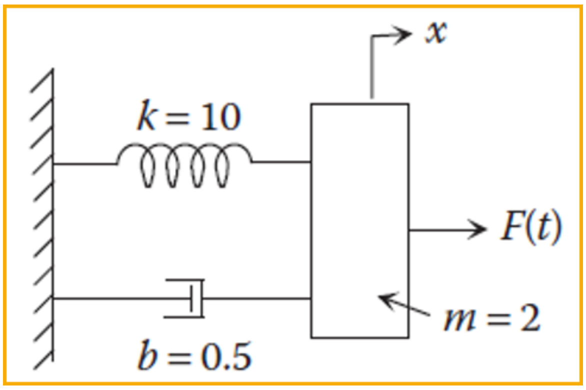
MODELARE ŞI SIMULARE - 2024

LABORATOR NR. 3 - SISTEME MECANICE (2), MATLAB (3), SIMULINK (2)

3.1 Modelați sistemul mecanic din figura de mai jos (modelul pe stare, forma matricială). Simulați dinamica sa în Simulink.



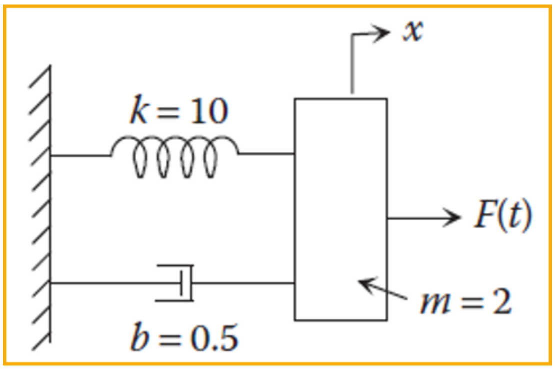
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$$

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{Bmatrix}_{n\times 1}, \quad \mathbf{u} = \begin{Bmatrix} u_1 \\ u_2 \\ \ldots \\ u_m \end{Bmatrix}_{m\times 1}, \quad \mathbf{f} = \begin{Bmatrix} f_1 \\ f_2 \\ \ldots \\ f_n \end{Bmatrix}_{n\times 1}$$

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + \cdots + a_{1n}x_n + b_{11}u_1 + \cdots + b_{1m}u_m \\ \dot{x}_2 = a_{21}x_1 + \cdots + a_{2n}x_n + b_{21}u_1 + \cdots + b_{2m}u_m \\ \ldots \\ \dot{x}_n = a_{n1}x_1 + \cdots + a_{nn}x_n + b_{n1}u_1 + \cdots + b_{nm}u_m \end{cases}$$

$$\begin{cases} y_1 = c_{11}x_1 + \cdots + c_{1n}x_n + d_{11}u_1 + \cdots + d_{1m}u_m \\ y_2 = c_{21}x_1 + \cdots + c_{2n}x_n + d_{21}u_1 + \cdots + d_{2m}u_m \\ \ldots \\ y_p = c_{p1}x_1 + \cdots + c_{pn}x_n + d_{p1}u_1 + \cdots + d_{pm}u_m \end{cases}$$

$$\boxed{\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}}$$



$$2\ddot{x} + 0.5\dot{x} + 10x = F(t)$$

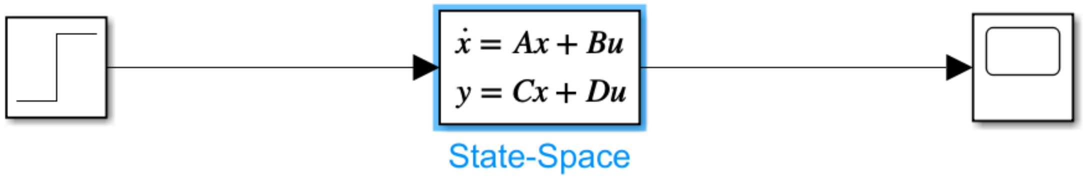
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{2}\left[-0.5x_2 - 10x_1 + F(t)\right] \end{cases}$$

