Performance Table Report

$\ensuremath{\mathbf{QQN}}$ Optimizer Benchmark

August 3, 2025

$1 \quad Problem: \ Sphere_2D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam	40.0	7.60e-3	4.96e-3	2432.4	0.049
GD-WeightDecay	100.0	4.67e-3	4.31e-3	51.6	0.002
Adam-Robust	0.0	5.86e-2	2.27e-2	2502.0	0.056
QQN-Bisection-1	100.0	0.00e0	0.00e0	15.0	0.000
GD-Momentum	100.0	4.85e-4	1.93e-4	16.9	0.001
Adam-Fast	30.0	8.78e-2	2.82e-4	33.5	0.001
Adam-AMSGrad	0.0	7.20e-2	2.82e-2	2502.0	0.055
Trust Region- Adaptive	35.0	6.68e2	1.21e-5	41.1	0.000
Trust Region- Standard	10.0	1.25e3	7.55e-4	27.1	0.000
QQN-StrongWolfe	100.0	0.00e0	0.00e0	11.0	0.000
Trust Region- Conservative	100.0	1.69e-3	2.11e-4	602.0	0.004
GD	100.0	4.93e-3	4.81e-3	152.6	0.004
QQN-Bisection-2	100.0	0.00e0	0.00e0	13.0	0.000
QQN- GoldenSection	100.0	3.22e-14	2.46e-14	46.0	0.000
Adam- WeightDecay	100.0	4.98e-3	4.94e-3	1113.1	0.024
Trust Region- Aggressive	30.0	7.95e3	2.26e-5	18.6	0.000
Trust Region- Precise	55.0	6.58e-1	5.18e-6	115.3	0.001
L-BFGS-Limited	100.0	1.24e-3	0.00e0	23.1	0.000
QQN-	100.0	0.00e0	0.00e0	12.0	0.000
CubicQuadraticInter		0.0000	0.0000	12.0	0.000
L-BFGS-	100.0	4.28e-3	2.80e-3	156.2	0.004
Conservative					
L-BFGS	100.0	8.79e-5	0.00e0	13.5	0.000
L-BFGS-	100.0	0.00e0	0.00e0	10.0	0.000
MoreThuente					
GD-Nesterov	100.0	2.51e-3	2.83e-4	16.4	0.000
L-BFGS-	100.0	0.00e0	0.00e0	10.0	0.000
Aggressive				2.0	
GD-	60.0	2.05e-1	2.68e-5	13.8	0.000
AdaptiveMomentum					

2 Problem: Sphere_10D

Optimizer Success Rate (%) Mean Final Value	Best Value Mean Func Evals Mean Tim	ne (s)
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L-BFGS-	100.0	0.00e0	0.00e0	10.0	0.000
MoreThuente					
GD	100.0	4.91e-3	4.82e-3	192.4	0.005
Adam-Robust	0.0	1.08e-1	4.84e-2	2502.0	0.060
Adam-AMSGrad	0.0	2.64e-1	1.95e-1	2502.0	0.059
Trust Region-	30.0	7.30e2	2.03e-4	139.3	0.001
Adaptive					
Adam-Fast	0.0	6.12e-1	5.73e-1	37.4	0.001
Adam	95.0	4.98e-3	4.94e-3	2391.8	0.053
QQN-	100.0	1.60e-13	1.45e-13	46.0	0.000
GoldenSection					
GD-Nesterov	65.0	1.59e-1	1.53e-5	19.9	0.001
QQN-StrongWolfe	100.0	0.00e0	0.00e0	11.0	0.000
GD-	60.0	5.51e-1	2.42e-4	18.4	0.001
AdaptiveMomentum					
Trust Region-	35.0	3.41e-1	1.07e-3	2968.8	0.020
Conservative					
Trust Region-	0.0	1.09e4	5.34e3	30.3	0.000
Aggressive					
Trust Region-	65.0	5.27e-1	1.01e-4	504.0	0.003
Precise					
QQN-	100.0	0.00e0	0.00e0	12.0	0.000
CubicQuadraticInterpola	ation				
L-BFGS	100.0	0.00e0	0.00e0	15.0	0.000
L-BFGS-	100.0	0.00e0	0.00e0	10.0	0.000
Aggressive					
QQN-Bisection-1	100.0	0.00e0	0.00e0	15.0	0.000
QQN-Bisection-2	100.0	0.00e0	0.00e0	13.0	0.000
L-BFGS-	100.0	4.51e-3	4.20e-3	197.5	0.005
Conservative					
GD-WeightDecay	100.0	4.61e-3	4.33e-3	62.5	0.002
Adam-	100.0	4.92e-3	4.88e-3	968.0	0.022
WeightDecay					
Trust Region-	15.0	6.75e2	2.86e-5	51.8	0.000
Standard					
GD-Momentum	75.0	1.70e-1	2.45e-5	19.4	0.001
L-BFGS-Limited	100.0	1.08e-3	5.57e-7	40.3	0.001

3 Problem: Rosenbrock_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD	0.0	1.23e0	7.46e-1	854.0	0.022
GD-Nesterov	0.0	1.49e0	5.05e-2	46.1	0.001
L-BFGS-	0.0	3.18e1	4.34e0	3852.0	0.027
Aggressive					
QQN-	40.0	3.73e-2	5.16e-3	1619.8	0.065
CubicQuadraticInter	polation				
GD-Momentum	0.0	5.61e0	4.79e-1	23.8	0.001
QQN-Bisection-1	5.0	4.65e-1	6.39e-3	2363.3	0.053
GD-	25.0	6.54 e-1	3.23e-3	47.7	0.002
AdaptiveMomentum					
Adam-Fast	60.0	2.13e0	8.12e-3	171.8	0.003
Trust Region-	0.0	4.12e0	3.83e0	494.4	0.003
Adaptive					
QQN-StrongWolfe	35.0	5.67e-2	4.58e-3	2004.3	0.059
L-BFGS-Limited	0.0	3.93e0	3.26e-2	2251.6	0.026
Adam-	0.0	4.11e0	1.55e-2	231.9	0.005
WeightDecay					

Adam-Ro	bust	0.0	4.04e0	1.90e0	419.2	0.009
Trust	Region-	0.0	4.66e0	4.01e0	27.6	0.000
Aggressive	e					
Trust	Region-	0.0	7.26e0	3.76e0	946.2	0.006
Precise						
Adam		0.0	1.22e0	4.86e-1	2502.0	0.050
Trust	Region-	0.0	4.18e0	3.95e0	89.6	0.001
Standard						
Adam-Al		0.0	3.83e0	4.66e-1	678.1	0.015
Trust	Region-	0.0	2.84e1	1.90e-1	2770.7	0.017
Conservat						
GD-Weigl	htDecay	0.0	3.65e0	3.75e-2	58.7	0.002
L-BFGS		0.0	1.36e2	8.12e-1	121.5	0.002
QQN-		10.0	1.26e-1	8.08e-5	4248.6	0.080
GoldenSe	ction					
L-BFGS-		100.0	6.53e-3	1.09e-3	985.0	0.016
Conservat						
QQN-Bise	ection-2	30.0	6.07e-2	4.36e-3	304.7	0.007
L-BFGS-		95.0	1.91e-2	1.40e-4	651.0	0.011
MoreThue	ente					

4 Problem: Rosenbrock_5D

GD-Nesterov 10.0 4.24e0 3.90e-1 335.4 0.011 Trust Region- 0.0 1.01e3 8.08e2 3002.0 0.019 Precise
Precise QQN-Bisection-1 85.0 6.94e-1 2.50e-1 1147.7 0.029 GD- 0.0 4.60e1 3.36e1 20.6 0.001 AdaptiveMomentum 0.0 3.92e0 2.83e0 2471.6 0.050 GD-Momentum 0.0 3.55e1 1.96e1 20.8 0.001 Adam-Robust 0.0 1.46e1 6.12e0 2502.0 0.059 Trust Region- 0.0 1.02e3 7.14e2 3002.0 0.019 Conservative QQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 CubicQuadraticInterpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019
QQN-Bisection-1 85.0 6.94e-1 2.50e-1 1147.7 0.029 GD- 0.0 4.60e1 3.36e1 20.6 0.001 AdaptiveMomentum 0.0 3.92e0 2.83e0 2471.6 0.050 GD-Momentum 0.0 3.55e1 1.96e1 20.8 0.001 Adam-Robust 0.0 1.46e1 6.12e0 2502.0 0.059 Trust Region- 0.0 1.02e3 7.14e2 3002.0 0.019 Conservative QQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 CubicQuadraticInterpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.09e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L L<
GD- 0.0 4.60e1 3.36e1 20.6 0.001 AdaptiveMomentum 0.0 3.92e0 2.83e0 2471.6 0.050 GD-Momentum 0.0 3.55e1 1.96e1 20.8 0.001 Adam-Robust 0.0 1.46e1 6.12e0 2502.0 0.059 Trust Region- 0.0 1.02e3 7.14e2 3002.0 0.019 Conservative QQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 CubicQuadraticInterpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032
AdaptiveMomentum Adam 0.0 3.92e0 2.83e0 2471.6 0.050 GD-Momentum 0.0 3.55e1 1.96e1 20.8 0.001 Adam-Robust 0.0 1.46e1 6.12e0 2502.0 0.059 Trust Region- 0.0 1.02e3 7.14e2 3002.0 0.019 Conservative CQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 CubicQuadraticInterpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
Adam 0.0 3.92e0 2.83e0 2471.6 0.050 GD-Momentum 0.0 3.55e1 1.96e1 20.8 0.001 Adam-Robust 0.0 1.46e1 6.12e0 2502.0 0.059 Trust Region- 0.0 1.02e3 7.14e2 3002.0 0.019 Conservative QQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 CubicQuadraticInterpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
GD-Momentum 0.0 3.55e1 1.96e1 20.8 0.001 Adam-Robust 0.0 1.46e1 6.12e0 2502.0 0.059 Trust Region- 0.0 1.02e3 7.14e2 3002.0 0.019 Conservative V QQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 CubicQuadraticInterpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
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Trust Region- 0.0 1.02e3 7.14e2 3002.0 0.019 Conservative QQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 Cubic Quadratic Interpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
Conservative QQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 CubicQuadraticInterpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
QQN- 70.0 4.25e-1 2.38e-1 1199.2 0.049 CubicQuadraticInterpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
Cubic Quadratic Interpolation Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
Trust Region- 0.0 6.23e1 4.66e0 2827.2 0.018 Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
Standard Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive 6D 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
Trust Region- 0.0 5.00e0 4.66e0 776.1 0.005 Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
Aggressive GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
GD 0.0 5.09e0 4.75e0 32.5 0.001 L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
L-BFGS- 70.0 9.01e-1 2.37e-1 1090.7 0.019 MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
MoreThuente L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
L-BFGS- 20.0 2.02e1 3.89e-1 3106.7 0.032 Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
Conservative QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
QQN- 55.0 6.13e-1 2.60e-1 3314.1 0.061
GoldenSection
QQN-StrongWolfe 100.0 3.45e-1 2.58e-1 792.6 0.024
L-BFGS 0.0 $1.50e2$ $1.98e1$ 135.3 0.002
L-BFGS- 0.0 $8.07e2$ $1.72e1$ 3851.6 0.028
Aggressive
GD-WeightDecay 60.0 7.30e-1 3.59e-1 72.1 0.002
L-BFGS-Limited 45.0 4.21e-1 3.92e-1 3855.4 0.044
QQN-Bisection-2 55.0 4.48e-1 2.15e-1 1588.3 0.038
Adam-AMSGrad 0.0 4.40e0 3.25e0 2442.0 0.057
Adam-Fast 5.0 1.44e1 3.48e-1 44.4 0.001

Adam-		60.0	2.07e0	3.93e-1	1128.9	0.025
WeightDe	cay					
Trust	Region-	0.0	8.41e2	5.05e2	3002.0	0.019
Adaptive						

5 Problem: Rosenbrock_10D

Optimize	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	30.0	1.00e1	9.59e0	1615.8	0.011
Aggressiv	e					
Trust	Region-	0.0	2.10e3	1.66e3	3002.0	0.021
Precise						
Adam-Ro	bust	0.0	3.49e1	1.61e1	2502.0	0.061
Trust	Region-	0.0	1.06e3	6.70e2	3002.0	0.022
Standard						
GD-Neste	rov	100.0	9.31e0	8.97e0	28.6	0.001
GD-Weigl	$\operatorname{ntDecay}$	100.0	9.58e0	9.45e0	35.7	0.001
L-BFGS-		0.0	1.64e2	5.82e1	3850.3	0.051
Aggressiv	e					
QQN-		100.0	9.45e0	9.03e0	277.2	0.005
GoldenSe	ction					
Adam		90.0	9.71e0	9.68e0	2293.1	0.050
QQN-		100.0	9.60e0	9.48e0	109.6	0.004
CubicQua	draticInter	polation				
L-BFGS		0.0	1.19e2	2.51e1	338.9	0.004
GD		75.0	9.75e0	9.59e0	41.5	0.001
GD-		0.0	1.02e2	8.45e1	23.1	0.001
Adaptivel	Momentum					
Trust	Region-	0.0	2.02e3	1.64e3	3002.0	0.021
Adaptive	Ü					
L-BFGS-		95.0	9.20e0	7.83e0	212.8	0.004
MoreThue	ente					
Adam-AN	ISGrad	100.0	9.69 e0	9.66e0	2353.0	0.059
QQN-Bise	ection-1	100.0	8.47e0	7.17e0	120.3	0.004
L-BFGS-		100.0	9.68e0	9.63e0	205.3	0.005
Conservat	ive					
QQN-Stro	ongWolfe	100.0	8.69e0	7.76e0	90.3	0.002
GD-Mom	_	55.0	3.53e1	9.32e0	35.0	0.001
QQN-Bise	ection-2	100.0	9.37e0	8.02e0	83.2	0.002
L-BFGS-I		100.0	9.59e0	9.51e0	62.3	0.001
Adam-Fas	st	45.0	1.94e1	9.18e0	61.6	0.001
Adam-		100.0	9.67e0	9.61e0	789.4	0.019
WeightDe	ecay					
Trust	Region-	0.0	2.16e3	1.76e3	3002.0	0.022
Conservat						

6 Problem: Michalewicz_2D_m10

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-2	0.0	-4.01e-2	-8.01e-1	1074.5	0.025
L-BFGS	10.0	-9.99e-2	-1.00e0	802.0	0.013
L-BFGS-	25.0	-2.50e-1	-1.00e0	752.4	0.011
MoreThuente					
Trust Region-	0.0	-5.33e-7	-8.89e-6	4.2	0.000
Aggressive					

QQN-	0.0	-7.52e-7	-9.60e-6	1573.8	0.026
GoldenSection					
Adam-	0.0	-3.49e-7	-2.33e-6	13.0	0.000
WeightDecay					
Adam	50.0	-4.98e-1	-9.97e-1	1642.0	0.032
GD	0.0	-1.16e-6	-7.38e-6	252.8	0.006
QQN-	0.0	-5.13e-7	-1.00e-5	19.1	0.000
CubicQuadraticInterpol	ation				
Trust Region-	0.0	-7.80e-7	-4.71e-6	4.5	0.000
Precise					
GD-	5.0	-5.00e-2	-9.99e-1	29.5	0.001
AdaptiveMomentum					
L-BFGS-	0.0	-8.97e-2	-8.97e-1	1145.5	0.015
Aggressive					
Trust Region-	0.0	-4.40e-7	-2.59e-6	4.3	0.000
Adaptive					
Adam-AMSGrad	30.0	-2.99e-1	-9.96e-1	192.4	0.004
Adam-Robust	0.0	-3.65e-7	-3.26e-6	13.0	0.000
$GD ext{-}WeightDecay$	0.0	-3.24e-6	-2.45e-5	423.0	0.013
L-BFGS-	0.0	-7.20e-7	-1.07e-5	838.7	0.014
Conservative					
L-BFGS-Limited	25.0	-2.50e-1	-1.00e0	466.0	0.006
Trust Region-	0.0	-6.09e-7	-9.91e-6	4.2	0.000
Standard					
Adam-Fast	40.0	-4.71e-1	-1.00e0	1058.0	0.021
QQN-StrongWolfe	0.0	-4.01e-2	-8.01e-1	1192.1	0.029
QQN-Bisection-1	0.0	-1.99e-2	-3.99e-1	1278.7	0.034
Trust Region-	0.0	-8.81e-7	-1.46e-5	4.3	0.000
Conservative					
GD-Momentum	0.0	-3.25e-2	-6.49e-1	42.9	0.001
GD-Nesterov	10.0	-9.98e-2	-9.99e-1	272.0	0.008

7 Problem: Michalewicz_5D_m10

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS	0.0	-2.16e-1	-8.96e-1	90.4	0.001
Adam	80.0	-2.46e0	-2.71e0	474.6	0.010
Trust Region-	0.0	-9.74e-1	-1.86e0	263.6	0.002
Conservative					
GD-Nesterov	5.0	-1.24e0	-2.69e0	159.9	0.005
Adam-	10.0	-1.75e0	-2.69e0	178.5	0.004
WeightDecay					
GD-	5.0	-1.09e0	-2.69e0	27.8	0.001
AdaptiveMomentum					
L-BFGS-	0.0	-4.98e-1	-1.25e0	3524.2	0.027
Aggressive					
QQN-StrongWolfe	25.0	-1.76e0	-2.79e0	1288.2	0.040
QQN-	5.0	-1.36e0	-2.69e0	1751.2	0.033
GoldenSection					
Trust Region-	0.0	-7.58e-1	-1.73e0	10.8	0.000
Adaptive					
Adam-AMSGrad	55.0	-2.17e0	-2.71e0	439.6	0.011
L-BFGS-	30.0	-1.78e0	-2.69e0	1302.2	0.027
Conservative					
L-BFGS-	5.0	-1.39e0	-2.73e0	519.5	0.010
MoreThuente					
Adam-Robust	10.0	-1.65e0	-2.69e0	83.1	0.002

Adam-Fast	65.0	-2.62e0	-3.29e0	70.8	0.002
GD	45.0	-2.00e0	-2.69e0	738.0	0.019
GD-WeightDecay	5.0	-1.24e0	-2.69e0	86.0	0.003
GD-Momentum	5.0	-1.11e0	-2.69e0	51.9	0.002
QQN-Bisection-2	35.0	-1.78e0	-2.85e0	1315.8	0.032
QQN-Bisection-1	20.0	-1.78e0	-2.74e0	1432.2	0.036
Trust Region	n- 0.0	-1.81e-1	-7.69e-1	5.0	0.000
Aggressive					
QQN-	0.0	-8.49e-1	-1.88e0	1726.7	0.055
CubicQuadraticIn	terpolation				
L-BFGS-Limited	0.0	-8.44e-1	-1.86e0	2003.6	0.025
Trust Region	n- 0.0	-3.29e-1	-1.53e0	5.5	0.000
Standard					
Trust Region	n- 0.0	-9.85e-1	-1.74e0	41.0	0.000
Precise					

