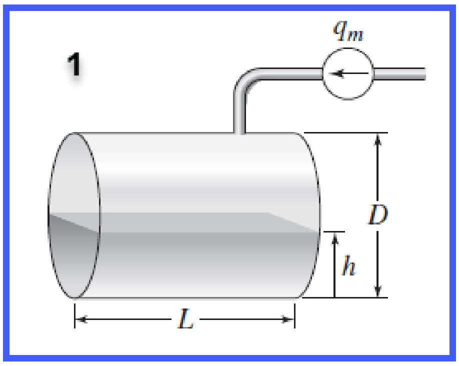
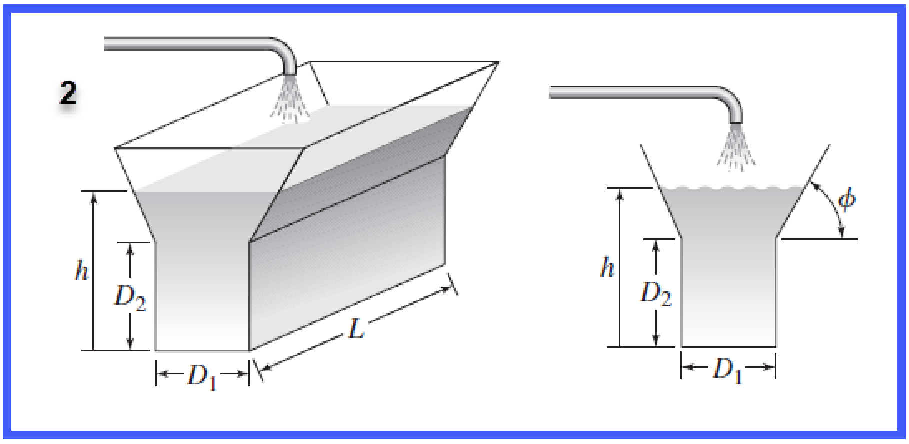
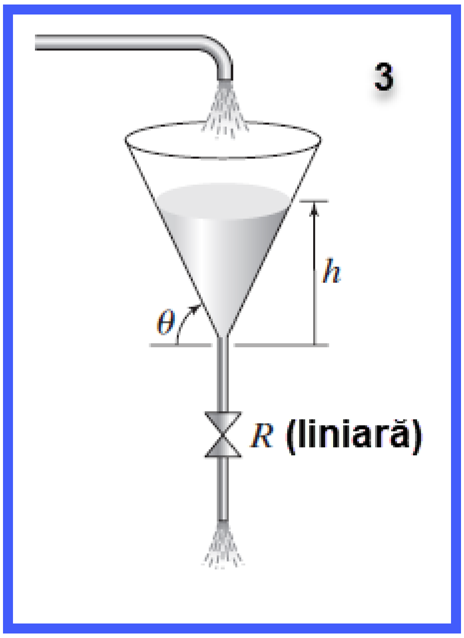
MODELARE ŞI SIMULARE- 2024

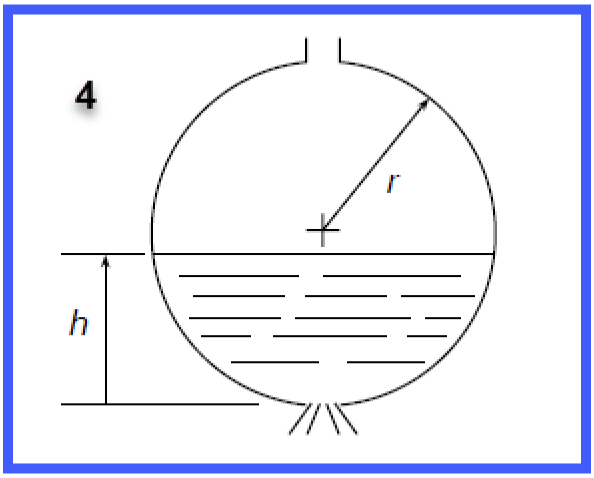
LABORATOR NR. 5 – SISTEME FLUIDICE (1), SIMULINK (4)

5.1 Deduceţi expresia capacitaţii fluidice pentru rezervoarele de mai jos, precum şi modelele dinamice (variabila este nivelul h), cunoscându-se la primele trei debitul de intrare $q_m(t)$. Realizati modelul Simulink pentru simularea unuia dintre cele 4 modele (la alegere).

