	1.91e0	200.0	0.004
ng wone					0.003
	40.0	2 26.1	1 16 1	100.2	0.003
~	0.06	2.33e1	1.82e1	45.7	0.000
					0.001
			- ·		0.001
	imited SGrad bust ntum draticInterpolar Region-	tDecay 50.0 Region- 50.0  Region- 50.0  10 40.0  10 40.0  10 40.0  10 40.0  10 40.0  10 40.0  10 40.0  10 50.0  11 50.0  12 50.0  13 50.0  14 50.0  15 60.0  16 60.0  17 60.0  18 60.0  18 60.0  19 60.0  10 60.0	tDecay 50.0 2.25e1 Region- 50.0 2.33e1  IngWolfe 40.0 2.36e1 35.0 2.31e1  Ition 0.0 7.97e1  40.0 2.34e1  ve imited 35.0 2.84e1 30.0 2.42e1 SGrad 40.0 2.38e1  pust 55.0 2.24e1 Intum 5.0 3.03e1 form 50.0 2.30e1 IdraticInterpolation Region- 45.0 2.31e1  Region- 0.0 1.02e2 Region- 0.0 6.91e1	tDecay 50.0 2.25e1 1.89e1 Region- 50.0 2.33e1 1.82e1  IngWolfe 40.0 2.36e1 1.16e1 35.0 2.31e1 7.97e0  tion 0.0 7.97e1 5.68e1  ve imited 35.0 2.84e1 1.70e1 30.0 2.42e1 2.03e1 SGrad 40.0 2.38e1 2.04e1 bust 55.0 2.24e1 1.99e1  ntum 5.0 3.03e1 2.04e1 bust 50.0 3.03e1 2.04e1 bust 50.0 2.30e1 4.91e0  draticInterpolation Region- 45.0 2.31e1 1.95e1  Region- 0.0 1.02e2 5.89e1 Region- 0.0 6.91e1 5.03e1	tDecay 50.0 2.25e1 1.89e1 37.8 Region- 50.0 2.33e1 1.82e1 45.7 mgWolfe 40.0 2.36e1 1.16e1 100.2 35.0 2.31e1 7.97e0 266.0 tion 0.0 7.97e1 5.68e1 3852.0    40.0 2.34e1 1.99e1 881.9 ve imited 35.0 2.84e1 1.70e1 115.2 30.0 2.42e1 2.03e1 745.3 SGrad 40.0 2.38e1 2.04e1 719.2 cust 55.0 2.24e1 1.99e1 103.5 mtum 5.0 3.03e1 2.04e1 28.9 50.0 2.30e1 4.91e0 106.2 chraticInterpolation Region- 45.0 2.31e1 1.95e1 168.5 Region- 0.0 6.91e1 5.03e1 3002.0

# 11 Problem: Rastrigin\_10D

Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
40.0	4.85e1	3.80e1	23.2	0.001
30.0	4.51e1	3.98e1	71.9	0.002
50.0	4.36e1	3.39e1	122.0	0.002
15.0	5.38e1	2.52e1	50.6	0.001
25.0	4.50 e1	4.02e1	249.0	0.008
40.0	5.16e1	3.24e1	118.5	0.003
60.0	4.52e1	3.21e1	102.5	0.003
olation				
55.0	4.48e1	4.17e1	718.0	0.018
20.0	4.68e1	4.06e1	2037.2	0.020
0.0	1.57e2	1.28e2	3002.0	0.021
45.0	4.36e1	3.87e1	93.5	0.001
0.0	9.70e1	6.33e1	3002.0	0.021
45.0	4.39e1	3.60 e1	61.3	0.002
35.0	$4.52\mathrm{e}1$	4.15e1	253.8	0.006
45.0	5.11e1	3.88e1	216.4	0.005
10.0	6.15e1	4.06e1	30.6	0.001
30.0	4.44e1	4.13e1	127.9	0.003
30.0	4.65e1	4.16e1	731.5	0.016
40.0	4.39e1	4.16e1	1397.8	0.010
	40.0  30.0 50.0 15.0 25.0 40.0 60.0 olation  55.0 20.0  0.0  45.0 35.0  45.0 30.0 30.0 30.0	40.0 4.85e1  30.0 4.51e1 50.0 4.36e1 15.0 5.38e1 25.0 4.50e1 40.0 5.16e1 60.0 4.52e1  olation  55.0 4.48e1 20.0 4.68e1  0.0 1.57e2  45.0 4.36e1  0.0 9.70e1  45.0 4.39e1 35.0 4.52e1  45.0 5.11e1 10.0 6.15e1 30.0 4.44e1 30.0 4.65e1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

QQN- GoldenSec	tion	50.0	4.53e1	2.11e1	275.4	0.005
L-BFGS	61011	0.0	1.85e2	1.42e2	102.0	0.001
L-BFGS-		0.0	1.63e2	1.40e2	3852.0	0.031
Aggressive						
GD		35.0	$4.47\mathrm{e}1$	3.89e1	19.4	0.001
Trust	Region-	25.0	4.59e1	4.10e1	346.6	0.002
Standard						
L-BFGS-		65.0	4.11e1	2.56e1	291.6	0.006
MoreThue	nte					

### $12 \quad Problem: \ Ackley\_2D\_a20\_b0.2\_c6.28e0$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-Momentum	0.0	3.70e0	3.57e0	14.7	0.000
QQN-StrongWolfe	15.0	3.49e0	3.00e0	62.8	0.001
L-BFGS-Limited	15.0	3.37e0	2.64e-1	176.7	0.002
GD-Nesterov	0.0	3.57e0	3.57e0	30.6	0.001
Trust Region-	0.0	3.58e0	3.57e0	171.2	0.001
Conservative					
L-BFGS-	20.0	3.74e0	2.80e0	3083.9	0.018
Aggressive					
Trust Region-	0.0	4.78e0	3.62e0	5.3	0.000
Aggressive					
Trust Region-	0.0	3.58e0	3.58e0	39.5	0.000
Precise					
L-BFGS-	0.0	3.57e0	3.57e0	183.6	0.004
Conservative					
GD-WeightDecay	0.0	3.59e0	3.57e0	49.2	0.002
L-BFGS-	45.0	3.34e0	1.83e0	42.2	0.001
MoreThuente					
Adam-AMSGrad	0.0	3.57e0	3.57e0	753.0	0.017
Adam-	0.0	3.58e0	3.57e0	119.4	0.003
WeightDecay					
Trust Region-	0.0	5.19e0	3.58e0	8.6	0.000
Standard					
L-BFGS	50.0	3.20e0	1.89e0	202.7	0.003
Adam	0.0	3.57e0	3.57e0	301.6	0.006
QQN-	35.0	3.18e0	1.58e0	137.5	0.002
GoldenSection					
GD-	0.0	3.57e0	3.57e0	15.2	0.001
AdaptiveMomentum					
GD	0.0	3.57e0	3.57e0	20.2	0.001
QQN-Bisection-1	60.0	2.85e0	1.12e-2	53.6	0.001
QQN-	40.0	3.44e0	1.95e0	496.8	0.015
CubicQuadraticInter	polation				
QQN-Bisection-2	40.0	2.75e0	3.79e-1	36.5	0.001
Adam-Fast	0.0	3.62e0	3.57e0	52.9	0.001
Adam-Robust	0.0	3.58e0	3.57e0	42.9	0.001
Trust Region-	5.0	3.66e0	3.32e0	14.8	0.000
Adaptive					

## $13 \quad \text{Problem: } Ackley\_5D\_a20\_b0.2\_c6.28e0$

QQN- CubicQuadraticInterpolation	5.0	3.56e0	3.14e0	206.9	0.006
Trust Region-	0.0	3.70e0	3.58e0	17.3	0.000
Adaptive Region-	0.0	3.7060	3.0000	17.0	0.000
GD-WeightDecay	0.0	3.61e0	3.58e0	15.3	0.000
Trust Region-	0.0	4.15e0	3.71e0	5.0	0.000
Aggressive	0.0	1.1000	0.1100	0.0	0.000
L-BFGS	25.0	3.46e0	1.87e0	121.3	0.002
GD-	0.0	3.58e0	3.57e0	34.5	0.001
AdaptiveMomentum	0.0	3.0000	0.0100	01.0	0.001
QQN-	5.0	3.51e0	2.30e0	193.7	0.003
GoldenSection		0.020			0.000
QQN-StrongWolfe	5.0	3.50e0	2.03e0	53.0	0.002
L-BFGS-	0.0	3.57e0	3.57e0	373.8	0.006
Conservative					
Trust Region-	0.0	4.49e0	3.60e0	9.1	0.000
Standard					
Adam-Fast	0.0	3.60 e0	3.57e0	83.3	0.002
Adam-Robust	0.0	3.58e0	3.57e0	59.0	0.001
L-BFGS-	10.0	3.50e0	2.54e0	51.0	0.001
MoreThuente					
Trust Region-	0.0	3.58e0	3.57e0	212.3	0.002
Conservative					
GD	0.0	3.57e0	3.57e0	51.8	0.001
L-BFGS-Limited	0.0	3.57e0	3.57e0	310.5	0.005
L-BFGS-	20.0	3.76e0	3.01e0	3085.2	0.020
Aggressive					
Trust Region-	0.0	3.62e0	3.57e0	45.5	0.000
Precise					
GD-Nesterov	0.0	3.60e0	3.57e0	17.0	0.001
Adam	0.0	3.57e0	3.57e0	548.5	0.012
Adam-	0.0	3.57e0	3.57e0	183.0	0.004
WeightDecay					
QQN-Bisection-2	35.0	3.34e0	2.48e0	50.8	0.001
QQN-Bisection-1	30.0	3.38e0	2.37e0	65.2	0.002
Adam-AMSGrad	0.0	3.57e0	3.57e0	1172.5	0.028
GD-Momentum	0.0	3.72e0	3.59e0	13.0	0.000

# 14 Problem: Ackley\_10D\_a20\_b0.2\_c6.28e0

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Fast	0.0	3.60e0	3.57e0	106.6	0.002
GD-WeightDecay	0.0	3.59e0	3.58e0	19.4	0.001
QQN-Bisection-1	10.0	3.54e0	3.08e0	96.2	0.002
L-BFGS	15.0	3.50e0	2.72e0	133.3	0.002
GD-Nesterov	0.0	3.66e0	3.62e0	14.0	0.000
Adam	0.0	3.57e0	3.57e0	745.8	0.017
QQN-Bisection-2	20.0	3.54e0	3.30e0	65.9	0.002
QQN-	0.0	3.57e0	3.57e0	186.9	0.003
GoldenSection					
GD-	0.0	3.60e0	3.57e0	16.1	0.001
AdaptiveMomentum					
Adam-AMSGrad	0.0	3.57e0	3.57e0	1241.4	0.031
Adam-	0.0	3.57e0	3.57e0	281.3	0.007
WeightDecay					
Trust Region-	0.0	4.41e0	3.59e0	18.2	0.000
Adaptive					

L-BFGS-Li	mited	0.0	3.57e0	3.57e0	142.2	0.003
QQN-Stron	gWolfe	0.0	3.57e0	3.57e0	79.2	0.002
QQN-		5.0	3.59e0	3.33e0	471.4	0.016
CubicQuad	raticInterpolation					
GD-Momen	ntum	0.0	3.81e0	3.64e0	14.5	0.000
L-BFGS-		0.0	3.57e0	3.57e0	170.7	0.004
Conservativ	<i>r</i> e					
L-BFGS-		5.0	3.57e0	3.52e0	55.9	0.001
MoreThuen	te					
Adam-Robu	ust	0.0	3.58e0	3.57e0	126.8	0.003
Trust	Region-	0.0	4.36e0	3.64e0	9.1	0.000
Standard						
Trust	Region-	0.0	3.58e0	3.57e0	205.8	0.002
Conservativ	<i>r</i> e					
Trust	Region-	0.0	3.61e0	3.58e0	42.5	0.000
Precise						
Trust	Region-	0.0	4.22e0	3.81e0	5.2	0.000
Aggressive						
GD		0.0	3.57e0	3.57e0	99.3	0.003
L-BFGS-		5.0	3.54e0	2.95e0	185.8	0.003
Aggressive						
Trust Conservativ Trust Precise Trust Aggressive GD L-BFGS-	ve Region-	0.0 0.0 0.0	3.61e0 4.22e0 3.57e0	3.58e0 3.81e0 3.57e0	42.5 5.2 99.3	0.000 0.000 0.003

# 15 Problem: StyblinskiTang\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	90.0	-7.69e1	-7.83e1	300.1	0.008
Conservative					
QQN-Bisection-1	70.0	-7.40e1	-7.83e1	385.6	0.007
Adam	0.0	-2.94e1	-3.47e1	2502.0	0.050
Adam-AMSGrad	0.0	-3.03e1	-3.29e1	2502.0	0.056
L-BFGS-	0.0	-5.13e1	-7.77e1	3848.1	0.025
Aggressive					
Trust Region-	40.0	6.01e1	-7.83e1	212.3	0.001
Standard					
Trust Region-	20.0	-6.43e1	-7.83e1	49.3	0.000
Aggressive					
GD-WeightDecay	35.0	-7.46e1	-7.83e1	39.9	0.001
QQN-StrongWolfe	65.0	-7.34e1	-7.83e1	284.4	0.008
GD-Momentum	15.0	-5.68e1	-7.83e1	35.9	0.001
GD	65.0	-7.34e1	-7.83e1	66.5	0.002
QQN-	90.0	-7.69e1	-7.83e1	159.8	0.002
GoldenSection					
GD-	5.0	-3.87e1	-7.83e1	24.4	0.001
AdaptiveMomentum					
QQN-Bisection-2	75.0	-7.48e1	-7.83e1	99.0	0.002
QQN-	70.0	-7.34e1	-7.83e1	74.5	0.002
CubicQuadraticInter	-				
Adam-Fast	35.0	-7.48e1	-7.83e1	67.3	0.001
Adam-Robust	0.0	-2.07e1	-2.44e1	2502.0	0.056
L-BFGS	50.0	-7.32e1	-7.83e1	132.3	0.002
L-BFGS-	70.0	-7.41e1	-7.83e1	54.1	0.001
MoreThuente					
L-BFGS-Limited	70.0	-7.41e1	-7.83e1	500.0	0.006
Trust Region-	70.0	-7.37e1	-7.83e1	885.1	0.005
Adaptive					
GD-Nesterov	10.0	-6.56e1	-7.83e1	29.1	0.001

Adam- WeightDe	cav	80.0	-7.55e1	-7.83e1	1893.5	0.040
Trust Precise	Region-	0.0	-6.26e1	-6.35e1	3002.0	0.019
Trust Conservat	Region- ive	0.0	-1.02e1	-1.15e1	3002.0	0.019

# $16 \quad Problem: \ Styblinski Tang\_5D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-StrongWolfe	40.0	-1.85e2	-1.96e2	261.1	0.009
Adam-Fast	55.0	-1.86e2	-1.96e2	65.0	0.001
Adam-AMSGrad	0.0	-7.50e1	-8.10e1	2502.0	0.059
GD-Momentum	15.0	-1.64e2	-1.95e2	57.8	0.002
QQN-	70.0	-1.90e2	-1.96e2	330.5	0.006
GoldenSection					
Adam-	65.0	-1.90e2	-1.95e2	1865.5	0.041
WeightDecay					
Trust Region-	0.0	-6.28e1	-6.40e1	3002.0	0.020
Precise					
QQN-	45.0	-1.82e2	-1.96e2	129.7	0.005
CubicQuadraticInterp	oolation				
Adam-Robust	0.0	-5.32e1	-6.12e1	2502.0	0.059
GD	50.0	-1.86e2	-1.95e2	95.7	0.003
Trust Region-	45.0	-1.90e2	-1.96e2	144.5	0.001
Aggressive					
Trust Region-	0.0	-1.05e1	-1.22e1	3002.0	0.019
Conservative					
GD-Nesterov	30.0	-1.82e2	-1.96e2	44.9	0.001
QQN-Bisection-1	60.0	-1.89e2	-1.96e2	413.4	0.008
GD-WeightDecay	60.0	-1.88e2	-1.96e2	53.1	0.002
GD-	10.0	-1.35e2	-1.95e2	34.4	0.001
${\bf Adaptive Momentum}$					
Trust Region-	55.0	-1.88e2	-1.96e2	568.8	0.004
Standard					
L-BFGS-	0.0	-1.04e2	-1.83e2	3847.9	0.028
Aggressive					
QQN-Bisection-2	60.0	-1.89e2	-1.96e2	177.8	0.005
L-BFGS	30.0	-1.86e2	-1.96e2	153.3	0.002
L-BFGS-	65.0	-1.89e2	-1.95e2	567.8	0.012
Conservative					
L-BFGS-	50.0	-1.89e2	-1.96e2	209.2	0.004
MoreThuente					
L-BFGS-Limited	50.0	-1.89e2	-1.96e2	839.9	0.011
Adam	0.0	-7.38e1	-8.30e1	2502.0	0.052
Trust Region- Adaptive	65.0	-1.88e2	-1.95e2	2249.5	0.014

### $17 \quad Problem: \ StyblinskiTang\_10D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-1	30.0	-3.74e2	-3.89e2	412.5	0.011
L-BFGS-Limited	35.0	-3.76e2	-3.86e2	550.6	0.009
GD-Momentum	15.0	-3.59e2	-3.82e2	86.2	0.003