8 Problem: Michalewicz_10D_m10

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMSGrad	30.0	-5.74e0	-6.28e0	450.1	0.012
Adam-Fast	35.0	-6.12e0	-6.77e0	106.2	0.002
L-BFGS-	0.0	-4.11e0	-5.98e0	925.2	0.020
MoreThuente					
Trust Region-	0.0	-2.49e0	-4.98e0	9.8	0.000
Standard					
QQN-	5.0	-4.14e0	-6.30e0	2197.4	0.045
GoldenSection					
GD-Momentum	0.0	-3.79e0	-5.24e0	37.3	0.001
GD	0.0	-3.59e0	-5.04e0	17.2	0.000
L-BFGS	0.0	-3.66e-1	-1.14e0	64.8	0.001
Trust Region-	0.0	-3.71e0	-5.42e0	644.0	0.005
Conservative					
GD-	0.0	-4.52e0	-6.23e0	27.6	0.001
AdaptiveMomentum					
GD-Nesterov	0.0	-3.74e0	-6.19e0	26.9	0.001
Adam-Robust	0.0	-5.03e0	-6.26e0	111.0	0.003
QQN-Bisection-2	5.0	-4.37e0	-6.26e0	1808.5	0.047
L-BFGS-	0.0	-1.30e0	-3.26e0	3050.9	0.021
Aggressive					
GD-WeightDecay	0.0	-4.56e0	-6.26e0	174.7	0.006
Trust Region-	0.0	-3.69e0	-5.30e0	114.2	0.001
Precise					
Trust Region-	0.0	-3.26e0	-5.27e0	28.7	0.000
Adaptive					
$\operatorname{QQN-StrongWolfe}$	5.0	-4.71e0	-6.27e0	1916.5	0.061
QQN-	0.0	-3.34e0	-6.26e0	1730.8	0.071
CubicQuadraticInterp	•				
L-BFGS-	0.0	-4.54e0	-6.26e0	2566.8	0.038
Conservative					
Adam-	0.0	-5.31e0	-6.25e0	179.3	0.005
WeightDecay					
Trust Region-	0.0	-1.68e0	-3.16e0	5.3	0.000
Aggressive					
QQN-Bisection-1	0.0	-4.24e0	-5.36e0	2030.0	0.056
Adam	20.0	-5.84e0	-6.27e0	508.9	0.012
L-BFGS-Limited	0.0	-3.52e0	-5.59e0	3009.3	0.039

9 Problem: Rastrigin_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	55.0	8.19e0	1.29e-1	156.0	0.002
GoldenSection					
Adam-Robust	15.0	9.22e0	7.96e0	111.5	0.003
Trust Region-	0.0	1.85e1	9.36e0	3002.0	0.019
Conservative					
Trust Region-	15.0	9.68e0	7.96e0	241.0	0.002
Adaptive					
Trust Region-	10.0	9.71e0	7.96e0	915.9	0.006
Precise					
GD-Momentum	0.0	1.04e1	7.98e0	22.7	0.001
Trust Region-	0.0	1.29e1	7.97e0	20.1	0.000
Aggressive					
L-BFGS	5.0	5.70e1	2.49e0	88.4	0.001
QQN-Bisection-1	65.0	8.59e0	1.03e0	125.3	0.002
GD-	0.0	8.98e0	7.97e0	21.5	0.001
AdaptiveMomentum	1				
Adam-	50.0	9.71e0	7.96e0	254.7	0.006
WeightDecay					
Trust Region-	0.0	9.02e0	7.96e0	63.8	0.000
Standard					
QQN-	80.0	7.75e0	2.17e0	64.2	0.002
CubicQuadraticInte	rpolation				
QQN-Bisection-2	30.0	8.57e0	2.07e0	23.8	0.000
GD-Nesterov	5.0	1.00e1	7.96e0	20.1	0.001
L-BFGS-	75.0	9.82e0	7.96e0	455.9	0.006
Conservative					
GD	5.0	$1.04\mathrm{e}1$	7.96e0	13.3	0.000
GD-WeightDecay	5.0	1.01e1	7.96e0	24.3	0.001
Adam-Fast	0.0	9.46e0	8.02e0	36.3	0.001
L-BFGS-	55.0	8.56e0	1.83e0	170.8	0.003
MoreThuente					
QQN-StrongWolfe	65.0	8.32e0	1.36e0	71.4	0.002
L-BFGS-	0.0	2.81e1	1.91e1	3851.8	0.024
Aggressive					
Adam-AMSGrad	55.0	1.02e1	7.96e0	770.2	0.018
L-BFGS-Limited	70.0	9.13e0	5.86e0	106.5	0.001
Adam	55.0	1.02e1	7.96e0	780.5	0.016

$10 \quad Problem: \ Rastrigin_5D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-1	25.0	2.48e1	1.80e1	186.6	0.004
QQN-	35.0	2.31e1	7.97e0	266.0	0.004
GoldenSection					
QQN-	50.0	2.30e1	4.91e0	106.2	0.003
CubicQuadraticInter	rpolation				
Trust Region-	0.0	6.91e1	5.03e1	3002.0	0.020
Conservative					
QQN-StrongWolfe	40.0	2.36e1	1.16e1	100.2	0.003
GD-Momentum	5.0	3.03e1	2.04e1	28.9	0.001
Adam-Robust	55.0	2.24e1	1.99e1	103.5	0.003
GD-Nesterov	45.0	2.37e1	1.99e1	41.2	0.001
GD-WeightDecay	50.0	2.25e1	1.89e1	37.8	0.002

GD	30.0	2.44e1	1.99e1	15.7	0.000
Trust Region-	35.0	2.39e1	2.03e1	2558.8	0.017
Precise	0010				0.021
L-BFGS-	50.0	2.77e1	7.71e0	165.1	0.003
MoreThuente					
QQN-Bisection-2	15.0	3.08e1	1.10e1	36.6	0.001
Adam-Fast	40.0	2.33e1	1.92e1	48.9	0.001
L-BFGS-	40.0	2.34e1	1.99e1	881.9	0.011
Conservative					
GD-	65.0	2.14e1	1.83e1	52.4	0.002
AdaptiveMomentum					
Adam-	55.0	$2.27\mathrm{e}1$	2.02e1	246.9	0.006
WeightDecay					
L-BFGS-Limited	35.0	2.84e1	1.70e1	115.2	0.002
Trust Region-	45.0	2.31e1	1.95e1	168.5	0.001
Standard					
L-BFGS	0.0	1.02e2	5.89e1	98.2	0.001
Trust Region-	50.0	2.33e1	1.82e1	45.7	0.000
Aggressive					
Adam	30.0	2.42e1	2.03e1	745.3	0.017
Trust Region-	60.0	2.34e1	2.02e1	646.1	0.004
Adaptive					
L-BFGS-	0.0	7.97e1	$5.68\mathrm{e}1$	3852.0	0.032
Aggressive					
Adam-AMSGrad	40.0	2.38e1	2.04e1	719.2	0.018

$11 \quad Problem: \ Rastrigin_10D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Robust	30.0	4.44e1	4.13e1	127.9	0.003
L-BFGS-	20.0	4.68e1	4.06e1	2037.2	0.020
Conservative					
Trust Region-	25.0	4.59e1	4.10e1	346.6	0.002
Standard					
Trust Region-	45.0	4.36e1	3.87e1	93.5	0.001
Aggressive					
Adam-AMSGrad	55.0	4.48e1	4.17e1	718.0	0.018
Trust Region-	40.0	4.39e1	4.16e1	1397.8	0.010
Adaptive					
GD-Nesterov	25.0	4.50e1	4.02e1	249.0	0.009
L-BFGS-Limited	50.0	4.36e1	3.39e1	122.0	0.002
Trust Region-	0.0	9.70e1	6.33e1	3002.0	0.021
Precise					
Adam-Fast	30.0	4.51e1	3.98e1	71.9	0.002
GD-WeightDecay	45.0	4.39e1	3.60e1	61.3	0.002
L-BFGS-	65.0	4.11e1	2.56e1	291.6	0.006
MoreThuente					
L-BFGS-	0.0	1.63e2	1.40e2	3852.0	0.031
Aggressive					
QQN-StrongWolfe	40.0	5.16e1	3.24e1	118.5	0.003
QQN-Bisection-1	45.0	5.11e1	3.88e1	216.4	0.005
GD	35.0	4.47e1	3.89e1	19.4	0.001
GD-Momentum	10.0	6.15e1	4.06e1	30.6	0.001
GD-	40.0	4.85e1	3.80e1	23.2	0.001
${\bf Adaptive Momentum}$					
Trust Region-	0.0	1.57e2	1.28e2	3002.0	0.021
Conservative					

QQN-Bisection-2	15.0	5.38e1	2.52e1	50.6	0.001
QQN-	60.0	4.52e1	3.21e1	102.5	0.003
CubicQuadraticInterpolation	on				
QQN-	50.0	4.53e1	2.11e1	275.4	0.005
GoldenSection					
L-BFGS	0.0	1.85e2	1.42e2	102.0	0.001
Adam	30.0	4.65e1	4.16e1	731.5	0.016
Adam-	35.0	4.52e1	4.15e1	253.8	0.006
WeightDecay					

$12 \quad Problem: \ Ackley_2D_a20_b0.2_c6.28e0$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-StrongWolfe	15.0	3.49e0	3.00e0	62.8	0.001
Adam-Robust	0.0	3.58e0	3.57e0	42.9	0.001
GD-WeightDecay	0.0	3.59e0	3.57e0	49.2	0.002
QQN-	40.0	3.44e0	1.95e0	496.8	0.014
CubicQuadraticInter	polation				
Adam-Fast	0.0	3.62e0	3.57e0	52.9	0.001
Adam-AMSGrad	0.0	3.57e0	3.57e0	753.0	0.017
GD	0.0	3.57e0	3.57e0	20.2	0.001
QQN-Bisection-2	40.0	2.75e0	3.79e-1	36.5	0.001
GD-	0.0	3.57e0	3.57e0	15.2	0.001
AdaptiveMomentum					
Trust Region-	5.0	3.66e0	3.32e0	14.8	0.000
Adaptive					
Trust Region-	0.0	4.78e0	3.62e0	5.3	0.000
Aggressive					
L-BFGS-	20.0	3.74e0	2.80e0	3083.9	0.018
Aggressive					
Trust Region-	0.0	5.19e0	3.58e0	8.6	0.000
Standard					
Trust Region-	0.0	3.58e0	3.57e0	171.2	0.001
Conservative					
Trust Region-	0.0	3.58e0	3.58e0	39.5	0.000
Precise					
GD-Nesterov	0.0	3.57e0	3.57e0	30.6	0.001
L-BFGS-	45.0	3.34e0	1.83e0	42.2	0.001
MoreThuente					
Adam-	0.0	3.58e0	3.57e0	119.4	0.003
WeightDecay					
L-BFGS	50.0	3.20e0	1.89e0	202.7	0.003
Adam	0.0	3.57e0	3.57e0	301.6	0.006
L-BFGS-Limited	15.0	3.37e0	2.64e-1	176.7	0.002
QQN-	35.0	3.18e0	1.58e0	137.5	0.002
GoldenSection					
QQN-Bisection-1	60.0	2.85e0	1.12e-2	53.6	0.001
L-BFGS-	0.0	3.57e0	3.57e0	183.6	0.004
Conservative					
GD-Momentum	0.0	3.70e0	3.57e0	14.7	0.000

$13\quad Problem:\ Ackley_5D_a20_b0.2_c6.28e0$

Trust Region-	0.0	4.15e0	3.71e0	5.0	0.000
Aggressive					
Adam	0.0	3.57e0	3.57e0	548.5	0.011
L-BFGS-Limited	0.0	3.57e0	3.57e0	310.5	0.005
L-BFGS-	20.0	3.76e0	3.01e0	3085.2	0.020
Aggressive					
Trust Region-	0.0	3.62e0	3.57e0	45.5	0.000
Precise					
QQN-Bisection-2	35.0	3.34e0	2.48e0	50.8	0.001
GD-WeightDecay	0.0	3.61e0	3.58e0	15.3	0.000
Trust Region-	0.0	3.58e0	3.57e0	212.3	0.002
Conservative					
GD-Momentum	0.0	3.72e0	3.59e0	13.0	0.000
QQN-StrongWolfe	5.0	3.50e0	2.03e0	53.0	0.001
Adam-AMSGrad	0.0	3.57e0	3.57e0	1172.5	0.028
L-BFGS-	0.0	3.57e0	3.57e0	373.8	0.006
Conservative					
Trust Region-	0.0	4.49e0	3.60 e0	9.1	0.000
Standard					
Adam-Fast	0.0	3.60e0	3.57e0	83.3	0.002
GD-	0.0	3.58e0	3.57e0	34.5	0.001
Adaptive Momentum					
GD	0.0	3.57e0	3.57e0	51.8	0.001
QQN-	5.0	3.56e0	3.14e0	206.9	0.006
CubicQuadraticInterpo	olation				
QQN-	5.0	3.51e0	2.30e0	193.7	0.003
GoldenSection					
L-BFGS	25.0	3.46e0	1.87e0	121.3	0.002
GD-Nesterov	0.0	3.60 e0	3.57e0	17.0	0.001
Adam-	0.0	3.57e0	3.57e0	183.0	0.004
WeightDecay					
Adam-Robust	0.0	3.58e0	3.57e0	59.0	0.001
QQN-Bisection-1	30.0	3.38e0	2.37e0	65.2	0.001
L-BFGS-	10.0	3.50e0	2.54e0	51.0	0.001
MoreThuente					
Trust Region-	0.0	3.70e0	3.58e0	17.3	0.000
Adaptive					

14 Problem: Ackley_10D_a20_b0.2_c6.28e0

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	5.0	3.57e0	3.52e0	55.9	0.001
MoreThuente					
QQN-	5.0	3.59e0	3.33e0	471.4	0.016
CubicQuadraticInter	rpolation				
QQN-	0.0	3.57e0	3.57e0	186.9	0.003
GoldenSection					
L-BFGS	15.0	3.50 e0	2.72e0	133.3	0.002
GD-WeightDecay	0.0	3.59e0	3.58e0	19.4	0.001
L-BFGS-	5.0	3.54e0	2.95e0	185.8	0.003
Aggressive					
GD-Momentum	0.0	3.81e0	3.64e0	14.5	0.000
GD-Nesterov	0.0	3.66e0	3.62e0	14.0	0.000
L-BFGS-	0.0	3.57e0	3.57e0	170.7	0.005
Conservative					
GD-	0.0	3.60 e0	3.57e0	16.1	0.001
AdaptiveMomentum	<u>l</u>				

Adam-Fast	0.0	3.60 e0	3.57e0	106.6	0.002
Adam-AMSGrad	0.0	3.57e0	3.57e0	1241.4	0.031
Adam-	0.0	3.57e0	3.57e0	281.3	0.007
WeightDecay					
Trust Region-	0.0	3.61e0	3.58e0	42.5	0.000
Precise					
Adam-Robust	0.0	3.58e0	3.57e0	126.8	0.003
Trust Region-	0.0	4.36e0	3.64e0	9.1	0.000
Standard					
L-BFGS-Limited	0.0	3.57e0	3.57e0	142.2	0.003
Trust Region-	0.0	4.22e0	3.81e0	5.2	0.000
Aggressive					
Adam	0.0	3.57e0	3.57e0	745.8	0.017
Trust Region-	0.0	4.41e0	3.59e0	18.2	0.000
Adaptive					
QQN-Bisection-2	20.0	3.54e0	3.30e0	65.9	0.001
Trust Region-	0.0	3.58e0	3.57e0	205.8	0.002
Conservative					
QQN-StrongWolfe	0.0	3.57e0	3.57e0	79.2	0.002
QQN-Bisection-1	10.0	3.54e0	3.08e0	96.2	0.002
GD	0.0	3.57e0	3.57e0	99.3	0.003

$15 \quad Problem: \ Styblinski Tang_2D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-StrongWolfe	65.0	-7.34e1	-7.83e1	284.4	0.008
Adam-AMSGrad	0.0	-3.03e1	-3.29e1	2502.0	0.058
GD-	5.0	-3.87e1	-7.83e1	24.4	0.001
AdaptiveMomentum					
Trust Region-	20.0	-6.43e1	-7.83e1	49.3	0.000
Aggressive					
Adam-	80.0	-7.55e1	-7.83e1	1893.5	0.042
WeightDecay					
Trust Region-	0.0	-6.26e1	-6.35e1	3002.0	0.019
Precise					
L-BFGS-	0.0	-5.13e1	-7.77e1	3848.1	0.025
Aggressive					
Adam	0.0	-2.94e1	-3.47e1	2502.0	0.049
QQN-Bisection-1	70.0	-7.40e1	-7.83e1	385.6	0.007
GD-Momentum	15.0	-5.68e1	-7.83e1	35.9	0.001
QQN-Bisection-2	75.0	-7.48e1	-7.83e1	99.0	0.002
Trust Region-	40.0	6.01e1	-7.83e1	212.3	0.001
Standard					
QQN-	90.0	-7.69e1	-7.83e1	159.8	0.002
GoldenSection					
L-BFGS	50.0	-7.32e1	-7.83e1	132.3	0.002
GD-Nesterov	10.0	-6.56e1	-7.83e1	29.1	0.001
L-BFGS-	90.0	-7.69e1	-7.83e1	300.1	0.007
Conservative					
Adam-Fast	35.0	-7.48e1	-7.83e1	67.3	0.001
QQN-	70.0	-7.34e1	-7.83e1	74.5	0.002
CubicQuadraticInter	polation				
Trust Region-	70.0	-7.37e1	-7.83e1	885.1	0.006
Adaptive					
Trust Region-	0.0	-1.02e1	-1.15e1	3002.0	0.019
Conservative					
GD	65.0	-7.34e1	-7.83e1	66.5	0.002

Adam-Robust L-BFGS-Limited	$0.0 \\ 70.0$	-2.07e1 -7.41e1	-2.44e1 -7.83e1	$2502.0 \\ 500.0$	$0.057 \\ 0.006$
L-BFGS-	70.0	-7.41e1	-7.83e1	54.1	0.001
MoreThuente GD-WeightDecay	35.0	-7.46e1	-7.83e1	39.9	0.001

$16 \quad Problem: \ Styblinski Tang_5D$

Optimize	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-		45.0	-1.82e2	-1.96e2	129.7	0.004
CubicQua	adraticInter	polation				
GD-Weigl	htDecay	60.0	-1.88e2	-1.96e2	53.1	0.002
Adam		0.0	-7.38e1	-8.30e1	2502.0	0.055
Trust	Region-	0.0	-6.28e1	-6.40e1	3002.0	0.020
Precise						
Trust	Region-	55.0	-1.88e2	-1.96e2	568.8	0.004
Standard						
QQN-Stro	ongWolfe	40.0	-1.85e2	-1.96e2	261.1	0.007
L-BFGS		30.0	-1.86e2	-1.96e2	153.3	0.002
QQN-		70.0	-1.90e2	-1.96e2	330.5	0.005
GoldenSe	ction					
L-BFGS-		50.0	-1.89e2	-1.96e2	209.2	0.004
MoreThue	ente					
GD-		10.0	-1.35e2	-1.95e2	34.4	0.001
	Momentum					
Adam-Al	ASGrad	0.0	-7.50e1	-8.10e1	2502.0	0.060
GD-Mom	entum	15.0	-1.64e2	-1.95e2	57.8	0.002
QQN-Bise	ection-2	60.0	-1.89e2	-1.96e2	177.8	0.004
L-BFGS-		0.0	-1.04e2	-1.83e2	3847.9	0.028
Aggressiv	e					
QQN-Bise	ection-1	60.0	-1.89e2	-1.96e2	413.4	0.008
Adam-		65.0	-1.90e2	-1.95e2	1865.5	0.041
WeightDe						
L-BFGS-I	Limited	50.0	-1.89e2	-1.96e2	839.9	0.011
Adam-Ro	bust	0.0	-5.32e1	-6.12e1	2502.0	0.058
L-BFGS-		65.0	-1.89e2	-1.95e2	567.8	0.012
Conservat	tive					
GD		50.0	-1.86e2	-1.95e2	95.7	0.003
GD-Neste		30.0	-1.82e2	-1.96e2	44.9	0.001
Trust	Region-	65.0	-1.88e2	-1.95e2	2249.5	0.014
Adaptive						
Adam-Fas	st	55.0	-1.86e2	-1.96e2	65.0	0.001
Trust	Region-	0.0	-1.05e1	-1.22e1	3002.0	0.019
Conservat						
Trust	Region-	45.0	-1.90e2	-1.96e2	144.5	0.001
Aggressiv	e					