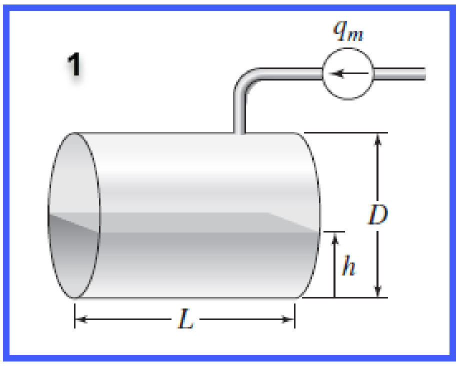








Pentru primul rezervor:



$$C = \frac{A(h)}{g}$$

$$A(h) = 2L\sqrt{Dh - h^2}$$

$$C = \frac{2L}{g}\sqrt{Dh - h^2}$$

$$\rho A(h)\frac{dh}{dt} = q_{mi} - q_{mo}$$

$$A(h) = 2L\sqrt{Dh - h^2}$$

$$\rho 2L\sqrt{Dh - h^2}\frac{dh}{dt} = q_m$$

Pentru al doilea:

$$C = \frac{A(h)}{g}$$

$$A = D_1 L \qquad h < D_2$$

$$A = \frac{2L(h - D_2)}{\tan\phi} + D_1 L \qquad h \geq D_2$$

$$C = \begin{cases} \frac{D_1 L}{g} & h < D_2 \\ \frac{2L(h-D_2)}{g\tan\phi} + \frac{D_1 L}{g} & h \geq D_2 \end{cases}$$

$$\rho A(h)\frac{dh}{dt} = q_{mi} - q_{mo}$$

$$A(h) = \begin{cases} D_1 L & h < D_2 \\ \frac{2L(h-D_2)}{\tan\phi} + D_1 L & h \geq D_2 \end{cases}$$

$$\rho D_1 L\frac{dh}{dt} = q_{mi} \qquad h < D_2$$

$$\rho\left[\frac{2L(h - D_2)}{\tan\phi} + D_1 L\right]\frac{dh}{dt} = q_{mi} \qquad h \geq D_2$$

Pentru al treilea:

$$C = \frac{A(h)}{g}$$

$$A(h) = \pi \left(\frac{h}{\tan\,\theta}\right)^2$$

$$C = \frac{\pi}{g}\left(\frac{h}{\tan\,\theta}\right)^2$$

$$\rho A(h)\frac{dh}{dt} = q_{mi} - q_{mo}$$

$$A(h) = \pi \left(\frac{h}{\tan\,\theta}\right)^2$$

$$q_{mo} = \frac{\rho g h}{R}$$

$$\pi\rho\left(\frac{h}{\tan\,\theta}\right)^2\frac{dh}{dt} = q_{mi} - \frac{\rho g h}{R}$$

Pentru al patrulea:

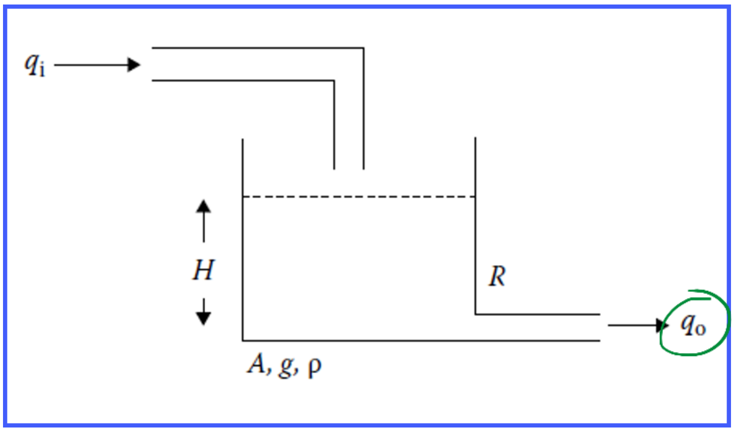
$$V(h) = \pi r h^2 - \pi\frac{h^3}{3} \qquad 1$$

$$q = C_d A\sqrt{2gh} \qquad 2$$

$$\frac{dV}{dt} = -q \qquad 3$$

$$4 \qquad \frac{dV}{dt} = 2\pi r h\frac{dh}{dt} - \pi h^2\frac{dh}{dt} = \pi h(2r - h)\frac{dh}{dt}$$