Adam-	35.0	-3.73e2	-3.78e2	1837.7	0.043
WeightDecay					
GD-WeightDecay	45.0	-3.75e2	-3.83e2	74.7	0.003
Trust Region-	40.0	-3.68e2	-3.79e2	1120.5	0.008
Standard					
Adam	0.0	-1.49e2	-1.65e2	2502.0	0.055
Trust Region-	0.0	-1.09e1	-1.25e1	3002.0	0.021
Conservative					
L-BFGS	45.0	-3.76e2	-3.90e2	132.2	0.002
Adam-Fast	30.0	-3.65e2	-3.85e2	67.5	0.002
Adam-AMSGrad	0.0	-1.51e2	-1.63e2	2502.0	0.061
Adam-Robust	0.0	-1.05e2	-1.13e2	2502.0	0.062
Trust Region-	50.0	-3.71e2	-3.81e2	284.1	0.002
Aggressive					
Trust Region-	0.0	-6.35e1	-6.56e1	3002.0	0.023
Precise					
L-BFGS-	0.0	-1.97e2	-3.07e2	3848.4	0.029
Aggressive					
QQN-StrongWolfe	15.0	-3.60e2	-3.86e2	362.2	0.012
L-BFGS-	30.0	-3.74e2	-3.82e2	543.3	0.015
Conservative					
GD-Nesterov	30.0	-3.75e2	-3.83e2	65.9	0.002
QQN-	15.0	-3.66e2	-3.91e2	172.1	0.006
${\bf Cubic Quadratic Interpolation}$					
L-BFGS-	35.0	-3.76e2	-3.91e2	125.9	0.002
MoreThuente					
GD-	20.0	-3.21e2	-3.82e2	53.4	0.002
${\bf Adaptive Momentum}$					
Trust Region-	0.0	-2.51e2	-2.54e2	3002.0	0.020
Adaptive					
GD	25.0	-3.71e2	-3.80e2	137.3	0.004
QQN-	45.0	-3.76e2	-3.91e2	452.4	0.010
GoldenSection					
QQN-Bisection-2	35.0	-3.72e2	-3.90e2	234.2	0.006

#### 18 Problem: Beale\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-Limited	100.0	1.00e-2	3.68e-4	110.5	0.002
QQN-StrongWolfe	90.0	4.59e-1	3.96e-3	786.0	0.024
Trust Region-	5.0	1.23e0	1.33e-3	17.8	0.000
Aggressive					
GD	100.0	1.49e-2	1.49e-2	229.2	0.006
Trust Region-	0.0	2.71e2	1.24e-1	50.2	0.000
Standard					
GD-WeightDecay	100.0	1.48e-2	1.46e-2	88.0	0.003
Trust Region-	0.0	1.27e-1	3.79e-2	179.6	0.001
Adaptive					
Adam	0.0	7.19e-2	3.04e-2	2502.0	0.049
L-BFGS-	100.0	8.71e-3	3.68e-4	78.1	0.001
MoreThuente					
GD-Momentum	5.0	1.32e0	1.20e-2	24.4	0.001
QQN-	95.0	4.97e-1	2.46e-3	218.8	0.008
CubicQuadraticInterp	oolation				
L-BFGS-	100.0	1.17e-2	5.27e-3	200.7	0.005
Conservative					
L-BFGS	65.0	4.55e-1	1.99e-3	122.0	0.002
GD-Nesterov	100.0	9.18e-3	4.69e-3	27.7	0.001
Adam L-BFGS- MoreThuente GD-Momentum QQN- CubicQuadraticInterp L-BFGS- Conservative L-BFGS	100.0 5.0 95.0 polation 100.0 65.0	8.71e-3 1.32e0 4.97e-1 1.17e-2 4.55e-1	3.68e-4 1.20e-2 2.46e-3 5.27e-3 1.99e-3	78.1 24.4 218.8 200.7 122.0	0.001 0.001 0.008 0.005 0.002

QQN-	100.0	6.90e-3	2.42e-3	347.2	0.005
GoldenSection					
L-BFGS-	0.0	1.39e1	8.86e0	3851.9	0.021
Aggressive					
GD-	80.0	3.09e-1	6.42e-4	25.5	0.001
Adaptive Momentum					
Adam-AMSGrad	0.0	2.17e-1	8.70e-2	2502.0	0.055
Adam-Fast	0.0	1.59e0	1.39e0	37.6	0.001
QQN-Bisection-2	70.0	2.61e0	3.70e-3	83.8	0.002
Trust Region-	5.0	4.33e0	9.92e-3	2999.7	0.018
Conservative					
Adam-	100.0	1.50e-2	1.49e-2	1539.9	0.031
WeightDecay					
Trust Region-	45.0	2.17e-2	3.20e-3	743.8	0.005
Precise					
Adam-Robust	0.0	4.58e-1	1.68e-1	2502.0	0.054
QQN-Bisection-1	95.0	4.13e-1	4.49e-3	277.4	0.006

### 19 Problem: Levi\_2D

GD-Momentum	Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Aggressive         QQN         45.0         9.83e-1         1.87e-2         262.0         0.004           GoldenSection         GORDENSCOTION         U         1.44e0         1.40e-1         450.4         0.014           L-BFGS         0.0         1.94e0         1.43e0         209.4         0.004           Conservative         GD-Nesterov         0.0         1.93e0         7.77e-1         15.2         0.000           Adam-Fast         0.0         2.09e0         1.47e0         47.2         0.001           Adam-AMSGrad         0.0         1.91e0         1.43e0         282.9         0.006           Adam-AMSGrad         0.0         1.88e0         1.08e0         42.6         0.001           L-BFGS         15.0         4.40e0         1.84e-1         107.2         0.002           Trust         Region-         0.0         1.33e2         1.02e0         6.3         0.000           Aggressive         V         V         V         V         0.001         V         V         0.001           QQN-Bisection-2         20.0         1.08e0         4.47e-2         77.5         0.001         0.001         V         V         V         0.001         0.001	GD-Momentum	0.0	2.02e0	7.01e-1	19.0	0.001
QQN-         45.0         9.83e-1         1.87e-2         262.0         0.004           GoldenSection         QQN-StrongWolfe         25.0         1.44e0         1.40e-1         450.4         0.014           L-BFGS-         0.0         1.94e0         1.43e0         209.4         0.004           Conservative         GD-Nesterov         0.0         1.93e0         7.77e-1         15.2         0.000           Adam-Fast         0.0         2.09e0         1.47e0         47.2         0.001           Adam-AMSGrad         0.0         1.91e0         1.43e0         282.9         0.006           Adam-Robust         0.0         1.88e0         1.08e0         42.6         0.001           L-BFGS         15.0         4.40e0         1.84e-1         107.2         0.002           Trust         Region-         0.0         2.18e0         1.43e0         10.7         0.000           Standard         Trust         Region-         0.0         1.33e2         1.02e0         6.3         0.000           Aggressive         QQN-Bisection-2         20.0         1.08e0         4.47e-2         77.5         0.001           QN-Bisection-1	L-BFGS-	0.0	2.87e0	1.00e0	3851.3	0.022
GoldenSection   QQN-StrongWolfe   25.0   1.44e0   1.40e-1   450.4   0.014   1.4FGS-   0.0   1.94e0   1.43e0   209.4   0.004   Conservative	Aggressive					
QQN-StrongWolfe         25.0         1.44e0         1.40e-1         450.4         0.014           L-BFGS         0.0         1.94e0         1.43e0         209.4         0.004           Conservative         GD-Nesterov         0.0         1.93e0         7.77e-1         15.2         0.000           Adam-Fast         0.0         2.09e0         1.47e0         47.2         0.001           Adam-AMSGrad         0.0         1.91e0         1.43e0         282.9         0.006           Adam-Robust         0.0         1.88e0         1.08e0         42.6         0.001           L-BFGS         15.0         4.40e0         1.84e-1         107.2         0.002           Trust         Region-         0.0         2.18e0         1.43e0         10.7         0.000           Aggressive         Trust         Region-         0.0         1.33e2         1.02e0         6.3         0.000           Aggressive         QQN-Bisection-2         20.0         1.08e0         4.47e-2         77.5         0.001           MoreThuente         QQN-Bisection-1         15.0         1.30e0         1.54e-1         200.7         0.004           QQN-Bisection-1	QQN-	45.0	9.83e-1	1.87e-2	262.0	0.004
Lagrange   Lagrange	GoldenSection					
Conservative   GD-Nesterov   0.0   1.93e0   7.77e-1   15.2   0.000   Adam-Fast   0.0   2.09e0   1.47e0   47.2   0.001   Adam-Fast   0.0   1.91e0   1.43e0   282.9   0.006   Adam-AMSGrad   0.0   1.91e0   1.43e0   282.9   0.006   Adam-Robust   0.0   1.88e0   1.08e0   42.6   0.001   L-BFGS   15.0   4.40e0   1.84e-1   107.2   0.002   Trust   Region   0.0   2.18e0   1.43e0   10.7   0.000   Standard   Trust   Region   0.0   1.33e2   1.02e0   6.3   0.000   Aggressive   QN-Bisection-2   20.0   1.08e0   4.47e-2   77.5   0.001   L-BFGS   15.0   1.48e0   1.36e-1   86.2   0.001   MoreThuente   QQN-Bisection-1   15.0   1.30e0   1.54e-1   200.7   0.004   QQN-   40.0   1.04e0   1.50e-1   189.6   0.007   CubicQuadraticInterpolation   GD-WeightDecay   0.0   2.04e0   1.43e0   76.8   0.001   Precise   GD   0.0   2.04e0   1.43e0   242.4   0.005   Trust   Region   0.0   2.04e0   1.43e0   242.4   0.005   Trust   Region   0.0   2.04e0   1.43e0   2.96   0.001   Precise   GD   0.0   2.04e0   1.43e0   2.96   0.000   Trust   Region   0.0   2.04e0   1.43e0   2.96   0.000   Adaptive   L-BFGS-Limited   25.0   9.89e-1   1.13e-1   828.4   0.009   GD-   0.0   2.06e0   1.28e0   18.7   0.001   AdaptiveMomentum   Adam-   0.0   1.95e0   1.44e0   99.6   0.002   0	QQN-StrongWolfe	25.0	1.44e0	1.40e-1	450.4	0.014
GD-Nesterov   0.0   1.93e0   7.77e-1   15.2   0.000   Adam-Fast   0.0   2.09e0   1.47e0   47.2   0.001   Adam-Fast   0.0   1.91e0   1.43e0   282.9   0.006   Adam-AMSGrad   0.0   1.88e0   1.08e0   42.6   0.001   L-BFGS   15.0   4.40e0   1.84e-1   107.2   0.002   Trust   Region   0.0   2.18e0   1.43e0   10.7   0.000   Standard   Trust   Region   0.0   1.33e2   1.02e0   6.3   0.000   Aggressive   USAGRIFICATION   0.001	L-BFGS-	0.0	1.94e0	1.43e0	209.4	0.004
Adam-Fast         0.0         2.09e0         1.47e0         47.2         0.001           Adam-AMSGrad         0.0         1.91e0         1.43e0         282.9         0.006           Adam-Robust         0.0         1.88e0         1.08e0         42.6         0.001           L-BFGS         15.0         4.40e0         1.84e-1         107.2         0.002           Trust         Region-         0.0         2.18e0         1.43e0         10.7         0.000           Standard         Trust         Region-         0.0         1.33e2         1.02e0         6.3         0.000           Aggressive         Trust         Region-         0.0         1.08e0         4.47e-2         77.5         0.001           L-BFGS-         15.0         1.48e0         1.36e-1         86.2         0.001           MoreThuente         QQN-Bisection-1         15.0         1.30e0         1.54e-1         200.7         0.004           QQN-Bisection-1         15.0         1.30e0         1.54e-1         200.7         0.004           QQN-Bisection-1         15.0         1.30e0         1.54e-1         200.7         0.004           QQN-Bisection-1         15.0         1.30e0         1.54e	Conservative					
Adam-AMSGrad         0.0         1.91e0         1.43e0         282.9         0.006           Adam-Robust         0.0         1.88e0         1.08e0         42.6         0.001           L-BFGS         15.0         4.40e0         1.84e-1         107.2         0.002           Trust         Region-         0.0         2.18e0         1.43e0         10.7         0.000           Standard         Trust         Region-         0.0         1.33e2         1.02e0         6.3         0.000           Aggressive         QQN-Bisection-2         20.0         1.08e0         4.47e-2         77.5         0.001           L-BFGS-         15.0         1.48e0         1.36e-1         86.2         0.001           MoreThuente         QQN-Bisection-1         15.0         1.30e0         1.54e-1         200.7         0.004           QQN-         40.0         1.04e0         1.50e-1         189.6         0.007           CubicQuadraticInterpolation         GD-WeightDecay         0.0         2.00e0         1.19e0         28.1         0.001           Adam         0.0         1.93e0         1.43e0         76.8         0.001           Precise <td< td=""><td>GD-Nesterov</td><td>0.0</td><td>1.93e0</td><td>7.77e-1</td><td>15.2</td><td>0.000</td></td<>	GD-Nesterov	0.0	1.93e0	7.77e-1	15.2	0.000
Adam-Robust         0.0         1.88e0         1.08e0         42.6         0.001           L-BFGS         15.0         4.40e0         1.84e-1         107.2         0.002           Trust         Region-         0.0         2.18e0         1.43e0         10.7         0.000           Standard         Trust         Region-         0.0         1.33e2         1.02e0         6.3         0.000           Aggressive         V           QQN-Bisection-2         20.0         1.08e0         4.47e-2         77.5         0.001           L-BFGS-         15.0         1.48e0         1.36e-1         86.2         0.001           MoreThuente         QQN-Bisection-1         15.0         1.30e0         1.54e-1         200.7         0.004           QQN-         40.0         1.04e0         1.50e-1         189.6         0.007           CubicQuadraticInterpolation         GD-WeightDecay         0.0         2.00e0         1.19e0         28.1         0.001           Adam         0.0         2.04e0         1.43e0         242.4         0.005           Trust         Region-         0.0         2.04e0         1.43e0         21.9         0.001	Adam-Fast	0.0	2.09e0	1.47e0	47.2	0.001
L-BFGS	Adam-AMSGrad	0.0	1.91e0	1.43e0	282.9	0.006
Trust         Region- Standard         0.0         2.18e0         1.43e0         10.7         0.000           Standard         Trust         Region- 0.0         1.33e2         1.02e0         6.3         0.000           Aggressive         QQN-Bisection-2         20.0         1.08e0         4.47e-2         77.5         0.001           L-BFGS-         15.0         1.48e0         1.36e-1         86.2         0.001           MoreThuente         QQN-Bisection-1         15.0         1.30e0         1.54e-1         200.7         0.004           QQN-         40.0         1.04e0         1.50e-1         189.6         0.007           CubicQuadraticInterpolation         GD-WeightDecay         0.0         2.00e0         1.19e0         28.1         0.001           Adam         0.0         1.93e0         1.43e0         242.4         0.005           Trust         Region-         0.0         2.04e0         1.43e0         242.4         0.001           Precise         GD         0.0         2.04e0         1.43e0         21.9         0.001           Trust         Region-         0.0         2.09e0         1.46e0         29.6         0.000<	Adam-Robust	0.0	1.88e0	1.08e0	42.6	0.001
Standard   Trust   Region-   0.0   1.33e2   1.02e0   6.3   0.000	L-BFGS	15.0	4.40e0	1.84e-1	107.2	0.002
Standard   Trust   Region-   0.0   1.33e2   1.02e0   6.3   0.000	Trust Region-	0.0	2.18e0	1.43e0	10.7	0.000
Aggressive       QQN-Bisection-2       20.0       1.08e0       4.47e-2       77.5       0.001         L-BFGS-       15.0       1.48e0       1.36e-1       86.2       0.001         MoreThuente       QQN-Bisection-1       15.0       1.30e0       1.54e-1       200.7       0.004         QQN-       40.0       1.04e0       1.50e-1       189.6       0.007         CubicQuadraticInterpolation       GD-WeightDecay       0.0       2.00e0       1.19e0       28.1       0.001         Adam       0.0       1.93e0       1.43e0       242.4       0.005         Trust       Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise       GD       0.0       2.04e0       1.43e0       21.9       0.001         Trust       Region-       0.0       2.09e0       1.43e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum       Adam-       0.0       1.95e0       1.44e0       99.6 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td></t<>						
QQN-Bisection-2       20.0       1.08e0       4.47e-2       77.5       0.001         L-BFGS-       15.0       1.48e0       1.36e-1       86.2       0.001         MoreThuente       QQN-Bisection-1       15.0       1.30e0       1.54e-1       200.7       0.004         QQN-       40.0       1.04e0       1.50e-1       189.6       0.007         CubicQuadraticInterpolation       GD-WeightDecay       0.0       2.00e0       1.19e0       28.1       0.001         Adam       0.0       1.93e0       1.43e0       242.4       0.005         Trust       Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise       GD       0.0       2.04e0       1.43e0       21.9       0.001         Trust       Region-       0.0       2.04e0       1.43e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum       Adam-       0.0       1.95e0       1.44e0       99.6       0.002    <	Trust Region-	0.0	1.33e2	1.02e0	6.3	0.000
L-BFGS- 15.0 1.48e0 1.36e-1 86.2 0.001  More Thuente  QQN-Bisection-1 15.0 1.30e0 1.54e-1 200.7 0.004  QQN- 40.0 1.04e0 1.50e-1 189.6 0.007  Cubic Quadratic Interpolation  GD-Weight Decay 0.0 2.00e0 1.19e0 28.1 0.001  Adam 0.0 1.93e0 1.43e0 242.4 0.005  Trust Region- 0.0 2.04e0 1.43e0 76.8 0.001  Precise  GD 0.0 2.04e0 1.43e0 21.9 0.001  Trust Region- 0.0 2.09e0 1.46e0 29.6 0.000  Adaptive  L-BFGS-Limited 25.0 9.89e-1 1.13e-1 828.4 0.009  GD- 0.0 2.06e0 1.28e0 18.7 0.001  Adaptive Momentum  Adam- 0.0 1.95e0 1.44e0 99.6 0.002	Aggressive					
MoreThuente         QQN-Bisection-1       15.0       1.30e0       1.54e-1       200.7       0.004         QQN-       40.0       1.04e0       1.50e-1       189.6       0.007         CubicQuadraticInterpolation         GD-WeightDecay       0.0       2.00e0       1.19e0       28.1       0.001         Adam       0.0       1.93e0       1.43e0       242.4       0.005         Trust       Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise       F       0.0       2.04e0       1.43e0       21.9       0.001         Trust       Region-       0.0       2.09e0       1.46e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum       Adam-       0.0       1.95e0       1.44e0       99.6       0.002	QQN-Bisection-2	20.0	1.08e0	4.47e-2	77.5	0.001
QQN-Bisection-1       15.0       1.30e0       1.54e-1       200.7       0.004         QQN-       40.0       1.04e0       1.50e-1       189.6       0.007         Cubic Quadratic Interpolation         GD-Weight Decay       0.0       2.00e0       1.19e0       28.1       0.001         Adam       0.0       1.93e0       1.43e0       242.4       0.005         Trust Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise         GD       0.0       2.04e0       1.43e0       21.9       0.001         Trust Region-       0.0       2.09e0       1.46e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum       Adam-       0.0       1.95e0       1.44e0       99.6       0.002	L-BFGS-	15.0	1.48e0	1.36e-1	86.2	0.001
QQN-       40.0       1.04e0       1.50e-1       189.6       0.007         CubicQuadraticInterpolation       CubicQuadraticInterpolation       0.0       2.00e0       1.19e0       28.1       0.001         Adam       0.0       1.93e0       1.43e0       242.4       0.005         Trust       Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise       FGD       0.0       2.04e0       1.43e0       21.9       0.001         Trust       Region-       0.0       2.09e0       1.46e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum       Adam-       0.0       1.95e0       1.44e0       99.6       0.002	MoreThuente					
Cubic Quadratic Interpolation         GD-Weight Decay       0.0       2.00e0       1.19e0       28.1       0.001         Adam       0.0       1.93e0       1.43e0       242.4       0.005         Trust       Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise       FGD       0.0       2.04e0       1.43e0       21.9       0.001         Trust       Region-       0.0       2.09e0       1.46e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum       Adam-       0.0       1.95e0       1.44e0       99.6       0.002	QQN-Bisection-1	15.0	1.30e0	1.54e-1	200.7	0.004
GD-WeightDecay       0.0       2.00e0       1.19e0       28.1       0.001         Adam       0.0       1.93e0       1.43e0       242.4       0.005         Trust       Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise       GD       0.0       2.04e0       1.43e0       21.9       0.001         Trust       Region-       0.0       2.09e0       1.46e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum       Adam-       0.0       1.95e0       1.44e0       99.6       0.002	QQN-	40.0	1.04e0	1.50e-1	189.6	0.007
Adam       0.0       1.93e0       1.43e0       242.4       0.005         Trust       Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise       GD       0.0       2.04e0       1.43e0       21.9       0.001         Trust       Region-       0.0       2.09e0       1.46e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum         Adam-       0.0       1.95e0       1.44e0       99.6       0.002	CubicQuadraticInter	polation				
Trust       Region-       0.0       2.04e0       1.43e0       76.8       0.001         Precise       GD       0.0       2.04e0       1.43e0       21.9       0.001         Trust       Region-       0.0       2.09e0       1.46e0       29.6       0.000         Adaptive       L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum         Adam-       0.0       1.95e0       1.44e0       99.6       0.002	GD-WeightDecay	0.0	2.00e0	1.19e0	28.1	0.001
Precise         GD       0.0       2.04e0       1.43e0       21.9       0.001         Trust Region-       0.0       2.09e0       1.46e0       29.6       0.000         Adaptive         L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum         Adam-       0.0       1.95e0       1.44e0       99.6       0.002	Adam	0.0	1.93e0	1.43e0	242.4	0.005
GD     0.0     2.04e0     1.43e0     21.9     0.001       Trust Region-     0.0     2.09e0     1.46e0     29.6     0.000       Adaptive       L-BFGS-Limited     25.0     9.89e-1     1.13e-1     828.4     0.009       GD-     0.0     2.06e0     1.28e0     18.7     0.001       AdaptiveMomentum       Adam-     0.0     1.95e0     1.44e0     99.6     0.002	Trust Region-	0.0	2.04e0	1.43e0	76.8	0.001
Trust     Region-     0.0     2.09e0     1.46e0     29.6     0.000       Adaptive       L-BFGS-Limited     25.0     9.89e-1     1.13e-1     828.4     0.009       GD-     0.0     2.06e0     1.28e0     18.7     0.001       AdaptiveMomentum       Adam-     0.0     1.95e0     1.44e0     99.6     0.002	Precise					
Adaptive         L-BFGS-Limited       25.0       9.89e-1       1.13e-1       828.4       0.009         GD-       0.0       2.06e0       1.28e0       18.7       0.001         AdaptiveMomentum         Adam-       0.0       1.95e0       1.44e0       99.6       0.002	GD	0.0	2.04e0	1.43e0	21.9	0.001
L-BFGS-Limited 25.0 9.89e-1 1.13e-1 828.4 0.009 GD- 0.0 2.06e0 1.28e0 18.7 0.001 AdaptiveMomentum Adam- 0.0 1.95e0 1.44e0 99.6 0.002	Trust Region-	0.0	2.09e0	1.46e0	29.6	0.000
GD- 0.0 2.06e0 1.28e0 18.7 0.001 AdaptiveMomentum Adam- 0.0 1.95e0 1.44e0 99.6 0.002	Adaptive					
AdaptiveMomentum         Adam-       0.0       1.95e0       1.44e0       99.6       0.002	L-BFGS-Limited	25.0	9.89 e-1	1.13e-1	828.4	0.009
Adam- 0.0 1.95e0 1.44e0 99.6 0.002	GD-	0.0	2.06e0	1.28e0	18.7	0.001
	AdaptiveMomentum					
WeightDecay	Adam-	0.0	1.95e0	1.44e0	99.6	0.002
	WeightDecay					

