

Table 1: Performance Results for Sphere_2D Problem

| Optimizer | Mean Final Value | Std Dev | Best Value | Worst Value | Mean Func Evals | Success Rate (%) | Mean Time (s) |
|---------------------------------|------------------|----------|------------|-------------|-----------------|------------------|---------------|
| Adam-Robust | 5.86e-2 | 2.41e-2 | 2.27e-2 | 1.25e-1 | 2502.0 | 0.0 | 0.055 |
| Adam-AMSGrad | 7.20e-2 | 3.21e-2 | 2.82e-2 | 1.30e-1 | 2502.0 | 0.0 | 0.054 |
| Adam | 7.60e-3 | 2.83e-3 | 4.96e-3 | 1.28e-2 | 2432.4 | 40.0 | 0.048 |
| Adam-WeightDecay | 4.98e-3 | 2.03e-5 | 4.94e-3 | 5.00e-3 | 1113.1 | 100.0 | 0.023 |
| GD | 4.93e-3 | 5.43e-5 | 4.81e-3 | 5.00e-3 | 152.6 | 100.0 | 0.004 |
| Trust Region-Conservative | 1.69e-3 | 1.41e-3 | 2.11e-4 | 4.46e-3 | 602.0 | 100.0 | 0.004 |
| L-BFGS-Conservative | 4.28e-3 | 6.13e-4 | 2.80e-3 | 4.97e-3 | 156.2 | 100.0 | 0.004 |
| GD-WeightDecay | 4.67e-3 | 1.71e-4 | 4.31e-3 | 4.95e-3 | 51.6 | 100.0 | 0.002 |
| Trust Region-Precise | 6.58e-1 | 7.91e-1 | 5.18e-6 | 2.79e0 | 115.3 | 55.0 | 0.001 |
| Adam-Fast | 8.78e-2 | 5.67e-2 | 2.82e-4 | 1.34e-1 | 33.5 | 30.0 | 0.001 |
| GD-Nesterov | 2.51e-3 | 2.02e-3 | 2.83e-4 | 4.93e-3 | 16.4 | 100.0 | 0.000 |
| GD-Momentum | 4.85e-4 | 1.02e-3 | 1.93e-4 | 4.95e-3 | 16.9 | 100.0 | 0.000 |
| GD-AdaptiveMomentum | 2.05e-1 | 2.50e-1 | 2.68e-5 | 5.53e-1 | 13.8 | 60.0 | 0.000 |
| L-BFGS-Limited | 1.24e-3 | 1.70e-3 | 0.00e0 | 4.92e-3 | 23.1 | 100.0 | 0.000 |
| Trust Region-Adaptive | 6.68e2 | 6.05e2 | 1.21e-5 | 1.22e3 | 41.1 | 35.0 | 0.000 |
| QQN-GoldenSection | 3.22e-14 | 4.92e-15 | 2.46e-14 | 3.96e-14 | 46.0 | 100.0 | 0.000 |
| QQN-CubicQuadraticInterpolation | 0.00e0 | 0.00e0 | 0.00e0 | 0.00e0 | 12.0 | 100.0 | 0.000 |
| Trust Region-Standard | 1.25e3 | 2.01e3 | 7.55e-4 | 6.04e3 | 27.1 | 10.0 | 0.000 |
| L-BFGS | 8.79e-5 | 3.32e-4 | 0.00e0 | 1.51e-3 | 13.5 | 100.0 | 0.000 |
| QQN-Bisection-2 | 0.00e0 | 0.00e0 | 0.00e0 | 0.00e0 | 13.0 | 100.0 | 0.000 |
| QQN-Bisection-1 | 0.00e0 | 0.00e0 | 0.00e0 | 0.00e0 | 15.0 | 100.0 | 0.000 |
| Trust Region-Aggressive | 7.95e3 | 5.21e3 | 2.26e-5 | 1.15e4 | 18.6 | 30.0 | 0.000 |
| QQN-StrongWolfe | 0.00e0 | 0.00e0 | 0.00e0 | 0.00e0 | 11.0 | 100.0 | 0.000 |
| L-BFGS-MoreThuente | 0.00e0 | 0.00e0 | 0.00e0 | 0.00e0 | 10.0 | 100.0 | 0.000 |
| L-BFGS-Aggressive | 0.00e0 | 0.00e0 | 0.00e0 | 0.00e0 | 10.0 | 100.0 | 0.000 |