$17 \quad Problem: \ Styblinski Tang_10D$

Optimize	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bise	ction-2	35.0	-3.72e2	-3.90e2	234.2	0.006
Trust	Region-	0.0	-2.51e2	-2.54e2	3002.0	0.020
Adaptive						
Trust	Region-	50.0	-3.71e2	-3.81e2	284.1	0.002
Aggressive)					

Trust Region-	0.0	-6.35e1	-6.56e1	3002.0	0.021
Precise					
QQN-Bisection-1	30.0	-3.74e2	-3.89e2	412.5	0.018
QQN-	45.0	-3.76e2	-3.91e2	452.4	0.012
GoldenSection					
Trust Region-	40.0	-3.68e2	-3.79e2	1120.5	0.008
Standard					
GD-	20.0	-3.21e2	-3.82e2	53.4	0.002
AdaptiveMomentum					
L-BFGS-	35.0	-3.76e2	-3.91e2	125.9	0.002
MoreThuente					
GD	25.0	-3.71e2	-3.80e2	137.3	0.004
L-BFGS	45.0	-3.76e2	-3.90e2	132.2	0.002
GD-WeightDecay	45.0	-3.75e2	-3.83e2	74.7	0.003
QQN-StrongWolfe	15.0	-3.60e2	-3.86e2	362.2	0.012
Adam-AMSGrad	0.0	-1.51e2	-1.63e2	2502.0	0.061
Adam-Robust	0.0	-1.05e2	-1.13e2	2502.0	0.062
Trust Region-	0.0	-1.09e1	-1.25e1	3002.0	0.021
Conservative					
L-BFGS-	30.0	-3.74e2	-3.82e2	543.3	0.015
Conservative					
L-BFGS-	0.0	-1.97e2	-3.07e2	3848.4	0.029
Aggressive					
L-BFGS-Limited	35.0	-3.76e2	-3.86e2	550.6	0.009
GD-Nesterov	30.0	-3.75e2	-3.83e2	65.9	0.002
GD-Momentum	15.0	-3.59e2	-3.82e2	86.2	0.003
Adam-Fast	30.0	-3.65e2	-3.85e2	67.5	0.002
QQN-	15.0	-3.66e2	-3.91e2	172.1	0.006
CubicQuadraticInterpola	ation				
Adam	0.0	-1.49e2	-1.65e2	2502.0	0.053
Adam-	35.0	-3.73e2	-3.78e2	1837.7	0.043
WeightDecay					

18 Problem: Beale_2D

Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
100.0	9.18e-3	4.69e-3	27.7	0.001
0.0	2.17e-1	8.70e-2	2502.0	0.056
100.0	1.00e-2	3.68e-4	110.5	0.002
0.0	2.71e2	1.24e-1	50.2	0.000
100.0	1.17e-2	5.27e-3	200.7	0.005
95.0	4.13e-1	4.49e-3	277.4	0.006
70.0	2.61e0	3.70e-3	83.8	0.002
100.0	1.48e-2	1.46e-2	88.0	0.003
5.0	1.23e0	1.33e-3	17.8	0.000
45.0	2.17e-2	3.20e-3	743.8	0.005
100.0	1.49e-2	1.49e-2	229.2	0.006
65.0	4.55e-1	1.99e-3	122.0	0.002
0.0	4.58e-1	1.68e-1	2502.0	0.054
100.0	6.90e-3	2.42e-3	347.2	0.005
95.0	4.97e-1	2.46e-3	218.8	0.008
olation				
5.0	1.32e0	1.20e-2	24.4	0.001
	100.0 0.0 100.0 0.0 100.0 95.0 70.0 100.0 5.0 45.0 100.0 65.0 0.0 100.0 95.0	100.0 9.18e-3 0.0 2.17e-1 100.0 1.00e-2 0.0 2.71e2 100.0 1.17e-2 95.0 4.13e-1 70.0 2.61e0 100.0 1.48e-2 5.0 1.23e0 45.0 2.17e-2 100.0 1.49e-2 65.0 4.55e-1 0.0 4.58e-1 100.0 6.90e-3 95.0 4.97e-1	100.0 9.18e-3 4.69e-3 0.0 2.17e-1 8.70e-2 100.0 1.00e-2 3.68e-4 0.0 2.71e2 1.24e-1 100.0 1.17e-2 5.27e-3 95.0 4.13e-1 4.49e-3 70.0 2.61e0 3.70e-3 100.0 1.48e-2 1.46e-2 5.0 1.23e0 1.33e-3 45.0 2.17e-2 3.20e-3 100.0 1.49e-2 1.49e-2 65.0 4.55e-1 1.99e-3 0.0 4.58e-1 1.68e-1 100.0 6.90e-3 2.42e-3 95.0 4.97e-1 2.46e-3	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

GD-	80.0	3.09e-1	6.42e-4	25.5	0.001
${\bf Adaptive Momentum}$					
L-BFGS-	100.0	8.71e-3	3.68e-4	78.1	0.001
MoreThuente					
Adam-Fast	0.0	1.59e0	1.39e0	37.6	0.001
Adam	0.0	7.19e-2	3.04e-2	2502.0	0.048
QQN-StrongWolfe	90.0	4.59e-1	3.96e-3	786.0	0.023
Adam-	100.0	1.50e-2	1.49e-2	1539.9	0.033
WeightDecay					
Trust Region-	0.0	1.27e-1	3.79e-2	179.6	0.001
Adaptive					
L-BFGS-	0.0	1.39e1	8.86e0	3851.9	0.021
Aggressive					
Trust Region-	5.0	4.33e0	9.92e-3	2999.7	0.017
Conservative					

19 Problem: Levi_2D

	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMSGrad	0.0	1.91e0	1.43e0	282.9	0.006
Trust Region-	0.0	2.09e0	1.46e0	29.6	0.000
Adaptive					
QQN-Bisection-1	15.0	1.30e0	1.54e-1	200.7	0.004
L-BFGS-	0.0	1.94e0	1.43e0	209.4	0.004
Conservative					
QQN-	40.0	1.04e0	1.50e-1	189.6	0.007
CubicQuadraticInterp	olation				
GD	0.0	2.04e0	1.43e0	21.9	0.001
Trust Region-	0.0	2.18e0	1.43e0	10.7	0.000
Standard					
Adam-Fast	0.0	2.09e0	1.47e0	47.2	0.001
Adam	0.0	1.93e0	1.43e0	242.4	0.005
GD-Momentum	0.0	2.02e0	7.01e-1	19.0	0.001
Trust Region-	0.0	2.04e0	1.43e0	76.8	0.001
Precise					
GD-WeightDecay	0.0	2.00e0	1.19e0	28.1	0.001
QQN-Bisection-2	20.0	1.08e0	4.47e-2	77.5	0.001
Trust Region-	0.0	1.33e2	1.02e0	6.3	0.000
Aggressive					
Adam-	0.0	1.95e0	1.44e0	99.6	0.002
WeightDecay					
Trust Region-	0.0	1.95e0	1.07e0	490.0	0.003
Conservative					
QQN-StrongWolfe	25.0	1.44e0	1.40e-1	450.4	0.013
Adam-Robust	0.0	1.88e0	1.08e0	42.6	0.001
L-BFGS-	0.0	2.87e0	1.00e0	3851.3	0.022
Aggressive					
L-BFGS-	15.0	1.48e0	1.36e-1	86.2	0.001
MoreThuente					
L-BFGS-Limited	25.0	9.89e-1	1.13e-1	828.4	0.010
GD-Nesterov	0.0	1.93e0	7.77e-1	15.2	0.000
L-BFGS	15.0	4.40e0	1.84e-1	107.2	0.001
QQN-	45.0	9.83e-1	1.87e-2	262.0	0.004
GoldenSection					
GD-	0.0	2.06e0	1.28e0	18.7	0.001
${\bf Adaptive Momentum}$					

$20 \quad Problem: \ Goldstein Price_2D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-WeightDecay	0.0	8.57e2	8.40e2	20.3	0.001
QQN-	35.0	8.40e1	8.40e1	383.2	0.006
GoldenSection					
L-BFGS-	0.0	8.60e2	8.43e2	3847.0	0.032
Aggressive					
QQN-	15.0	2.71e2	8.40e1	291.6	0.011
CubicQuadraticInterp	polation				
GD-Momentum	0.0	8.98e2	9.88e1	17.9	0.000
Trust Region-	0.0	2.12e3	8.97e2	3002.0	0.018
Conservative					
QQN-StrongWolfe	15.0	8.34e1	7.11e1	552.0	0.014
Adam-	0.0	8.40e2	8.40e2	209.7	0.004
WeightDecay					
L-BFGS-Limited	0.0	8.41e1	8.40e1	4259.4	0.037
L-BFGS-	0.0	8.44e2	8.40e2	346.4	0.005
Conservative					
Adam	0.0	8.40e2	8.40e2	597.1	0.012
Adam-Robust	0.0	8.41e2	8.40e2	81.0	0.002
Trust Region-	0.0	2.09e3	8.40e2	2649.8	0.016
Adaptive					
L-BFGS-	5.0	1.19e2	2.24e1	621.9	0.010
MoreThuente					
Trust Region-	0.0	1.22e3	8.40e2	1684.8	0.010
Standard					
Trust Region-	0.0	9.54e2	8.40e2	725.5	0.004
Aggressive					
Adam-Fast	0.0	8.55e2	8.40e2	35.3	0.001
L-BFGS	5.0	8.00e2	6.35e1	176.8	0.003
GD	0.0	8.50e2	8.41e2	24.6	0.001
QQN-Bisection-1	20.0	8.40e1	8.40e1	362.8	0.010
Adam-AMSGrad	0.0	8.40e2	8.40e2	603.6	0.013
GD-	0.0	8.45e2	8.41e2	19.9	0.001
AdaptiveMomentum	0.0	5 5 5 -	510 -	_3.0	0.002
Trust Region-	0.0	2.20e3	8.40e2	2943.8	0.018
Precise	0.0				0.0-0
QQN-Bisection-2	5.0	1.58e2	8.40e1	21.2	0.001
GD-Nesterov	0.0	8.57e2	8.40e2	24.6	0.001

21 Problem: Matyas_2D

Optimiz	zer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	6.67 e-1	2.86e-2	6.3	0.000
Aggressiv	ve					
L-BFGS-	Limited	100.0	1.73e-2	7.16e-3	24.6	0.000
QQN-Bis	section-2	100.0	1.49e-2	8.19e-4	40.9	0.001
GD-Weig	ghtDecay	100.0	2.50e-2	2.50e-2	223.7	0.007
Trust	Region-	0.0	4.71e-2	3.00e-2	5.0	0.000
Precise						
QQN-		100.0	1.07e-2	2.70e-30	34.0	0.001
CubicQu	adraticInter	polation				
Adam		100.0	2.50e-2	2.49e-2	624.2	0.012
Adam-Al	MSGrad	100.0	2.50e-2	2.50e-2	680.7	0.015
L-BFGS		100.0	1.47e-2	5.88e-4	20.1	0.000

L-BFGS-	95.0	2.44e-2	2.16e-2	217.4	0.002
Aggressive					
QQN-Bisection-1	100.0	1.91e-2	2.83e-3	34.2	0.001
QQN-	100.0	1.55e-2	1.12e-3	138.9	0.002
GoldenSection					
L-BFGS-	100.0	2.30e-2	2.07e-2	39.0	0.001
Conservative					
GD-Momentum	100.0	2.49e-2	2.48e-2	67.2	0.002
GD-	95.0	2.52e-2	2.42e-2	29.2	0.001
${\bf Adaptive Momentum}$					
Adam-Fast	100.0	2.30e-2	1.98e-2	12.9	0.000
Adam-Robust	100.0	2.48e-2	2.46e-2	79.0	0.002
Trust Region-	0.0	6.88e-2	2.83e-2	7.0	0.000
Adaptive					
L-BFGS-	100.0	1.72e-2	2.59e-5	20.8	0.000
MoreThuente					
GD	100.0	2.50e-2	2.50e-2	629.6	0.015
QQN-StrongWolfe	100.0	2.73e-29	1.77e-32	24.0	0.000
Adam-	100.0	2.49e-2	2.48e-2	221.4	0.004
WeightDecay					
GD-Nesterov	100.0	2.49e-2	2.48e-2	71.0	0.002
Trust Region-	0.0	2.41e-1	3.14e-2	7.2	0.000
Standard					
Trust Region-	0.0	3.93e-2	2.66e-2	7.0	0.000
Conservative					

22 Problem: Himmelblau_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust Region-	80.0	1.50e-1	3.76e-3	516.7	0.003
Standard					
Adam-	100.0	2.41e-1	2.34e-1	1744.1	0.037
WeightDecay					
GD-Nesterov	40.0	9.49e0	3.27e-2	26.7	0.001
L-BFGS-Limited	100.0	9.58e-2	7.81e-3	60.9	0.001
Adam	0.0	8.20e1	7.20e1	2502.0	0.048
Adam-Robust	0.0	1.05e2	9.10e1	2502.0	0.055
Trust Region-	100.0	1.32e-1	7.13e-2	2048.1	0.013
Adaptive					
QQN-	100.0	7.91e-2	1.17e-4	102.8	0.001
GoldenSection					
GD-WeightDecay	75.0	3.93e-1	8.90e-2	29.6	0.001
L-BFGS-	0.0	$3.44\mathrm{e}1$	4.95e0	3850.9	0.022
Aggressive					
GD	100.0	1.53e-1	2.80e-2	42.9	0.001
Trust Region-	0.0	1.60e2	1.52e2	3002.0	0.018
Conservative					
L-BFGS-	100.0	2.03e-1	1.42e-1	264.2	0.006
Conservative					
QQN-Bisection-2	100.0	8.17e-2	2.27e-3	55.1	0.001
Trust Region-	25.0	6.71e-1	1.80e-2	133.1	0.001
Aggressive					
QQN-Bisection-1	100.0	1.09e-1	2.06e-2	107.9	0.002
Adam-Fast	0.0	5.29e0	4.98e0	69.3	0.001
Trust Region-	0.0	1.07e2	1.02e2	3002.0	0.018
Precise					
L-BFGS	35.0	6.22e0	1.56e-2	93.7	0.001
QQN-StrongWolfe	100.0	9.18e-2	9.60e-3	61.5	0.001

L-BFGS-	100.0	1.02e-1	8.92e-3	36.3	0.000
MoreThuente					
GD-Momentum	0.0	4.37e1	3.43e0	24.3	0.001
Adam-AMSGrad	0.0	8.17e1	7.19e1	2502.0	0.055
QQN-	100.0	9.46e-2	5.19e-3	68.0	0.002
CubicQuadraticInter	polation				
GD-	0.0	6.94e1	5.87e1	22.8	0.001
${\bf Adaptive Momentum}$					

23 Problem: Booth_2D

Optimiz	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	1.05e1	2.82e0	3002.0	0.018
Precise						
Trust	Region-	0.0	1.49e0	4.00e-1	63.5	0.000
Aggressiv	e					
QQN-Str	$_{ m ongWolfe}$	100.0	1.13e-2	9.93e-3	26.0	0.000
QQN-Bis	ection-1	100.0	1.87e-2	1.21e-2	162.4	0.003
QQN-		100.0	2.56e-6	2.81e-7	56.0	0.001
CubicQua	draticInter	polation				
QQN-Bis	ection-2	100.0	1.49e-2	1.15e-2	100.0	0.002
L-BFGS-		100.0	3.86e-2	5.15e-4	29.7	0.000
MoreThu	ente					
L-BFGS-	Limited	100.0	6.14e-2	2.04e-3	56.1	0.001
GD		100.0	1.17e-1	1.15e-1	86.0	0.002
GD-Mom	$_{ m entum}$	0.0	7.27e0	6.20e0	21.2	0.001
GD-Weig	htDecay	15.0	1.29e0	1.15e-1	27.0	0.001
Adam-Fa	st	0.0	3.09e0	2.74e0	56.3	0.001
Adam-Ro	bust	0.0	2.03e1	1.43e1	2502.0	0.054
L-BFGS-		0.0	7.43e1	6.24e1	3852.0	0.021
Aggressiv	e					
Adam-AN	4SGrad	0.0	1.19e1	8.69e0	2502.0	0.055
Adam		0.0	1.20e1	8.76e0	2502.0	0.048
L-BFGS-		100.0	1.10e-1	1.03e-1	208.0	0.005
Conservat	tive					
GD-Neste	erov	0.0	2.56e0	2.10e0	20.6	0.001
Trust	Region-	0.0	6.49e1	5.04e1	3002.0	0.018
Conservat	tive					
Adam-		100.0	1.19e-1	1.19e-1	1886.3	0.038
WeightDe	ecay					
Trust	Region-	90.0	6.12e-2	2.38e-2	919.9	0.005
Adaptive						
Trust	Region-	0.0	5.27e-1	3.46e-1	235.1	0.001
Standard						
L-BFGS		100.0	4.05e-2	1.17e-2	42.4	0.001
QQN-		100.0	3.39e-2	1.60e-2	92.0	0.001
GoldenSe	ction					
GD-		0.0	9.51e0	8.56e0	21.0	0.001
Adaptive	Momentum					

24 Problem: Griewank_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS	0.0	4.91e0	4.91e0	295.6	0.006
QQN-Bisection-1	0.0	4.91e0	4.91e0	76.3	0.002

GD-Nesterov	(0.0	4.91e0	4.91e0	125.0	0.004
Adam-Fast		0.0	4.93e0	4.92e0	67.0	0.004
L-BFGS-		0.0	4.91e0	4.91e0	73.0	0.001
MoreThuente	().U	4.0100	4.5100	10.0	0.001
	gion- (0.0	5.99e0	5.90e0	5.0	0.000
Standard Res	31011-).0	5.9960	5.9000	5.0	0.000
	gion- (0.0	4.92e0	4.91e0	336.5	0.002
Conservative	31011-).0	4.9200	4.9160	550.5	0.002
	gion- (0.0	6.00e0	5.90e0	5.0	0.000
Aggressive Aggressive	gion- ().0	0.00e0	5.90e0	5.0	0.000
GD-WeightDec		0.0	4.91e0	4.91e0	406.6	0.013
			4.74e0	3.76e0	347.6	0.013
QQN-StrongWo						
L-BFGS-	C	0.0	4.91e0	4.91e0	2427.6	0.056
Aggressive			4.01.0	4.01.0	050 0	0.004
QQN-	C	0.0	4.91e0	4.91e0	258.3	0.004
GoldenSection			7.04.0	F 10 0	2502.0	0.040
Adam		0.0	5.24e0		2502.0	0.049
	gion- (0.0	4.95e0	4.93e0	58.7	0.000
Precise						
Adam-AMSGra		0.0	5.24e0		2502.0	0.056
GD		0.0	4.91e0		1668.0	0.042
Adam-	(0.0	4.92e0	4.92e0	2231.2	0.047
WeightDecay						
GD-		0.0	4.99e0	4.97e0	32.0	0.001
AdaptiveMome						
Adam-Robust		0.0	5.36e0		2502.0	0.055
QQN-Bisection			4.66e0	4.25e0	94.6	0.002
Trust Reg	gion- (0.0	5.11e0	5.00e0	16.4	0.000
Adaptive						
QQN-	(0.0	4.80e0	2.74e0	77.3	0.002
CubicQuadratic	cInterpolation					
GD-Momentum	n (0.0	4.91e0	4.91e0	100.2	0.003
L-BFGS-Limite	ed 0	0.0	4.91e0	4.91e0	927.3	0.021
L-BFGS-	(0.0	4.91e0	4.91e0	504.6	0.012
Conservative						