#### 20 Problem: GoldsteinPrice\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-	0.0	8.40e2	8.40e2	209.7	0.005
WeightDecay					
QQN-	15.0	2.71e2	8.40e1	291.6	0.011
CubicQuadraticInter	polation				
Adam-AMSGrad	0.0	8.40e2	8.40e2	603.6	0.014
GD	0.0	8.50e2	8.41e2	24.6	0.001
Trust Region-	0.0	1.22e3	8.40e2	1684.8	0.010
Standard					
L-BFGS	5.0	8.00e2	6.35e1	176.8	0.003
QQN-Bisection-2	5.0	1.58e2	8.40e1	21.2	0.001
L-BFGS-	0.0	8.60e2	8.43e2	3847.0	0.033
Aggressive					
QQN-Bisection-1	20.0	8.40e1	8.40e1	362.8	0.010
L-BFGS-	0.0	8.44e2	8.40e2	346.4	0.005
Conservative					
GD-Momentum	0.0	8.98e2	9.88e1	17.9	0.000
GD-WeightDecay	0.0	8.57e2	8.40e2	20.3	0.001
QQN-	35.0	8.40e1	8.40e1	383.2	0.006
GoldenSection					
GD-	0.0	8.45e2	8.41e2	19.9	0.001
AdaptiveMomentum					
Adam-Fast	0.0	$8.55\mathrm{e}2$	8.40e2	35.3	0.001
Adam-Robust	0.0	8.41e2	8.40e2	81.0	0.002
L-BFGS-Limited	0.0	8.41e1	8.40e1	4259.4	0.038
Trust Region-	0.0	2.20e3	8.40e2	2943.8	0.018
Precise					
QQN-StrongWolfe	15.0	8.34e1	7.11e1	552.0	0.014
Adam	0.0	8.40e2	8.40e2	597.1	0.012
GD-Nesterov	0.0	8.57e2	8.40e2	24.6	0.001
Trust Region-	0.0	2.09e3	8.40e2	2649.8	0.016
Adaptive					
L-BFGS-	5.0	1.19e2	2.24e1	621.9	0.010
MoreThuente					
Trust Region-	0.0	2.12e3	8.97e2	3002.0	0.018
Conservative					
Trust Region-	0.0	9.54e2	8.40e2	725.5	0.005
Aggressive					

#### 21 Problem: Matyas\_2D

Optimiz	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	4.71e-2	3.00e-2	5.0	0.000
Precise						
Adam-Ro	bust	100.0	2.48e-2	2.46e-2	79.0	0.002
L-BFGS-		95.0	2.44e-2	2.16e-2	217.4	0.002
Aggressiv	<i>'</i> e					
$\operatorname{Adam}$		100.0	2.50e-2	2.49e-2	624.2	0.012
L-BFGS		100.0	1.47e-2	5.88e-4	20.1	0.000
QQN-Bis	ection-1	100.0	1.91e-2	2.83e-3	34.2	0.001

GD	100.0	2.50e-2	2.50e-2	629.6	0.015
GD-Momentum	100.0	2.49e-2	2.48e-2	67.2	0.002
Adam-Fast	100.0	2.30e-2	1.98e-2	12.9	0.000
QQN-StrongWolfe	100.0	2.73e-29	1.77e-32	24.0	0.001
Adam-	100.0	2.49e-2	2.48e-2	221.4	0.005
WeightDecay					
L-BFGS-Limited	100.0	1.73e-2	7.16e-3	24.6	0.000
GD-WeightDecay	100.0	2.50e-2	2.50e-2	223.7	0.007
GD-	95.0	2.52e-2	2.42e-2	29.2	0.001
AdaptiveMomentum					
GD-Nesterov	100.0	2.49e-2	2.48e-2	71.0	0.002
Trust Region-	0.0	2.41e-1	3.14e-2	7.2	0.000
Standard					
QQN-	100.0	1.55e-2	1.12e-3	138.9	0.002
GoldenSection					
QQN-	100.0	1.07e-2	2.70e-30	34.0	0.001
CubicQuadraticInterpolat	ion				
Trust Region-	0.0	3.93e-2	2.66e-2	7.0	0.000
Conservative					
L-BFGS-	100.0	1.72e-2	2.59e-5	20.8	0.000
MoreThuente					
Trust Region-	0.0	6.88e-2	2.83e-2	7.0	0.000
Adaptive					
QQN-Bisection-2	100.0	1.49e-2	8.19e-4	40.9	0.001
L-BFGS-	100.0	2.30e-2	2.07e-2	39.0	0.001
Conservative					
Adam-AMSGrad	100.0	2.50e-2	2.50e-2	680.7	0.015
Trust Region-	0.0	6.67e-1	2.86e-2	6.3	0.000
Aggressive					

#### 22 Problem: Himmelblau\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	100.0	9.46e-2	5.19e-3	68.0	0.002
CubicQuadraticInter	rpolation				
L-BFGS-	0.0	3.44e1	4.95e0	3850.9	0.022
Aggressive					
GD-Momentum	0.0	4.37e1	3.43e0	24.3	0.001
Trust Region-	100.0	1.32e-1	7.13e-2	2048.1	0.012
Adaptive					
Trust Region-	80.0	1.50e-1	3.76e-3	516.7	0.003
Standard					
QQN-	100.0	7.91e-2	1.17e-4	102.8	0.001
GoldenSection					
GD-WeightDecay	75.0	3.93e-1	8.90e-2	29.6	0.001
L-BFGS-	100.0	1.02e-1	8.92e-3	36.3	0.000
MoreThuente					
Adam-Robust	0.0	1.05e2	9.10e1	2502.0	0.055
QQN-StrongWolfe	100.0	9.18e-2	9.60e-3	61.5	0.001
GD-	0.0	6.94e1	5.87e1	22.8	0.001
AdaptiveMomentum					
Adam	0.0	8.20e1	7.20e1	2502.0	0.049
Adam-AMSGrad	0.0	8.17e1	7.19e1	2502.0	0.055
Trust Region-	0.0	1.60e2	1.52e2	3002.0	0.018
Conservative					
Trust Region- Aggressive	25.0	6.71e-1	1.80e-2	133.1	0.001

Trust R	legion-	0.0	1.07e2	1.02e2	3002.0	0.018
Precise						
Adam-Fast		0.0	5.29e0	4.98e0	69.3	0.001
L-BFGS-Limi	ited	100.0	9.58e-2	7.81e-3	60.9	0.001
GD		100.0	1.53e-1	2.80e-2	42.9	0.001
QQN-Bisection	on-2	100.0	8.17e-2	2.27e-3	55.1	0.001
Adam-		100.0	2.41e-1	2.34e-1	1744.1	0.036
WeightDecay						
QQN-Bisection	on-1	100.0	1.09e-1	2.06e-2	107.9	0.002
L-BFGS-		100.0	2.03e-1	1.42e-1	264.2	0.006
Conservative						
L-BFGS		35.0	6.22e0	1.56e-2	93.7	0.001
GD-Nesterov		40.0	9.49e0	3.27e-2	26.7	0.001

#### Problem: Booth\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMSGrad	0.0	1.19e1	8.69e0	2502.0	0.055
Trust Region-	90.0	6.12e-2	2.38e-2	919.9	0.006
Adaptive					
Trust Region-	0.0	5.27e-1	3.46e-1	235.1	0.001
Standard					
L-BFGS-Limited	100.0	6.14e-2	2.04e-3	56.1	0.001
Adam-Fast	0.0	3.09e0	2.74e0	56.3	0.001
QQN-StrongWolfe	100.0	1.13e-2	9.93e-3	26.0	0.000
L-BFGS	100.0	4.05e-2	1.17e-2	42.4	0.001
GD	100.0	1.17e-1	1.15e-1	86.0	0.002
L-BFGS-	0.0	7.43e1	6.24e1	3852.0	0.021
Aggressive					
Trust Region-	0.0	6.49e1	5.04e1	3002.0	0.018
Conservative					
Trust Region-	0.0	1.49e0	4.00e-1	63.5	0.000
Aggressive					
Trust Region-	0.0	1.05e1	2.82e0	3002.0	0.018
Precise					
QQN-Bisection-2	100.0	1.49e-2	1.15e-2	100.0	0.002
GD-Momentum	0.0	7.27e0	6.20e0	21.2	0.001
Adam-	100.0	1.19e-1	1.19e-1	1886.3	0.039
WeightDecay					
Adam-Robust	0.0	2.03e1	1.43e1	2502.0	0.055
GD-WeightDecay	15.0	1.29e0	1.15e-1	27.0	0.001
QQN-	100.0	3.39e-2	1.60e-2	92.0	0.001
GoldenSection					
QQN-Bisection-1	100.0	1.87e-2	1.21e-2	162.4	0.003
L-BFGS-	100.0	1.10e-1	1.03e-1	208.0	0.005
Conservative					
L-BFGS-	100.0	3.86e-2	5.15e-4	29.7	0.000
MoreThuente					
GD-	0.0	9.51e0	8.56e0	21.0	0.001
AdaptiveMomentum					
GD-Nesterov	0.0	2.56e0	2.10e0	20.6	0.001
QQN-	100.0	2.56e-6	2.81e-7	56.0	0.001
CubicQuadraticInterp	polation				
Adam	0.0	1.20e1	8.76e0	2502.0	0.049

#### 24 Problem: Griewank\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	0.0	4.91e0	4.91e0	73.0	0.001
MoreThuente					
QQN-	0.0	4.80e0	2.74e0	77.3	0.002
CubicQuadraticInter	polation				
L-BFGS	0.0	4.91e0	4.91e0	295.6	0.006
L-BFGS-	0.0	4.91e0	4.91e0	504.6	0.012
Conservative					
Adam-Fast	0.0	4.93e0	4.92e0	67.0	0.001
Adam-Robust	0.0	5.36e0	5.26e0	2502.0	0.056
Adam-AMSGrad	0.0	5.24e0	5.16e0	2502.0	0.057
QQN-Bisection-2	0.0	4.66e0	4.25e0	94.6	0.002
Adam-	0.0	4.92e0	4.92e0	2231.2	0.047
WeightDecay					
Trust Region-	0.0	4.95e0	4.93e0	58.7	0.000
Precise					
Trust Region-	0.0	5.11e0	5.00e0	16.4	0.000
Adaptive					
QQN-	0.0	4.91e0	4.91e0	258.3	0.004
GoldenSection					
L-BFGS-Limited	0.0	4.91e0	4.91e0	927.3	0.021
GD-Momentum	0.0	4.91e0	4.91e0	100.2	0.003
Trust Region-	0.0	5.99e0	5.90e0	5.0	0.000
Standard					
Trust Region-	0.0	4.92e0	4.91e0	336.5	0.002
Conservative					
L-BFGS-	0.0	4.91e0	4.91e0	2427.6	0.055
Aggressive					
GD-WeightDecay	0.0	4.91e0	4.91e0	406.6	0.012
Trust Region-	0.0	6.00e0	5.90e0	5.0	0.000
Aggressive					
Adam	0.0	5.24e0	5.16e0	2502.0	0.050
GD-Nesterov	0.0	4.91e0	4.91e0	125.0	0.004
QQN-StrongWolfe	0.0	4.74e0	3.76e0	347.6	0.015
QQN-Bisection-1	0.0	4.91e0	4.91e0	76.3	0.002
GD	0.0	4.91e0	4.91e0	1668.0	0.041
GD-	0.0	4.99e0	4.97e0	32.0	0.001
AdaptiveMomentum					

#### 25 Problem: Griewank\_5D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Robust	0.0	1.29e1	1.28e1	2502.0	0.059
QQN-	0.0	1.12e1	1.62e0	1154.2	0.023
GoldenSection					
Adam	0.0	$1.25\mathrm{e}1$	1.25e1	2502.0	0.052
L-BFGS-	0.0	1.09e1	5.15e0	1097.7	0.015
Aggressive					
Trust Region-	0.0	1.23e1	1.23e1	380.4	0.003
Conservative					
GD-WeightDecay	0.0	1.22e1	1.22e1	915.3	0.030
Trust Region-	0.0	$1.35\mathrm{e}1$	1.34e1	5.0	0.000
Adaptive					
Trust Region-	0.0	$1.35\mathrm{e}1$	1.35e1	5.0	0.000
Aggressive					

L-BFGS-	0.0	1.22e1	1.22e1	118.5	0.003
MoreThuente					
QQN-	0.0	1.28e1	1.22e1	115.1	0.004
Cubic Quadratic Interpolati	on				
QQN-StrongWolfe	0.0	1.09e1	5.74e0	1535.0	0.071
Adam-	0.0	1.23e1	1.23e1	1651.7	0.037
WeightDecay					
Trust Region-	0.0	1.35e1	1.34e1	5.0	0.000
Standard					
QQN-Bisection-1	0.0	1.06e1	2.18e0	872.0	0.020
GD-Momentum	0.0	1.22e1	1.22e1	387.2	0.012
GD	0.0	1.22e1	1.22e1	1668.0	0.044
L-BFGS	0.0	1.22e1	1.22e1	259.7	0.006
QQN-Bisection-2	0.0	1.21e1	9.76e0	984.9	0.025
Adam-Fast	0.0	1.23e1	1.22e1	67.1	0.001
GD-	0.0	1.23e1	1.22e1	133.1	0.005
AdaptiveMomentum					
Trust Region-	0.0	1.34e1	1.23e1	11.4	0.000
Precise					
GD-Nesterov	0.0	1.22e1	1.22e1	394.2	0.013
L-BFGS-Limited	0.0	1.22e1	1.22e1	632.5	0.015
Adam-AMSGrad	0.0	$1.25\mathrm{e}1$	1.25e1	2502.0	0.059
L-BFGS-	0.0	1.22e1	1.22e1	705.6	0.019
Conservative					