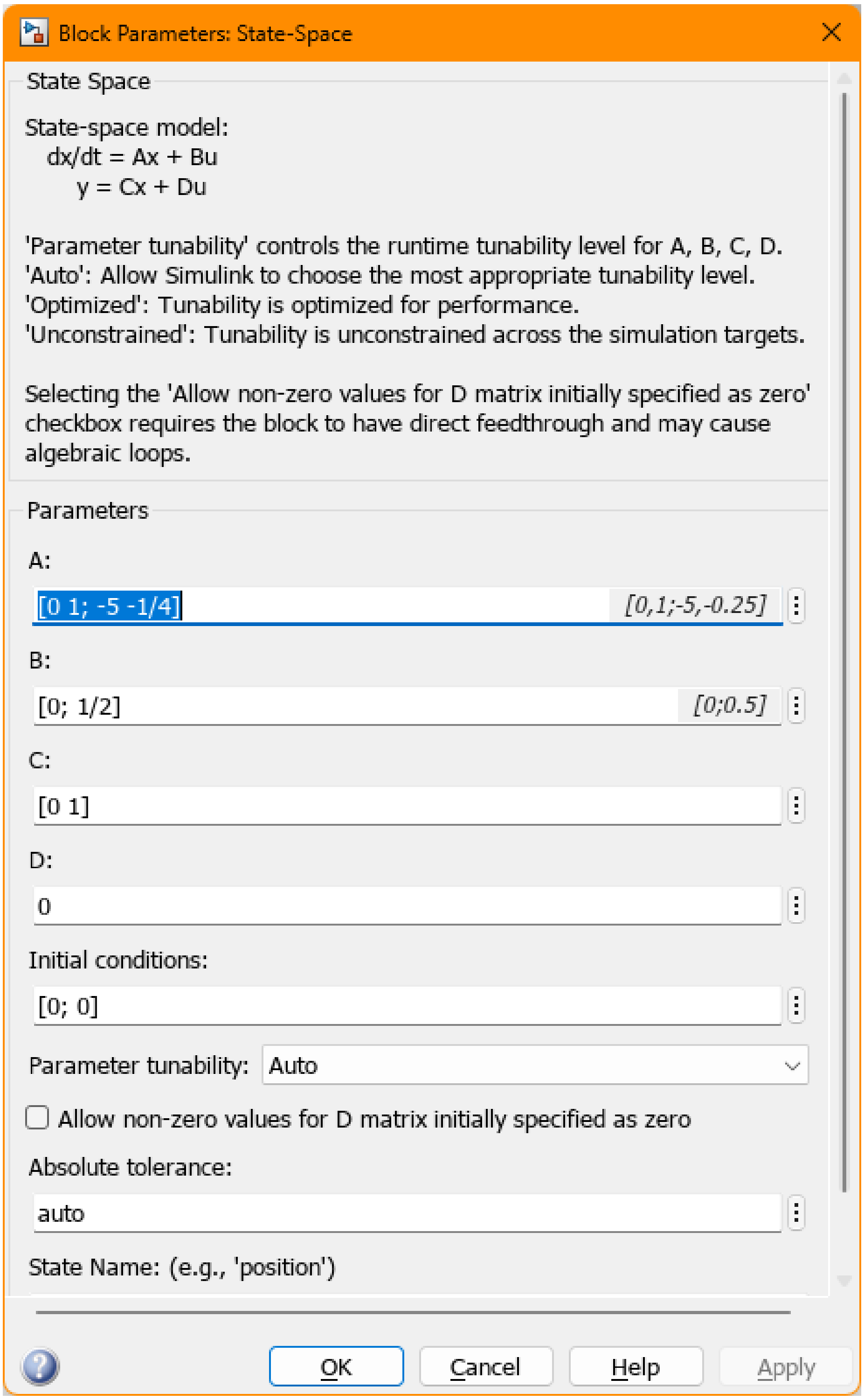
$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{1}{4} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} F(t)$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \ \mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}, \ \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{1}{4} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}, \ u = F(t)$$

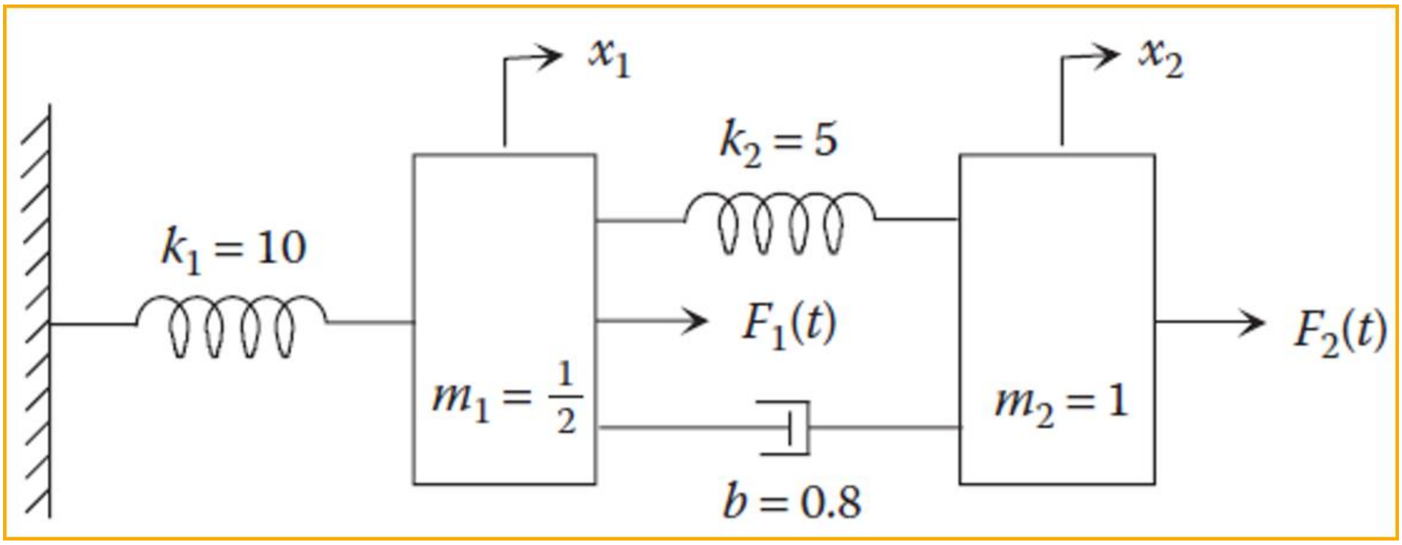
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + 0 \cdot u$$


$\dot{x} = Ax + Bu$
$y = Cx + Du$
State-Space


Block Parameters: State-Space
State Space
State-space model:
dx/dt = Ax + Bu
y = Cx + Du
'Parameter tunability' controls the runtime tunability level for A, B, C, D.
'Auto': Allow Simulink to choose the most appropriate tunability level.
'Optimized': Tunability is optimized for performance.
'Unconstrained': Tunability is unconstrained across the simulation targets.
Selecting the 'Allow non-zero values for D matrix initially specified as zero' checkbox requires the block to have direct feedthrough and may cause algebraic loops.
Parameters
A:
[0 1; -5 -1/4]
[0,1;-5,-0.25]
B:
[0; 1/2]
[0;0.5]
C:
[0 1]
D:
0
Initial conditions:
[0; 0]
Parameter tunability:
Auto
Allow non-zero values for D matrix initially specified as zero
Absolute tolerance:
auto
State Name: (e.g., 'position')
OK
Cancel
Help
Apply

3.2 Deduceți modelul dinamic al sistemului mecanic de mai jos. Scrieți-l și în forma pe stare. Simulați în MATLAB dinamica sa, atunci când cele două intrări sunt semnale de tip treaptă unitară.