5.2 Deduceţi modelul matematic (funcţia de transfer) pentru sistemul fluidic de mai jos (debitele sunt volumetrice). Realizaţi modelul Simulink corespunzător. Realizati si o varianta a diagramei Simulink, in care graficul lui *h(t)* se va realiza in MATLAB.



$$q_i = A\frac{dh}{dt} + q_o$$

1

$$q_i - q_o = \frac{A}{\rho g}\frac{dp}{dt}$$

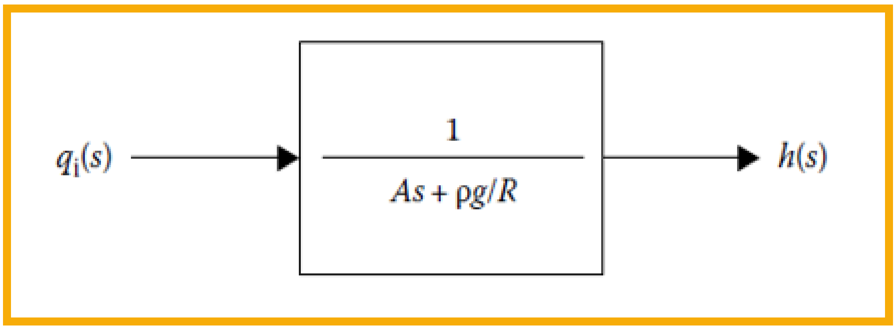
$$q_i = \frac{A}{\rho g}\frac{dp}{dt} + q_o$$

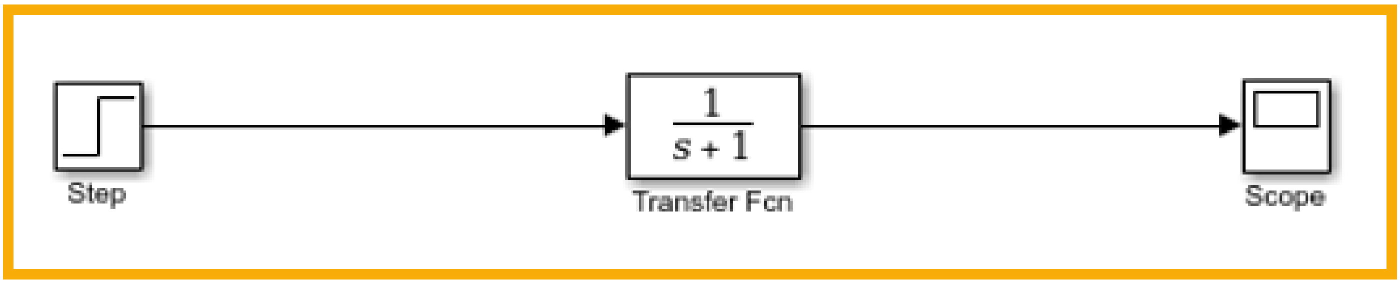
2 $$q_o = \frac{p_1 - p_2}{R} = \frac{h\rho g}{R}$$