25 Problem: Griewank_5D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	0.0	1.22e1	1.22e1	705.6	0.018
Conservative					
L-BFGS-	0.0	1.22e1	1.22e1	118.5	0.002
MoreThuente					
GD-Momentum	0.0	1.22e1	1.22e1	387.2	0.012
L-BFGS-Limited	0.0	1.22e1	1.22e1	632.5	0.016
GD	0.0	1.22e1	1.22e1	1668.0	0.044
L-BFGS-	0.0	1.09e1	5.15e0	1097.7	0.015
Aggressive					
QQN-	0.0	1.28e1	1.22e1	115.1	0.004
CubicQuadraticInter	polation				
GD-Nesterov	0.0	1.22e1	1.22e1	394.2	0.013
QQN-	0.0	1.12e1	1.62e0	1154.2	0.023
GoldenSection					
QQN-Bisection-1	0.0	1.06e1	2.18e0	872.0	0.019
GD-	0.0	1.23e1	1.22e1	133.1	0.005
AdaptiveMomentum					
Adam	0.0	1.25e1	$1.25\mathrm{e}1$	2502.0	0.052

Adam-Fast	0.0	1.23e1	1.22e1	67.1	0.001
QQN-StrongWolfe	0.0	1.09e1	5.74e0	1535.0	0.067
GD-WeightDecay	0.0	1.22e1	1.22e1	915.3	0.029
Adam-	0.0	1.23e1	1.23e1	1651.7	0.036
WeightDecay					
Trust Region-	0.0	$1.35\mathrm{e}1$	1.34e1	5.0	0.000
Adaptive					
Adam-AMSGrad	0.0	$1.25\mathrm{e}1$	1.25e1	2502.0	0.058
Trust Region-	0.0	1.35e1	1.34e1	5.0	0.000
Standard					
Trust Region-	0.0	1.23e1	1.23e1	380.4	0.003
Conservative					
Trust Region-	0.0	1.34e1	1.23e1	11.4	0.000
Precise					
L-BFGS	0.0	1.22e1	1.22e1	259.7	0.006
Adam-Robust	0.0	1.29e1	1.28e1	2502.0	0.058
Trust Region-	0.0	1.35e1	1.35e1	5.0	0.000
Aggressive					
QQN-Bisection-2	0.0	1.21e1	9.76e0	984.9	0.024

26 Problem: Griewank_10D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	0.0	2.45e1	9.67e-1	108.6	0.003
CubicQuadraticInter	polation				
L-BFGS-	0.0	1.42e1	1.93e0	499.3	0.011
MoreThuente					
QQN-Bisection-1	5.0	2.19e1	5.93e-10	2381.1	0.064
QQN-StrongWolfe	0.0	6.06e0	9.86e-3	2281.2	0.089
QQN-	0.0	2.35e1	2.19e0	4343.6	0.085
GoldenSection					
Trust Region-	0.0	2.60e1	2.60e1	5.0	0.000
Aggressive					
L-BFGS-Limited	0.0	1.95e1	8.48e0	2256.7	0.056
L-BFGS	0.0	1.93e1	1.20e1	483.9	0.012
Adam-AMSGrad	0.0	2.54e1	2.53e1	2502.0	0.063
QQN-Bisection-2	0.0	2.33e1	1.67e0	3260.0	0.083
GD-Nesterov	0.0	2.21e1	2.15e1	1668.0	0.060
Adam-Fast	0.0	1.17e1	2.89e0	950.2	0.021
GD-WeightDecay	0.0	2.48e1	2.48e1	1668.0	0.060
Adam-	0.0	2.42e1	2.41e1	2502.0	0.059
WeightDecay					
Trust Region-	0.0	2.60e1	2.60e1	5.0	0.000
Adaptive					
Trust Region-	0.0	2.60 e1	2.60e1	5.0	0.000
Precise					
GD-Momentum	0.0	2.20e1	2.16e1	1668.0	0.053
L-BFGS-	0.0	6.09e0	1.01e0	3817.8	0.054
Aggressive					
Adam	0.0	2.54e1	2.53e1	2502.0	0.056
L-BFGS-	0.0	2.33e1	2.01e1	1668.3	0.048
Conservative					
GD	0.0	2.56e1	$2.55\mathrm{e}1$	1668.0	0.047
Adam-Robust	0.0	2.55e1	2.55e1	2502.0	0.062
Trust Region-	0.0	2.60e1	2.60e1	5.0	0.000
Standard					

GD-		0.0	1.90e1	1.39e1	920.4	0.035
Adaptive	Momentum					
Trust	Region-	0.0	2.08e1	1.60e1	1557.3	0.012
Conserva	tive					

27 Problem: Schwefel_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam	0.0	9.35e2	9.33e2	2502.0	0.050
Trust Region-	0.0	7.17e2	7.16e2	179.0	0.001
Aggressive					
Trust Region-	0.0	9.37e2	9.35e2	3002.0	0.019
Conservative					
QQN-Bisection-1	0.0	7.11e2	7.11e2	47.6	0.001
GD-WeightDecay	0.0	7.11e2	7.11e2	1115.0	0.033
QQN-	0.0	7.11e2	7.11e2	145.1	0.002
GoldenSection					
Adam-AMSGrad	0.0	9.35e2	9.34e2	2502.0	0.056
Trust Region-	0.0	8.84e2	8.83e2	3002.0	0.019
Precise					
L-BFGS-	0.0	7.11e2	7.11e2	147.6	0.003
MoreThuente					
GD-Momentum	0.0	7.11e2	7.11e2	277.4	0.008
GD-	0.0	7.12e2	7.12e2	89.0	0.003
AdaptiveMomentum					
L-BFGS	0.0	7.11e2	7.11e2	292.1	0.007
L-BFGS-	0.0	7.11e2	7.11e2	101.0	0.001
Aggressive					
Adam-Robust	0.0	9.37e2	9.36e2	2502.0	0.056
Adam-	0.0	9.10e2	9.09e2	2502.0	0.053
WeightDecay					
${\bf QQN\text{-}StrongWolfe}$	50.0	-7.26e2	-1.07e4	379.3	0.016
Trust Region-	0.0	7.11e2	7.11e2	2834.8	0.018
Adaptive					
L-BFGS-Limited	0.0	7.11e2	7.11e2	513.3	0.013
Trust Region-	0.0	7.12e2	7.12e2	711.8	0.004
Standard					
QQN-	0.0	7.11e2	7.11e2	57.9	0.001
CubicQuadraticInter	-				
GD	0.0	7.11e2	7.11e2	1668.0	0.040
Adam-Fast	0.0	7.11e2	7.11e2	1972.4	0.037
QQN-Bisection-2	0.0	7.11e2	7.11e2	50.9	0.001
L-BFGS-	0.0	7.11e2	7.11e2	2095.3	0.055
Conservative					
GD-Nesterov	0.0	7.11e2	7.11e2	292.6	0.009

$28 \quad Problem: \ Schwefel_5D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-1	0.0	1.78e3	1.78e3	52.0	0.001
QQN-	0.0	1.78e3	1.78e3	149.4	0.002
GoldenSection					
Adam-Fast	0.0	1.78e3	1.78e3	1907.4	0.039
Adam-	0.0	2.28e3	2.27e3	2502.0	0.055
WeightDecay					

L-BFGS	0.0	1.78e3	1.78e3	325.7	0.008
Trust Region-	0.0	2.12e3	2.11e3	3002.0	0.020
Adaptive					
_	0.0	2.34e3	2.33e3	2502.0	0.053
QQN-StrongWolfe	50.0	-1.74e3	-3.30e4	349.1	0.015
GD-Nesterov	0.0	1.78e3	1.78e3	339.1	0.011
L-BFGS-Limited	0.0	1.79e3	1.78e3	661.0	0.019
GD-Momentum	0.0	1.78e3	1.78e3	323.9	0.010
Trust Region-	0.0	1.78e3	1.78e3	1774.8	0.012
Standard					
Trust Region-	0.0	2.36e3	2.35e3	3002.0	0.020
Conservative					
QQN-Bisection-2	0.0	1.78e3	1.78e3	52.0	0.001
Adam-Robust	0.0	2.34e3	2.34e3	2502.0	0.059
Trust Region-	0.0	1.78e3	1.78e3	446.0	0.003
Aggressive					
L-BFGS-	0.0	1.78e3	1.78e3	2163.4	0.059
Conservative					
GD-	0.0	1.78e3	1.78e3	124.3	0.004
AdaptiveMomentum					
9	0.0	2.30e3	2.30e3	3002.0	0.020
Precise					
v v	0.0	1.78e3	1.78e3	1124.1	0.037
	0.0	1.78e3	1.78e3	101.0	0.001
Aggressive					
	0.0	1.78e3	1.78e3	158.2	0.003
MoreThuente					
• •	0.0	1.78e3	1.78e3	57.3	0.001
${\bf Cubic Quadratic Interpolation}$					
	0.0	1.78e3	1.78e3	1668.0	0.043
Adam-AMSGrad	0.0	2.34e3	2.34e3	2502.0	0.059

29 Problem: Schwefel_10D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD	0.0	3.55e3	3.55e3	1668.0	0.047
GD-WeightDecay	0.0	3.55e3	3.55e3	1194.6	0.040
Trust Region-	0.0	4.48e3	4.48e3	3002.0	0.022
Adaptive					
L-BFGS	0.0	3.55e3	3.55e3	300.9	0.008
QQN-StrongWolfe	50.0	8.35e2	-2.28e3	395.5	0.017
QQN-Bisection-2	0.0	3.55e3	3.55e3	52.0	0.001
QQN-	0.0	3.55e3	3.55e3	56.9	0.001
CubicQuadraticInter	polation				
QQN-Bisection-1	0.0	3.55e3	3.55e3	52.0	0.001
L-BFGS-	0.0	3.55e3	3.55e3	159.3	0.004
MoreThuente					
L-BFGS-	0.0	3.55e3	3.55e3	101.0	0.002
Aggressive					
Trust Region-	0.0	4.72e3	4.72e3	3002.0	0.022
Conservative					
Trust Region-	0.0	4.67e3	4.67e3	3002.0	0.022
Precise					
GD-Nesterov	0.0	3.55e3	3.55e3	392.1	0.013
QQN-	0.0	3.55e3	3.55e3	147.8	0.002
GoldenSection					

GD-	0.0	3.55e3	3.55e3	164.1	0.006
Adaptive Momentum					
Adam-	0.0	4.55e3	4.55e3	2502.0	0.060
WeightDecay					
Adam-Robust	0.0	4.69e3	4.68e3	2502.0	0.062
Trust Region-	0.0	3.73e3	3.73e3	3002.0	0.022
Standard					
L-BFGS-	0.0	3.55e3	3.55e3	2188.9	0.063
Conservative					
Adam	0.0	4.67e3	4.67e3	2502.0	0.054
GD-Momentum	0.0	3.55e3	3.55e3	380.3	0.012
Adam-Fast	0.0	3.55e3	3.55e3	1628.7	0.035
Trust Region-	0.0	3.56e3	3.56e3	888.8	0.007
Aggressive					
L-BFGS-Limited	0.0	3.55e3	3.42e3	577.7	0.017
Adam-AMSGrad	0.0	4.67e3	4.67e3	2502.0	0.061

30 Problem: Levy_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust Region-	0.0	1.01e1	6.77e-2	27.2	0.000
Adaptive					
Adam-	100.0	9.83e-7	9.67e-7	2229.0	0.048
WeightDecay					
Trust Region-	0.0	3.36e0	1.74e-3	18.2	0.000
Standard					
GD-Nesterov	0.0	1.31e-1	9.69e-2	22.4	0.001
Adam	0.0	5.69e-3	1.60e-3	2502.0	0.051
Adam-Robust	0.0	4.34e-2	1.83e-2	2502.0	0.058
QQN-	100.0	1.81e-7	1.48e-10	300.9	0.005
GoldenSection					
QQN-	100.0	8.29e-8	5.12e-10	87.6	0.003
CubicQuadraticInter	polation				
GD	0.0	2.39e-4	1.85e-4	1668.0	0.042
GD-Momentum	0.0	1.57e-1	1.15e-1	22.5	0.001
L-BFGS-	70.0	6.19e-4	6.80e-8	1303.1	0.017
Aggressive					
L-BFGS	80.0	7.38e-3	1.57e-7	285.0	0.007
Adam-Fast	0.0	7.87e-2	7.26e-2	36.5	0.001
Trust Region-	0.0	1.31e0	9.21e-1	5.0	0.000
Aggressive					
Trust Region-	0.0	6.16e-2	3.16e-2	70.6	0.001
Precise					
Adam-AMSGrad	0.0	2.61e-2	1.24e-2	2502.0	0.057
QQN-Bisection-2	100.0	1.80e-7	2.53e-9	93.0	0.002
QQN-StrongWolfe	100.0	2.98e-7	1.89e-10	79.0	0.003
L-BFGS-	95.0	5.43e-6	7.84e-9	286.5	0.006
MoreThuente					
GD-	0.0	3.36e-1	2.35e-1	16.4	0.001
${\bf Adaptive Momentum}$					
Trust Region-	0.0	1.34e-2	9.88e-3	385.2	0.003
Conservative					
GD-WeightDecay	100.0	9.96e-7	9.93e-7	1453.5	0.046
QQN-Bisection-1	100.0	1.15e-7	2.81e-9	102.2	0.002
L-BFGS-Limited	95.0	1.05e-6	1.49e-7	583.7	0.013
L-BFGS-	85.0	9.25 e-7	8.80e-8	626.6	0.016
Conservative					

31 Problem: Levy_5D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-1	100.0	3.74e-7	4.32e-9	103.4	0.003
L-BFGS-	75.0	2.73e-2	6.19e-7	825.3	0.012
Aggressive					
GD-WeightDecay	100.0	9.96e-7	9.93e-7	1448.8	0.048
Adam	0.0	7.73e-3	3.13e-3	2502.0	0.054
QQN-	100.0	2.57e-7	1.77e-10	382.3	0.007
GoldenSection					
L-BFGS-	100.0	6.91e-7	1.12e-9	296.9	0.006
MoreThuente					
GD-Nesterov	25.0	9.57e-2	9.78e-7	102.2	0.003
GD	0.0	2.32e-4	1.98e-4	1668.0	0.044
QQN-	100.0	2.65e-7	2.28e-9	96.0	0.003
${\bf Cubic Quadratic Inter}$	rpolation				
GD-Momentum	10.0	1.72e-1	9.75e-7	56.5	0.002
GD-	0.0	4.77e-1	3.80e-1	19.2	0.001
${\bf Adaptive Momentum}$					
Adam-Robust	0.0	6.80e-2	3.18e-2	2502.0	0.061
Trust Region-	0.0	2.49e0	4.43e-4	26.4	0.000
Standard					
Adam-	60.0	1.43e-6	9.65e-7	2087.2	0.050
WeightDecay					
L-BFGS-Limited	70.0	2.87e-4	2.75e-7	1375.8	0.024
Trust Region-	0.0	1.23e-2	8.28e-3	996.6	0.007
Conservative					
QQN-Bisection-2	100.0	2.16e-7	1.51e-10	101.3	0.002
Adam-AMSGrad	0.0	7.64e-2	5.66e-2	2502.0	0.061
L-BFGS	80.0	4.35e-2	2.47e-7	173.6	0.004
Trust Region-	0.0	4.37e-1	1.82e-1	8.2	0.000
Aggressive					
L-BFGS-	100.0	6.68e-7	1.69e-8	332.8	0.008
Conservative					
QQN-StrongWolfe	100.0	3.31e-7	1.45e-9	69.2	0.002
Adam-Fast	0.0	1.67e-1	1.54e-1	37.0	0.001
Trust Region-	0.0	1.06e1	4.60e-2	46.9	0.000
Adaptive					
Trust Region-	0.0	3.62e-2	1.69e-2	168.1	0.001
Precise					

$32 \quad Problem: \ Levy_10D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-Momentum	0.0	2.58e-1	1.93e-1	28.1	0.001
Trust Region-	0.0	1.21e-2	9.38e-3	1976.9	0.016
Conservative					
Adam	0.0	6.46e-3	2.32e-3	2502.0	0.060
QQN-	100.0	2.74e-7	1.87e-10	399.3	0.007
GoldenSection					
QQN-	100.0	2.83e-7	2.06e-9	98.6	0.003
CubicQuadraticInter	rpolation				
GD-	0.0	6.50 e-1	5.54e-1	21.9	0.001
AdaptiveMomentum	L				
QQN-Bisection-1	100.0	2.49e-7	1.67e-8	109.7	0.003

Trust R	legion-	0.0	3.14e-2	1.56e-2	325.6	0.003
Precise						
Adam-Fast		0.0	3.12e-1	2.89e-1	36.9	0.001
L-BFGS		80.0	3.96e-3	3.36e-7	223.2	0.005
QQN-StrongV	Wolfe	100.0	1.46e-7	7.28e-10	70.1	0.002
L-BFGS-		25.0	1.27e0	4.01e-7	2637.9	0.034
Aggressive						
GD		0.0	2.28e-4	1.85e-4	1668.0	0.050
Adam-		0.0	2.80e-6	1.41e-6	1897.2	0.048
WeightDecay						
	legion-	0.0	7.36e-2	1.96e-2	84.5	0.001
Adaptive						
Adam-AMSG	Frad	0.0	1.51e-1	1.12e-1	2502.0	0.066
Trust R	legion-	0.0	2.04e0	8.10e-2	29.6	0.000
Standard						
QQN-Bisection	on-2	100.0	2.70e-7	1.96e-10	111.1	0.003
Adam-Robust	t	0.0	9.82e-2	6.13e-2	2502.0	0.067
Trust R	legion-	0.0	6.17e-1	2.22e-1	11.8	0.000
Aggressive						
L-BFGS-		100.0	6.63e-7	7.05e-8	365.9	0.010
Conservative						
L-BFGS-Limi	ited	80.0	2.41e-5	5.25 e-7	1051.8	0.022
L-BFGS-		100.0	7.24e-7	8.40e-8	165.1	0.004
MoreThuente)					
GD-WeightDe	ecay	100.0	9.97e-7	9.93e-7	1451.7	0.052
GD-Nesterov	•	0.0	1.74e-1	1.40e-1	28.4	0.001