$\boxed{m_1}$ $\boxed{m_2}$

$$\begin{cases} x_1 \\ x_2 \\ \dot{x}_1 = x_3 \\ x_4 = \dot{x}_2 \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = -30x_1 + 10x_2 + 1.6x_4 - 1.6x_3 + 2F_1 \\ \dot{x}_4 = -5x_2 + 5x_1 - 0.8x_4 + 0.8x_3 + F_2 \end{cases}$$

$\dot{X} = AX + Bu$

$y = \begin{Bmatrix} x_1 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} x \\ \ddot{x} \end{Bmatrix}$

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = -30x_1 + 10x_2 + 1.6x_4 - 1.6x_3 + 2F_1 \\ \dot{x}_4 = -5x_2 + 5x_1 - 0.8x_4 + 0.8x_3 + F_2 \end{cases}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}_{4\times 1}, \quad \mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -30 & 10 & -1.6 & 1.6 \\ 5 & -5 & 0.8 & -0.8 \end{bmatrix}_{4\times 4}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 0 \\ 0 & 1 \end{bmatrix}_{4\times 2}, \quad \mathbf{u} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}_{2\times 1}$$

$$\mathbf{y} = \begin{Bmatrix} x_1 \\ \dot{x}_1 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_3 \end{Bmatrix}$$

$$\mathbf{y}_{2\times 1} = \mathbf{C}_{2\times 4}\mathbf{x}_{4\times 1} + \mathbf{D}_{2\times 2}\mathbf{u}_{2\times 1}$$

Implementare MATLAB – folosim functia *ss*

```
>> help ss
 ss  Construct state-space model or convert model to state space.

  Construction:
   SYS = ss(A,B,C,D) creates an object SYS representing the continuous-
   time state-space model
        dx/dt = Ax(t) + Bu(t)
        y(t) = Cx(t) + Du(t)
```

```
% Introducem matricile A,B,C si D
A = [0 0 1 0;0 0 0 1;-30 10 -1.6 1.6;5 -5 0.8 -0.8];
B = [0 0;0 0;2 0;0 1]; C = [ 1 0 0 0;0 0 1 0]; D = [0 0;0 0];
sys = ss(A,B,C,D) % forma pe stare
```

step(sys)

```
sys =

  A =
          x1    x2    x3    x4
   x1      0     0     1     0
   x2      0     0     0     1
   x3    -30    10  -1.6   1.6
   x4      5    -5   0.8  -0.8

  B =
       u1  u2
   x1   0   0
   x2   0   0
   x3   2   0
   x4   0   1

  C =
       x1  x2  x3  x4
   y1   1   0   0   0
   y2   0   0   1   0

  D =
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time state-space model.
```

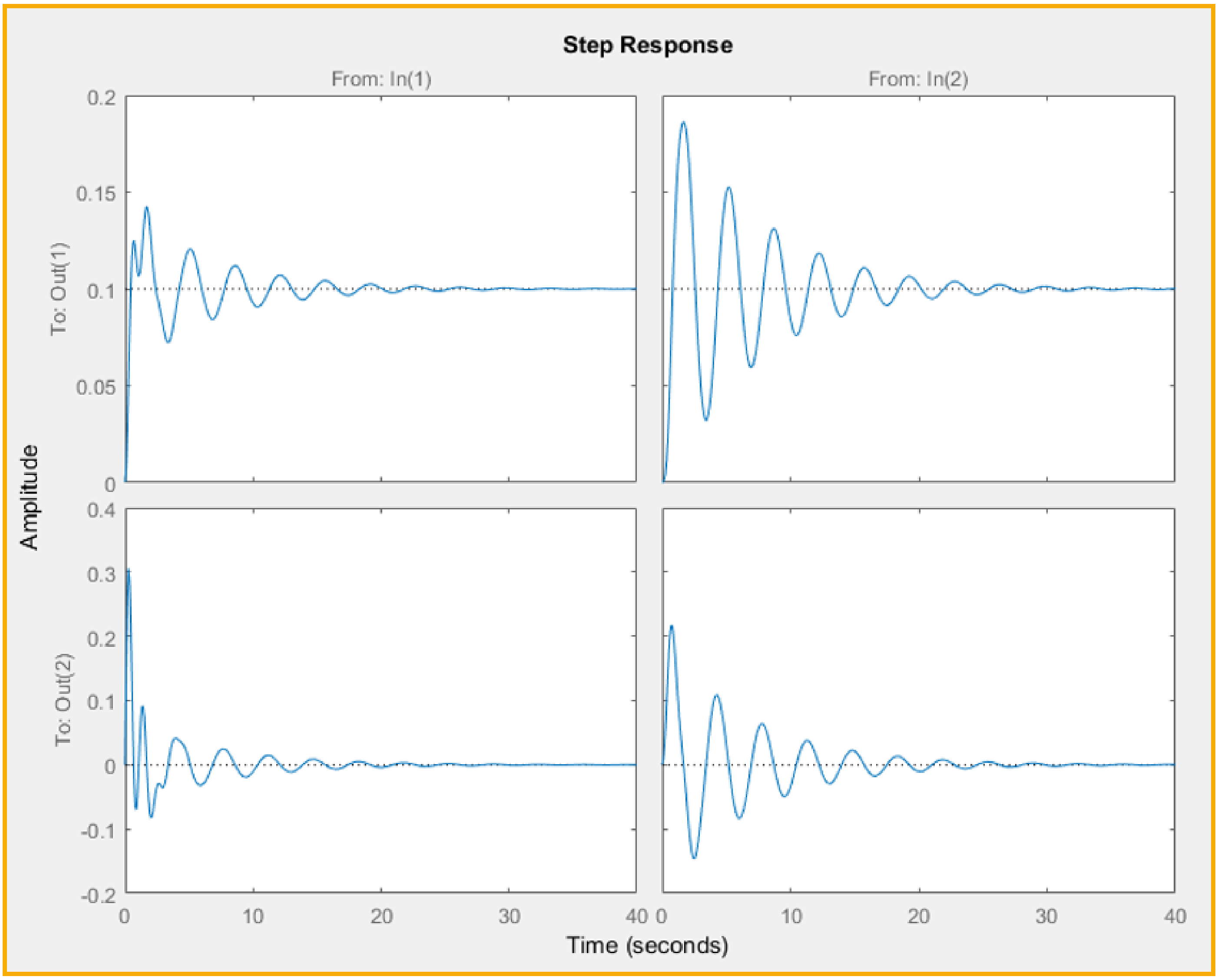
A se vedea şi funcţia ssdata.