3 $$q_i = A\frac{dh}{dt} + \frac{\rho g}{R}h$$

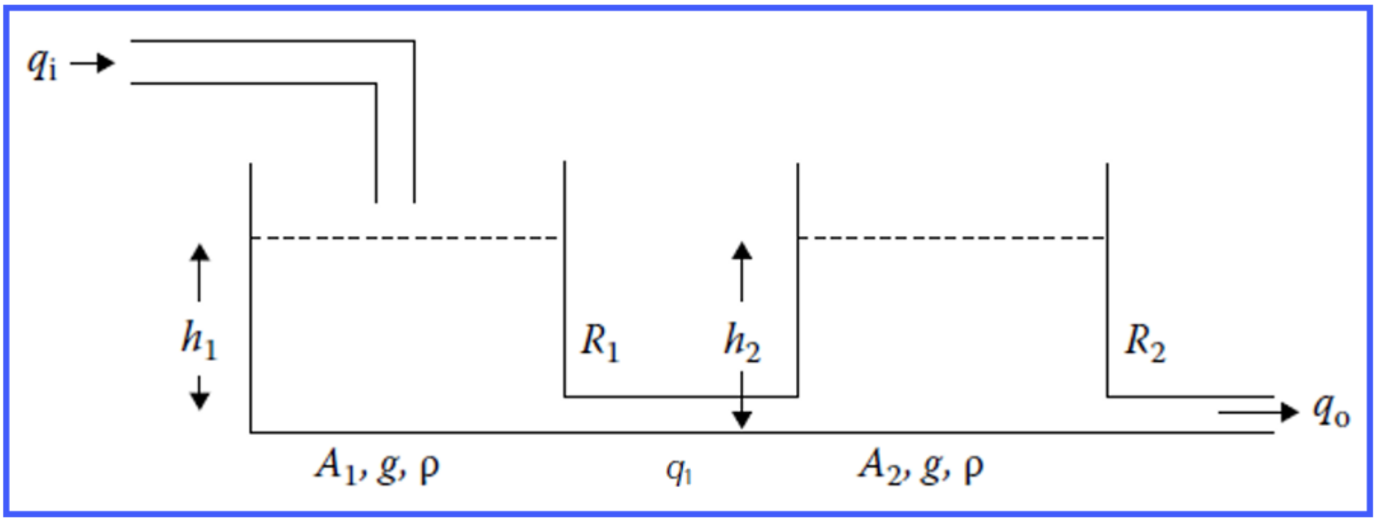
$$q_i(s) = Ash(s) + \frac{\rho g}{R}h(s)$$

$$\frac{h(s)}{q_i(s)} = \frac{1}{As + \frac{\rho g}{R}}$$





5.3 Modelaţi matematic sistemul fluidic cu două rezervoare prezentat în figura de mai jos. Pornind de la diagrama bloc, realizati diagrama Simulink corespunzatoare. Realizati si diagrama Simulink pe baza modelului pe stare.



REZERVOR 1

$$q_i - q_1 = \frac{A_1}{\rho g}\frac{dp}{dt}$$

$$q_i = \frac{A_1}{\rho g}\frac{dp}{dt} + q_1$$

$$P = h\rho g,$$

$$q_i = A_1\frac{dh_1}{dt} + q_1$$

$$p_1 - p_2 = R_1 q_1$$

$$q_1 = \frac{p_1 - p_2}{R_1} = \frac{h_1\rho g - h_2\rho g}{R_1}$$

$$q_i = A_1\frac{dh_1}{dt} + \frac{h_1\rho g - h_2\rho g}{R_1}$$

REZERVOR 2

$$q_1 - q_o = \frac{A_2}{\rho g}\frac{dp}{dt}$$

$$q_1 - q_o = A_2 \frac{dh_2}{dt}$$

$$q_1 = \frac{p_1 - p_2}{R_1} \qquad q_o = \frac{p_2 - p_3}{R_2}$$

$$q_1 - q_o = \frac{p_1 - p_2}{R_1} - \frac{p_2 - p_3}{R_2} = \frac{h_1\rho g - h_2\rho g}{R_1} - \frac{h_2\rho g}{R_2}$$

$$A_2 \frac{dh_2}{dt} - \frac{\rho g h_1}{R_1} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\rho g h_2 = 0$$

$$\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \frac{d}{dt}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{\rho g}{R_1} & \frac{-\rho g}{R_1} \\ \frac{-\rho g}{R_1} & \frac{\rho g}{R_1} + \frac{\rho g}{R_2} \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} q_i \\ 0 \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \frac{q_i}{A_1} \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{\rho g}{A_1 R_1} & \frac{-\rho g}{A_1 R_1} \\ \frac{-\rho g}{A_2 R_1} & \frac{\rho g}{A_2 R_1} + \frac{\rho g}{A_2 R_2} \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