#### 26 Problem: Griewank\_10D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust Region-	0.0	2.60e1	2.60e1	5.0	0.000
Precise					
QQN-StrongWolfe	0.0	6.06e0	9.86e-3	2281.2	0.091
L-BFGS-	0.0	2.33e1	2.01e1	1668.3	0.049
Conservative					
QQN-	0.0	2.45e1	9.67e-1	108.6	0.003
CubicQuadraticInterp					
Adam-AMSGrad	0.0	2.54e1	2.53e1	2502.0	0.064
Adam-Robust	0.0	2.55e1	2.55e1	2502.0	0.063
Trust Region-	0.0	2.60e1	2.60e1	5.0	0.000
Aggressive					
GD	0.0	2.56e1	2.55e1	1668.0	0.049
GD-WeightDecay	0.0	2.48e1	2.48e1	1668.0	0.059
Adam-Fast	0.0	1.17e1	2.89e0	950.2	0.022
GD-Momentum	0.0	2.20e1	2.16e1	1668.0	0.054
QQN-Bisection-1	5.0	2.19e1	5.93e-10	2381.1	0.065
QQN-	0.0	2.35e1	2.19e0	4343.6	0.085
GoldenSection					
L-BFGS-	0.0	6.09e0	1.01e0	3817.8	0.054
Aggressive					
Adam	0.0	2.54e1	2.53e1	2502.0	0.057
Trust Region-	0.0	2.60e1	2.60e1	5.0	0.000
Adaptive					
Trust Region-	0.0	2.60e1	2.60e1	5.0	0.000
Standard					
Trust Region-	0.0	2.08e1	1.60e1	1557.3	0.012
Conservative					
Adam-	0.0	2.42e1	2.41e1	2502.0	0.061
WeightDecay					

QQN-Bisection-2	0.0	2.33e1	1.67e0	3260.0	0.083
L-BFGS	0.0	1.93e1	1.20e1	483.9	0.012
L-BFGS-	0.0	$1.42\mathrm{e}1$	1.93e0	499.3	0.011
MoreThuente					
L-BFGS-Limited	0.0	1.95e1	8.48e0	2256.7	0.059
GD-Nesterov	0.0	2.21e1	2.15e1	1668.0	0.058
GD-	0.0	1.90e1	1.39e1	920.4	0.035
AdaptiveMomentum					

#### 27 Problem: Schwefel\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS	0.0	7.11e2	7.11e2	292.1	0.008
GD	0.0	7.11e2	7.11e2	1668.0	0.041
GD-	0.0	7.12e2	7.12e2	89.0	0.003
AdaptiveMomentum					
QQN-	0.0	7.11e2	7.11e2	145.1	0.002
GoldenSection					
Trust Region-	0.0	7.12e2	7.12e2	711.8	0.005
Standard					
Adam-Fast	0.0	7.11e2	7.11e2	1972.4	0.039
Trust Region-	0.0	8.84e2	8.83e2	3002.0	0.019
Precise					
Adam-AMSGrad	0.0	9.35e2	9.34e2	2502.0	0.057
QQN-	0.0	7.11e2	7.11e2	57.9	0.001
CubicQuadraticInter	polation				
L-BFGS-	0.0	7.11e2	7.11e2	2095.3	0.057
Conservative					
Adam	0.0	9.35e2	9.33e2	2502.0	0.050
Adam-	0.0	9.10e2	9.09e2	2502.0	0.054
WeightDecay					
Adam-Robust	0.0	9.37e2	9.36e2	2502.0	0.057
QQN-Bisection-2	0.0	7.11e2	7.11e2	50.9	0.001
L-BFGS-	0.0	7.11e2	7.11e2	147.6	0.003
MoreThuente					
Trust Region-	0.0	7.11e2	7.11e2	2834.8	0.018
Adaptive					
Trust Region-	0.0	9.37e2	9.35e2	3002.0	0.019
Conservative					
Trust Region-	0.0	7.17e2	7.16e2	179.0	0.001
Aggressive					
QQN-Bisection-1	0.0	7.11e2	7.11e2	47.6	0.001
GD-Momentum	0.0	7.11e2	7.11e2	277.4	0.008
${\bf QQN\text{-}StrongWolfe}$	50.0	-7.26e2	-1.07e4	379.3	0.016
L-BFGS-Limited	0.0	7.11e2	7.11e2	513.3	0.013
L-BFGS-	0.0	7.11e2	7.11e2	101.0	0.001
Aggressive					
GD-Nesterov	0.0	7.11e2	7.11e2	292.6	0.009
GD-WeightDecay	0.0	7.11e2	7.11e2	1115.0	0.034

#### 28 Problem: Schwefel\_5D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam	0.0	2.34e3	2.33e3	2502.0	0.052
GD	0.0	1.78e3	1.78e3	1668.0	0.044

Trust	Region-	0.0	2.12e3	2.11e3	3002.0	0.020
Adaptive			4 = 0 0			
Adam-Fas		0.0	1.78e3	1.78e3	1907.4	0.039
QQN-Stro	ongWolfe	50.0	-1.74e3	-3.30e4	349.1	0.015
QQN-		0.0	1.78e3	1.78e3	149.4	0.002
GoldenSec						
QQN-Bise		0.0	1.78e3	1.78e3	52.0	0.001
GD-Weigl		0.0	1.78e3	1.78e3	1124.1	0.036
L-BFGS-I	Limited	0.0	1.79e3	1.78e3	661.0	0.020
GD-		0.0	1.78e3	1.78e3	124.3	0.004
Adaptivel	Momentum					
Adam-		0.0	2.28e3	2.27e3	2502.0	0.056
WeightDe	cay					
L-BFGS-	·	0.0	1.78e3	1.78e3	2163.4	0.061
Conservat	ive					
Adam-AN	ISGrad	0.0	2.34e3	2.34e3	2502.0	0.059
QQN-		0.0	1.78e3	1.78e3	57.3	0.001
	draticInterp					
L-BFGS-		0.0	1.78e3	1.78e3	158.2	0.004
MoreThue	ente					
Adam-Ro		0.0	2.34e3	2.34e3	2502.0	0.059
GD-Mome		0.0	1.78e3	1.78e3	323.9	0.010
L-BFGS		0.0	1.78e3	1.78e3	325.7	0.008
Trust	Region-	0.0	1.78e3	1.78e3	1774.8	0.012
Standard	10081011	0.0	2.,000	11,000	1,,1,0	0.012
Trust	Region-	0.0	2.30e3	2.30e3	3002.0	0.020
Precise	region	0.0	2.0000	2.0000	9002.0	0.020
L-BFGS-		0.0	1.78e3	1.78e3	101.0	0.001
Aggressive	3	0.0	1.1009	1.1009	101.0	0.001
Trust	Region-	0.0	2.36e3	2.35e3	3002.0	0.020
Conservat	~	0.0	2.3003	2.5505	3002.0	0.020
Trust	Region-	0.0	1.78e3	1.78e3	446.0	0.003
Aggressive		0.0	1.1069	1.1069	440.0	0.000
GD-Neste		0.0	1.78e3	1.78e3	339.1	0.011
					52.0	
QQN-Bise	ecuon-1	0.0	1.78e3	1.78e3	5Z.U	0.001

#### 29 Problem: Schwefel\_10D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-	0.0	4.55e3	4.55e3	2502.0	0.060
WeightDecay					
QQN-	0.0	3.55e3	3.55e3	56.9	0.001
CubicQuadraticInter	rpolation				
Trust Region-	0.0	4.48e3	4.48e3	3002.0	0.022
Adaptive					
Trust Region-	0.0	4.72e3	4.72e3	3002.0	0.022
Conservative					
GD-	0.0	3.55e3	3.55e3	164.1	0.007
AdaptiveMomentum	[				
L-BFGS	0.0	3.55e3	3.55e3	300.9	0.008
QQN-StrongWolfe	50.0	8.35e2	-2.28e3	395.5	0.019
Trust Region-	0.0	3.56e3	3.56e3	888.8	0.007
Aggressive					
GD-Nesterov	0.0	3.55e3	3.55e3	392.1	0.013
Adam-Robust	0.0	4.69e3	4.68e3	2502.0	0.064
QQN-	0.0	3.55e3	3.55e3	147.8	0.002
GoldenSection					

Trust	Region-	0.0	4.67e3	4.67e3	3002.0	0.022
Precise						
L-BFGS-		0.0	3.55e3	3.55e3	159.3	0.004
MoreThue	nte					
GD-Mome	$_{ m ntum}$	0.0	3.55e3	3.55e3	380.3	0.012
Trust	Region-	0.0	3.73e3	3.73e3	3002.0	0.022
Standard						
L-BFGS-		0.0	3.55e3	3.55e3	101.0	0.002
Aggressive	<b>)</b>					
QQN-Bise	ction-1	0.0	3.55e3	3.55e3	52.0	0.001
L-BFGS-		0.0	3.55e3	3.55e3	2188.9	0.064
Conservati	ive					
GD		0.0	3.55e3	3.55e3	1668.0	0.046
GD-Weigh	tDecay	0.0	3.55e3	3.55e3	1194.6	0.040
Adam-Fast	t	0.0	3.55e3	3.55e3	1628.7	0.035
$\operatorname{Adam}$		0.0	4.67e3	4.67e3	2502.0	0.056
QQN-Bise	ction-2	0.0	3.55e3	3.55e3	52.0	0.002
L-BFGS-L	imited	0.0	3.55e3	3.42e3	577.7	0.020
Adam-AM	SGrad	0.0	4.67e3	4.67e3	2502.0	0.063

# 30 Problem: Levy\_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-	100.0	9.83e-7	9.67e-7	2229.0	0.048
WeightDecay					
GD-Momentum	0.0	1.57e-1	1.15e-1	22.5	0.001
Adam-AMSGrad	0.0	2.61e-2	1.24e-2	2502.0	0.058
QQN-Bisection-1	100.0	1.15e-7	2.81e-9	102.2	0.003
QQN-	100.0	8.29e-8	5.12e-10	87.6	0.003
CubicQuadraticInterp	polation				
QQN-StrongWolfe	100.0	2.98e-7	1.89e-10	79.0	0.003
L-BFGS-Limited	95.0	1.05e-6	1.49e-7	583.7	0.013
GD-	0.0	3.36e-1	2.35e-1	16.4	0.001
AdaptiveMomentum					
Trust Region-	0.0	1.01e1	6.77e-2	27.2	0.000
Adaptive					
Adam-Robust	0.0	4.34e-2	1.83e-2	2502.0	0.058
L-BFGS-	85.0	9.25 e-7	8.80e-8	626.6	0.016
Conservative					
GD	0.0	2.39e-4	1.85e-4	1668.0	0.043
GD-WeightDecay	100.0	9.96e-7	9.93e-7	1453.5	0.046
Trust Region-	0.0	3.36e0	1.74e-3	18.2	0.000
Standard					
L-BFGS-	95.0	5.43e-6	7.84e-9	286.5	0.006
MoreThuente					
GD-Nesterov	0.0	1.31e-1	9.69e-2	22.4	0.001
Adam-Fast	0.0	7.87e-2	7.26e-2	36.5	0.001
Trust Region-	0.0	1.34e-2	9.88e-3	385.2	0.003
Conservative					
L-BFGS	80.0	7.38e-3	1.57e-7	285.0	0.007
Trust Region-	0.0	6.16e-2	3.16e-2	70.6	0.001
Precise					
QQN-	100.0	1.81e-7	1.48e-10	300.9	0.005
GoldenSection					
L-BFGS-	70.0	6.19e-4	6.80e-8	1303.1	0.017
Aggressive					

Trust	Region-	0.0	1.31e0	9.21e-1	5.0	0.000
Aggressive						
Adam		0.0	5.69e-3	1.60e-3	2502.0	0.052
QQN-Bisec	tion-2	100.0	1.80e-7	2.53e-9	93.0	0.002

## 31 Problem: Levy\_5D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMSGrad	0.0	7.64e-2	5.66e-2	2502.0	0.060
L-BFGS	80.0	4.35e-2	2.47e-7	173.6	0.004
L-BFGS-	75.0	2.73e-2	6.19e-7	825.3	0.012
Aggressive					
L-BFGS-Limited	70.0	2.87e-4	2.75e-7	1375.8	0.025
Adam-Robust	0.0	6.80e-2	3.18e-2	2502.0	0.060
Trust Region-	0.0	2.49e0	4.43e-4	26.4	0.000
Standard					
Trust Region-	0.0	4.37e-1	1.82e-1	8.2	0.000
Aggressive					
Adam-Fast	0.0	1.67e-1	1.54e-1	37.0	0.001
GD	0.0	2.32e-4	1.98e-4	1668.0	0.045
GD-Momentum	10.0	1.72e-1	9.75e-7	56.5	0.002
L-BFGS-	100.0	6.68e-7	1.69e-8	332.8	0.008
Conservative					
QQN-	100.0	2.65e-7	2.28e-9	96.0	0.003
CubicQuadraticInt	erpolation				
Trust Region-	0.0	1.06e1	4.60e-2	46.9	0.000
Adaptive					
L-BFGS-	100.0	6.91e-7	1.12e-9	296.9	0.006
MoreThuente					
QQN-Bisection-2	100.0	2.16e-7	1.51e-10	101.3	0.003
GD-WeightDecay	100.0	9.96e-7	9.93e-7	1448.8	0.048
Trust Region-	- 0.0	3.62e-2	1.69e-2	168.1	0.001
Precise					
Trust Region-	- 0.0	1.23e-2	8.28e-3	996.6	0.007
Conservative					
Adam	0.0	7.73e-3	3.13e-3	2502.0	0.054
GD-Nesterov	25.0	9.57e-2	9.78e-7	102.2	0.003
QQN-Bisection-1	100.0	3.74e-7	4.32e-9	103.4	0.003
GD-	0.0	4.77e-1	3.80e-1	19.2	0.001
AdaptiveMomentur					
Adam-	60.0	1.43e-6	9.65e-7	2087.2	0.048
WeightDecay					
QQN-StrongWolfe	100.0	3.31e-7	1.45e-9	69.2	0.002
QQN-	100.0	2.57e-7	1.77e-10	382.3	0.007
GoldenSection					

## $32 \quad Problem: \ Levy\_10D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS	80.0	3.96e-3	3.36e-7	223.2	0.005
GD-WeightDecay	100.0	9.97e-7	9.93e-7	1451.7	0.052
L-BFGS-	25.0	1.27e0	4.01e-7	2637.9	0.034
Aggressive					
QQN-Bisection-2	100.0	2.70e-7	1.96e-10	111.1	0.003
L-BFGS-Limited	80.0	2.41e-5	5.25e-7	1051.8	0.021

GD-Momentum	0.0	2.58e-1	1.93e-1	28.1	0.001
QQN-	100.0	2.74e-7	1.87e-10	399.3	0.007
GoldenSection					
Adam	0.0	6.46e-3	2.32e-3	2502.0	0.058
Adam-	0.0	2.80e-6	1.41e-6	1897.2	0.047
WeightDecay					
QQN-Bisection-1	100.0	2.49e-7	1.67e-8	109.7	0.003
Adam-Robust	0.0	9.82e-2	6.13e-2	2502.0	0.065
Trust Region-	0.0	7.36e-2	1.96e-2	84.5	0.001
Adaptive					
QQN-	100.0	2.83e-7	2.06e-9	98.6	0.003
CubicQuadraticInterpol	ation				
Trust Region-	0.0	3.14e-2	1.56e-2	325.6	0.003
Precise					
L-BFGS-	100.0	6.63e-7	7.05e-8	365.9	0.010
Conservative					
$\operatorname{GD}$	0.0	2.28e-4	1.85e-4	1668.0	0.049
Trust Region-	0.0	2.04e0	8.10e-2	29.6	0.000
Standard					
Trust Region-	0.0	6.17e-1	2.22e-1	11.8	0.000
Aggressive					
QQN-StrongWolfe	100.0	1.46e-7	7.28e-10	70.1	0.002
GD-Nesterov	0.0	1.74e-1	1.40e-1	28.4	0.001
L-BFGS-	100.0	7.24e-7	8.40e-8	165.1	0.004
MoreThuente					
Adam-AMSGrad	0.0	1.51e-1	1.12e-1	2502.0	0.065
Adam-Fast	0.0	3.12e-1	2.89e-1	36.9	0.001
Trust Region-	0.0	1.21e-2	9.38e-3	1976.9	0.016
Conservative					
GD-	0.0	6.50 e-1	5.54e-1	21.9	0.001
AdaptiveMomentum					

#### 33 Problem: $Zakharov_2D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Robust	0.0	3.83e-2	6.91e-3	2502.0	0.056
QQN-	100.0	1.52e-9	1.85e-11	89.2	0.003
CubicQuadraticInter	polation				
GD-Nesterov	0.0	6.00e-1	5.01e-1	16.7	0.001
Trust Region-	0.0	6.91e-2	8.62e-3	503.2	0.003
Precise					
QQN-Bisection-1	100.0	7.75e-14	9.76e-18	114.6	0.002
L-BFGS-	70.0	3.75e0	4.91e-10	1213.0	0.007
Aggressive					
GD-WeightDecay	25.0	8.53e-2	8.63e-9	47.9	0.001
Adam-Fast	0.0	2.81e-1	2.54e-1	37.5	0.001
QQN-StrongWolfe	100.0	4.83e-9	1.51e-9	95.0	0.003
L-BFGS-	75.0	6.39e-7	5.41e-10	863.5	0.022
Conservative					
L-BFGS-	100.0	7.07e-9	3.76e-9	67.3	0.001
MoreThuente					
GD	100.0	9.76e-9	9.61e-9	417.4	0.011
GD-Momentum	0.0	1.08e0	8.96e-1	17.1	0.000
Adam-	100.0	9.88e-9	9.74e-9	2040.6	0.043
WeightDecay					
Trust Region- Adaptive	0.0	4.96e3	3.14e-2	121.0	0.001