ssdata

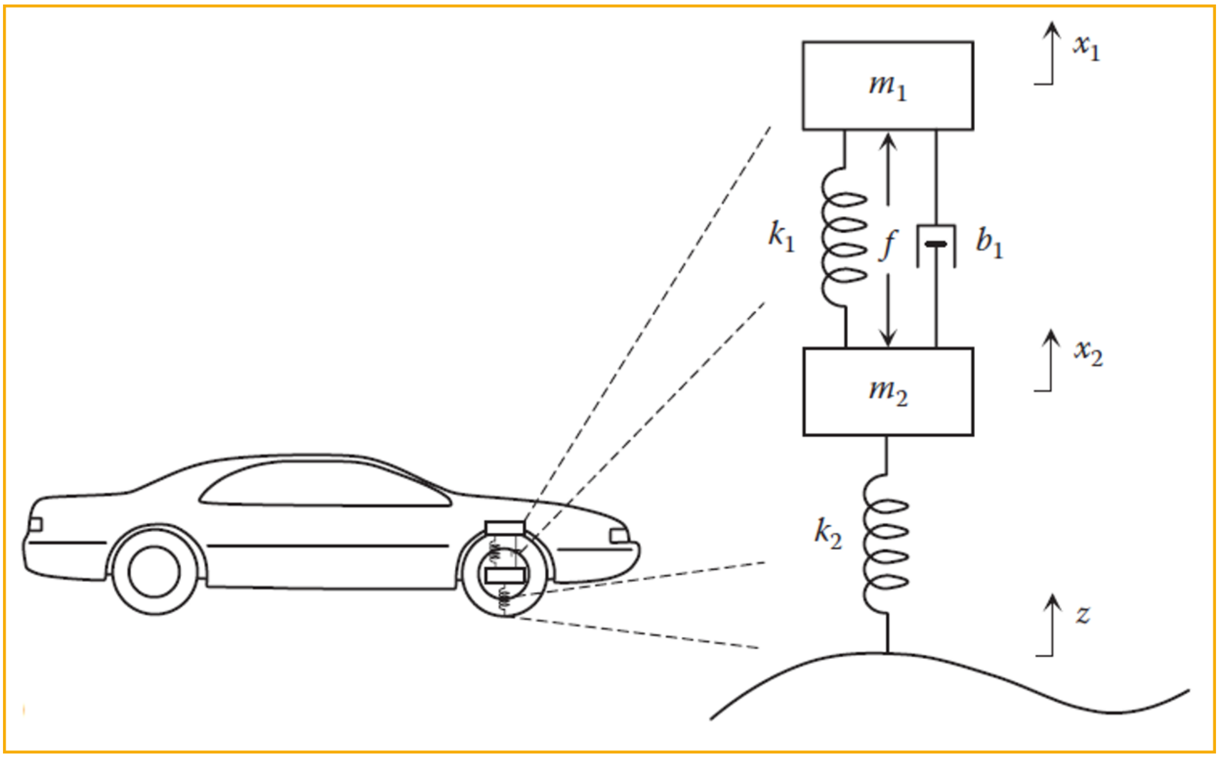
Access state-space model data

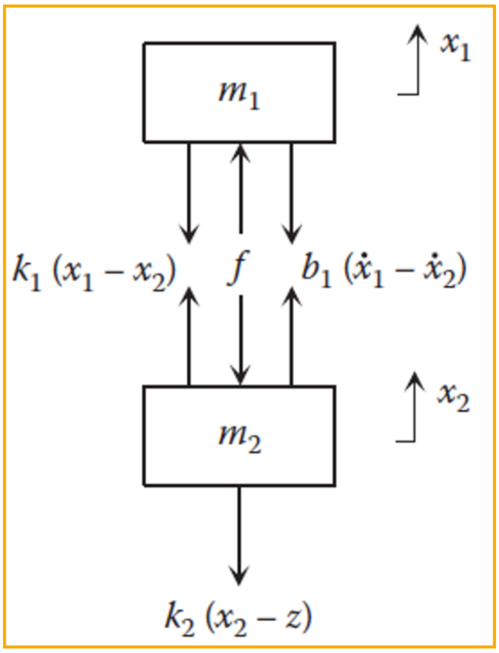
Syntax

```
[a,b,c,d] = ssdata(sys)
[a,b,c,d,Ts] = ssdata(sys)
```


Step Response
From: In(1)
From: In(2)
Amplitude
To: Out(1)
To: Out(2)
0.2
0.15
0.1
0.05
0
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
10
20
30
40
Time (seconds)

3.3 Deduceți modelul matematic pentru suspensia din figură, inclusiv modelul pe stare. Nu se ia în considerare efectul greutății. Simulați modelul pe stare în MATLAB.





$$f - k_1(x_1 - x_2) - b_1(\dot{x}_1 - \dot{x}_2) = m_1\ddot{x}_1$$

$$-f + k_1(x_1 - x_2) + b_1(\dot{x}_1 - \dot{x}_2) - k_2(x_2 - z) = m_2\ddot{x}_2$$