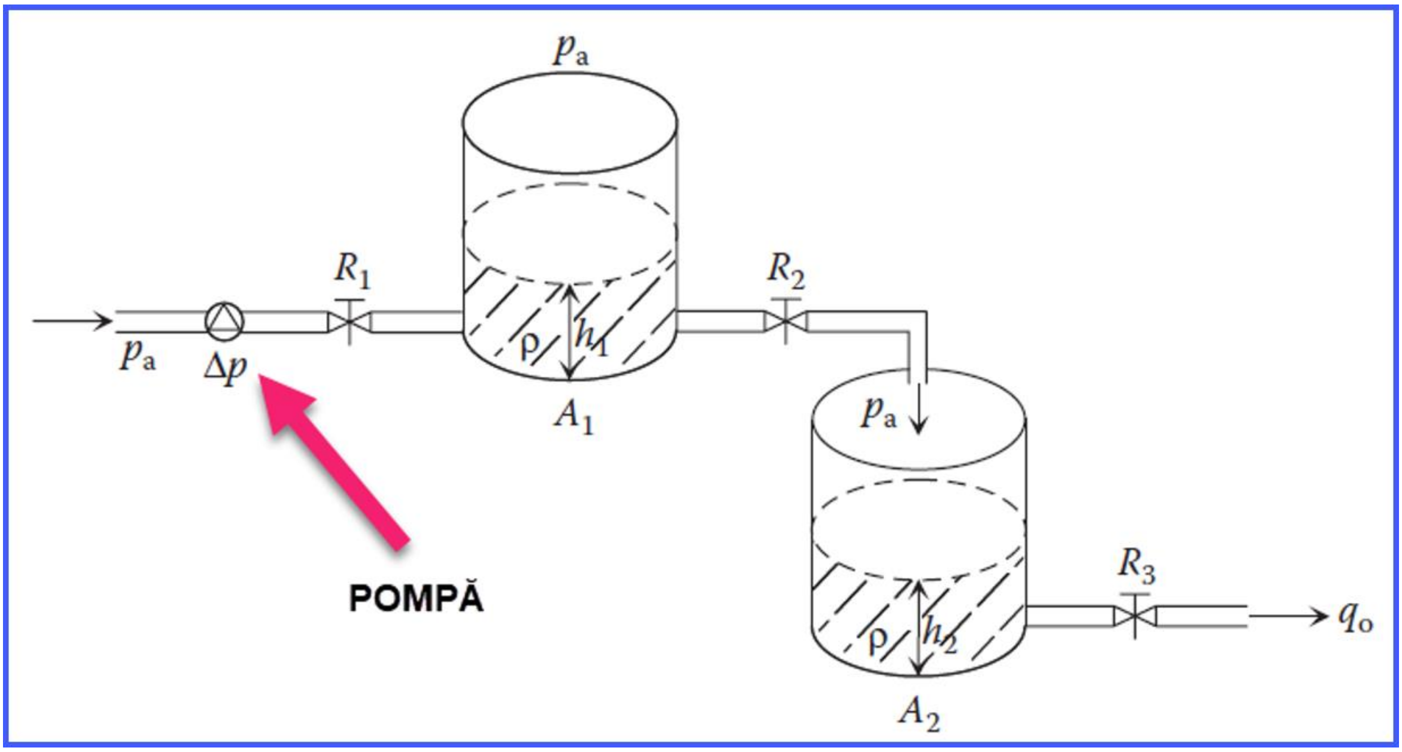
5.4 Se dă sistemul fluidic de mai jos. O pompă este conectată la intrarea rezervorului 1 printr-o valvă cu rezistenţa liniară R_1. Fluidul curge din rezervorul 1 în rezervorul 2 printr-o valvă cu rezistenţa liniară R_2. Densitatea fluidului este constantă. Se cer:

1. Ecuaţiile diferenţiale pentru h_1 şi h_2.

2. Dacă presiunea furnizată de pompă, Δp, este intrarea şi nivelurile h_1, h_2 sunt ieşirile, determinaţi modelul pe stare al sistemului.