Trust	Region-	0.0	2.38e0	1.22e-1	38.3	0.000
Standard						
L-BFGS		60.0	2.88e-2	5.69e-10	158.7	0.003
L-BFGS-	Limited	90.0	6.46 e-7	1.04e-9	658.2	0.012
QQN-Bis	ection-2	100.0	9.02e-14	4.37e-17	84.0	0.002
Trust	Region-	0.0	8.84e-1	1.05e-2	2569.1	0.016
Conserva	tive					
Trust	Region-	0.0	5.16e4	5.33e-1	18.9	0.000
Aggressiv	re					
Adam-Al	MSGrad	0.0	9.29e-2	4.85e-2	2502.0	0.057
QQN-		100.0	7.14e-11	5.21e-21	180.9	0.003
GoldenSe	ection					
$\operatorname{Adam}$		0.0	1.77e-3	5.24e-5	2502.0	0.050
GD-		0.0	7.05e-1	4.12e-1	16.6	0.001
Adaptive	Momentum					

#### 34 Problem: Zakharov\_5D

Optimizer		Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMS	Grad	0.0	2.23e-2	4.59e-3	2502.0	0.059
Trust	Region-	0.0	2.28e3	6.92e2	3002.0	0.020
Standard						
Trust	Region-	0.0	$5.65\mathrm{e}1$	6.02e-1	2415.8	0.016
Aggressive						
Adam		0.0	2.82e-3	2.65e-4	2502.0	0.052
GD-		0.0	$6.97\mathrm{e}1$	4.83e1	18.5	0.001
AdaptiveMo	mentum					
Trust	Region-	0.0	3.28e3	2.05e3	3002.0	0.020
Conservative						
Trust	Region-	0.0	3.50e3	2.04e3	3002.0	0.020
Precise						
Adam-Robu	ıst	0.0	7.06e-2	6.18e-3	2502.0	0.059
L-BFGS		0.0	2.69e1	2.58e-1	93.0	0.002
QQN-Strong	gWolfe	100.0	2.41e-9	2.91e-14	100.3	0.003
QQN-		100.0	2.83e-12	2.45e-13	138.0	0.002
GoldenSecti	on					
QQN-		100.0	1.52e-9	3.50e-12	205.0	0.008
CubicQuadr						
QQN-Bisect	ion-1	100.0	1.12e-12	5.59e-14	118.0	0.002
L-BFGS-		0.0	4.22e1	2.82e1	3309.5	0.061
Aggressive						
Adam-		60.0	8.64e-3	9.76e-9	1586.3	0.035
WeightDeca						
GD-Moment	$\operatorname{tum}$	0.0	5.45e1	3.48e1	18.9	0.001
Adam-Fast		0.0	9.66e0	7.78e0	34.8	0.001
	Region-	0.0	3.10e3	1.77e3	3002.0	0.020
Adaptive						
GD-Nestero		0.0	5.05e0	1.42e0	18.1	0.001
GD-WeightI		25.0	7.62e-1	9.24e-9	52.5	0.002
L-BFGS-Lin	$\operatorname{nited}$	95.0	1.94e-8	7.85e-9	1078.6	0.016
L-BFGS-		100.0	7.32e-9	2.42e-10	492.9	0.012
Conservative						
QQN-Bisect	ion-2	100.0	1.50e-9	2.56e-15	114.7	0.003
L-BFGS-		100.0	9.32e-9	8.60e-9	634.6	0.013
MoreThuent	te					
GD		100.0	9.78e-9	9.63e-9	477.1	0.012

#### $35 \quad Problem: \ Zakharov\_10D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Fast	0.0	1.13e3	3.62e2	34.7	0.001
Trust Region-	0.0	$5.56\mathrm{e}5$	4.08e5	3002.0	0.022
Adaptive					
QQN-Bisection-1	100.0	1.96e-9	4.68e-16	161.5	0.003
QQN-	100.0	1.75e-9	4.53e-10	312.8	0.013
CubicQuadraticInterp	polation				
L-BFGS-Limited	55.0	9.29 e-2	5.26e-9	2588.7	0.034
QQN-StrongWolfe	95.0	9.67e-4	2.12e-14	281.4	0.009
L-BFGS-	0.0	7.82e23	1.68e5	68.5	0.002
Conservative					
L-BFGS-	0.0	1.67e-6	4.87e-7	2886.3	0.059
MoreThuente					
GD-Momentum	0.0	6.41e3	1.95e3	21.1	0.001
GD-	0.0	6.98e3	4.52e3	20.9	0.001
AdaptiveMomentum					
Adam-Robust	0.0	3.12e0	2.92e-2	2502.0	0.062
GD-Nesterov	0.0	8.62e2	4.52e1	20.2	0.001
Trust Region-	0.0	5.73e5	4.07e5	3002.0	0.022
Precise					
QQN-	100.0	1.50e-9	3.14e-19	233.1	0.004
GoldenSection					
GD	10.0	3.37e-1	9.65e-9	686.0	0.020
L-BFGS-	0.0	9.79e-1	1.37e-2	3811.6	0.052
Aggressive					
L-BFGS	0.0	2.07e4	6.23e3	3286.7	0.060
Adam-AMSGrad	0.0	1.09e-1	3.67e-2	2016.2	0.050
Adam-	0.0	1.79e-1	1.29e-1	683.1	0.016
WeightDecay					
Trust Region-	0.0	5.85e5	4.55e5	3002.0	0.022
Conservative		0.0000		000210	0.0
Trust Region-	0.0	5.60e5	4.02e5	3002.0	0.022
Aggressive		0.0000		000210	0.0
QQN-Bisection-2	100.0	1.58e-9	1.13e-10	159.2	0.004
Trust Region-	0.0	5.72e5	3.74e5	3002.0	0.022
Standard	···	0., <b>2</b> 00	3., 200	5002.0	V.V
Adam	0.0	4.52e-2	1.69e-3	2253.9	0.050
GD-WeightDecay	30.0	1.51e1	8.77e-9	67.5	0.002

### $36 \quad Problem: \ Ill Conditioned Rosenbrock\_2D\_alpha 100$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	65.0	1.52e-2	3.79e-9	1487.5	0.027
MoreThuente					
Adam-AMSGrad	0.0	3.83e0	4.66e-1	678.1	0.015
Adam-	0.0	4.11e0	1.55e-2	231.9	0.005
WeightDecay					
Trust Region-	0.0	4.66e0	4.01e0	27.6	0.000
Aggressive					
Trust Region-	0.0	7.26e0	3.76e0	946.2	0.006
Precise					
GD-	0.0	8.18e-1	4.17e-2	49.2	0.002
AdaptiveMomentum					

Trust Region-	0.0	2.84e1	1.90e-1	2770.7	0.017
Conservative					
Adam	0.0	1.22e0	4.86e-1	2502.0	0.049
L-BFGS-Limited	0.0	3.93e0	3.26e-2	2251.6	0.025
Adam-Robust	0.0	4.04e0	1.90e0	419.2	0.009
GD-Momentum	0.0	5.61e0	4.79e-1	23.8	0.001
L-BFGS	0.0	1.36e2	8.12e-1	121.5	0.002
L-BFGS-	0.0	3.18e1	4.34e0	3852.0	0.027
Aggressive					
Trust Region-	0.0	4.18e0	3.95e0	89.6	0.001
Standard					
Trust Region-	0.0	4.12e0	3.83e0	494.4	0.003
Adaptive					
QQN-Bisection-1	5.0	4.65e-1	1.58e-8	2369.2	0.052
QQN-	35.0	3.48e-2	4.34e-10	1722.9	0.070
CubicQuadraticInterpo	olation				
QQN-Bisection-2	25.0	5.88e-2	6.64e-9	479.6	0.012
GD	0.0	1.23e0	7.46e-1	854.0	0.021
GD-WeightDecay	0.0	3.65e0	3.75e-2	58.7	0.002
QQN-StrongWolfe	20.0	5.46e-2	9.29e-9	2343.8	0.072
Adam-Fast	0.0	2.13e0	2.59e-5	313.6	0.006
QQN-	0.0	1.25e-1	6.99e-5	4459.4	0.081
GoldenSection					
L-BFGS-	80.0	2.69e-4	2.05e-9	1800.6	0.030
Conservative					
GD-Nesterov	0.0	1.49e0	5.05e-2	46.1	0.001

# ${\bf 37}\quad Problem:\ Ill Conditioned Rosenbrock\_5D\_alpha 100$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMSGrad	0.0	4.40e0	3.25e0	2442.0	0.056
Trust Region-	0.0	6.23e1	4.66e0	2827.2	0.018
Standard					
L-BFGS	0.0	1.50e2	1.98e1	135.3	0.002
Adam	0.0	3.92e0	2.83e0	2471.6	0.050
L-BFGS-	20.0	1.74e8	3.07e-7	2320.8	0.041
MoreThuente					
QQN-	35.0	4.32e-1	5.69e-8	3805.3	0.070
GoldenSection					
L-BFGS-Limited	0.0	4.05e-1	3.00e-1	4035.2	0.045
GD-Nesterov	0.0	4.24e0	3.82e-1	372.4	0.012
GD-Momentum	0.0	3.55e1	1.96e1	20.8	0.001
QQN-Bisection-2	15.0	2.99e-1	4.91e-8	1647.8	0.041
QQN-Bisection-1	80.0	4.19e-1	2.00e-9	1511.2	0.040
GD-	0.0	4.60e1	3.36e1	20.6	0.001
AdaptiveMomentum					
Adam-	0.0	1.84e0	1.84e-3	1797.3	0.038
WeightDecay					
$\operatorname{GD}$	0.0	5.09e0	4.75e0	32.5	0.001
Trust Region-	0.0	8.41e2	5.05e2	3002.0	0.019
Adaptive					
L-BFGS-	0.0	8.07e2	1.72e1	3851.6	0.029
Aggressive					
Trust Region-	0.0	1.02e3	7.14e2	3002.0	0.019
Conservative					
Trust Region-	0.0	5.00e0	4.66e0	776.1	0.005
Aggressive					

Trust Region-	0.0	1.01e3	8.08e2	3002.0	0.019
Precise					
Adam-Robust	0.0	1.46e1	6.12e0	2502.0	0.058
GD-WeightDecay	0.0	7.32e-1	1.46e-1	459.3	0.015
L-BFGS-	0.0	2.02e1	2.46e-1	3386.6	0.034
Conservative					
QQN-StrongWolfe	100.0	1.59e-7	2.70e-9	1191.3	0.037
QQN-	65.0	1.97e-1	4.07e-9	1403.3	0.062
CubicQuadraticInterp	olation				
Adam-Fast	0.0	1.43e1	4.36e-2	49.9	0.001

### $38 \quad Problem: \ Ill Conditioned Rosenbrock\_10D\_alpha 100$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-1	65.0	1.22e0	1.19e-7	1844.9	0.052
GD-WeightDecay	0.0	1.34e0	1.70e-1	1217.3	0.042
GD-Momentum	0.0	3.04e1	5.05e-1	517.6	0.016
Trust Region-	0.0	2.16e3	1.76e3	3002.0	0.020
Conservative					
L-BFGS-	0.0	3.41e0	3.35e-5	2853.3	0.053
MoreThuente					
Adam-Fast	0.0	1.53e1	1.91e-1	149.2	0.003
QQN-	0.0	4.56e0	4.63e-1	4477.5	0.086
GoldenSection					
L-BFGS	0.0	1.19e2	2.51e1	338.9	0.005
L-BFGS-	0.0	1.90e4	1.03e0	3278.5	0.038
Conservative					
GD-	0.0	1.02e2	8.45e1	23.1	0.001
AdaptiveMomentum					
QQN-Bisection-2	5.0	4.40e0	1.07e-7	1646.8	0.042
GD-Nesterov	0.0	1.17e0	1.12e0	1514.9	0.051
Adam	0.0	9.11e0	7.98e0	2475.8	0.054
L-BFGS-Limited	0.0	3.86e0	2.71e0	4036.8	0.048
Adam-AMSGrad	0.0	9.31e0	8.08e0	2490.9	0.061
GD	0.0	9.96e0	9.64e0	46.6	0.001
Adam-	0.0	2.38e0	6.46e-6	2074.9	0.048
WeightDecay					
Adam-Robust	0.0	3.49e1	1.61e1	2502.0	0.061
QQN-	75.0	6.00e-1	1.13e-7	1666.2	0.073
CubicQuadraticInter	-				
Trust Region-	0.0	1.06e3	6.70e2	3002.0	0.021
Standard					
Trust Region-	0.0	2.02e3	1.64e3	3002.0	0.020
Adaptive					
Trust Region-	0.0	6.55e1	9.60e0	1618.0	0.011
Aggressive					
Trust Region-	0.0	2.10e3	1.66e3	3002.0	0.021
Precise					
QQN-StrongWolfe	70.0	5.85e-1	9.35e-8	1847.0	0.059
L-BFGS-	0.0	1.64e2	5.82e1	3850.3	0.051
Aggressive					

### ${\bf 39 \quad Problem: \ Trigonometric\_2D}$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
o p cimino ci	2 4 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	TITOGET E IIIGE TOLLO	2000 / 00100	11100011 1 01110 11 10110	1110011 111110 (0)

Adaptive         Trust       Region-       0.0       2.50e1       3.36e-2       18.3       0.000         Aggressive         Trust       Region-       0.0       1.22e1       2.19e-1       13.5       0.000         Standard
Aggressive         Trust         Region-         0.0         1.22e1         2.19e-1         13.5         0.000
Trust Region- 0.0 1.22e1 2.19e-1 13.5 0.000
Standard
Adam- 75.0 1.76e-5 3.77e-7 362.9 0.008
WeightDecay
Trust Region- 0.0 1.11e-1 1.34e-2 16.8 0.000
Precise
QQN- 100.0 3.11e-7 6.47e-10 95.8 0.003
CubicQuadraticInterpolation
GD 100.0 9.80e-7 9.61e-7 361.1 0.009
QQN-Bisection-2 100.0 2.06e-7 7.96e-10 207.1 0.005
GD-WeightDecay 85.0 2.11e-4 8.75e-7 102.4 0.003
L-BFGS 90.0 4.06e-3 8.58e-10 93.3 0.002
GD-Momentum 0.0 3.43e-2 2.30e-4 22.1 0.001
QQN- 95.0 1.28e-3 6.12e-10 462.6 0.008
GoldenSection
QQN-Bisection-1 100.0 2.08e-7 1.49e-9 220.8 0.004
L-BFGS- 80.0 3.62e-4 7.20e-8 843.0 0.009
Aggressive
L-BFGS- 100.0 4.44e-7 1.25e-8 85.4 0.001
MoreThuente
L-BFGS-Limited 90.0 7.02e-6 2.15e-8 433.9 0.008
L-BFGS- 100.0 5.22e-7 3.92e-8 129.4 0.003
Conservative
QQN-StrongWolfe 100.0 4.90e-7 2.93e-9 107.8 0.003
Adam 100.0 9.77e-7 9.40e-7 1269.5 0.025
Adam-Fast 5.0 6.08e-3 1.89e-8 47.6 0.001
Adam-Robust 5.0 3.10e-3 5.08e-7 442.4 0.010
GD-Nesterov 0.0 2.31e-2 5.79e-4 24.2 0.001
GD- $0.0$ $7.65e-3$ $5.90e-5$ $23.2$ $0.001$
AdaptiveMomentum
Adam-AMSGrad 85.0 1.08e-6 9.47e-7 1241.2 0.028
Trust Region- 0.0 8.94e-3 1.87e-3 43.2 0.000
Conservative

## 40 Problem: Trigonometric\_5D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	0.0	1.22e0	1.48e-1	3851.9	0.025
Aggressive					
QQN-StrongWolfe	90.0	7.54e-7	1.51e-8	1274.7	0.042
L-BFGS-	75.0	5.44e-4	1.15e-7	1087.8	0.022
MoreThuente					
GD-WeightDecay	75.0	1.36e-2	8.18e-7	105.4	0.004
Trust Region-	0.0	1.41e0	6.56e-1	5.2	0.000
Aggressive					
Adam	85.0	1.12e-2	8.42e-7	1728.1	0.038
Trust Region-	0.0	5.59e-2	1.47e-2	84.8	0.001
Precise					
L-BFGS-Limited	95.0	2.14e-3	7.20e-7	1331.0	0.019
GD-	0.0	1.79e-2	1.30e-4	30.6	0.001
AdaptiveMomentum					
GD-Momentum	0.0	1.57e-1	4.87e-3	25.0	0.001

Trust	Region-	0.0	1.25 e-1	2.86e-2	26.1	0.000
Adaptive						
Adam-Fas	t	0.0	6.17e-2	7.00e-4	54.8	0.001
QQN-		100.0	2.34e-7	3.03e-9	1123.5	0.021
GoldenSed	etion					
QQN-Bise	ection-1	90.0	8.71e-4	1.72e-10	664.0	0.015
Trust	Region-	0.0	4.09e-1	1.21e-1	9.8	0.000
Standard						
QQN-		100.0	4.16e-7	1.09e-9	389.1	0.016
CubicQua	draticInterpol	lation				
Trust	Region-	0.0	1.91e-2	8.29e-4	425.6	0.003
Conservat	ive					
QQN-Bise	ection-2	80.0	3.22e-7	6.17e-10	305.6	0.008
GD-Neste	rov	0.0	1.60e-2	4.85e-4	33.5	0.001
L-BFGS-		90.0	2.73e-3	1.66e-7	887.4	0.014
Conservat	ive					
GD		100.0	9.83e-7	9.62e-7	399.2	0.011
Adam-Ro	bust	20.0	1.43e-2	2.34e-7	978.1	0.024
L-BFGS		0.0	4.77e0	1.26e-2	233.9	0.004
Adam-AN	ISGrad	35.0	2.94e-2	9.71e-7	2192.1	0.053
Adam-		70.0	1.95e-2	3.70e-7	784.6	0.018
WeightDe	cay					