Forma intrare-ieşire standard:

$$m_1\ddot{x}_1 + b_1\dot{x}_1 - b_1\dot{x}_2 + k_1x_1 - k_1x_2 = f$$

$$m_2\ddot{x}_2 - b_1\dot{x}_1 + b_1\dot{x}_2 - k_1x_1 + (k_1+k_2)x_2 = -f + k_2z$$

Obtinem forma pe stare:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} b_1 & -b_1 \\ -b_1 & b_1 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & k_2 \end{bmatrix}\begin{Bmatrix} f \\ z \end{Bmatrix}$$

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}, \quad \mathbf{u} = \begin{Bmatrix} f \\ z \end{Bmatrix}, \quad \mathbf{y} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = \ddot{x}_1 = -\frac{k_1}{m_1}x_1 + \frac{k_1}{m_1}x_2 - \frac{b_1}{m_1}\dot{x}_1 + \frac{b_1}{m_1}\dot{x}_2 + \frac{f}{m_1}$$

$$= -\frac{k_1}{m_1}x_1 + \frac{k_1}{m_1}x_2 - \frac{b_1}{m_1}x_3 + \frac{b_1}{m_1}x_4 + \frac{1}{m_1}u_1$$

$$\dot{x}_4 = \ddot{x}_2 = \frac{k_1}{m_2}x_1 - \frac{k_1+k_2}{m_2}x_2 + \frac{b_1}{m_2}\dot{x}_1 - \frac{b_1}{m_2}\dot{x}_2 - \frac{f}{m_2} + \frac{k_2}{m_2}z$$

$$= \frac{k_1}{m_2}x_1 - \frac{k_1+k_2}{m_2}x_2 + \frac{b_1}{m_2}x_3 - \frac{b_1}{m_2}x_4 - \frac{1}{m_2}u_1 + \frac{k_2}{m_2}u_2$$

$$\mathbf{y} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$$
\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \overset{\textbf{A}}{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1}{m_1} & \dfrac{k_1}{m_1} & -\dfrac{b_1}{m_1} & \dfrac{b_1}{m_1} \\ \dfrac{k_1}{m_2} & -\dfrac{k_1+k_2}{m_2} & \dfrac{b_1}{m_2} & -\dfrac{b_1}{m_2} \end{bmatrix}} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \overset{\textbf{B}}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{m_1} & 0 \\ -\dfrac{1}{m_2} & \dfrac{k_2}{m_2} \end{bmatrix}} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}
$$

$$
\mathbf{y} = \overset{\textbf{C}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \overset{\textbf{D}}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}
$$

% programul 4.3

```
m1 = 290;
m2 = 59;
b1 = 1000;
k1 = 16182;
k2 = 19000;
A = [0 0 1 0;
0 0 0 1;
-k1/m1 k1/m1 -b1/m1 b1/m1;
k1/m2 -(k1+k2)/m2 b1/m2 -b1/m2];
B = [0 0; 0 0; 1/m1 0; -1/m2 k2/m2];
C = [1 0 0 0; 0 1 0 0];
D = zeros(2,2);
sys_ss = ss(A,B,C,D);
sys_tf = tf(sys_ss) % atentie
step(sys_ss)
```

```
sys_ss =

  A =
            x1       x2       x3       x4
   x1        0        0        1        0
   x2        0        0        0        1
   x3    -55.8     55.8   -3.448    3.448
   x4    274.3   -596.3    16.95   -16.95

  B =
              u1         u2
   x1          0          0
   x2          0          0
   x3   0.003448          0
   x4   -0.01695        322

  C =
       x1  x2  x3  x4
   y1   1   0   0   0
   y2   0   1   0   0

  D =
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time state-space model.
```

```
sys_tf =

  From input 1 to output...
             0.003448 s^2 + 1.087e-17 s + 1.11
   1:  ----------------------------------------------
       s^4 + 20.4 s^3 + 652.1 s^2 + 1110 s + 1.797e04

           -0.01695 s^2 + 1.087e-17 s - 1.703e-16
   2:  ----------------------------------------------
       s^4 + 20.4 s^3 + 652.1 s^2 + 1110 s + 1.797e04

  From input 2 to output...
                     1110 s + 1.797e04
   1:  ----------------------------------------------
       s^4 + 20.4 s^3 + 652.1 s^2 + 1110 s + 1.797e04

                322 s^2 + 1110 s + 1.797e04
   2:  ----------------------------------------------
       s^4 + 20.4 s^3 + 652.1 s^2 + 1110 s + 1.797e04

Continuous-time transfer function.
Model Properties
```