33 Problem: Zakharov_2D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	100.0	7.14e-11	5.21e-21	180.9	0.003
GoldenSection					
L-BFGS	60.0	2.88e-2	5.69e-10	158.7	0.003
L-BFGS-	70.0	3.75e0	4.91e-10	1213.0	0.007
Aggressive					
Trust Region-	0.0	2.38e0	1.22e-1	38.3	0.000
Standard					
QQN-StrongWolfe	100.0	4.83e-9	1.51e-9	95.0	0.003
QQN-	100.0	1.52e-9	1.85e-11	89.2	0.003
CubicQuadraticInter	polation				
L-BFGS-Limited	90.0	6.46e-7	1.04e-9	658.2	0.012
Trust Region-	0.0	5.16e4	5.33e-1	18.9	0.000
Aggressive					
QQN-Bisection-2	100.0	9.02e-14	4.37e-17	84.0	0.002
L-BFGS-	75.0	6.39e-7	5.41e-10	863.5	0.022
Conservative					
L-BFGS-	100.0	7.07e-9	3.76e-9	67.3	0.001
MoreThuente					
Adam-Fast	0.0	2.81e-1	2.54e-1	37.5	0.001
Adam	0.0	1.77e-3	5.24e-5	2502.0	0.050
Trust Region-	0.0	8.84e-1	1.05e-2	2569.1	0.016
Conservative					
GD	100.0	9.76e-9	9.61e-9	417.4	0.011
GD-WeightDecay	25.0	8.53e-2	8.63e-9	47.9	0.002
Trust Region-	0.0	4.96e3	3.14e-2	121.0	0.001
Adaptive					
GD-Momentum	0.0	1.08e0	8.96e-1	17.1	0.000

Trust Region- Precise	0.0	6.91e-2	8.62e-3	503.2	0.003
Adam-AMSGrad	0.0	9.29e-2	4.85e-2	2502.0	0.056
Adam-Robust	0.0	3.83e-2	6.91e-3	2502.0	0.058
QQN-Bisection-1	100.0	7.75e-14	9.76e-18	114.6	0.002
Adam-	100.0	9.88e-9	9.74e-9	2040.6	0.043
WeightDecay					
GD-	0.0	7.05e-1	4.12e-1	16.6	0.001
AdaptiveMomentum					
GD-Nesterov	0.0	6.00e-1	5.01e-1	16.7	0.001

34 Problem: Zakharov_5D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	0.0	4.22e1	2.82e1	3309.5	0.062
Aggressive					
QQN-	100.0	2.83e-12	2.45e-13	138.0	0.002
GoldenSection					
Adam-	60.0	8.64e-3	9.76e-9	1586.3	0.036
WeightDecay					
Adam-AMSGrad	0.0	2.23e-2	4.59e-3	2502.0	0.059
QQN-Bisection-2	100.0	1.50e-9	2.56e-15	114.7	0.002
Trust Region-	0.0	2.28e3	6.92e2	3002.0	0.020
Standard					
L-BFGS	0.0	2.69e1	2.58e-1	93.0	0.001
L-BFGS-	100.0	9.32e-9	8.60e-9	634.6	0.013
MoreThuente					
L-BFGS-Limited	95.0	1.94e-8	7.85e-9	1078.6	0.016
Trust Region-	0.0	3.28e3	2.05e3	3002.0	0.020
Conservative					
GD-Momentum	0.0	5.45e1	3.48e1	18.9	0.001
Adam	0.0	2.82e-3	2.65e-4	2502.0	0.054
Trust Region-	0.0	$5.65\mathrm{e}1$	6.02e-1	2415.8	0.017
Aggressive					
QQN-	100.0	1.52e-9	3.50e-12	205.0	0.007
CubicQuadraticInter					
QQN-StrongWolfe	100.0	2.41e-9	2.91e-14	100.3	0.003
GD	100.0	9.78e-9	9.63e-9	477.1	0.013
GD-WeightDecay	25.0	7.62e-1	9.24e-9	52.5	0.002
GD-	0.0	$6.97\mathrm{e}1$	4.83e1	18.5	0.001
AdaptiveMomentum					
GD-Nesterov	0.0	5.05e0	1.42e0	18.1	0.001
QQN-Bisection-1	100.0	1.12e-12	5.59e-14	118.0	0.002
L-BFGS-	100.0	7.32e-9	2.42e-10	492.9	0.012
Conservative					
Adam-Robust	0.0	7.06e-2	6.18e-3	2502.0	0.059
Trust Region-	0.0	3.10e3	1.77e3	3002.0	0.021
Adaptive					
Trust Region-	0.0	3.50e3	2.04e3	3002.0	0.021
Precise					
Adam-Fast	0.0	9.66e0	7.78e0	34.8	0.001

35 Problem: Zakharov_10D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
o p cimino ci	2 4 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	TITOGET E IIIGE TOLLO	2000 / 00100	11100011 1 01110 12 10110	1110011 111110 (0)

	Region-	0.0	5.60e5	4.02e5	3002.0	0.022
Aggressive						
GD-Momen		0.0	6.41e3	1.95e3	21.1	0.001
	Region-	0.0	5.73e5	4.07e5	3002.0	0.022
Precise						
Adam-Fast		0.0	1.13e3	3.62e2	34.7	0.001
QQN-		100.0	1.50e-9	3.14e-19	233.1	0.004
GoldenSect	ion					
L-BFGS-		0.0	9.79e-1	1.37e-2	3811.6	0.052
Aggressive						
Trust	Region-	0.0	5.56e5	4.08e5	3002.0	0.022
Adaptive						
GD-		0.0	6.98e3	4.52e3	20.9	0.001
AdaptiveMe	omentum					
L-BFGS		0.0	2.07e4	6.23e3	3286.7	0.060
Adam-Robi	ust	0.0	3.12e0	2.92e-2	2502.0	0.062
GD-Weight	Decay	30.0	1.51e1	8.77e-9	67.5	0.002
L-BFGS-		0.0	1.67e-6	4.87e-7	2886.3	0.059
MoreThuen	ite					
Adam-AMS	GGrad	0.0	1.09e-1	3.67e-2	2016.2	0.050
Adam-		0.0	1.79e-1	1.29e-1	683.1	0.016
WeightDeca	ay					
Trust	Region-	0.0	5.72e5	3.74e5	3002.0	0.022
Standard						
GD		10.0	3.37e-1	9.65e-9	686.0	0.020
QQN-		100.0	1.75e-9	4.53e-10	312.8	0.013
CubicQuad	raticInterpo	lation				
Trust	Region-	0.0	5.85e5	4.55e5	3002.0	0.021
Conservativ	<i>r</i> e					
QQN-Bisec	tion-2	100.0	1.58e-9	1.13e-10	159.2	0.004
QQN-Bisec	tion-1	100.0	1.96e-9	4.68e-16	161.5	0.003
QQN-Stron	gWolfe	95.0	9.67e-4	2.12e-14	281.4	0.009
L-BFGS-Li	mited	55.0	9.29e-2	5.26e-9	2588.7	0.034
L-BFGS-		0.0	7.82e23	1.68e5	68.5	0.002
Conservativ	<i>r</i> e					
GD-Nestero	ov	0.0	8.62e2	4.52e1	20.2	0.001
Adam		0.0	4.52e-2	1.69e-3	2253.9	0.050

$36 \quad Problem: Ill Conditioned Rosenbrock_2D_alpha 100$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	35.0	3.48e-2	4.34e-10	1722.9	0.070
CubicQuadraticInter	polation				
Trust Region-	0.0	4.18e0	3.95e0	89.6	0.001
Standard					
QQN-Bisection-1	5.0	4.65e-1	1.58e-8	2369.2	0.052
Adam-Fast	0.0	2.13e0	2.59e-5	313.6	0.006
Trust Region-	0.0	4.66e0	4.01e0	27.6	0.000
Aggressive					
Trust Region-	0.0	7.26e0	3.76e0	946.2	0.006
Precise					
QQN-	0.0	1.25e-1	6.99e-5	4459.4	0.083
GoldenSection					
Adam-AMSGrad	0.0	3.83e0	4.66e-1	678.1	0.015
L-BFGS-	65.0	1.52e-2	3.79e-9	1487.5	0.027
MoreThuente					
${\bf QQN\text{-}StrongWolfe}$	20.0	5.46e-2	9.29e-9	2343.8	0.068
${\bf QQN\text{-}StrongWolfe}$	20.0	5.46e-2	9.29e-9	2343.8	0.068

L-BFGS	0.0	1.36e2	8.12e-1	121.5	0.002
Adam-Robust	0.0	4.04e0	1.90e0	419.2	0.009
Adam	0.0	1.22e0	4.86e-1	2502.0	0.049
L-BFGS-	0.0	3.18e1	4.34e0	3852.0	0.028
Aggressive					
L-BFGS-	80.0	2.69e-4	2.05e-9	1800.6	0.029
Conservative					
Trust Region-	0.0	2.84e1	1.90e-1	2770.7	0.017
Conservative					
GD-Nesterov	0.0	1.49e0	5.05e-2	46.1	0.001
GD	0.0	1.23e0	7.46e-1	854.0	0.021
GD-	0.0	8.18e-1	4.17e-2	49.2	0.002
AdaptiveMomentum					
GD-WeightDecay	0.0	3.65e0	3.75e-2	58.7	0.002
Adam-	0.0	4.11e0	1.55e-2	231.9	0.005
WeightDecay					
Trust Region-	0.0	4.12e0	3.83e0	494.4	0.003
Adaptive					
L-BFGS-Limited	0.0	3.93e0	3.26e-2	2251.6	0.025
QQN-Bisection-2	25.0	5.88e-2	6.64e-9	479.6	0.011
GD-Momentum	0.0	5.61e0	4.79e-1	23.8	0.001

37 Problem: IllConditionedRosenbrock_5D_alpha100

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	20.0	1.74e8	3.07e-7	2320.8	0.040
MoreThuente					
QQN-	65.0	1.97e-1	4.07e-9	1403.3	0.059
CubicQuadraticInter	polation				
L-BFGS-	0.0	2.02e1	2.46e-1	3386.6	0.034
Conservative					
Adam	0.0	3.92e0	2.83e0	2471.6	0.049
Adam-	0.0	1.84e0	1.84e-3	1797.3	0.038
WeightDecay					
QQN-Bisection-2	15.0	2.99e-1	4.91e-8	1647.8	0.040
Trust Region-	0.0	8.41e2	5.05e2	3002.0	0.019
Adaptive					
Adam-AMSGrad	0.0	4.40e0	3.25e0	2442.0	0.055
Trust Region-	0.0	1.01e3	8.08e2	3002.0	0.019
Precise					
L-BFGS-Limited	0.0	4.05e-1	3.00e-1	4035.2	0.045
GD-WeightDecay	0.0	7.32e-1	1.46e-1	459.3	0.015
Trust Region-	0.0	6.23e1	4.66e0	2827.2	0.018
Standard					
Adam-Fast	0.0	1.43e1	4.36e-2	49.9	0.001
L-BFGS	0.0	1.50e2	1.98e1	135.3	0.002
GD	0.0	5.09e0	4.75e0	32.5	0.001
GD-Nesterov	0.0	4.24e0	3.82e-1	372.4	0.012
QQN-	35.0	4.32e-1	5.69e-8	3805.3	0.072
GoldenSection					
L-BFGS-	0.0	8.07e2	1.72e1	3851.6	0.029
Aggressive					
Trust Region-	0.0	5.00e0	4.66e0	776.1	0.005
Aggressive					
GD-Momentum	0.0	3.55e1	1.96e1	20.8	0.001
Adam-Robust	0.0	1.46e1	6.12e0	2502.0	0.056

Trust	Region-	0.0	1.02e3	7.14e2	3002.0	0.019
Conservati	ve					
QQN-Stroi	ngWolfe	100.0	1.59e-7	2.70e-9	1191.3	0.036
GD-		0.0	4.60e1	3.36e1	20.6	0.001
AdaptiveM	Iomentum					
QQN-Bised	ction-1	80.0	4.19e-1	2.00e-9	1511.2	0.039

$38 \quad Problem: \ Ill Conditioned Rosenbrock_10D_alpha 100$

Optimizer		Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-StrongW	olfe	70.0	5.85e-1	9.35e-8	1847.0	0.058
L-BFGS-Limite	ed	0.0	3.86e0	2.71e0	4036.8	0.048
QQN-		0.0	4.56e0	4.63e-1	4477.5	0.086
GoldenSection						
QQN-		75.0	6.00e-1	1.13e-7	1666.2	0.070
CubicQuadrati	cInterp	oolation				
GD-	-	0.0	1.02e2	8.45e1	23.1	0.001
AdaptiveMome	entum					
Adam-Fast		0.0	1.53e1	1.91e-1	149.2	0.003
GD-Momentum	n	0.0	3.04e1	5.05e-1	517.6	0.016
Adam-AMSGra	ad	0.0	9.31e0	8.08e0	2490.9	0.061
L-BFGS-		0.0	1.64e2	5.82e1	3850.3	0.049
Aggressive						
Adam-Robust		0.0	3.49e1	1.61e1	2502.0	0.061
QQN-Bisection	1-2	5.0	4.40e0	1.07e-7	1646.8	0.042
$\overline{\mathrm{GD}}$		0.0	9.96e0	9.64e0	46.6	0.002
QQN-Bisection	n-1	65.0	1.22e0	1.19e-7	1844.9	0.051
L-BFGS		0.0	1.19e2	2.51e1	338.9	0.004
L-BFGS-		0.0	1.90e4	1.03e0	3278.5	0.037
Conservative						
L-BFGS-		0.0	3.41e0	3.35e-5	2853.3	0.053
MoreThuente						
GD-WeightDec	cay	0.0	1.34e0	1.70e-1	1217.3	0.042
_	gion-	0.0	2.02e3	1.64e3	3002.0	0.021
Adaptive	0					
	gion-	0.0	1.06e3	6.70e2	3002.0	0.021
Standard						
Trust Res	gion-	0.0	$6.55\mathrm{e}1$	9.60e0	1618.0	0.011
Aggressive	0					
Adam		0.0	9.11e0	7.98e0	2475.8	0.053
GD-Nesterov		0.0	1.17e0	1.12e0	1514.9	0.056
Adam-		0.0	2.38e0	6.46e-6	2074.9	0.047
WeightDecay						
0	gion-	0.0	2.10e3	1.66e3	3002.0	0.021
Precise	_					
	gion-	0.0	2.16e3	1.76e3	3002.0	0.021
Conservative	~					

${\bf 39 \quad Problem: \ Trigonometric_2D}$

Optimiz	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Ro	bust	5.0	3.10e-3	5.08e-7	442.4	0.010
Trust	Region-	0.0	1.22e1	2.19e-1	13.5	0.000
Standard						

L-BFGS-	80.0	3.62e-4	7.20e-8	843.0	0.009
Aggressive					
Trust Region-	0.0	2.50e1	3.36e-2	18.3	0.000
Aggressive					
QQN-Bisection-2	100.0	2.06e-7	7.96e-10	207.1	0.005
L-BFGS-Limited	90.0	7.02e-6	2.15e-8	433.9	0.008
QQN-	95.0	1.28e-3	6.12e-10	462.6	0.008
GoldenSection					
L-BFGS-	100.0	4.44e-7	1.25e-8	85.4	0.002
MoreThuente					
Trust Region-	0.0	8.94e-3	1.87e-3	43.2	0.000
Conservative					
GD-	0.0	7.65e-3	5.90e-5	23.2	0.001
AdaptiveMomentum					
Adam-AMSGrad	85.0	1.08e-6	9.47e-7	1241.2	0.028
GD	100.0	9.80e-7	9.61e-7	361.1	0.010
Trust Region-	0.0	8.77e0	1.18e-2	12.8	0.000
Adaptive					
L-BFGS	90.0	4.06e-3	8.58e-10	93.3	0.002
L-BFGS-	100.0	5.22e-7	3.92e-8	129.4	0.003
Conservative					
QQN-StrongWolfe	100.0	4.90e-7	2.93e-9	107.8	0.004
GD-Momentum	0.0	3.43e-2	2.30e-4	22.1	0.001
Trust Region-	0.0	1.11e-1	1.34e-2	16.8	0.000
Precise					
Adam	100.0	9.77e-7	9.40e-7	1269.5	0.026
GD-Nesterov	0.0	2.31e-2	5.79e-4	24.2	0.001
QQN-	100.0	3.11e-7	6.47e-10	95.8	0.003
CubicQuadraticInterpolation	on				
GD-WeightDecay	85.0	2.11e-4	8.75e-7	102.4	0.003
Adam-Fast	5.0	6.08e-3	1.89e-8	47.6	0.001
Adam-	75.0	1.76e-5	3.77e-7	362.9	0.008
WeightDecay					
QQN-Bisection-1	100.0	2.08e-7	1.49e-9	220.8	0.005

$40 \quad Problem: \ Trigonometric_5D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMSGrad	35.0	2.94e-2	9.71e-7	2192.1	0.054
GD-Momentum	0.0	1.57e-1	4.87e-3	25.0	0.001
Adam-Fast	0.0	6.17e-2	7.00e-4	54.8	0.001
Adam	85.0	1.12e-2	8.42e-7	1728.1	0.039
Trust Region-	0.0	1.25e-1	2.86e-2	26.1	0.000
Adaptive					
Adam-Robust	20.0	1.43e-2	2.34e-7	978.1	0.024
L-BFGS-	75.0	5.44e-4	1.15e-7	1087.8	0.023
MoreThuente					
Trust Region-	0.0	1.91e-2	8.29e-4	425.6	0.004
Conservative					
QQN-	100.0	4.16e-7	1.09e-9	389.1	0.017
CubicQuadraticInter	polation				
Trust Region-	0.0	5.59e-2	1.47e-2	84.8	0.001
Precise					
GD	100.0	9.83e-7	9.62e-7	399.2	0.011
L-BFGS-	0.0	1.22e0	1.48e-1	3851.9	0.025
Aggressive					

GD-	0.0	1.79e-2	1.30e-4	30.6	0.001
AdaptiveMomentum					
L-BFGS-Limited	95.0	2.14e-3	7.20e-7	1331.0	0.019
GD-WeightDecay	75.0	1.36e-2	8.18e-7	105.4	0.004
QQN-Bisection-1	90.0	8.71e-4	1.72e-10	664.0	0.015
L-BFGS	0.0	4.77e0	1.26e-2	233.9	0.004
QQN-Bisection-2	80.0	3.22e-7	6.17e-10	305.6	0.009
GD-Nesterov	0.0	1.60e-2	4.85e-4	33.5	0.001
Trust Region-	0.0	4.09e-1	1.21e-1	9.8	0.000
Standard					
QQN-	100.0	2.34e-7	3.03e-9	1123.5	0.021
GoldenSection					
Adam-	70.0	1.95e-2	3.70e-7	784.6	0.018
WeightDecay					
L-BFGS-	90.0	2.73e-3	1.66e-7	887.4	0.014
Conservative					
Trust Region-	0.0	1.41e0	6.56e-1	5.2	0.000
Aggressive					
QQN-StrongWolfe	90.0	7.54e-7	1.51e-8	1274.7	0.046