# 41 Problem: Trigonometric\_10D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-	70.0	4.25e-2	8.69e-7	803.8	0.023
WeightDecay					
QQN-	100.0	4.48e-7	9.99e-9	329.2	0.016
CubicQuadraticInter	polation				
GD-Nesterov	0.0	1.85e-1	7.60e-2	45.1	0.002
Adam	35.0	9.70e-2	9.68e-7	2080.5	0.056
QQN-	80.0	4.35e-4	1.12e-7	1834.2	0.039
GoldenSection					
Trust Region-	0.0	1.22e0	3.82e-3	2651.3	0.028
Conservative					
GD	55.0	4.39e-3	9.77e-7	958.0	0.035
Trust Region-	0.0	6.02e-1	1.37e-1	34.1	0.000
Standard					
L-BFGS	0.0	4.76e1	8.87e0	77.5	0.001
Adam-Robust	10.0	7.13e-2	9.71e-7	1825.9	0.054
Trust Region-	0.0	1.90e0	9.79e-1	12.2	0.000
Aggressive					
L-BFGS-	25.0	9.45e0	7.52e-7	3123.2	0.042
Conservative					
L-BFGS-	60.0	1.34e-2	4.78e-7	1949.3	0.048
MoreThuente					
L-BFGS-Limited	0.0	7.31e-3	9.82e-6	3943.6	0.057
GD-Momentum	0.0	1.17e0	2.40e-3	34.0	0.001
L-BFGS-	0.0	1.00e1	2.42e0	3852.0	0.032
Aggressive					
GD-WeightDecay	85.0	8.48e-3	8.63e-7	134.4	0.006
${\bf QQN\text{-}StrongWolfe}$	100.0	2.92e-7	9.27e-9	347.3	0.015
Adam-AMSGrad	0.0	3.00e-2	7.20e-6	2502.0	0.074
Trust Region-	0.0	1.86e-1	2.37e-2	118.4	0.001
Adaptive					

	Region-	0.0	1.07e-1	2.28e-2	455.3	0.005
Precise						
QQN-Bisect	tion-1	85.0	7.53e-4	2.91e-8	858.0	0.027
Adam-Fast		0.0	6.24e-2	2.65e-4	102.7	0.003
GD-		0.0	1.41e-1	4.06e-2	48.0	0.002
AdaptiveMo	omentum					
QQN-Bisect	tion-2	75.0	3.56e-7	8.26e-8	357.8	0.011

## ${\bf 42 \quad Problem:\ Penalty I\_2D\_alpha1e6}$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust Region-	0.0	1.39e5	8.35e3	3002.0	0.019
Conservative					
QQN-	0.0	1.12e0	1.12e0	38.0	0.001
CubicQuadraticInter	polation				
L-BFGS-Limited	0.0	1.18e0	1.13e0	4341.4	0.030
Adam-	0.0	1.59e0	1.14e0	226.2	0.005
WeightDecay					
L-BFGS-	0.0	4.78e0	1.18e0	2878.8	0.043
MoreThuente					
Trust Region-	0.0	1.39e5	1.23e4	3002.0	0.019
Aggressive					
GD-WeightDecay	0.0	5.08e0	1.34e0	13.2	0.000
GD-	0.0	7.74e2	1.15e0	17.6	0.001
AdaptiveMomentum					
QQN-StrongWolfe	0.0	1.14e0	1.12e0	3162.1	0.086
Adam-AMSGrad	0.0	1.16e0	1.13e0	647.9	0.015
L-BFGS-	0.0	4.69e4	1.17e0	2664.3	0.051
Aggressive					
L-BFGS-	0.0	1.13e0	1.12e0	1183.8	0.015
Conservative					
QQN-	0.0	1.13e0	1.12e0	2567.9	0.046
GoldenSection					
L-BFGS	0.0	7.22e4	1.52e0	109.5	0.002
Adam-Fast	0.0	1.01e2	1.26e0	22.0	0.000
Trust Region-	0.0	1.43e5	3.60e4	3002.0	0.019
Standard					
GD	0.0	1.37e1	1.28e0	14.9	0.000
Trust Region-	0.0	1.67e5	2.57e4	3002.0	0.019
Precise					
QQN-Bisection-1	0.0	1.12e0	1.12e0	1032.0	0.032
QQN-Bisection-2	0.0	2.42e0	1.17e0	299.3	0.007
Adam	0.0	1.17e0	1.13e0	625.9	0.013
GD-Momentum	0.0	$6.67\mathrm{e}1$	1.26e0	13.0	0.000
GD-Nesterov	0.0	1.46e2	1.38e0	13.9	0.000
Adam-Robust	0.0	2.11e0	1.13e0	107.1	0.003
Trust Region-	0.0	1.32e5	1.57e4	3002.0	0.019
Adaptive					

## $43 \quad Problem: \ Penalty I\_5D\_alpha 1e6$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS- Aggressive	0.0	2.82e5	3.49e0	1110.7	0.022
GD	0.0	8.04e1	3.17e0	17.1	0.000

Trust Region-	0.0	3.90e5	1.47e5	3002.0	0.020
Conservative					
QQN-	0.0	2.81e0	2.81e0	3026.4	0.057
GoldenSection					
$\operatorname{GD-WeightDecay}$	0.0	3.50e1	3.50e0	14.5	0.000
Trust Region-	0.0	3.71e5	9.78e4	3002.0	0.020
Adaptive					
GD-Nesterov	0.0	3.89e2	3.28e0	14.1	0.000
Adam-Fast	0.0	4.69e2	3.30e0	29.0	0.001
L-BFGS-Limited	0.0	2.96e0	2.84e0	4259.2	0.037
Adam-	0.0	7.65e0	2.86e0	260.1	0.006
WeightDecay					
Adam	0.0	2.90e0	2.82e0	733.5	0.015
QQN-Bisection-2	0.0	1.03e1	3.91e0	469.8	0.011
L-BFGS-	0.0	1.29e1	3.57e0	2860.6	0.037
MoreThuente					
GD-	0.0	5.87e2	3.36e0	13.9	0.000
AdaptiveMomentum					
Trust Region-	0.0	4.20e5	2.36e5	3002.0	0.020
Precise					
GD-Momentum	0.0	9.24e1	3.99e0	14.4	0.000
Trust Region-	0.0	3.74e5	1.16e5	3002.0	0.020
Standard					
QQN-Bisection-1	0.0	2.81e0	2.81e0	2178.7	0.072
L-BFGS-	0.0	2.85e0	2.81e0	3087.5	0.033
Conservative					
Trust Region-	0.0	3.61e5	1.76e5	3002.0	0.020
Aggressive					
QQN-StrongWolfe	0.0	4.21e0	2.86e0	3121.1	0.087
L-BFGS	0.0	7.00e4	4.88e0	95.2	0.002
Adam-Robust	0.0	2.16e1	2.82e0	120.4	0.003
QQN-	0.0	2.81e0	2.81e0	38.0	0.001
Cubic Quadratic Interpolation					
Adam-AMSGrad	0.0	2.85e0	2.82e0	747.3	0.018

# $44 \quad Problem: \ Penalty I\_10 D\_alpha 1e6$

Optimiz	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	7.20e5	3.91e5	3002.0	0.021
Aggressiv	e					
L-BFGS		0.0	1.71e5	8.62e0	101.3	0.002
Trust	Region-	0.0	7.40e5	2.96e5	3002.0	0.021
Standard						
QQN-Bis	ection-1	0.0	5.63e0	5.63e0	2161.3	0.075
GD-		0.0	1.60e2	7.22e0	15.0	0.000
Adaptive	Momentum					
Adam-Ro	$_{ m bust}$	0.0	2.18e1	5.69e0	129.1	0.003
Trust	Region-	0.0	7.20e5	3.49e5	3002.0	0.021
Adaptive						
Trust	Region-	0.0	7.72e5	5.48e5	3002.0	0.021
Conserva	tive					
L-BFGS-	Limited	0.0	5.89e0	5.78e0	4213.4	0.043
$\operatorname{Adam}$		0.0	5.68e0	5.64e0	764.5	0.017
L-BFGS-		0.0	5.73e0	5.63e0	2611.7	0.034
Conserva	tive					
QQN-Str	$\operatorname{ongWolfe}$	0.0	1.10e1	8.12e0	3057.8	0.090

L-BFGS-	0.0	7.30e5	2.94e5	11.0	0.001
Aggressive					
GD	0.0	6.02e0	5.96e0	19.4	0.001
GD-Momentum	0.0	2.58e2	7.18e0	15.4	0.000
Adam-Fast	0.0	1.17e3	6.26e0	42.3	0.001
Trust Region-	0.0	7.75e5	4.16e5	3002.0	0.021
Precise					
GD-WeightDecay	0.0	6.01e1	6.73e0	15.8	0.000
QQN-	0.0	5.62e0	5.62e0	38.0	0.001
CubicQuadraticInterpola	ition				
QQN-	0.0	5.63e0	5.62e0	4382.9	0.085
GoldenSection					
QQN-Bisection-2	0.0	1.86e1	9.12e0	1070.2	0.027
L-BFGS-	0.0	2.89e1	1.06e1	2859.4	0.034
MoreThuente					
Adam-	0.0	9.99e0	5.72e0	274.1	0.007
WeightDecay					
GD-Nesterov	0.0	4.81e2	6.89e0	14.9	0.000
Adam-AMSGrad	0.0	5.66e0	5.64e0	761.0	0.019

#### 45 Problem: Barrier\_2D\_mu0.1

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Robust	0.0	4.09e-1	4.00e-1	2502.0	0.056
QQN-	0.0	$\inf$	$\inf$	0.0	0.000
GoldenSection					
L-BFGS-	0.0	4.00e-1	4.00e-1	880.4	0.023
Conservative					
L-BFGS-	0.0	$\inf$	$\inf$	0.0	0.000
Aggressive					
L-BFGS-	0.0	$\inf$	$\inf$	0.0	0.000
MoreThuente					
GD-WeightDecay	0.0	4.00e-1	4.00e-1	49.0	0.002
GD-Nesterov	0.0	5.03e-1	4.85e-1	20.4	0.001
Trust Region-	0.0	4.54e-1	4.00e-1	24.1	0.000
Adaptive					
L-BFGS	0.0	$\inf$	$\inf$	0.0	0.000
Trust Region-	0.0	2.25e0	5.85e-1	3.4	0.000
Aggressive					
QQN-Bisection-2	0.0	$\inf$	$\inf$	0.0	0.000
L-BFGS-Limited	0.0	4.00e-1	4.00e-1	140.3	0.003
Adam-Fast	0.0	5.15e-1	4.74e-1	31.4	0.001
Adam-AMSGrad	0.0	4.13e-1	4.01e-1	2502.0	0.056
Adam-	0.0	4.00e-1	4.00e-1	1498.4	0.032
WeightDecay					
Trust Region-	0.0	7.17e-1	4.00e-1	10.2	0.000
Standard					
GD-	0.0	4.65e-1	4.60e-1	1.6	0.000
AdaptiveMomentum	0.0	4.00	4.00	25000	0.054
Adam	0.0	4.00e-1	4.00e-1	2502.0	0.051
GD-Momentum	0.0	5.05e-1	4.82e-1	20.9	0.001
Trust Region-	0.0	4.61e-1	4.00e-1	494.1	0.003
Conservative					
Trust Region-	0.0	4.55e-1	4.00e-1	96.7	0.001
Precise	0.0			0.0	0.000
QQN-Bisection-1	0.0	$\inf_{i \in C}$	inf	0.0	0.000
QQN-StrongWolfe	0.0	$\inf$	$\inf$	0.0	0.000

QQN-	0.0	$\inf$	$\inf$	0.0	0.000
CubicQuadraticInterpolation					
GD	0.0	4.00e-1	4.00e-1	292.1	0.007

#### 46 Problem: Barrier\_5D\_mu0.1

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust Region	1- 0.0	1.02e0	1.00e0	220.3	0.002
Precise					
L-BFGS-	0.0	$\inf$	$\inf$	0.0	0.000
Aggressive					
Trust Region	0.0	1.10e0	9.99e-1	56.7	0.000
Adaptive					
Trust Region	0.0	1.86e0	1.53e0	6.8	0.000
Aggressive					
L-BFGS-	0.0	$\inf$	$\inf$	0.0	0.000
MoreThuente					
Adam-	0.0	9.99e-1	9.99e-1	1316.6	0.029
WeightDecay					
L-BFGS-	0.0	9.99e-1	9.99e-1	599.6	0.015
Conservative					
L-BFGS-Limited	0.0	1.00e0	9.99e-1	182.4	0.004
QQN-Bisection-1	0.0	$\inf$	$\inf$	0.0	0.000
GD-Nesterov	0.0	1.26e0	1.22e0	20.4	0.001
Trust Region	0.0	1.32e0	9.99e-1	18.4	0.000
Standard					
Adam-AMSGrad	0.0	1.03e0	1.01e0	2502.0	0.058
QQN-	0.0	$\inf$	$\inf$	0.0	0.000
GoldenSection					
QQN-StrongWolfe		$\inf$	$\inf$	0.0	0.000
GD	0.0	9.99e-1	9.99e-1	304.1	0.008
Adam	0.0	9.99e-1	9.99e-1	2502.0	0.052
QQN-Bisection-2	0.0	$\inf$	$\inf$	0.0	0.000
L-BFGS	0.0	$\inf$	$\inf$	0.0	0.000
GD-WeightDecay	0.0	9.99e-1	9.99e-1	49.2	0.002
Adam-Robust	0.0	1.01e0	1.00e0	2502.0	0.058
Trust Region	0.0	1.04e0	9.99e-1	1250.3	0.009
Conservative					
QQN-	0.0	$\inf$	$\inf$	0.0	0.000
CubicQuadraticInt	-	4.05.0	4.00.0	24.0	0.004
GD-Momentum	0.0	1.37e0	1.28e0	21.0	0.001
GD-	0.0	$\inf$	$\inf$	0.0	0.000
AdaptiveMomentu		1.00.0	1 10 0	0.1 💆	0.001
Adam-Fast	0.0	1.28e0	1.19e0	31.7	0.001

#### 47 Problem: Barrier\_10D\_mu0.1

Optimiz	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	2.34e0	2.00e0	30.9	0.000
Standard						
QQN-Bis	ection-1	0.0	$\inf$	$\inf$	0.0	0.000
QQN-Str	ongWolfe	0.0	$\inf$	$\inf$	0.0	0.000
L-BFGS-		0.0	$\inf$	$\inf$	0.0	0.000
MoreThu	ente					

GD-	0.0	$\inf$	$\inf$	0.0	0.000
AdaptiveMomentum					
Adam-	0.0	2.00e0	2.00e0	1144.5	0.027
WeightDecay					
Trust Region-	0.0	2.05e0	2.00e0	2487.1	0.018
Conservative					
$GD ext{-}WeightDecay$	0.0	2.00e0	2.00e0	49.4	0.002
$\operatorname{GD}$	0.0	2.00e0	2.00e0	305.0	0.008
GD-Nesterov	0.0	2.50e0	2.43e0	21.1	0.001
L-BFGS	0.0	$\inf$	$\inf$	0.0	0.000
L-BFGS-Limited	0.0	2.00e0	2.00e0	195.4	0.004
QQN-	0.0	$\inf$	$\inf$	0.0	0.000
GoldenSection					
Adam-Fast	0.0	2.58e0	2.45e0	31.5	0.001
Adam-Robust	0.0	2.00e0	2.00e0	2502.0	0.062
Trust Region-	0.0	2.17e0	2.00e0	106.3	0.001
Adaptive					
L-BFGS-	0.0	2.00e0	2.00e0	555.1	0.015
Conservative					
QQN-Bisection-2	0.0	$\inf$	$\inf$	0.0	0.000
GD-Momentum	0.0	3.39e0	2.91e0	21.2	0.001
Adam	0.0	2.00e0	2.00e0	2502.0	0.056
Adam-AMSGrad	0.0	2.04e0	2.02e0	2502.0	0.062
L-BFGS-	0.0	$\inf$	$\inf$	0.0	0.000
Aggressive					
QQN-	0.0	$\inf$	$\inf$	0.0	0.000
CubicQuadraticInterpolation	on				
Trust Region-	0.0	3.43e0	2.00e0	10.9	0.000
Aggressive					
Trust Region-	0.0	2.04e0	2.00e0	413.8	0.003
Precise					

# 48 Problem: NoisySphere\_2D\_sigma0.01

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-1	35.0	1.33e0	2.16e-1	41.2	0.003
GD-	30.0	2.39e0	1.45e0	6.6	0.001
${\bf Adaptive Momentum}$					
Adam-Robust	0.0	2.10e0	1.76e0	16.9	0.001
Trust Region-	0.0	2.06e0	1.68e0	5.0	0.000
Conservative					
Trust Region-	0.0	2.09e0	1.70e0	4.2	0.000
Aggressive					
GD-Momentum	35.0	2.45e0	1.43e0	6.8	0.001
Trust Region-	0.0	2.10e0	1.70e0	4.0	0.000
Standard					
Adam-	0.0	2.18e0	1.76e0	27.6	0.002
WeightDecay					
L-BFGS-	80.0	1.57e0	1.46e0	97.0	0.002
Conservative					
GD-WeightDecay	25.0	2.30e0	1.61e0	6.8	0.001
QQN-Bisection-2	0.0	1.97e0	1.96e0	2.4	0.000
L-BFGS-	15.0	1.84e0	1.12e-1	18.7	0.001
MoreThuente					
Adam-AMSGrad	5.0	2.10e0	1.65e0	25.7	0.002
L-BFGS-Limited	45.0	1.64e0	7.63e-1	31.4	0.001

Trust	Region-	0.0	2.03e0	1.67e0	4.2	0.000
Adaptive						
L-BFGS-		0.0	2.05e0	1.69e0	5.5	0.000
Aggressive Adam-Fast		35.0	2.26e0	1.40e0	12.3	0.001
					_	
Trust	Region-	0.0	2.19e0	1.76e0	5.3	0.000
Precise						
QQN-Stro	ngWolfe	60.0	9.23e-1	2.83e-2	30.5	0.001
GD		35.0	2.20e0	1.42e0	8.9	0.001
$\operatorname{Adam}$		0.0	2.13e0	1.72e0	21.4	0.001
GD-Nester	ov	25.0	2.82e0	1.53e0	7.0	0.001
QQN-		5.0	2.04e0	1.45e0	27.1	0.001
GoldenSec	tion					
L-BFGS		40.0	1.02e1	7.26e-2	33.2	0.001
QQN-		0.0	2.04e0	1.70e0	9.2	0.001
CubicQuae	draticInterpo	lation				