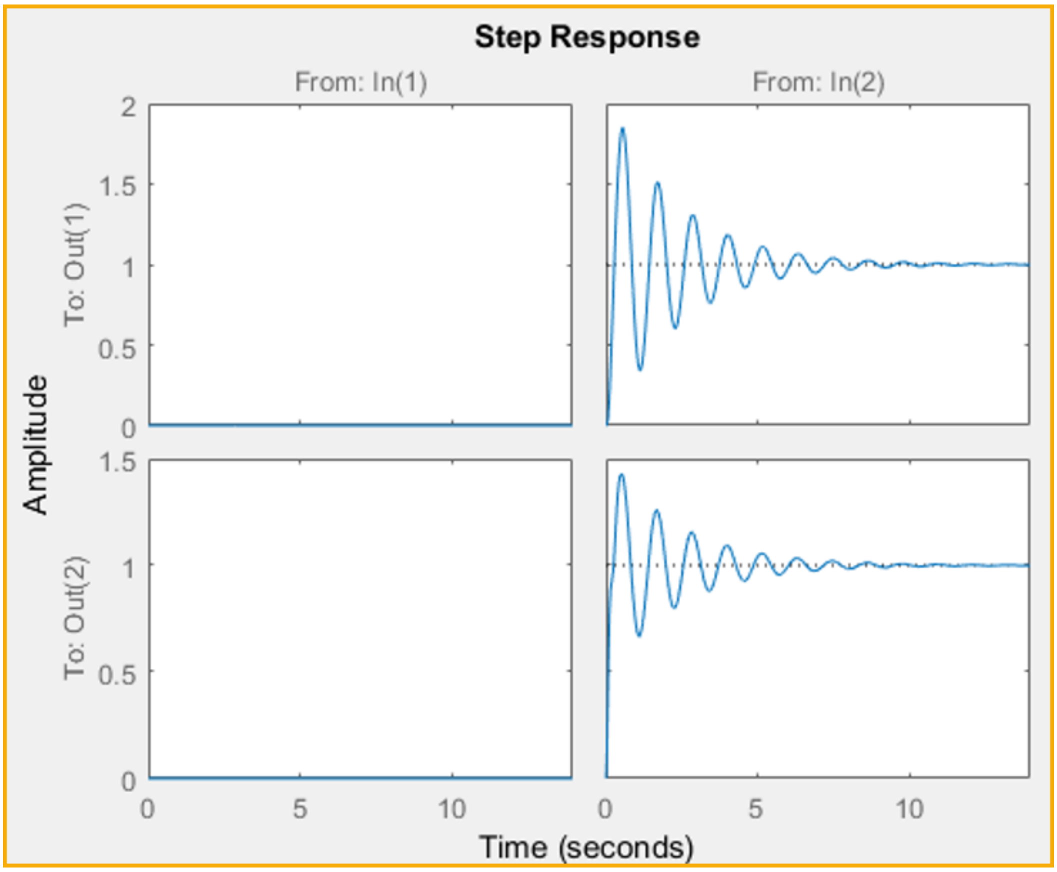
$$\frac{X_1(s)}{F(s)} = \frac{0.003448s^2 + 1.11}{s^4 + 20.4s^3 + 652.1s^2 + 1110s + 17{,}970}$$

$$\frac{X_2(s)}{F(s)} = \frac{-0.01695s^2}{s^4 + 20.4s^3 + 652.1s^2 + 1110s + 17{,}970}$$

$$\frac{X_1(s)}{Z(s)} = \frac{1110s + 17{,}970}{s^4 + 20.4s^3 + 652.1s^2 + 1110s + 17{,}970}$$

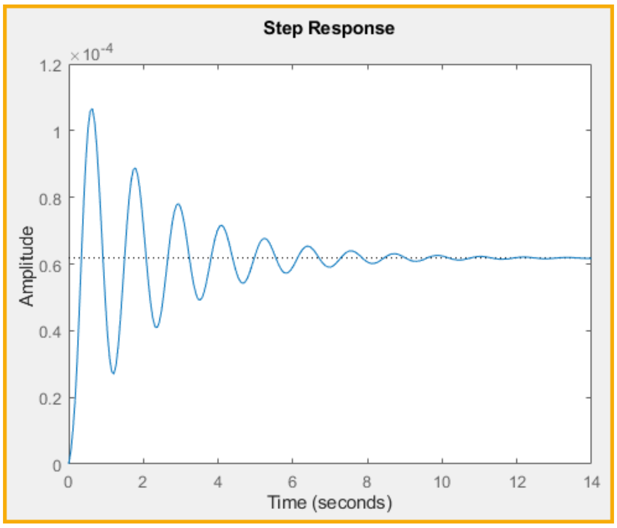
$$\frac{X_2(s)}{Z(s)} = \frac{322s^2 + 1110s + 17{,}970}{s^4 + 20.4s^3 + 652.1s^2 + 1110s + 17{,}970}$$

$$\mathbf{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{X_1(s)}{F(s)} & \dfrac{X_1(s)}{Z(s)} \\ \dfrac{X_2(s)}{F(s)} & \dfrac{X_2(s)}{Z(s)} \end{bmatrix}$$

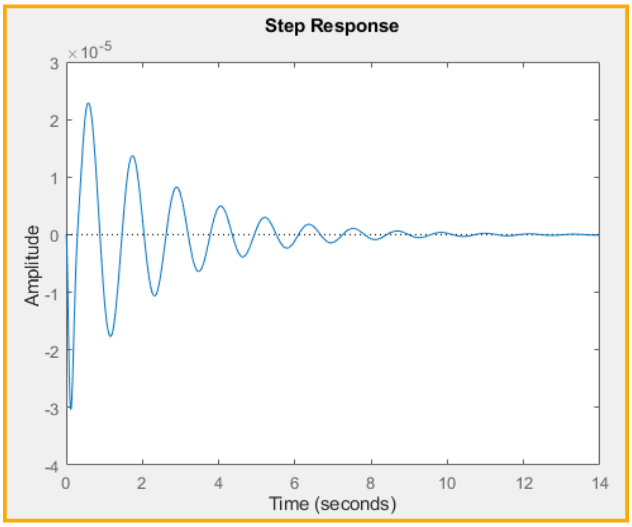


Pentru a vedea in detaliu cele doua raspunsuri de pe coloana din stanga (de la intrarea 1 la iesirile 1 si respectiv, 2), folosim sys_tf generat anterior.