41 Problem: Trigonometric_10D

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Fast	0.0	6.24e-2	2.65e-4	102.7	0.003
QQN-	80.0	4.35e-4	1.12e-7	1834.2	0.040
GoldenSection					
GD-Momentum	0.0	1.17e0	2.40e-3	34.0	0.001
Trust Region-	0.0	1.86e-1	2.37e-2	118.4	0.001
Adaptive					
Trust Region-	0.0	1.90e0	9.79e-1	12.2	0.000
Aggressive					
GD-Nesterov	0.0	1.85e-1	7.60e-2	45.1	0.002
L-BFGS-	0.0	1.00e1	2.42e0	3852.0	0.032
Aggressive					
QQN-Bisection-1	85.0	7.53e-4	2.91e-8	858.0	0.027
Adam-	70.0	4.25e-2	8.69e-7	803.8	0.022
WeightDecay					
Trust Region-	0.0	1.22e0	3.82e-3	2651.3	0.029
Conservative					
Adam-Robust	10.0	7.13e-2	9.71e-7	1825.9	0.057
L-BFGS-	60.0	1.34e-2	4.78e-7	1949.3	0.050
MoreThuente					
QQN-	100.0	4.48e-7	9.99e-9	329.2	0.016
CubicQuadraticInterp	•				
QQN-StrongWolfe	100.0	2.92e-7	9.27e-9	347.3	0.015
Adam-AMSGrad	0.0	3.00e-2	7.20e-6	2502.0	0.074
GD-WeightDecay	85.0	8.48e-3	8.63e-7	134.4	0.006
GD	55.0	4.39e-3	9.77e-7	958.0	0.034
Adam	35.0	9.70e-2	9.68e-7	2080.5	0.054
Trust Region-	0.0	6.02e-1	1.37e-1	34.1	0.000
Standard					
L-BFGS-	25.0	9.45e0	7.52e-7	3123.2	0.043
Conservative					
GD-	0.0	1.41e-1	4.06e-2	48.0	0.002
AdaptiveMomentum					
QQN-Bisection-2	75.0	3.56e-7	8.26e-8	357.8	0.011
L-BFGS	0.0	4.76e1	8.87e0	77.5	0.001

Trust	Region-	0.0	1.07e-1	2.28e-2	455.3	0.005
Precise						
L-BFGS-L	$_{ m imited}$	0.0	7.31e-3	9.82e-6	3943.6	0.058

$42 \quad Problem: \ Penalty I_2D_alpha 1e6$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-2	0.0	2.42e0	1.17e0	299.3	0.007
Adam-	0.0	1.59e0	1.14e0	226.2	0.005
WeightDecay					
GD-	0.0	7.74e2	1.15e0	17.6	0.001
AdaptiveMomentum					
QQN-Bisection-1	0.0	1.12e0	1.12e0	1032.0	0.033
Adam	0.0	1.17e0	1.13e0	625.9	0.012
L-BFGS-	0.0	4.69e4	1.17e0	2664.3	0.049
Aggressive					
GD-Nesterov	0.0	1.46e2	1.38e0	13.9	0.000
GD	0.0	1.37e1	1.28e0	14.9	0.000
GD-Momentum	0.0	$6.67\mathrm{e}1$	1.26e0	13.0	0.000
QQN-	0.0	1.13e0	1.12e0	2567.9	0.048
GoldenSection					
QQN-StrongWolfe	0.0	1.14e0	1.12e0	3162.1	0.083
Adam-Robust	0.0	2.11e0	1.13e0	107.1	0.002
Adam-Fast	0.0	1.01e2	1.26e0	22.0	0.000
Trust Region-	0.0	1.32e5	1.57e4	3002.0	0.019
Adaptive					
L-BFGS-Limited	0.0	1.18e0	1.13e0	4341.4	0.029
L-BFGS-	0.0	4.78e0	1.18e0	2878.8	0.042
MoreThuente					
L-BFGS-	0.0	1.13e0	1.12e0	1183.8	0.015
Conservative					
GD-WeightDecay	0.0	5.08e0	1.34e0	13.2	0.000
Trust Region-	0.0	1.39e5	8.35e3	3002.0	0.019
Conservative					
Trust Region-	0.0	1.39e5	1.23e4	3002.0	0.019
Aggressive					
Trust Region-	0.0	1.67e5	2.57e4	3002.0	0.018
Precise					
Trust Region-	0.0	1.43e5	3.60e4	3002.0	0.019
Standard					
L-BFGS	0.0	7.22e4	1.52e0	109.5	0.002
QQN-	0.0	1.12e0	1.12e0	38.0	0.001
CubicQuadraticInter	oolation				
Adam-AMSGrad	0.0	1.16e0	1.13e0	647.9	0.015

${\bf 43 \quad Problem:\ Penalty I_5D_alpha 1e6}$

Optimiz	er	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	3.61e5	1.76e5	3002.0	0.020
Aggressiv	/e					
Adam-		0.0	7.65e0	2.86e0	260.1	0.006
WeightD	ecay					
QQN-Bis	ection-1	0.0	2.81e0	2.81e0	2178.7	0.071
L-BFGS		0.0	7.00e4	4.88e0	95.2	0.001
Adam-Fa	st	0.0	4.69e2	3.30e0	29.0	0.001

QQN-Bisection-2	0.0	1.03e1	3.91e0	469.8	0.011
$\overline{\mathrm{GD}}$	0.0	8.04e1	3.17e0	17.1	0.000
L-BFGS-	0.0	2.85e0	2.81e0	3087.5	0.033
Conservative					
GD-	0.0	5.87e2	3.36e0	13.9	0.000
AdaptiveMomentum					
Trust Region-	0.0	3.71e5	9.78e4	3002.0	0.019
Adaptive					
L-BFGS-	0.0	2.82e5	3.49e0	1110.7	0.022
Aggressive					
L-BFGS-	0.0	1.29e1	3.57e0	2860.6	0.036
MoreThuente					
GD-Nesterov	0.0	$3.89\mathrm{e}2$	3.28e0	14.1	0.000
Adam-AMSGrad	0.0	2.85e0	2.82e0	747.3	0.017
Trust Region-	0.0	3.74e5	1.16e5	3002.0	0.019
Standard					
Trust Region-	0.0	3.90e5	1.47e5	3002.0	0.022
Conservative					
QQN-	0.0	2.81e0	2.81e0	38.0	0.001
CubicQuadraticInterpola	tion				
L-BFGS-Limited	0.0	2.96e0	2.84e0	4259.2	0.036
Trust Region-	0.0	4.20e5	2.36e5	3002.0	0.020
Precise					
GD-WeightDecay	0.0	3.50e1	3.50e0	14.5	0.000
Adam	0.0	2.90e0	2.82e0	733.5	0.015
QQN-StrongWolfe	0.0	4.21e0	2.86e0	3121.1	0.086
GD-Momentum	0.0	9.24e1	3.99e0	14.4	0.000
QQN-	0.0	2.81e0	2.81e0	3026.4	0.056
GoldenSection					
Adam-Robust	0.0	2.16e1	2.82e0	120.4	0.003

44 Problem: PenaltyI_10D_alpha1e6

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD	0.0	6.02e0	5.96e0	19.4	0.001
L-BFGS-	0.0	7.30e5	2.94e5	11.0	0.000
Aggressive					
QQN-Bisection-2	0.0	1.86e1	9.12e0	1070.2	0.027
L-BFGS-	0.0	5.73e0	5.63e0	2611.7	0.034
Conservative					
Adam-	0.0	9.99e0	5.72e0	274.1	0.007
WeightDecay					
Adam-Robust	0.0	2.18e1	5.69e0	129.1	0.003
QQN-Bisection-1	0.0	5.63e0	5.63e0	2161.3	0.074
Adam-AMSGrad	0.0	5.66e0	5.64e0	761.0	0.019
QQN-	0.0	5.63e0	5.62e0	4382.9	0.086
GoldenSection					
QQN-	0.0	5.62e0	5.62e0	38.0	0.001
CubicQuadraticInter	polation				
GD-WeightDecay	0.0	6.01e1	6.73e0	15.8	0.000
Trust Region-	0.0	7.20e5	3.49e5	3002.0	0.021
Adaptive					
GD-Momentum	0.0	2.58e2	7.18e0	15.4	0.000
${ m QQN} ext{-StrongWolfe}$	0.0	1.10e1	8.12e0	3057.8	0.088
Trust Region-	0.0	7.72e5	5.48e5	3002.0	0.021
Conservative					

Trust	Region-	0.0	7.20e5	3.91e5	3002.0	0.021
Aggressive						
Trust	Region-	0.0	7.75e5	4.16e5	3002.0	0.021
Precise						
GD-Neste	rov	0.0	4.81e2	6.89e0	14.9	0.000
Adam-Fas	t	0.0	1.17e3	6.26e0	42.3	0.001
Trust	Region-	0.0	7.40e5	2.96e5	3002.0	0.021
Standard						
L-BFGS		0.0	1.71e5	8.62e0	101.3	0.002
GD-		0.0	1.60e2	7.22e0	15.0	0.000
Adaptive	Momentum					
Adam		0.0	5.68e0	5.64e0	764.5	0.017
L-BFGS-I	Limited	0.0	5.89e0	5.78e0	4213.4	0.042
L-BFGS-		0.0	2.89e1	1.06e1	2859.4	0.033
MoreThue	ente					

45 Problem: Barrier_2D_mu0.1

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-Nesterov	0.0	5.03e-1	4.85e-1	20.4	0.001
Adam-AMSGrad	0.0	4.13e-1	4.01e-1	2502.0	0.055
GD-WeightDecay	0.0	4.00e-1	4.00e-1	49.0	0.002
Trust Region-	0.0	1.50e0	5.85e-1	5.0	0.000
Aggressive					
Adam-Fast	0.0	5.15e-1	4.74e-1	31.4	0.001
L-BFGS-	0.0	\inf	\inf	0.0	0.000
MoreThuente					
Adam	0.0	4.00e-1	4.00e-1	2502.0	0.049
QQN-	0.0	4.00e-1	4.00e-1	83.8	0.002
CubicQuadraticInterp	polation				
QQN-	0.0	4.00e-1	4.00e-1	144.2	0.002
GoldenSection					
L-BFGS-	0.0	4.00e-1	4.00e-1	880.4	0.023
Conservative					
Adam-	0.0	4.00e-1	4.00e-1	1498.4	0.032
WeightDecay					
Adam-Robust	0.0	4.09e-1	4.00e-1	2502.0	0.055
Trust Region-	0.0	5.57e-1	4.86e-1	2.8	0.000
Standard					
Trust Region-	0.0	4.71e-1	4.01e-1	421.4	0.003
Conservative					
Trust Region-	0.0	4.72e-1	4.07e-1	72.3	0.001
Precise					
QQN-Bisection-2	0.0	4.00e-1	4.00e-1	125.5	0.003
QQN-Bisection-1	0.0	4.00e-1	4.00e-1	65.8	0.002
L-BFGS-Limited	0.0	4.00e-1	4.00e-1	150.8	0.004
GD-	0.0	4.65e-1	4.60e-1	1.6	0.000
AdaptiveMomentum					
GD	0.0	4.00e-1	4.00e-1	292.1	0.008
Trust Region-	0.0	5.15e-1	4.42e-1	6.2	0.000
Adaptive					
L-BFGS-	0.0	4.00e-1	4.00e-1	65.7	0.001
Aggressive					
QQN-StrongWolfe	0.0	4.00e-1	4.00e-1	108.3	0.003
L-BFGS	0.0	4.00e-1	4.00e-1	163.9	0.004
GD-Momentum	0.0	5.05e-1	4.82e-1	20.9	0.001

46 Problem: Barrier_5D_mu0.1

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	0.0	9.99e-1	9.99e-1	78.7	0.002
CubicQuadraticInter	polation				
GD-Nesterov	0.0	1.26e0	1.22e0	20.4	0.001
GD-	0.0	\inf	\inf	0.0	0.000
AdaptiveMomentum					
Adam-Fast	0.0	1.28e0	1.19e0	31.7	0.001
Adam-	0.0	9.99e-1	9.99e-1	1316.6	0.028
WeightDecay					
QQN-Bisection-2	0.0	9.99e-1	9.99e-1	126.0	0.003
Adam	0.0	9.99e-1	9.99e-1	2502.0	0.051
Trust Region-	0.0	1.17e0	1.03e0	28.0	0.000
Adaptive	0.0	1.1.00	1.0000	20.0	0.000
Trust Region-	0.0	1.99e0	1.58e0	5.7	0.000
Aggressive		2.00.00		• • • • • • • • • • • • • • • • • • • •	0.000
QQN-Bisection-1	0.0	9.99e-1	9.99e-1	77.2	0.002
GD-WeightDecay	0.0	9.99e-1	9.99e-1	49.2	0.002
L-BFGS	0.0	9.99e-1	9.99e-1	89.1	0.002
L-BFGS-	0.0	9.99e-1	9.99e-1	599.6	0.015
Conservative	0.0	0.000 1	0.000 1	000.0	0.010
GD-Momentum	0.0	1.37e0	1.28e0	21.0	0.001
L-BFGS-Limited	0.0	9.99e-1	9.99e-1	190.6	0.004
Adam-Robust	0.0	1.01e0	1.00e0	2502.0	0.057
Adam-AMSGrad	0.0	1.03e0	1.01e0	2502.0	0.057
QQN-StrongWolfe	0.0	9.99e-1	9.99e-1	112.1	0.004
GD	0.0	9.99e-1	9.99e-1	304.1	0.004
QQN-	0.0	9.99e-1	9.99e-1	144.7	0.002
GoldenSection	0.0	0.000-1	J.JJC-1	111.1	0.002
L-BFGS-	0.0	9.99e-1	9.99e-1	63.8	0.001
Aggressive	0.0	3.33C-1	9.99C-1	05.0	0.001
L-BFGS-	0.0	\inf	\inf	0.0	0.000
MoreThuente	0.0	1111	1111	0.0	0.000
Trust Region-	0.0	1.05e0	1.00e0	984.2	0.007
Conservative	0.0	1.0960	1.0060	304.4	0.007
Trust Region-	0.0	1.23e0	1.14e0	5.5	0.000
Standard Region-	0.0	1.2560	1.1460	0.0	0.000
	0.0	1.02-0	1.01-0	143.6	0.001
0	0.0	1.02e0	1.01e0	145.0	0.001
Precise					

47 Problem: Barrier_10D_mu0.1

Optimiz	zer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Trust	Region-	0.0	2.06e0	2.00e0	2239.1	0.017
Conserva	ative					
Trust	Region-	0.0	3.00e0	2.33e0	8.0	0.000
Aggressiv	ve					
QQN-Bis	section-1	0.0	2.00e0	2.00e0	84.3	0.002
L-BFGS-	-	0.0	\inf	\inf	0.0	0.000
MoreThu	$_{ m iente}$					
L-BFGS-	Limited	0.0	2.00e0	2.00e0	195.4	0.004
QQN-		0.0	2.00e0	2.00e0	148.2	0.002
GoldenSe	ection					
QQN-Str	ongWolfe	0.0	2.00e0	2.00e0	112.0	0.004

Weight Decay Adam-Robust 0.0 2.00e0 2.00e0 2502.0 0.061 Adam-AMSGrad 0.0 2.04e0 2.02e0 2502.0 0.060 L-BFGS- 0.0 2.00e0 2.00e0 62.2 0.001 Aggressive L-BFGS 0.0 2.00e0 2.00e0 102.2 0.002 GD-Momentum 0.0 3.39e0 2.91e0 21.2 0.001 GD-Mosterov 0.0 2.50e0 2.43e0 21.1 0.001 GD-Nosterov 0.0 2.58e0 2.45e0 31.5 0.001 L-BFGS- 0.0 2.00e0 2.00e0 555.1 0.014 L-BFGS- 0.0 2.00e0 2.00e0 555.1 0.014 Conservative GD- 0.0 inf inf 0.0 0.00 GD- 0.0 2.00e0 2.00e0 2502.0 0.054 GD-WeightDecay 0.0 2.00e0 2.00e0 326.6 0.002 <	Adam-	0.0	2.00e0	2.00e0	1144.5	0.026
Adam-Robust 0.0 2.00e0 2.00e0 2502.0 0.061 Adam-AMSGrad 0.0 2.04e0 2.02e0 2502.0 0.060 L-BFGS- 0.0 2.00e0 2.00e0 62.2 0.001 Aggressive L-BFGS 0.0 2.00e0 2.00e0 102.2 0.002 GD-Momentum 0.0 3.39e0 2.91e0 21.2 0.001 GD-Nesterov 0.0 2.50e0 2.43e0 21.1 0.001 Adam-Fast 0.0 2.58e0 2.45e0 31.5 0.001 L-BFGS- 0.0 2.00e0 2.00e0 555.1 0.014 Conservative GD- 0.0 inf inf 0.0 0.000 Adam 0.0 2.00e0 2.00e0 2502.0 0.054 GD-WeightDecay 0.0 2.00e0 2.00e0 326.6 0.002 Trust Region- 0.0 2.00e0 2.00e0 83.8 0.002 C					-	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.0	2.00e0	2.00e0	2502.0	0.061
Aggressive L-BFGS 0.0 2.00e0 2.00e0 102.2 0.002 GD-Momentum 0.0 3.39e0 2.91e0 21.2 0.001 GD-Nesterov 0.0 2.50e0 2.43e0 21.1 0.001 Adam-Fast 0.0 2.58e0 2.45e0 31.5 0.001 L-BFGS- 0.0 2.00e0 2.00e0 555.1 0.014 Conservative GD- 0.0 inf inf 0.0 0.000 Adam 0.0 2.00e0 2.00e0 2502.0 0.054 GD-Weight Decay 0.0 2.00e0 2.00e0 49.4 0.002 Trust Region- 0.0 2.05e0 2.00e0 326.6 0.002 Precise QQN- 0.0 2.00e0 2.00e0 83.8 0.002 CubicQuadraticInterpolation QQN-Bisection-2 0.0 2.00e0 2.00e0 305.0 0.008 Trust Region- 0.0 2.28e0	Adam-AMSGrad	0.0	2.04e0	2.02e0	2502.0	0.060
L-BFGS 0.0 2.00e0 2.00e0 102.2 0.002 GD-Momentum 0.0 3.39e0 2.91e0 21.2 0.001 GD-Nesterov 0.0 2.50e0 2.43e0 21.1 0.001 Adam-Fast 0.0 2.58e0 2.45e0 31.5 0.001 L-BFGS- 0.0 2.00e0 2.00e0 555.1 0.014 Conservative GD- 0.0 inf inf 0.0 0.000 AdaptiveMomentum Value 0.0 2.00e0 2.00e0 2502.0 0.054 GD-WeightDecay 0.0 2.00e0 2.00e0 49.4 0.002 Trust Region- 0.0 2.05e0 2.00e0 326.6 0.002 Precise Value Value 0.002 0.002 0.002 0.002 QN-Bisection-2 0.0 2.00e0 2.00e0 305.0 0.003 GD 0.0 2.00e0 2.00e0 305.0 0.008 Tru	L-BFGS-	0.0	2.00e0	2.00e0	62.2	0.001
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Aggressive					
GD-Nesterov 0.0 2.50e0 2.43e0 21.1 0.001 Adam-Fast 0.0 2.58e0 2.45e0 31.5 0.001 L-BFGS- 0.0 2.00e0 2.00e0 555.1 0.014 Conservative GD- 0.0 inf inf 0.0 0.000 Adam	L-BFGS	0.0	2.00e0	2.00e0	102.2	0.002
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	GD-Momentum	0.0	3.39e0	2.91e0	21.2	0.001
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	GD-Nesterov	0.0	2.50e0	2.43e0	21.1	0.001
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.0	2.58e0	2.45e0	31.5	0.001
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	L-BFGS-	0.0	2.00e0	2.00e0	555.1	0.014
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						
Adam 0.0 2.00e0 2.00e0 2502.0 0.054 GD-WeightDecay 0.0 2.00e0 2.00e0 49.4 0.002 Trust Region- 0.0 2.05e0 2.00e0 326.6 0.002 Precise V 0.0 2.00e0 2.00e0 83.8 0.002 CubicQuadraticInterpolation V V V V V V QQN-Bisection-2 0.0 2.00e0 2.00e0 126.0 0.003 GD 0.0 2.00e0 2.00e0 305.0 0.008 Trust Region- 0.0 2.28e0 2.05e0 54.1 0.000 Adaptive Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000	GD-	0.0	\inf	\inf	0.0	0.000
GD-WeightDecay 0.0 2.00e0 2.00e0 49.4 0.002 Trust Region- 0.0 2.05e0 2.00e0 326.6 0.002 Precise Value Value Value Value Value Value 0.002 CubicQuadraticInterpolation Value Value	AdaptiveMomentum	1				
Trust Region- 0.0 2.05e0 2.00e0 326.6 0.002 Precise QQN- 0.0 2.00e0 2.00e0 83.8 0.002 CubicQuadraticInterpolation V	Adam	0.0	2.00e0	2.00e0	2502.0	0.054
Precise QQN- 0.0 2.00e0 2.00e0 83.8 0.002 CubicQuadraticInterpolation 0.0 2.00e0 2.00e0 126.0 0.003 QQN-Bisection-2 0.0 2.00e0 2.00e0 305.0 0.008 GD 0.0 2.28e0 2.05e0 54.1 0.000 Adaptive Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000	GD-WeightDecay	0.0	2.00e0	2.00e0	49.4	0.002
QQN- 0.0 2.00e0 2.00e0 83.8 0.002 CubicQuadraticInterpolation 0.0 2.00e0 2.00e0 126.0 0.003 QQN-Bisection-2 0.0 2.00e0 2.00e0 305.0 0.008 GD 0.0 2.28e0 2.05e0 54.1 0.000 Adaptive Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000	Trust Region-	0.0	2.05e0	2.00e0	326.6	0.002
CubicQuadraticInterpolation QQN-Bisection-2 0.0 2.00e0 2.00e0 126.0 0.003 GD 0.0 2.00e0 2.00e0 305.0 0.008 Trust Region- 0.0 2.28e0 2.05e0 54.1 0.000 Adaptive Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000						
QQN-Bisection-2 0.0 2.00e0 2.00e0 126.0 0.003 GD 0.0 2.00e0 2.00e0 305.0 0.008 Trust Region- 0.0 2.28e0 2.05e0 54.1 0.000 Adaptive Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000	QQN-	0.0	2.00e0	2.00e0	83.8	0.002
GD 0.0 2.00e0 2.00e0 305.0 0.008 Trust Region- 0.0 2.28e0 2.05e0 54.1 0.000 Adaptive Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000	CubicQuadraticInte	rpolation				
Trust Region- 0.0 2.28e0 2.05e0 54.1 0.000 Adaptive Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000	QQN-Bisection-2	0.0	2.00e0	2.00e0	126.0	0.003
Adaptive Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000	GD	0.0	2.00e0	2.00e0	305.0	0.008
Trust Region- 0.0 2.23e0 2.11e0 9.4 0.000	Trust Region-	0.0	2.28e0	2.05e0	54.1	0.000
	Adaptive					
Standard	Trust Region-	0.0	2.23e0	2.11e0	9.4	0.000
Standard	Standard					