# 49 Problem: NoisySphere\_5D\_sigma0.01

		Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-Nesterov	30.0	5.61e0	4.26e0	6.8	0.001
Adam-Fast	10.0	5.99e0	4.17e0	11.7	0.001
Adam-AMSGrad	0.0	5.42e0	4.94e0	18.6	0.002
Adam-	0.0	5.22e0	4.62e0	16.6	0.002
WeightDecay					
Adam-Robust	0.0	5.11e0	4.60e0	17.1	0.002
Trust Region-	0.0	5.32e0	4.65e0	3.5	0.000
Adaptive					
QQN-Bisection-2	0.0	$\inf$	$\inf$	0.0	0.000
L-BFGS-	0.0	5.18e0	4.66e0	4.2	0.000
Aggressive					
Trust Region-	0.0	5.20e0	4.59e0	5.7	0.000
Conservative					
Trust Region-	0.0	5.27e0	4.65e0	3.5	0.000
Aggressive					
L-BFGS-	25.0	4.61e0	3.27e0	12.3	0.001
MoreThuente					
$\operatorname{GD}$	20.0	5.52e0	4.36e0	9.2	0.002
GD-	15.0	5.68e0	4.36e0	7.0	0.001
Adaptive Momentum					
$\operatorname{Adam}$	0.0	5.42e0	4.73e0	26.1	0.003
Trust Region-	0.0	5.20e0	4.73e0	4.2	0.000
Standard					
Trust Region-	0.0	5.28e0	4.65e0	6.5	0.001
Precise					
QQN-StrongWolfe	45.0	3.67e0	1.11e0	18.9	0.001
GD-Momentum	25.0	5.71e0	4.41e0	7.9	0.001
QQN-	0.0	5.26e0	4.75e0	7.2	0.001
CubicQuadraticInterpol	lation				
L-BFGS	20.0	1.44e1	2.91e0	38.4	0.002
L-BFGS-	85.0	4.42e0	4.14e0	120.3	0.004
Conservative					
QQN-	10.0	4.88e0	3.49e0	21.2	0.001
GoldenSection					
GD-WeightDecay	25.0	5.35e0	4.47e0	6.5	0.001
L-BFGS-Limited	55.0	3.90e0	2.63e0	35.4	0.001
QQN-Bisection-1	45.0	4.46e0	2.43e0	75.7	0.011

### $50 \quad Problem: \ NoisySphere\_10D\_sigma0.01$

Optimize	r	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	1.06e1	9.80e0	3.2	0.000
Standard	Ü					
QQN-		5.0	1.03e1	9.18e0	22.1	0.002
GoldenSec	tion					
GD-		35.0	$1.03\mathrm{e}1$	9.45e0	8.0	0.002
AdaptiveN	Iomentum					
Trust	Region-	0.0	$1.04\mathrm{e}1$	9.82e0	6.4	0.001
Conservati	ve					
Trust	Region-	0.0	$1.04\mathrm{e}1$	9.77e0	3.5	0.001
Aggressive						
Trust	Region-	0.0	1.06e1	9.93e0	4.8	0.001
Precise						
QQN-Bise	ction-1	45.0	9.08e0	4.69e0	55.0	0.016
L-BFGS-L	$_{ m imited}$	45.0	9.06e0	6.79e0	32.2	0.002
L-BFGS		20.0	2.28e1	7.46e0	31.4	0.002
L-BFGS-		0.0	1.06e1	9.75e0	5.4	0.001
Aggressive						
GD		10.0	1.10e1	9.54e0	10.2	0.003
GD-Mome	$\operatorname{ntum}$	10.0	1.11e1	9.66e0	6.8	0.002
QQN-Bise	ction-2	0.0	$\inf$	$\inf$	0.0	0.000
QQN-		0.0	$1.04\mathrm{e}1$	1.03e1	1.9	0.000
CubicQuad	draticInter	polation				
L-BFGS-		85.0	9.49e0	9.16e0	93.8	0.006
Conservati						
GD-Nester		15.0	1.09e1	9.63e0	7.5	0.002
Adam-AM	$\operatorname{SGrad}$	0.0	$1.04\mathrm{e}1$	9.75e0	18.9	0.004
Adam-		0.0	$1.04\mathrm{e}1$	9.79e0	28.9	0.006
WeightDec	eay					
Adam		0.0	1.07e1	9.87e0	18.2	0.003
L-BFGS-		10.0	9.77e0	6.61e0	12.4	0.002
MoreThue	nte					
Adam-Fast		15.0	1.11e1	9.46e0	9.7	0.002
GD-Weigh		5.0	1.10e1	9.66e0	6.0	0.002
Adam-Rob		5.0	$1.05\mathrm{e}1$	9.71e0	18.1	0.003
Trust	Region-	0.0	1.04e1	9.73e0	2.8	0.000
Adaptive						
QQN-Stro	ngWolfe	25.0	9.41e0	7.23e0	19.9	0.002

### ${\bf 51}\quad {\bf Problem:\ SparseRosenbrock\_4D}$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	45.0	5.85e-2	6.10e-9	1743.3	0.073
CubicQuadraticInter	polation				
Trust Region-	0.0	8.31e0	7.78e0	954.8	0.006
Adaptive					
GD	0.0	1.88e0	9.10e-1	510.1	0.013
GD-Momentum	0.0	1.10e1	4.39e-2	21.8	0.001
Adam-Robust	0.0	7.53e0	3.60e0	672.2	0.015
GD-Nesterov	0.0	3.38e0	1.58e-1	61.4	0.002
QQN-StrongWolfe	25.0	3.39e-1	2.24e-8	2429.1	0.081
L-BFGS	0.0	9.62e1	7.18e0	143.9	0.002
QQN-Bisection-1	15.0	4.56e-1	9.12e-8	2290.2	0.057

Adam-	0.0	6.07e0	1.75e-2	843.5	0.018
WeightDecay					
L-BFGS-Limited	0.0	2.33e0	7.38e-2	3916.8	0.042
QQN-	5.0	3.33e-1	6.04 e-7	4306.3	0.085
GoldenSection					
L-BFGS-	0.0	6.34e1	8.59e0	3852.0	0.029
Aggressive					
Trust Region-	0.0	4.72e1	3.37e-1	2998.1	0.019
Conservative					
L-BFGS-	0.0	1.27e2	3.78e-5	3413.7	0.039
Conservative					
GD-WeightDecay	0.0	3.04e0	4.73e-2	105.6	0.003
GD-	0.0	1.31e0	2.41e-2	56.2	0.002
Adaptive Momentum					
QQN-Bisection-2	5.0	1.42e0	5.00e-7	193.1	0.005
Adam	0.0	2.06e0	1.19e0	2502.0	0.050
Trust Region-	0.0	8.25e0	7.68e0	168.3	0.001
Standard					
L-BFGS-	20.0	1.75e0	2.55e-7	2601.9	0.046
MoreThuente					
Trust Region-	0.0	8.72e0	7.92e0	57.2	0.000
Aggressive					
Adam-Fast	0.0	1.92e0	1.14e-3	243.3	0.005
Trust Region-	0.0	2.16e1	7.75e0	1739.2	0.011
Precise					
Adam-AMSGrad	0.0	3.39e0	1.65e0	2394.2	0.054

# ${\bf 52}\quad {\bf Problem:\ SparseRosenbrock\_10D}$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-StrongWolfe	45.0	5.37e-1	1.90e-9	2419.7	0.076
GD-	0.0	1.31e1	5.08e-2	43.6	0.002
AdaptiveMomentum					
L-BFGS-	0.0	7.68e0	2.91e-4	2749.9	0.052
MoreThuente					
GD	0.0	3.27e0	1.20e0	388.9	0.011
Adam	0.0	4.91e0	3.14e0	2502.0	0.054
GD-Momentum	0.0	3.15e1	3.12e-1	21.1	0.001
Trust Region-	0.0	2.11e1	1.97e1	122.0	0.001
Aggressive					
Trust Region-	0.0	1.04e2	1.18e0	3002.0	0.020
Precise					
L-BFGS	0.0	2.80e2	2.70e1	136.4	0.002
L-BFGS-	0.0	1.81e2	8.09e1	3852.0	0.029
Aggressive					
Adam-AMSGrad	0.0	5.73e0	4.14e0	2502.0	0.060
Adam-	0.0	3.27e0	1.06e-2	2150.9	0.049
WeightDecay					
Trust Region-	0.0	2.92e1	2.02e1	2139.5	0.014
Adaptive					
QQN-	5.0	5.84e-1	5.69e-7	4401.4	0.085
GoldenSection					
QQN-Bisection-1	20.0	6.70e-1	1.26e-7	2281.2	0.067
GD-Nesterov	0.0	7.90e-1	5.84e-2	100.3	0.003
Adam-Fast	0.0	1.90e0	5.27e-2	206.0	0.005
Adam-Robust	0.0	1.19e1	9.15e0	2380.8	0.058

0.0	2.05e1	1.95e1	439.7	0.003
0.0	3.21e0	3.42e-1	842.9	0.022
55.0	1.42e-1	1.89e-8	1670.9	0.073
ation				
0.0	1.28e0	3.17e-1	3999.6	0.051
0.0	4.84e0	1.18e-1	127.2	0.004
0.0	1.56e2	$6.91\mathrm{e}1$	3002.0	0.020
0.0	1.05e7	2.96e-2	3686.9	0.039
	0.0 55.0 ation 0.0 0.0 0.0	0.0 3.21e0 55.0 1.42e-1 ation 0.0 1.28e0 0.0 4.84e0 0.0 1.56e2	0.0 3.21e0 3.42e-1 55.0 1.42e-1 1.89e-8 ation  0.0 1.28e0 3.17e-1 0.0 4.84e0 1.18e-1 0.0 1.56e2 6.91e1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

### 53 Problem: SparseQuadratic\_5D\_pattern[1, 3]

Optimize	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	1.23e3	5.86e-2	90.2	0.001
Adaptive	O					
Trust	Region-	0.0	4.92e-1	3.14e-2	306.4	0.002
Precise	O					
GD-Weigh	tDecay	100.0	9.21e-7	8.42e-7	99.8	0.003
QQN-Stro		100.0	2.22e-7	4.00e-8	29.1	0.001
Trust	Region-	0.0	1.81e2	1.67e2	40.6	0.000
Standard						
L-BFGS-		100.0	2.84e-7	1.15e-10	314.8	0.008
Conservat	ive					
GD		100.0	9.77e-7	9.59e-7	352.9	0.010
GD-Nester	rov	0.0	3.19e-1	2.73e-1	22.0	0.001
QQN-		100.0	4.31e-7	3.60e-7	57.0	0.001
CubicQua	draticInter	polation				
L-BFGS-L	imited	100.0	4.57e-7	4.26e-9	117.5	0.003
L-BFGS-		100.0	2.84e-7	2.36e-7	31.0	0.000
Aggressive	)					
QQN-Bise	ction-2	100.0	1.58e-7	1.83e-9	37.5	0.001
L-BFGS-		100.0	8.81e-8	1.12e-8	23.6	0.000
MoreThue	nte					
GD-		0.0	9.75 e-1	8.63e-1	19.4	0.001
Adaptive	Iomentum					
QQN-		100.0	1.09e-7	2.49e-9	132.7	0.002
GoldenSec	tion					
$\operatorname{Adam}$		0.0	3.67e-3	5.91e-4	2502.0	0.052
Adam-		100.0	9.85e-7	9.73e-7	1727.8	0.038
WeightDe	cay					
L-BFGS		100.0	4.77e-7	1.77e-8	48.4	0.001
Adam-Rol	oust	0.0	6.93 e-2	2.83e-2	2502.0	0.058
QQN-Bise	ction-1	100.0	1.79e-7	4.95e-9	38.0	0.001
Adam-Fas		0.0	3.56e-1	3.36e-1	37.5	0.001
GD-Mome	$_{ m entum}$	0.0	5.03e-1	4.06e-1	22.0	0.001
Adam-AM	[SGrad	0.0	1.95e-1	1.19e-1	2502.0	0.058
Trust	Region-	0.0	4.06e0	8.95e-3	1737.1	0.012
Conservat						
Trust	Region-	0.0	1.82e4	1.51e4	27.0	0.000
Aggressive						

#### 54 Problem: $SparseQuadratic_10D_pattern[1, 3]$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS	90.0	1.93e-3	1.06e-8	75.7	0.002
Adam-	100.0	9.83e-7	9.70e-7	1532.3	0.037
WeightDecay					
Trust Region-	0.0	1.83e0	6.04e-1	3002.0	0.024
Conservative					
Adam-AMSGrad	0.0	2.33e-1	1.70e-1	2502.0	0.064
QQN-StrongWolfe	100.0	2.19e-7	8.82e-8	43.1	0.001
Trust Region-	0.0	2.01e4	8.89e-1	31.6	0.000
Aggressive					
Trust Region-	0.0	1.80e-1	2.31e-3	589.1	0.005
Precise					
QQN-Bisection-2	100.0	3.81e-7	2.03e-8	46.4	0.001
QQN-	100.0	3.11e-7	7.74e-9	156.2	0.002
GoldenSection					
GD-Nesterov	0.0	6.03 e-1	5.68e-1	23.1	0.001
L-BFGS-	100.0	2.03e-7	2.80e-9	339.5	0.009
Conservative					
Adam	0.0	2.03e-3	3.96e-4	2502.0	0.057
L-BFGS-	100.0	2.16e-7	1.72e-7	38.0	0.001
Aggressive					
Adam-Fast	0.0	7.28e-1	6.71e-1	37.3	0.001
Adam-Robust	0.0	1.03e-1	4.23e-2	2502.0	0.064
L-BFGS-	100.0	2.20e-7	2.06e-8	28.2	0.001
MoreThuente					
GD-WeightDecay	100.0	9.31e-7	8.75e-7	103.2	0.004
QQN-Bisection-1	100.0	5.68e-7	2.29e-8	55.8	0.001
GD	100.0	9.80e-7	9.60e-7	360.1	0.010
L-BFGS-Limited	100.0	4.74e-7	5.73e-8	58.5	0.001
Trust Region-	0.0	1.28e3	5.42e-2	162.1	0.001
Adaptive					
QQN-	100.0	2.38e-8	2.26e-9	60.4	0.002
CubicQuadraticInter	polation				
Trust Region-	0.0	1.02e2	2.21e-1	57.5	0.000
Standard					
GD-Momentum	0.0	9.15e-1	8.60e-1	23.1	0.001
GD-	0.0	1.57e0	1.41e0	21.9	0.001
AdaptiveMomentum					

### $55 \quad Problem: \ Logistic Regression\_100 samples\_5 features\_reg 0.01$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-2	0.0	3.15e-1	3.15e-1	98.5	0.033
L-BFGS-	0.0	3.15e-1	3.15e-1	106.7	0.035
MoreThuente					
GD-	0.0	3.15e-1	3.15e-1	426.1	0.235
AdaptiveMomentum					
Trust Region-	0.0	6.95 e-1	6.37e-1	5.0	0.002
Adaptive					
Trust Region-	0.0	6.98e-1	6.53e-1	5.0	0.002
Aggressive					
Trust Region-	0.0	6.78e-1	6.48e-1	5.5	0.002
Precise					
GD-WeightDecay	0.0	3.27e-1	3.26e-1	1668.0	0.913
L-BFGS	0.0	3.16e-1	3.15e-1	2673.8	0.800

Adam-	0.0	3.22e-1	3.20e-1	2502.0	0.899
WeightDecay					
Trust Region-	0.0	4.24e-1	4.20e-1	86.6	0.025
Conservative					
GD-Nesterov	0.0	3.15e-1	3.15e-1	1668.0	0.909
QQN-	0.0	3.15e-1	3.15e-1	91.5	0.036
CubicQuadraticInterpolation					
Adam-AMSGrad	0.0	4.05e-1	3.87e-1	2502.0	0.900
QQN-StrongWolfe	0.0	3.15e-1	3.15e-1	74.3	0.031
QQN-Bisection-1	0.0	3.15e-1	3.15e-1	98.5	0.037
Adam	0.0	4.02e-1	3.92e-1	2502.0	0.892
L-BFGS-	0.0	3.15e-1	3.15e-1	1776.5	0.515
Aggressive					
QQN-	0.0	3.15e-1	3.15e-1	307.2	0.069
GoldenSection					
Trust Region-	0.0	6.98e-1	6.55e-1	5.0	0.002
Standard					
GD	0.0	3.77e-1	3.72e-1	1668.0	0.906
Adam-Robust	0.0	4.32e-1	4.13e-1	2502.0	0.901
L-BFGS-Limited	0.0	3.15e-1	3.15e-1	3043.5	0.895
Adam-Fast	0.0	3.16e-1	3.16e-1	90.8	0.033
L-BFGS-	0.0	3.15e-1	3.15e-1	559.5	0.189
Conservative					
GD-Momentum	0.0	3.15e-1	3.15e-1	1668.0	0.906