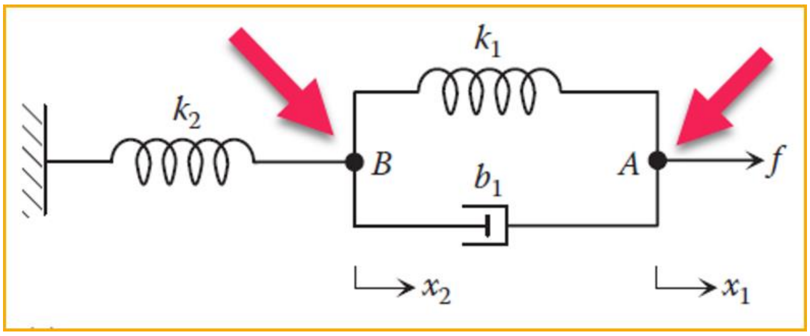
>> step(sys_tf(1,1))

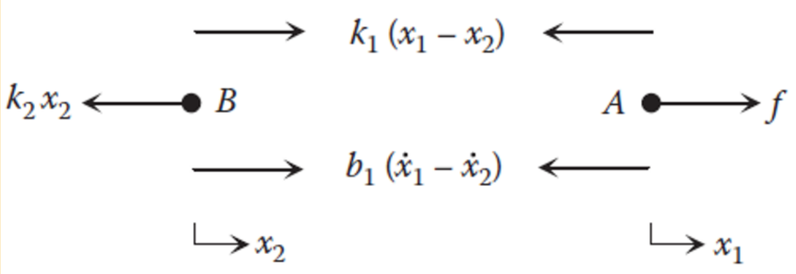


>> step(sys_tf(2,1))



3.4 Deduceţi modelul matematic pentru sistemul mecanic de mai jos.





$$\text{A:}\ -k_1(x_1 - x_2) - b_1(\dot{x}_1 - \dot{x}_2) + f = 0$$

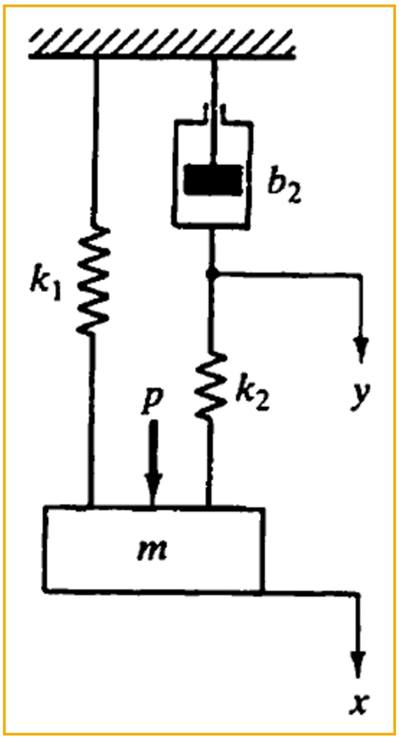
$$\text{B:}\ k_1(x_1 - x_2) - k_2 x_2 + b_1(\dot{x}_1 - \dot{x}_2) = 0$$

$$b_1\dot{x}_1 - b_1\dot{x}_2 + k_1 x_1 - k_1 x_2 = f$$

$$-b_1\dot{x}_1 + b_1\dot{x}_2 - k_1 x_1 + (k_1 + k_2) x_2 = 0$$

$$\begin{bmatrix} b_1 & -b_1 \\ -b_1 & b_1 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix}$$

3.5 Se dă sistemul mecanic de mai jos cu intrarea p(t)=10N şi ieşirea x(t). Obtineţi forma <u>analitică</u> pentru x(t), dacă: m=0.1 kg, b_2=0.4 N*s/m, k_1=6 N/m, k_2=4 N/m. Nu se ia în considerare efectul greutății.



$$m\ddot{x} + k_1 x + k_2(x - y) = p$$

$$k_2(x - y) = b_2\dot{y}$$

$\mathcal{L}$

$$(ms^2 + k_1 + k_2)X(s) = k_2Y(s) + P(s)$$

$$k_2X(s) = (k_2 + b_2s)Y(s)$$

$$(ms^2 + k_1 + k_2)X(s) = \frac{k_2^{\,2}}{k_2 + b_2s}X(s) + P(s)$$

$$[(ms^2 + k_1 + k_2)(k_2 + b_2 s) - k_2^2]X(s) = (k_2 + b_2 s)P(s)$$

$$\frac{X(s)}{P(s)} = \frac{b_2 s + k_2}{mb_2 s^3 + mk_2 s^2 + (k_1 + k_2)b_2 s + k_1 k_2}$$

$$\frac{X(s)}{P(s)} = \frac{0.4s + 4}{0.04s^3 + 0.4s^2 + 4s + 24} = \frac{10s + 100}{s^3 + 10s^2 + 100s + 600}$$

$$P(s) = \frac{10}{s}$$

$$X(s) = \frac{10s + 100}{s^3 + 10s^2 + 100s + 600} \frac{10}{s}$$

```
% Program 3.5
num = [100 1000];
den = [1 10 100 600 0];
[r,p,k] = residue(num, den)
```

Folosim funcția MATLAB **residue** pentru descompunerea în fracții simple.

residue

Partial fraction expansion (partial fraction decomposition)

Syntax

```
[r,p,k] = residue(b,a)
[b,a] = residue(r,p,k)
```

$$\frac{b(s)}{a(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} = \frac{r_n}{s - p_n} + \ldots + \frac{r_2}{s - p_2} + \frac{r_1}{s - p_1} + k(s).$$

```
r =

  -0.6845 + 0.2233i
  -0.6845 - 0.2233i
  -0.2977 + 0.0000i
   1.6667 + 0.0000i


p =

  -1.2898 + 8.8991i
  -1.2898 - 8.8991i
  -7.4204 + 0.0000i
   0.0000 + 0.0000i


k =

     []
```

$$X(s) = \frac{-0.6845 + j0.2233}{s + 1.2898 - j8.8991} + \frac{-0.6845 - j0.2233}{s + 1.2898 + j8.8991} + \frac{-0.2977}{s + 7.4204} + \frac{1.6667}{s}$$
$$= \frac{-1.3690(s + 1.2898) - 3.9743}{(s + 1.2898)^2 + 8.8991^2} - \frac{0.2977}{s + 7.4204} + \frac{1.6667}{s}$$