48 Problem: NoisySphere_2D_sigma0.01

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-1	35.0	1.33e0	2.16e-1	41.2	0.003
L-BFGS-Limited	45.0	1.64e0	7.63e-1	31.4	0.001
Adam-Robust	0.0	2.10e0	1.76e0	16.9	0.001
L-BFGS	40.0	1.02e1	7.26e-2	33.2	0.001
L-BFGS-	80.0	1.57e0	1.46e0	97.0	0.002
Conservative					
L-BFGS-	0.0	2.05e0	1.69e0	5.5	0.000
Aggressive					
QQN-StrongWolfe	60.0	9.23e-1	2.83e-2	30.5	0.001
Adam-	0.0	2.18e0	1.76e0	27.6	0.002
WeightDecay					
Trust Region-	0.0	2.19e0	1.76e0	5.3	0.000
Precise					
QQN-	5.0	2.04e0	1.45e0	27.1	0.001
GoldenSection					
GD	35.0	2.20e0	1.42e0	8.9	0.001
GD-WeightDecay	25.0	2.30e0	1.61e0	6.8	0.001
Adam-Fast	35.0	2.26e0	1.40e0	12.3	0.001
Trust Region-	0.0	2.03e0	1.67e0	4.2	0.000
Adaptive					
Trust Region-	0.0	2.06e0	1.68e0	5.0	0.000
Conservative					
Trust Region-	0.0	2.09e0	1.70e0	4.2	0.000
Aggressive					
GD-	30.0	2.39e0	1.45e0	6.6	0.001
AdaptiveMomentum					

QQN-	0.0	2.04e0	1.70e0	9.2	0.001
CubicQuadratic	Interpolation				
GD-Momentum	35.0	2.45e0	1.43e0	6.8	0.001
GD-Nesterov	25.0	2.82e0	1.53e0	7.0	0.001
Adam	0.0	2.13e0	1.72e0	21.4	0.001
Adam-AMSGrad	d 5.0	2.10e0	1.65e0	25.7	0.002
QQN-Bisection-	0.0	1.97e0	1.96e0	2.4	0.000
L-BFGS-	15.0	1.84e0	1.12e-1	18.7	0.001
MoreThuente					
Trust Regi	on- 0.0	2.10e0	1.70e0	4.0	0.000
Standard					

${\bf 49 \quad Problem:\ NoisySphere_5D_sigma0.01}$

Optimize	r	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam		0.0	5.42e0	4.73e0	26.1	0.003
Trust	Region-	0.0	5.20e0	4.59e0	5.7	0.000
Conservati	ve					
Trust	Region-	0.0	5.28e0	4.65e0	6.5	0.001
Precise						
QQN-		10.0	4.88e0	3.49e0	21.2	0.001
GoldenSec	tion					
QQN-Bise	ction-2	0.0	\inf	\inf	0.0	0.000
L-BFGS-		0.0	5.18e0	4.66e0	4.2	0.000
Aggressive						
L-BFGS-L		55.0	3.90e0	2.63e0	35.4	0.001
L-BFGS		20.0	$1.44\mathrm{e}1$	2.91e0	38.4	0.002
QQN-		0.0	5.26e0	4.75e0	7.2	0.001
CubicQuad	draticInter	polation				
GD		20.0	5.52e0	4.36e0	9.2	0.002
QQN-Bise	ction-1	45.0	4.46e0	2.43e0	75.7	0.011
Adam-		0.0	5.22e0	4.62e0	16.6	0.002
WeightDec	ay					
Adam-Rob	oust	0.0	5.11e0	4.60e0	17.1	0.002
Trust	Region-	0.0	5.32e0	4.65e0	3.5	0.000
Adaptive	Ü					
Trust	Region-	0.0	5.20e0	4.73e0	4.2	0.000
Standard						
Adam-Fast	-	10.0	5.99e0	4.17e0	11.7	0.001
Trust	Region-	0.0	5.27e0	4.65e0	3.5	0.000
Aggressive	_					
QQN-Stro	ngWolfe	45.0	3.67e0	1.11e0	18.9	0.001
L-BFGS-		85.0	4.42e0	4.14e0	120.3	0.004
Conservati	ve					
GD-Mome	ntum	25.0	5.71e0	4.41e0	7.9	0.001
L-BFGS-		25.0	4.61e0	3.27e0	12.3	0.001
MoreThue	nte					
GD-Nester	ov	30.0	5.61e0	4.26e0	6.8	0.001
GD-		15.0	5.68e0	4.36e0	7.0	0.001
AdaptiveN	Iomentum					
GD-Weigh		25.0	5.35e0	4.47e0	6.5	0.001
Adam-AM		0.0	5.42e0	4.94e0	18.6	0.002

$50 \quad Problem: \ NoisySphere_10D_sigma0.01$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	0.0	1.04e1	1.03e1	1.9	0.000
CubicQuadraticInter	polation				
GD-Momentum	10.0	1.11e1	9.66e0	6.8	0.002
QQN-Bisection-1	45.0	9.08e0	4.69e0	55.0	0.016
L-BFGS-Limited	45.0	9.06e0	6.79e0	32.2	0.002
L-BFGS-	10.0	9.77e0	6.61e0	12.4	0.002
MoreThuente					
Adam-AMSGrad	0.0	1.04e1	9.75e0	18.9	0.004
Adam-	0.0	1.04e1	9.79e0	28.9	0.005
WeightDecay					
Trust Region-	0.0	1.04e1	9.82e0	6.4	0.001
Conservative					
GD-Nesterov	15.0	1.09e1	9.63e0	7.5	0.002
GD-WeightDecay	5.0	1.10e1	9.66e0	6.0	0.002
Adam	0.0	1.07e1	9.87e0	18.2	0.003
Trust Region-	0.0	1.06e1	9.80e0	3.2	0.000
Standard					
Trust Region-	0.0	1.06e1	9.93e0	4.8	0.001
Precise					
Adam-Fast	15.0	1.11e1	9.46e0	9.7	0.002
QQN-Bisection-2	0.0	\inf	\inf	0.0	0.000
QQN-StrongWolfe	25.0	9.41e0	7.23e0	19.9	0.002
L-BFGS	20.0	2.28e1	7.46e0	31.4	0.002
L-BFGS-	85.0	9.49e0	9.16e0	93.8	0.006
Conservative					
GD	10.0	1.10e1	9.54e0	10.2	0.003
Adam-Robust	5.0	1.05e1	9.71e0	18.1	0.003
Trust Region-	0.0	1.04e1	9.77e0	3.5	0.001
Aggressive					
Trust Region-	0.0	1.04e1	9.73e0	2.8	0.000
Adaptive					
L-BFGS-	0.0	1.06e1	9.75e0	5.4	0.001
Aggressive					
QQN-	5.0	1.03e1	9.18e0	22.1	0.002
GoldenSection					
GD-	35.0	1.03e1	9.45e0	8.0	0.002
AdaptiveMomentum					
F					

$51 \quad Problem: \ SparseRosenbrock_4D$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	5.0	3.33e-1	6.04e-7	4306.3	0.085
GoldenSection					
GD-	0.0	1.31e0	2.41e-2	56.2	0.002
AdaptiveMomentum					
QQN-Bisection-1	15.0	4.56e-1	9.12e-8	2290.2	0.056
Adam-Fast	0.0	1.92e0	1.14e-3	243.3	0.005
QQN-StrongWolfe	25.0	3.39e-1	2.24e-8	2429.1	0.076
QQN-Bisection-2	5.0	1.42e0	5.00e-7	193.1	0.005
GD-Momentum	0.0	1.10e1	4.39e-2	21.8	0.001
Adam	0.0	2.06e0	1.19e0	2502.0	0.049
GD-Nesterov	0.0	3.38e0	1.58e-1	61.4	0.002
GD	0.0	1.88e0	9.10e-1	510.1	0.013
Adam-	0.0	6.07e0	1.75e-2	843.5	0.018
WeightDecay					

Trust Adaptive	Region-	0.0	8.31e0	7.78e0	954.8	0.006
Trust Standard	Region-	0.0	8.25e0	7.68e0	168.3	0.001
L-BFGS-L	imited	0.0	2.33e0	7.38e-2	3916.8	0.041
Trust	Region-	0.0	8.72e0	7.92e0	57.2	0.000
Aggressive	•					
QQN-		45.0	5.85e-2	6.10e-9	1743.3	0.071
	draticInterpo					
L-BFGS-		20.0	1.75e0	2.55e-7	2601.9	0.045
MoreThue						
Trust	Region-	0.0	2.16e1	7.75e0	1739.2	0.011
Precise						
Trust	Region-	0.0	4.72e1	3.37e-1	2998.1	0.018
Conservati	ive					
L-BFGS		0.0	9.62e1	7.18e0	143.9	0.002
Adam-Rol	oust	0.0	7.53e0	3.60e0	672.2	0.015
L-BFGS-		0.0	6.34e1	8.59e0	3852.0	0.029
Aggressive	;					
GD-Weigh		0.0	3.04e0	4.73e-2	105.6	0.003
Adam-AM	SGrad	0.0	3.39e0	1.65e0	2394.2	0.052
L-BFGS-		0.0	1.27e2	3.78e-5	3413.7	0.039
Conservati	ive					

${\bf 52}\quad {\bf Problem:\ SparseRosenbrock_10D}$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMSGrad	0.0	5.73e0	4.14e0	2502.0	0.058
L-BFGS-	0.0	7.68e0	2.91e-4	2749.9	0.051
MoreThuente					
L-BFGS-Limited	0.0	1.28e0	3.17e-1	3999.6	0.050
GD-WeightDecay	0.0	4.84e0	1.18e-1	127.2	0.004
Adam-	0.0	3.27e0	1.06e-2	2150.9	0.048
WeightDecay					
QQN-StrongWolfe	45.0	5.37e-1	1.90e-9	2419.7	0.074
QQN-Bisection-2	0.0	3.21e0	3.42e-1	842.9	0.021
Trust Region-	0.0	2.05e1	1.95e1	439.7	0.004
Standard					
GD-	0.0	1.31e1	5.08e-2	43.6	0.002
${\bf Adaptive Momentum}$					
Adam-Fast	0.0	1.90e0	5.27e-2	206.0	0.004
Adam-Robust	0.0	1.19e1	9.15e0	2380.8	0.056
Trust Region-	0.0	2.92e1	2.02e1	2139.5	0.014
Adaptive					
GD-Nesterov	0.0	7.90e-1	5.84e-2	100.3	0.003
Adam	0.0	4.91e0	3.14e0	2502.0	0.053
Trust Region-	0.0	1.56e2	6.91e1	3002.0	0.020
Conservative					
GD-Momentum	0.0	3.15e1	3.12e-1	21.1	0.001
Trust Region-	0.0	2.11e1	1.97e1	122.0	0.001
Aggressive					
L-BFGS	0.0	2.80e2	2.70e1	136.4	0.002
Trust Region-	0.0	1.04e2	1.18e0	3002.0	0.020
Precise					
QQN-	5.0	5.84e-1	5.69e-7	4401.4	0.086
GoldenSection					

L-BFGS-	0.0	1.81e2	8.09e1	3852.0	0.028
Aggressive L-BFGS-	0.0	1.05e7	2.96e-2	3686.9	0.038
Conservative					
GD	0.0	3.27e0	1.20e0	388.9	0.011
QQN-	55.0	1.42e-1	1.89e-8	1670.9	0.071
CubicQuadraticInterpolation	l				
QQN-Bisection-1	20.0	6.70e-1	1.26e-7	2281.2	0.061

$53 \quad Problem: \ SparseQuadratic_5D_pattern[1,\ 3]$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
GD-Momentum	0.0	5.03e-1	4.06e-1	22.0	0.001
L-BFGS-	100.0	8.81e-8	1.12e-8	23.6	0.000
MoreThuente					
GD	100.0	9.77e-7	9.59 e-7	352.9	0.009
L-BFGS-	100.0	2.84e-7	2.36e-7	31.0	0.000
Aggressive					
Trust Region- Standard	0.0	1.81e2	1.67e2	40.6	0.000
GD-WeightDecay	100.0	9.21e-7	8.42e-7	99.8	0.003
QQN-Bisection-2	100.0	1.58e-7	1.83e-9	37.5	0.001
Adam-Fast	0.0	3.56e-1	3.36e-1	37.5	0.001
Adam-AMSGrad	0.0	1.95e-1	1.19e-1	2502.0	0.057
Adam-Robust	0.0	6.93e-2	2.83e-2	2502.0	0.057
Trust Region-	0.0	1.82e4	1.51e4	27.0	0.000
Aggressive	0.0	1.0204	1.0104	21.0	0.000
L-BFGS	100.0	4.77e-7	1.77e-8	48.4	0.001
QQN-	100.0	4.31e-7	3.60e-7	57.0	0.001
CubicQuadraticInter		1.010 1	0.000 1	31.0	0.001
Adam-	100.0	9.85e-7	9.73e-7	1727.8	0.037
WeightDecay	100.0	0.000	0.1.55	112110	0.001
Trust Region-	0.0	4.06e0	8.95e-3	1737.1	0.012
Conservative	0.0	2,0000	0.000	1101.1	0.012
QQN-	100.0	1.09e-7	2.49e-9	132.7	0.002
GoldenSection	100.0	1.000 ,	2.1000	102	0.002
QQN-Bisection-1	100.0	1.79e-7	4.95e-9	38.0	0.001
GD-Nesterov	0.0	3.19e-1	2.73e-1	22.0	0.001
Adam	0.0	3.67e-3	5.91e-4	2502.0	0.051
Trust Region-	0.0	1.23e3	5.86e-2	90.2	0.001
Adaptive	0.0				0.00-
Trust Region-	0.0	4.92e-1	3.14e-2	306.4	0.002
Precise					
GD-	0.0	9.75 e-1	8.63e-1	19.4	0.001
AdaptiveMomentum				-	
L-BFGS-Limited	100.0	4.57e-7	4.26e-9	117.5	0.003
L-BFGS-	100.0	2.84e-7	1.15e-10	314.8	0.008
Conservative				-	
QQN-StrongWolfe	100.0	2.22e-7	4.00e-8	29.1	0.001

$54 \quad Problem: \ SparseQuadratic_10D_pattern[1,\ 3]$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	100.0	3.11e-7	7.74e-9	156.2	0.002
GoldenSection					

QQN-Bisection-1	100.0	5.68e-7	2.29e-8	55.8	0.001
GD	100.0	9.80e-7	9.60e-7	360.1	0.010
L-BFGS-	100.0	2.03e-7	2.80e-9	339.5	0.009
Conservative					
Adam	0.0	2.03e-3	3.96e-4	2502.0	0.056
QQN-Bisection-2	100.0	3.81e-7	2.03e-8	46.4	0.001
Adam-Fast	0.0	7.28e-1	6.71e-1	37.3	0.001
QQN-StrongWolfe	100.0	2.19e-7	8.82e-8	43.1	0.001
GD-	0.0	1.57e0	1.41e0	21.9	0.001
AdaptiveMomentum					
Adam-AMSGrad	0.0	2.33e-1	1.70e-1	2502.0	0.061
L-BFGS-	100.0	2.16e-7	1.72e-7	38.0	0.001
Aggressive					
QQN-	100.0	2.38e-8	2.26e-9	60.4	0.002
CubicQuadraticInterpol	ation				
L-BFGS-Limited	100.0	4.74e-7	5.73e-8	58.5	0.001
L-BFGS-	100.0	2.20e-7	2.06e-8	28.2	0.000
MoreThuente					
Adam-	100.0	9.83e-7	9.70e-7	1532.3	0.036
WeightDecay					
Trust Region-	0.0	1.28e3	5.42e-2	162.1	0.001
Adaptive					
Trust Region-	0.0	1.02e2	2.21e-1	57.5	0.000
Standard					
GD-WeightDecay	100.0	9.31e-7	8.75e-7	103.2	0.004
Trust Region-	0.0	2.01e4	8.89e-1	31.6	0.000
Aggressive					
Trust Region-	0.0	1.83e0	6.04e-1	3002.0	0.023
Conservative					
GD-Momentum	0.0	9.15e-1	8.60e-1	23.1	0.001
GD-Nesterov	0.0	6.03e-1	5.68e-1	23.1	0.001
Adam-Robust	0.0	1.03e-1	4.23e-2	2502.0	0.062
Trust Region-	0.0	1.80e-1	2.31e-3	589.1	0.005
Precise					
L-BFGS	90.0	1.93e-3	1.06e-8	75.7	0.001