# $56 \quad Problem: \ Logistic Regression\_200 samples\_10 features\_reg0.01$

GD-       0.0       3.23e-1       3.23e-1       479.1       0.478         AdaptiveMomentum       L-BFGS-       0.0       3.23e-1       3.23e-1       86.2       0.050         MoreThuente       BO       0.0       3.97e-1       3.93e-1       1668.0       1.640         QQN-       0.0       3.23e-1       3.23e-1       360.0       0.143         GoldenSection       QQN-       0.0       3.23e-1       3.23e-1       105.4       0.072         CubicQuadraticInterpolation       L-BFGS-Limited       0.0       3.23e-1       3.23e-1       3146.0       1.621         Adam       0.0       3.92e-1       3.75e-1       2502.0       1.617         Adam-       0.0       3.27e-1       3.25e-1       2502.0       1.618         WeightDecay
L-BFGS-       0.0       3.23e-1       3.23e-1       86.2       0.050         MoreThuente       0.0       3.97e-1       3.93e-1       1668.0       1.640         QQN-       0.0       3.23e-1       3.23e-1       360.0       0.143         GoldenSection       0.0       3.23e-1       3.23e-1       105.4       0.072         CubicQuadraticInterpolation       0.0       3.23e-1       3.23e-1       3146.0       1.621         Adam       0.0       3.92e-1       3.75e-1       2502.0       1.617         Adam-       0.0       3.27e-1       3.25e-1       2502.0       1.618
MoreThuente         GD       0.0       3.97e-1       3.93e-1       1668.0       1.640         QQN-       0.0       3.23e-1       3.23e-1       360.0       0.143         GoldenSection       CQN-       0.0       3.23e-1       3.23e-1       105.4       0.072         CubicQuadraticInterpolation       L-BFGS-Limited       0.0       3.23e-1       3.23e-1       3146.0       1.621         Adam       0.0       3.92e-1       3.75e-1       2502.0       1.617         Adam-       0.0       3.27e-1       3.25e-1       2502.0       1.618
GD 0.0 3.97e-1 3.93e-1 1668.0 1.640 QQN- 0.0 3.23e-1 3.23e-1 360.0 0.143 GoldenSection QQN- 0.0 3.23e-1 3.23e-1 105.4 0.072 CubicQuadraticInterpolation L-BFGS-Limited 0.0 3.23e-1 3.23e-1 3146.0 1.621 Adam 0.0 3.92e-1 3.75e-1 2502.0 1.617 Adam- 0.0 3.27e-1 3.25e-1 2502.0 1.618
QQN-       0.0       3.23e-1       3.23e-1       360.0       0.143         GoldenSection       QQN-       0.0       3.23e-1       3.23e-1       105.4       0.072         CubicQuadraticInterpolation       Use of the control of the contr
GoldenSection         QQN-       0.0       3.23e-1       3.23e-1       105.4       0.072         CubicQuadraticInterpolation         L-BFGS-Limited       0.0       3.23e-1       3.23e-1       3146.0       1.621         Adam       0.0       3.92e-1       3.75e-1       2502.0       1.617         Adam-       0.0       3.27e-1       3.25e-1       2502.0       1.618
QQN-       0.0       3.23e-1       3.23e-1       105.4       0.072         CubicQuadraticInterpolation
Cubic Quadratic Interpolation       L-BFGS-Limited     0.0     3.23e-1     3.23e-1     3146.0     1.621       Adam     0.0     3.92e-1     3.75e-1     2502.0     1.617       Adam-     0.0     3.27e-1     3.25e-1     2502.0     1.618
L-BFGS-Limited       0.0       3.23e-1       3.23e-1       3146.0       1.621         Adam       0.0       3.92e-1       3.75e-1       2502.0       1.617         Adam-       0.0       3.27e-1       3.25e-1       2502.0       1.618
Adam       0.0       3.92e-1       3.75e-1       2502.0       1.617         Adam-       0.0       3.27e-1       3.25e-1       2502.0       1.618
Adam- 0.0 3.27e-1 3.25e-1 2502.0 1.618
WeightDecay
QQN-Bisection-2 0.0 3.23e-1 3.23e-1 110.7 0.066
Trust Region- $0.0$ $7.01e-1$ $6.45e-1$ $5.0$ $0.003$
Standard
Trust Region- 0.0 4.44e-1 4.39e-1 82.4 0.044
Conservative
GD-Nesterov 0.0 3.23e-1 3.23e-1 1668.0 1.649
GD-WeightDecay 0.0 3.39e-1 3.38e-1 1668.0 1.649
L-BFGS- 0.0 3.25e-1 3.23e-1 3432.2 1.407
Aggressive
L-BFGS 0.0 3.23e-1 3.23e-1 2311.4 1.256
Trust Region- 0.0 7.06e-1 6.56e-1 5.0 0.003
Adaptive
QQN-Bisection-1 0.0 3.23e-1 3.23e-1 110.7 0.072
Trust Region- 0.0 7.03e-1 6.65e-1 5.0 0.003
Aggressive
Adam-AMSGrad 0.0 3.95e-1 3.84e-1 2502.0 1.620

Trust Precise	Region-	0.0	7.01e-1	6.59e-1	5.0	0.003
QQN-Stron	gWolfe	0.0	3.23e-1	3.23e-1	80.3	0.059
L-BFGS-		0.0	3.23e-1	3.23e-1	545.5	0.324
Conservativ	re					
Adam-Robu	$\operatorname{ist}$	0.0	4.12e-1	3.96e-1	2502.0	1.621
Adam-Fast		0.0	3.24e-1	3.24e-1	77.0	0.050
GD-Momen	tum	0.0	3.23e-1	3.23e-1	1668.0	1.648

### $57 \quad Problem: \ Linear Regression\_100 samples\_5 features\_reg 0.01$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-	0.0	7.18e-2	7.16e-2	2502.0	0.830
WeightDecay					
GD-Momentum	0.0	1.36e0	1.22e0	24.1	0.012
QQN-	0.0	7.15e-2	7.15e-2	325.7	0.063
GoldenSection					
GD-	0.0	2.00e0	1.90e0	23.0	0.012
AdaptiveMomentum					
Trust Region-	0.0	1.55e3	1.22e-1	234.8	0.062
Adaptive					
L-BFGS-	0.0	7.15e-2	7.15e-2	1306.5	0.406
Conservative					
Adam-Fast	0.0	1.90e-1	7.25e-2	102.4	0.035
QQN-	0.0	7.15e-2	7.15e-2	118.9	0.041
CubicQuadraticInter	polation				
GD-WeightDecay	0.0	7.15e-2	7.15e-2	165.2	0.088
L-BFGS-Limited	0.0	7.15e-2	7.15e-2	540.6	0.149
QQN-StrongWolfe	0.0	7.15e-2	7.15e-2	109.2	0.041
L-BFGS-	0.0	7.16e-2	7.15e-2	796.0	0.229
MoreThuente					
L-BFGS	0.0	1.38e-1	7.15e-2	234.7	0.060
GD	0.0	7.15e-2	7.15e-2	512.3	0.268
QQN-Bisection-1	0.0	7.15e-2	7.15e-2	123.0	0.048
QQN-Bisection-2	0.0	7.15e-2	7.15e-2	103.0	0.038
Adam	0.0	2.88e0	2.36e0	2502.0	0.833
Adam-AMSGrad	0.0	3.06e0	2.47e0	2502.0	0.839
Adam-Robust	0.0	4.89e0	4.01e0	2502.0	0.836
GD-Nesterov	0.0	9.19e-1	8.36e-1	23.9	0.012
Trust Region-	0.0	1.10e2	2.92e-1	76.2	0.020
Standard					
Trust Region-	0.0	3.62e4	9.10e-1	34.0	0.009
Aggressive					
Trust Region-	0.0	1.38e-1	7.38e-2	851.2	0.221
Precise					
L-BFGS-	0.0	7.15e-2	7.15e-2	153.1	0.036
Aggressive					
Trust Region-	0.0	7.87e0	6.26e0	3002.0	0.769
Conservative					

### $58 \quad Problem: \ Linear Regression\_200 samples\_10 features\_reg 0.01$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Fast	20.0	5.14e-1	4.82e-1	189.9	0.115
GD-Momentum	0.0	2.46e0	2.41e0	38.1	0.035

L-BFGS-Limited	100.0	4.82e-1	4.82e-1	159.2	0.069
Adam	0.0	5.34e1	4.86e1	2502.0	1.496
Trust Region-	0.0	2.56e3	7.13e-1	372.2	0.178
Standard					
Trust Region-	0.0	5.39e1	4.85e1	3002.0	1.422
Precise					
GD-Nesterov	0.0	1.80e0	1.78e0	37.9	0.035
L-BFGS-	100.0	4.82e-1	4.82e-1	205.7	0.086
Aggressive					
GD-	0.0	5.05e0	4.95e0	35.0	0.032
AdaptiveMomentum					
QQN-	100.0	4.82e-1	4.82e-1	54.1	0.035
CubicQuadraticInterpola	ation				
L-BFGS-	100.0	4.82e-1	4.82e-1	436.2	0.244
Conservative					
L-BFGS-	100.0	4.82e-1	4.82e-1	72.8	0.038
MoreThuente					
L-BFGS	90.0	5.09e-1	4.82e-1	77.5	0.039
Adam-AMSGrad	0.0	5.39e1	5.00e1	2502.0	1.511
Adam-Robust	0.0	6.34e1	5.92e1	2502.0	1.508
QQN-Bisection-1	100.0	4.82e-1	4.82e-1	89.0	0.066
QQN-StrongWolfe	100.0	4.82e-1	4.82e-1	62.0	0.039
QQN-	100.0	4.82e-1	4.82e-1	180.0	0.059
GoldenSection					
GD	100.0	4.82e-1	4.82e-1	343.6	0.323
QQN-Bisection-2	100.0	4.82e-1	4.82e-1	69.0	0.050
GD-WeightDecay	100.0	4.82e-1	4.82e-1	116.7	0.111
Trust Region-	0.0	1.08e3	5.40e-1	1420.7	0.673
Adaptive					
Trust Region-	0.0	3.92e4	1.28e0	111.1	0.054
Aggressive					
Trust Region-	0.0	1.06e2	1.03e2	3002.0	1.421
Conservative					
Adam-	0.0	2.42e0	1.66e0	2502.0	1.499
WeightDecay					

# $59 \quad Problem: \ Neural Network\_100 samples\_layers\_5\_10\_3$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	5.0	1.50e-1	1.40e-1	2776.7	1.320
Aggressive					
Adam-Robust	0.0	1.67e-1	1.48e-1	2502.0	2.086
GD	0.0	2.03e-1	1.98e-1	1668.0	2.339
GD-Nesterov	0.0	2.27e-1	2.20e-1	21.6	0.029
Adam	10.0	1.53e-1	1.40e-1	2473.1	2.047
Trust Region-	0.0	4.01e-1	2.08e-1	35.5	0.023
Standard					
Trust Region-	0.0	7.98e-1	2.20e-1	23.0	0.015
Aggressive					
Trust Region-	0.0	8.19e-1	1.95e-1	46.5	0.029
Adaptive					
QQN-	75.0	1.41e-1	1.38e-1	2749.2	0.957
GoldenSection					
L-BFGS	0.0	2.39e-1	1.41e-1	1556.7	0.904
Trust Region-	0.0	2.20e-1	2.18e-1	159.9	0.100
Conservative					
QQN-Bisection-2	90.0	1.40e-1	1.39e-1	864.2	0.629

L-BFGS-Limited	45.0	1.47e-1	1.40e-1	2641.3	1.307
Trust Region-	0.0	2.23e-1	1.97e-1	42.9	0.027
Precise					
Adam-	85.0	1.41e-1	1.40e-1	1794.2	1.486
WeightDecay					
GD-Momentum	0.0	2.34e-1	2.25e-1	21.2	0.028
Adam-AMSGrad	0.0	1.54e-1	1.40e-1	2502.0	2.087
QQN-Bisection-1	95.0	1.40e-1	1.38e-1	727.2	0.643
QQN-StrongWolfe	45.0	1.43e-1	1.38e-1	1646.3	1.542
L-BFGS-	45.0	1.48e-1	1.40e-1	1970.8	1.252
Conservative					
GD-	0.0	2.56e-1	2.32e-1	15.9	0.021
AdaptiveMomentum					
L-BFGS-	15.0	1.51e-1	1.40e-1	2755.1	1.880
MoreThuente					
GD-WeightDecay	0.0	1.84e-1	1.80e-1	1668.0	2.360
QQN-	80.0	1.41e-1	1.37e-1	764.9	0.831
CubicQuadraticInterpolation					
Adam-Fast	60.0	1.43e-1	1.39e-1	149.3	0.125

# $60 \quad Problem: \ Neural Network\_100 samples\_layers\_10\_20\_5$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-WeightDecay	0.0	9.58e-2	9.05e-2	1668.0	4.432
Trust Region-	0.0	1.76e-1	1.36e-1	26.9	0.032
Adaptive					
Trust Region-	0.0	1.61e-1	1.54e-1	23.4	0.028
Precise					
QQN-StrongWolfe	70.0	4.07e-2	3.75e-2	1111.5	2.001
QQN-	100.0	3.80e-2	3.77e-2	959.5	1.947
CubicQuadraticInter	polation				
GD-	0.0	1.74e-1	1.66e-1	15.2	0.037
AdaptiveMomentum					
Adam-Fast	50.0	4.24e-2	3.71e-2	168.4	0.263
GD-Nesterov	0.0	1.23e-1	4.04e-2	596.8	1.586
L-BFGS-Limited	0.0	5.06e-2	4.16e-2	3290.1	3.444
L-BFGS	0.0	1.36e-1	4.42e-2	1846.8	2.005
Adam-AMSGrad	5.0	4.96e-2	3.82e-2	2495.8	3.943
Trust Region-	0.0	7.57e-1	3.26e-1	7.8	0.010
Aggressive					
QQN-Bisection-2	100.0	3.80e-2	3.77e-2	883.2	1.296
QQN-Bisection-1	100.0	3.80e-2	3.78e-2	1029.0	1.615
Trust Region-	0.0	1.48e-1	1.28e-1	122.0	0.142
Conservative					
Adam-Robust	0.0	5.97e-2	4.85e-2	2502.0	3.922
QQN-	85.0	3.83e-2	3.79e-2	3686.0	2.351
GoldenSection					
Trust Region-	0.0	2.74e-1	2.05e-1	5.3	0.007
Standard					
Adam-	100.0	3.82e-2	3.82e-2	1278.0	2.066
WeightDecay					
L-BFGS-	0.0	5.60e-2	4.81e-2	3655.3	3.155
Aggressive					
L-BFGS- 0.0		6.08e-2	5.29e-2	2831.2	3.733
MoreThuente					
GD	0.0	1.28e-1	1.18e-1	1668.0	4.458
Adam	10.0	4.42e-2	3.82e-2	2469.9	3.834

#### $61 \quad Problem: \ SVM\_100 samples\_5 features\_C1$

Optimizer		Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	3.47e2	7.26e-1	15.4	0.004
Standard						
L-BFGS-Limited 90.0		90.0	6.43e-1	6.43e-1	444.7	0.095
QQN-		100.0	6.43e-1	6.43e-1	61.6	0.026
CubicQu	adraticInter	polation				
L-BFGS		100.0	6.43e-1	6.43e-1	42.0	0.012
GD-Mom	entum	0.0	6.73e-1	6.53e-1	24.9	0.013
QQN-		90.0	6.43e-1	6.43e-1	228.9	0.046
GoldenSe	ection					
GD-Weig	htDecay	100.0	6.43e-1	6.43e-1	97.2	0.055
QQN-Bis		100.0	6.43e-1	6.43e-1	68.8	0.030
L-BFGS-		100.0	6.43e-1	6.43e-1	46.0	0.011
Aggressiv	<i>'</i> e					
Adam-Ro	bust	50.0	6.43e-1	6.43e-1	1414.2	0.499
L-BFGS-			6.43 e-1	6.43e-1	41.3	0.013
MoreThu	ente					
Adam		100.0	6.43e-1	6.43e-1	1826.7	0.646
Trust	Region-	0.0	4.68e0	7.07e-1	19.0	0.005
Adaptive						
Trust	Region-	0.0	1.00e3	6.57e-1	16.3	0.005
Aggressiv	_					
Adam-		100.0	6.43 e-1	6.43e-1	628.6	0.225
WeightDe	ecay					
GD-Nest		0.0	6.64 e-1	6.50e-1	25.1	0.014
GD		100.0	6.43e-1	6.43e-1	274.4	0.153
GD-		0.0	7.53e-1	7.12e-1	17.6	0.009
Adaptive	Momentum					
Adam-Fa		0.0	6.84 e-1	6.44e-1	32.8	0.012
QQN-Str	ongWolfe	100.0	6.43 e-1	6.43e-1	57.5	0.022
Trust	Region-	0.0	2.82e0	6.45e-1	134.5	0.038
Conserva	_					
Trust	Region-	0.0	9.38e-1	6.73e-1	36.2	0.010
Precise	5					
QQN-Bisection-1		100.0	6.43e-1	6.43e-1	83.4	0.040
L-BFGS-		100.0	6.43e-1	6.43e-1	133.7	0.045
Conserva						
Adam-Al		90.0	6.43e-1	6.43e-1	1984.2	0.702

#### ${\bf 62 \quad Problem: \ SVM\_200 samples\_10 features\_C1}$

Optimizer		Success Rate (%)	Rate (%) Mean Final Value Best Value Mean		Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	2.49e0	6.88e-1	130.4	0.066
Conservative						
Adam-Robust		0.0	6.86e-1	6.86e-1	547.7	0.348
Adam-AMSGrad		0.0	6.86 e-1	6.86e-1	1996.0	1.254
$\operatorname{Adam}$		0.0	6.86e-1	6.86e-1	1755.8	1.097
QQN-		0.0	6.86e-1	6.86e-1	925.2	0.317
GoldenSe	ection					

QQN-StrongWolfe	0.0	6.86e-1	6.86e-1	1065.5	0.585
GD-Nesterov	0.0	7.03e-1	6.94e-1	25.7	0.024
L-BFGS-	0.0	5.16e3	6.86e-1	555.0	0.311
MoreThuente	0.0	0.1000	0.000 1	000.0	0.011
QQN-Bisection-1	0.0	6.86e-1	6.86e-1	1454.6	1.043
L-BFGS-	0.0	6.86e-1	6.86e-1	124.0	0.051
Aggressive	0.0	0.000 1	0.000 1	1-110	0.001
L-BFGS-Limited	0.0	6.86e-1	6.86e-1	770.6	0.264
L-BFGS-	0.0	6.86e-1	6.86e-1	624.9	0.263
Conservative					0.200
GD-Momentum	0.0	7.08e-1	6.99e-1	25.8	0.024
GD-	0.0	7.83e-1	7.62e-1	18.0	0.017
AdaptiveMomentum					
Trust Region-	0.0	6.77e0	6.97e-1	24.4	0.012
Adaptive					
Trust Region-	0.0	1.84e0	7.56e-1	11.8	0.006
Aggressive					
Trust Region-	0.0	9.09e-1	7.00e-1	36.1	0.019
Precise					
GD-WeightDecay	0.0	6.86e-1	6.86e-1	177.0	0.172
QQN-Bisection-2	0.0	6.86e-1	6.86e-1	911.8	0.458
QQN-	0.0	6.86e-1	6.86e-1	289.4	0.223
CubicQuadraticInterpolation	n				
GD	0.0	6.86e-1	6.86e-1	450.0	0.437
L-BFGS	0.0	7.68e-1	6.86e-1	321.6	0.143
Adam-Fast	0.0	7.22e-1	6.87e-1	42.1	0.027
Adam-	0.0	6.86e-1	6.86e-1	668.9	0.421
WeightDecay					
Trust Region-	0.0	8.76e-1	6.90e-1	20.8	0.011
Standard					