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)] = F(s)$		$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
1	$\frac{1}{s}$	(1)	$\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$	(19)
$e^{at} f(t)$	$F(s-a)$	(2)	te^{at}	$\frac{1}{(s-a)^2}$	(20)
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$	(3)	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	(21)
$f(t-a)\mathcal{U}(t-a)$	$e^{-as} F(s)$	(4)	$e^{at} \sin kt$	$\frac{k}{(s-a)^2 + k^2}$	(22)
$\delta(t)$	1	(5)	$e^{at} \cos kt$	$\frac{s-a}{(s-a)^2 + k^2}$	(23)
$\delta(t-t_0)$	e^{-st_0}	(6)	$e^{at} \sinh kt$	$\frac{k}{(s-a)^2 - k^2}$	(24)
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$	(7)	$e^{at} \cosh kt$	$\frac{s-a}{(s-a)^2 - k^2}$	(25)
$f'(t)$	$sF(s) - f(0)$	(8)	$t \sin kt$	$\frac{2ks}{(s^2+k^2)^2}$	(26)
$f^n(t)$	$s^n F(s) - s^{(n-1)} f(0) - \cdots - f^{(n-1)}(0)$	(9)	$t \cos kt$	$\frac{s^2 - k^2}{(s^2+k^2)^2}$	(27)
$\int_0^t f(x) g(t-x) dx$	$F(s)G(s)$	(10)	$t \sinh kt$	$\frac{2ks}{(s^2-k^2)^2}$	(28)
$t^n \ (n = 0, 1, 2, \dots)$	$\frac{n!}{s^{n+1}}$	(11)			

$$x(t) = -1.3690 e^{-1.2898t} \cos(8.8991t) - 0.4466 e^{-1.2898t} \sin(8.8991t) - 0.2977 e^{-7.4204t} + 1.6667$$

Dar dacă ieşirea este y(t)? În acest caz, obțineți forma analitică a lui *y(t)*.

$$\frac{X(s)}{P(s)} = \frac{b_2 s + k_2}{m b_2 s^3 + m k_2 s^2 + (k_1 + k_2) b_2 s + k_1 k_2}$$

$$k_2 X(s) = (k_2 + b_2 s) Y(s)$$

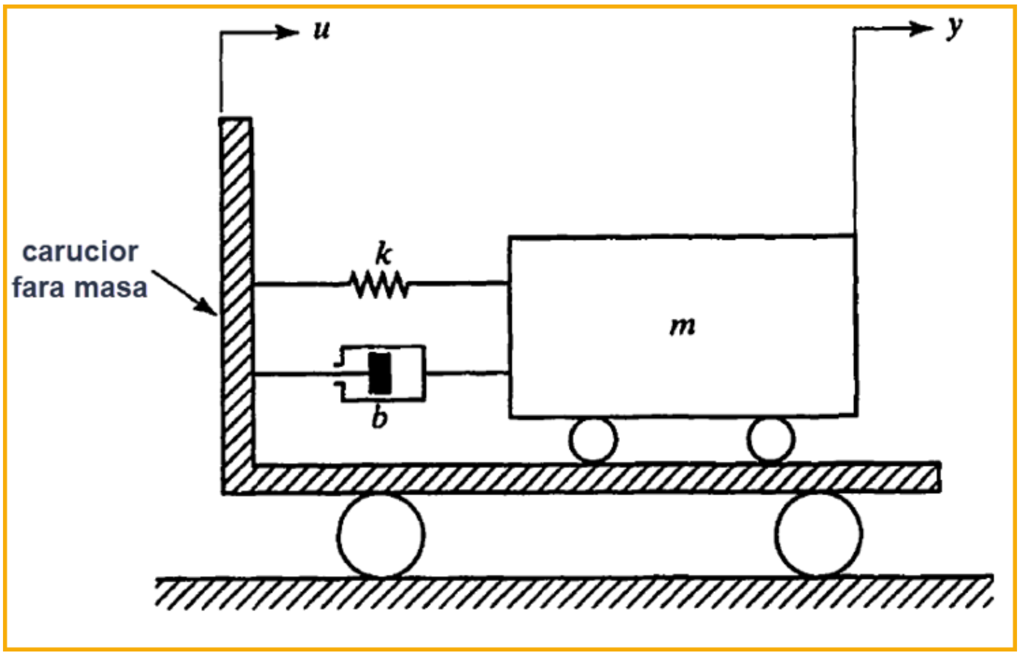
$$\frac{Y(s)}{X(s)} = \frac{k_2}{b_2 s + k_2}$$

$$\frac{Y(s)}{P(s)} = \frac{Y(s)}{X(s)} \frac{X(s)}{P(s)} = \frac{k_2}{m b_2 s^3 + m k_2 s^2 + (k_1 + k_2) b_2 s + k_1 k_2}$$

$$\frac{X(s)}{P(s)} = \frac{0.4s + 4}{0.04s^3 + 0.4s^2 + 4s + 24} = \frac{10s + 100}{s^3 + 10s^2 + 100s + 600}$$

$$\frac{Y(s)}{P(s)} = \frac{4}{0.04s^3 + 0.4s^2 + 4s + 24} = \frac{100}{s^3 + 10s^2 + 100s + 600}$$

PRO 3.6 Deduceţi modelul matematic al sistemului mecanic de mai jos, precum şi răspunsul y(t), dacă intrarea u(t) este o treaptă unitară, iar m=10kg, b=20Ns/m, k=100 N/m. Trasaţi grafic răspunsul sistemului la treaptă unitară.



PRO 3.7 În cazul problemei 3.5, obțineți graficele lui x(t) și y(t) în MATLAB și în Simulink. Reprezentați x(t) și y(t) pe <u>același</u> grafic.