$55 \quad Problem: \ Logistic Regression_100 samples_5 features_reg 0.01$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS-	0.0	3.15e-1	3.15e-1	559.5	0.185
Conservative					
Trust Region-	0.0	6.98e-1	6.53 e-1	5.0	0.002
Aggressive					
L-BFGS-Limited	0.0	3.15e-1	3.15e-1	3043.5	0.886
Adam	0.0	4.02e-1	3.92e-1	2502.0	0.889
QQN-Bisection-1	0.0	3.15e-1	3.15e-1	98.5	0.036
Trust Region-	0.0	4.24e-1	4.20e-1	86.6	0.025
Conservative					
QQN-StrongWolfe	0.0	3.15e-1	3.15e-1	74.3	0.031
QQN-	0.0	3.15e-1	3.15e-1	91.5	0.036
CubicQuadraticInter	polation				
QQN-Bisection-2	0.0	3.15e-1	3.15e-1	98.5	0.032
L-BFGS-	0.0	3.15e-1	3.15e-1	1776.5	0.508
Aggressive					
L-BFGS	0.0	3.16e-1	3.15e-1	2673.8	0.796
GD-Nesterov	0.0	3.15e-1	3.15e-1	1668.0	0.902
GD	0.0	3.77e-1	3.72e-1	1668.0	0.897

GD-Momer	ntum	0.0	3.15e-1	3.15e-1	1668.0	0.901
GD-Weight	tDecay	0.0	3.27e-1	3.26e-1	1668.0	0.905
L-BFGS-		0.0	3.15e-1	3.15e-1	106.7	0.034
MoreThuer	nte					
Adam-AMS	SGrad	0.0	4.05e-1	3.87e-1	2502.0	0.891
Adam-		0.0	3.22e-1	3.20e-1	2502.0	0.890
WeightDec	ay					
Adam-Rob	ust	0.0	4.32e-1	4.13e-1	2502.0	0.892
Adam-Fast	i	0.0	3.16e-1	3.16e-1	90.8	0.033
QQN-		0.0	3.15e-1	3.15e-1	307.2	0.067
GoldenSect	tion					
Trust	Region-	0.0	6.95 e-1	6.37e-1	5.0	0.002
Adaptive						
GD-		0.0	3.15e-1	3.15e-1	426.1	0.232
AdaptiveM	Iomentum					
Trust	Region-	0.0	6.98e-1	6.55 e-1	5.0	0.002
Standard						
Trust	Region-	0.0	6.78e-1	6.48e-1	5.5	0.002
Precise						
Trust Adaptive GD- AdaptiveM Trust Standard Trust	Region- Iomentum Region-	0.0	3.15e-1 6.98e-1	3.15e-1 6.55e-1	426.1 5.0	0.232 0.002

$56 \quad Problem: \ Logistic Regression_200 samples_10 features_reg 0.01$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-AMSGrad	0.0	3.95e-1	3.84e-1	2502.0	1.619
L-BFGS-	0.0	3.25 e-1	3.23e-1	3432.2	1.395
Aggressive					
Trust Region-	0.0	7.01e-1	6.45e-1	5.0	0.003
Standard					
Trust Region-	0.0	7.01e-1	6.59e-1	5.0	0.003
Precise					
QQN-Bisection-2	0.0	3.23e-1	3.23e-1	110.7	0.065
Adam-Robust	0.0	4.12e-1	3.96e-1	2502.0	1.621
QQN-Bisection-1	0.0	3.23e-1	3.23e-1	110.7	0.071
GD-WeightDecay	0.0	3.39e-1	3.38e-1	1668.0	1.650
QQN-	0.0	3.23e-1	3.23e-1	360.0	0.139
GoldenSection					
GD-Nesterov	0.0	3.23e-1	3.23e-1	1668.0	1.649
Adam-Fast	0.0	3.24e-1	3.24e-1	77.0	0.050
Trust Region-	0.0	7.06e-1	6.56e-1	5.0	0.003
Adaptive					
L-BFGS-	0.0	3.23e-1	3.23e-1	86.2	0.049
MoreThuente					
Trust Region-	0.0	4.44e-1	4.39e-1	82.4	0.044
Conservative					
L-BFGS	0.0	3.23e-1	3.23e-1	2311.4	1.246
GD	0.0	3.97e-1	3.93e-1	1668.0	1.635
GD-	0.0	3.23e-1	3.23e-1	479.1	0.478
AdaptiveMomentum					
L-BFGS-Limited	0.0	3.23e-1	3.23e-1	3146.0	1.606
Adam-	0.0	3.27e-1	3.25e-1	2502.0	1.623
WeightDecay					
Adam	0.0	3.92e-1	3.75e-1	2502.0	1.611
Trust Region-	0.0	7.03e-1	6.65e-1	5.0	0.003
Aggressive			0.05	406	
GD-Momentum	0.0	3.23e-1	3.23e-1	1668.0	1.648
QQN-StrongWolfe	0.0	3.23e-1	3.23e-1	80.3	0.058

L-BFGS-	0.0	3.23e-1	3.23e-1	545.5	0.317
Conservative QQN-	0.0	3.23e-1	3.23e-1	105.4	0.071
${\bf Cubic Quadratic Interpolation}$					

$57 \quad Problem: \ Linear Regression_100 samples_5 features_reg 0.01$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-	0.0	7.15e-2	7.15e-2	118.9	0.041
CubicQuadraticInter	polation				
L-BFGS-Limited	0.0	7.15e-2	7.15e-2	540.6	0.147
Trust Region-	0.0	1.10e2	2.92e-1	76.2	0.020
Standard					
QQN-StrongWolfe	0.0	7.15e-2	7.15e-2	109.2	0.040
GD	0.0	7.15e-2	7.15e-2	512.3	0.267
QQN-Bisection-2	0.0	7.15e-2	7.15e-2	103.0	0.038
Trust Region-	0.0	1.38e-1	7.38e-2	851.2	0.220
Precise					
GD-Nesterov	0.0	9.19e-1	8.36e-1	23.9	0.012
Adam-Fast	0.0	1.90e-1	7.25e-2	102.4	0.035
QQN-	0.0	7.15e-2	7.15e-2	325.7	0.063
GoldenSection					
L-BFGS-	0.0	7.15e-2	7.15e-2	1306.5	0.407
Conservative					
GD-	0.0	2.00e0	1.90e0	23.0	0.012
AdaptiveMomentum					
Adam	0.0	2.88e0	2.36e0	2502.0	0.829
Adam-	0.0	7.18e-2	7.16e-2	2502.0	0.828
WeightDecay					
Adam-AMSGrad	0.0	3.06e0	2.47e0	2502.0	0.833
GD-WeightDecay	0.0	7.15e-2	7.15e-2	165.2	0.087
L-BFGS	0.0	1.38e-1	7.15e-2	234.7	0.060
GD-Momentum	0.0	1.36e0	1.22e0	24.1	0.012
L-BFGS-	0.0	7.15e-2	7.15e-2	153.1	0.036
Aggressive					
L-BFGS-	0.0	7.16e-2	7.15e-2	796.0	0.229
MoreThuente					
Adam-Robust	0.0	4.89e0	4.01e0	2502.0	0.830
QQN-Bisection-1	0.0	7.15e-2	7.15e-2	123.0	0.047
Trust Region-	0.0	1.55e3	1.22e-1	234.8	0.062
Adaptive					
Trust Region-	0.0	7.87e0	6.26e0	3002.0	0.769
Conservative					
Trust Region-	0.0	3.62e4	9.10e-1	34.0	0.009
Aggressive					

$58 \quad Problem: \ Linear Regression_200 samples_10 features_reg0.01$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-1	100.0	4.82e-1	4.82e-1	89.0	0.066
Adam	0.0	5.34e1	4.86e1	2502.0	1.495
QQN-	100.0	4.82e-1	4.82e-1	180.0	0.059
GoldenSection					
L-BFGS-Limited	100.0	4.82e-1	4.82e-1	159.2	0.068
GD-Nesterov	0.0	1.80e0	1.78e0	37.9	0.035

Adam-Fast	20.0	5.14e-1	4.82e-1	189.9	0.115
QQN-StrongWolfe	100.0	4.82e-1	4.82e-1	62.0	0.039
Adam-AMSGrad	0.0	5.39e1	5.00e1	2502.0	1.502
GD-Momentum	0.0	2.46e0	2.41e0	38.1	0.035
L-BFGS	90.0	5.09e-1	4.82e-1	77.5	0.039
GD	100.0	4.82e-1	4.82e-1	343.6	0.321
Trust Region-	0.0	3.92e4	1.28e0	111.1	0.054
Aggressive					
Adam-	0.0	2.42e0	1.66e0	2502.0	1.496
WeightDecay					
Trust Region-	0.0	5.39e1	4.85e1	3002.0	1.419
Precise					
Trust Region-	0.0	1.08e3	5.40e-1	1420.7	0.673
Adaptive					
GD-Weight $Decay$	100.0	4.82e-1	4.82e-1	116.7	0.109
GD-	0.0	5.05e0	4.95e0	35.0	0.032
${\bf Adaptive Momentum}$					
QQN-Bisection-2	100.0	4.82e-1	4.82e-1	69.0	0.050
L-BFGS-	100.0	4.82e-1	4.82e-1	72.8	0.038
MoreThuente					
L-BFGS-	100.0	4.82e-1	4.82e-1	205.7	0.088
Aggressive					
Adam-Robust	0.0	6.34e1	5.92e1	2502.0	1.501
Trust Region-	0.0	1.06e2	1.03e2	3002.0	1.417
Conservative					
Trust Region-	0.0	2.56e3	7.13e-1	372.2	0.178
Standard					
QQN-	100.0	4.82e-1	4.82e-1	54.1	0.035
CubicQuadraticInterpola					0.040
L-BFGS-	100.0	4.82e-1	4.82e-1	436.2	0.242
Conservative					

$59 \quad Problem: \ Neural Network_100 samples_layers_5_10_3$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
QQN-Bisection-2	85.0	1.40e-1	1.38e-1	750.6	0.542
QQN-Bisection-1	90.0	1.40e-1	1.39e-1	995.0	0.864
QQN-StrongWolfe	60.0	1.43e-1	1.39e-1	1340.8	1.277
GD-WeightDecay	0.0	1.83e-1	1.75e-1	1668.0	2.353
GD-Momentum	0.0	2.35e-1	2.26e-1	21.2	0.028
Adam-Fast	85.0	1.40e-1	1.38e-1	145.4	0.121
Adam-	85.0	1.43e-1	1.40e-1	1555.7	1.287
WeightDecay					
Trust Region-	0.0	4.50e-1	1.89e-1	39.2	0.025
Standard					
GD-Nesterov	0.0	2.27e-1	2.19e-1	21.8	0.029
Trust Region-	0.0	2.19e-1	2.14e-1	157.6	0.098
Conservative					
Trust Region-	0.0	1.30e0	2.02e-1	25.4	0.016
Aggressive					
$\overline{\mathrm{GD}}$	0.0	2.01e-1	1.95e-1	1668.0	2.332
L-BFGS-Limited	40.0	1.48e-1	1.40e-1	2753.6	1.375
L-BFGS-	15.0	1.50e-1	1.40e-1	2900.4	1.375
Aggressive					
QQN-	90.0	1.40e-1	1.38e-1	617.1	0.666
CubicQuadraticInter	polation				

L-BFGS-		30.0	1.49e-1	1.39e-1	2286.9	1.481
Conservative						
GD-		0.0	2.59e-1	2.33e-1	15.8	0.020
Adaptive	Iomentum					
Adam		0.0	1.58e-1	1.45e-1	2502.0	2.063
Adam-AM	ISGrad	5.0	1.55e-1	1.40e-1	2490.8	2.069
L-BFGS		10.0	2.01e-1	1.40e-1	1471.2	0.854
L-BFGS-		0.0	1.51e-1	1.41e-1	2855.0	1.945
MoreThue	ente					
Adam-Rol	oust	0.0	1.69e-1	1.49e-1	2502.0	2.074
Trust	Region-	0.0	2.63e-1	1.83e-1	56.9	0.036
Adaptive						
Trust	Region-	0.0	2.20e-1	2.08e-1	34.4	0.022
Precise						
QQN- 85.0		1.41e-1	1.39e-1	2473.0	0.852	
GoldenSection						

$60 \quad Problem: \ NeuralNetwork_100 samples_layers_10_20_5$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
L-BFGS	0.0	1.83e-1	3.89e-2	2108.6	2.360
GD-WeightDecay	0.0	9.46e-2	8.67e-2	1668.0	4.605
L-BFGS-	20.0	4.85e-2	3.81e-2	2547.8	3.325
Conservative					
QQN-StrongWolfe	55.0	4.29e-2	3.79e-2	1389.3	2.526
Trust Region-	0.0	2.36e-1	2.04e-1	5.2	0.007
Standard					
QQN-Bisection-1	90.0	3.87e-2	3.78e-2	1132.3	1.786
GD-Nesterov	0.0	1.18e-1	3.95e-2	679.1	1.835
QQN-	100.0	3.80e-2	3.77e-2	1028.8	2.119
CubicQuadraticInter	polation				
L-BFGS-	0.0	5.55e-2	4.81e-2	3657.3	3.210
Aggressive					
Adam	15.0	4.60e-2	3.82e-2	2484.8	3.902
L-BFGS-	0.0	5.96e-2	5.34e-2	2833.2	3.804
MoreThuente					
GD-Momentum	0.0	1.63e-1	4.45e-2	102.8	0.276
Adam-AMSGrad	0.0	4.96e-2	3.99e-2	2502.0	3.917
QQN-Bisection-2	95.0	3.81e-2	3.78e-2	1033.4	1.492
Adam-Robust	0.0	6.03e-2	4.99e-2	2502.0	4.003
Adam-Fast	40.0	4.46e-2	3.70e-2	184.6	0.295
Trust Region-	0.0	1.60e-1	1.52e-1	25.1	0.031
Precise					
Trust Region-	0.0	1.48e-1	1.28e-1	113.2	0.141
Conservative					
Adam-	100.0	3.82e-2	3.81e-2	1312.7	2.082
WeightDecay					
Trust Region-	0.0	1.94e-1	1.56e-1	29.0	0.036
Adaptive					
QQN-	70.0	3.85e-2	3.78e-2	3690.9	2.355
GoldenSection					
Trust Region-	0.0	7.84e-1	3.46e-1	7.8	0.010
Aggressive					
GD-	0.0	1.74e-1	1.66e-1	15.2	0.037
AdaptiveMomentum					
GD	0.0	1.28e-1	1.19e-1	1668.0	4.558
L-BFGS-Limited	0.0	5.79e-2	3.92e-2	3421.6	3.299

${\bf 61 \quad Problem: \ SVM_100 samples_5 features_C1}$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)	
QQN-StrongWolfe	100.0	6.43e-1	6.43e-1	57.5	0.023	
Trust Region-	0.0	9.38e-1	6.73e-1	36.2	0.011	
Precise						
GD-WeightDecay	100.0	6.43e-1	6.43e-1	97.2	0.055	
Adam-	100.0	6.43e-1	6.43e-1	628.6	0.231	
WeightDecay						
Adam-Robust	50.0	6.43e-1	6.43e-1	1414.2	0.517	
L-BFGS-Limited	90.0	6.43e-1	6.43e-1	444.7	0.098	
QQN-Bisection-2	100.0	6.43e-1	6.43e-1	68.8	0.030	
GD-Momentum	0.0	6.73e-1	6.53e-1	24.9	0.014	
QQN-Bisection-1	100.0	6.43e-1	6.43e-1	83.4	0.039	
GD	100.0	6.43e-1	6.43e-1	274.4	0.156	
Adam-AMSGrad	90.0	6.43e-1	6.43e-1	1984.2	0.726	
L-BFGS-	100.0	6.43e-1	6.43e-1	46.0	0.012	
Aggressive						
L-BFGS-	100.0	6.43e-1	6.43e-1	41.3	0.013	
MoreThuente						
L-BFGS	100.0	6.43e-1	6.43e-1	42.0	0.012	
GD-Nesterov	0.0	6.64 e-1	6.50e-1	25.1	0.014	
Trust Region-	0.0	4.68e0	7.07e-1	19.0	0.006	
Adaptive						
GD-	0.0	7.53e-1	7.12e-1	17.6	0.010	
AdaptiveMomentum						
L-BFGS-	100.0	6.43e-1	6.43e-1	133.7	0.046	
Conservative						
Adam	100.0	6.43e-1	6.43e-1	1826.7	0.672	
QQN-	100.0	6.43e-1	6.43e-1	61.6	0.028	
CubicQuadraticInterp	oolation					
Trust Region-	0.0	3.47e2	7.26e-1	15.4	0.005	
Standard						
QQN-	90.0	6.43e-1	6.43e-1	228.9	0.046	
GoldenSection						
Adam-Fast	0.0	6.84e-1	6.44e-1	32.8	0.012	
Trust Region-	0.0	2.82e0	6.45 e-1	134.5	0.039	
Conservative						
Trust Region-	0.0	1.00e3	6.57e-1	16.3	0.005	
Aggressive						

${\bf 62 \quad Problem: \ SVM_200 samples_10 features_C1}$

Optimizer	Success Rate (%)	Mean Final Value	Best Value	Mean Func Evals	Mean Time (s)
Adam-Fast	0.0	7.22e-1	6.87e-1	42.1	0.026
Adam	0.0	6.86e-1	6.86e-1	1755.8	1.112
Trust Region-	0.0	2.49e0	6.88e-1	130.4	0.068
Conservative					
Adam-AMSGrad	0.0	6.86 e-1	6.86e-1	1996.0	1.280
Trust Region-	0.0	1.84e0	7.56e-1	11.8	0.006
Aggressive					
Trust Region-	0.0	9.09e-1	7.00e-1	36.1	0.019
Precise					
GD-	0.0	7.83e-1	7.62e-1	18.0	0.016
AdaptiveMomentum					
L-BFGS	0.0	7.68e-1	6.86e-1	321.6	0.146

Adam-	0.0	6.86e-1	6.86 e-1	668.9	0.428
WeightDecay					
QQN-Bisection-1	0.0	6.86 e-1	6.86e-1	1454.6	1.051
Trust Region-	0.0	8.76e-1	6.90e-1	20.8	0.011
Standard					
QQN-	0.0	6.86 e-1	6.86e-1	289.4	0.221
CubicQuadraticInter	polation				
GD-Momentum	0.0	7.08e-1	6.99e-1	25.8	0.024
L-BFGS-	0.0	6.86 e-1	6.86e-1	624.9	0.274
Conservative					
GD	0.0	6.86e-1	6.86e-1	450.0	0.442
Trust Region-	0.0	6.77e0	6.97e-1	24.4	0.013
Adaptive					
Adam-Robust	0.0	6.86e-1	6.86e-1	547.7	0.357
GD-WeightDecay	0.0	6.86e-1	6.86e-1	177.0	0.172
GD-Nesterov	0.0	7.03e-1	6.94e-1	25.7	0.024
QQN-StrongWolfe	0.0	6.86e-1	6.86e-1	1065.5	0.586
QQN-Bisection-2	0.0	6.86e-1	6.86e-1	911.8	0.461
L-BFGS-	0.0	5.16e3	6.86e-1	555.0	0.325
MoreThuente					
L-BFGS-	0.0	6.86e-1	6.86e-1	124.0	0.050
Aggressive					
QQN-	0.0	6.86e-1	6.86e-1	925.2	0.322
GoldenSection					
L-BFGS-Limited	0.0	6.86e-1	6.86e-1	770.6	0.264