3. Modelul Simulink, dacă se cunosc: $\rho = 1000\frac{kg}{m^3}, g = 9.81\frac{m}{s^s}, A_1 = 2m^2, A_2 = 3m^2, R_1 = R_2 = R_3 = 400\ \frac{N*s}{kg*m^2}, h_1(0) = 1\ m, h_2(0) = 0\ m, \Delta p = 130\ kPa.$

4. Se cer graficele lui h_1 şi h_2 realizate in Matlab.



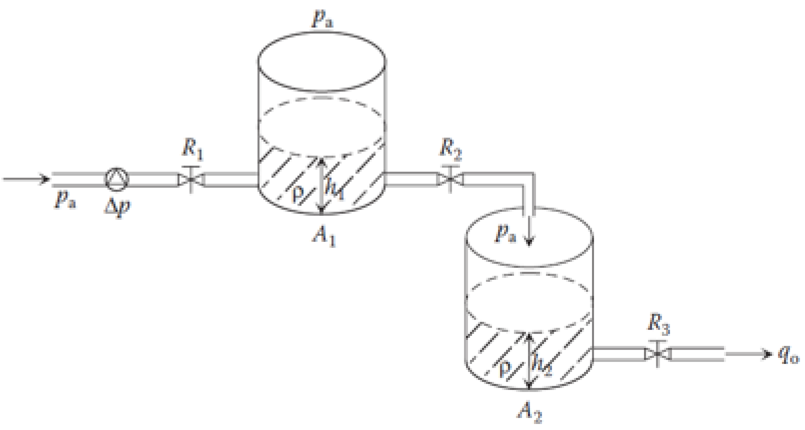
Legea de conservare a masei pentru rezervorul 1:

$$\boxed{\begin{gathered}\frac{\mathrm{d}m}{\mathrm{d}t}=q_{\mathrm{mi}}-q_{\mathrm{mo}} \\ \frac{\mathrm{d}m}{\mathrm{d}t}=\rho A_1\frac{\mathrm{d}h_1}{\mathrm{d}t}\end{gathered}}$$

$$\boxed{\begin{gathered}q_{\mathrm{mi}}=\frac{(p_{\mathrm{a}}+\Delta p)-(p_{\mathrm{a}}+\rho g h_1)}{R_1}=\frac{\Delta p-\rho g h_1}{R_1} \\ q_{\mathrm{mo}}=\frac{(p_{\mathrm{a}}+\rho g h_1)-p_{\mathrm{a}}}{R_2}=\frac{\rho g h_1}{R_2}\end{gathered}}$$

$$\boxed{\begin{gathered}\rho A_1\frac{\mathrm{d}h_1}{\mathrm{d}t}=\frac{\Delta p-\rho g h_1}{R_1}-\frac{\rho g h_1}{R_2} \\ \rho A_1\frac{\mathrm{d}h_1}{\mathrm{d}t}+\rho g h_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{\Delta p}{R_1}\end{gathered}}$$

Pentru rezervorul 2:



$$\frac{dm}{dt} = q_{mi} - q_{mo}$$

$$\frac{dm}{dt} = \rho A_2 \frac{dh_2}{dt}$$

$$q_{mi} = \frac{(p_a + \rho g h_1) - p_a}{R_2} = \frac{\rho g h_1}{R_2}$$

$$q_{mo} = \frac{(p_a + \rho g h_2) - p_a}{R_3} = \frac{\rho g h_2}{R_3}$$

$$\rho A_2 \frac{dh_2}{dt} = \frac{\rho g h_1}{R_2} - \frac{\rho g h_2}{R_3}$$

$$\rho A_2 \frac{dh_2}{dt} - \frac{\rho g h_1}{R_2} + \frac{\rho g h_2}{R_3} = 0$$

$$\begin{bmatrix} \rho A_1 & 0 \\ 0 & \rho A_2 \end{bmatrix} \begin{Bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{Bmatrix} + \begin{bmatrix} \frac{\rho g}{R_1} + \frac{\rho g}{R_2} & 0 \\ -\frac{\rho g}{R_2} & \frac{\rho g}{R_3} \end{bmatrix} \begin{Bmatrix} h_1 \\ h_2 \end{Bmatrix} = \begin{Bmatrix} \frac{\Delta p}{R_1} \\ 0 \end{Bmatrix}$$

b.

$$\boxed{\mathbf{x}=\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} h_1 \\ h_2 \end{Bmatrix}, \quad u=\Delta p, \quad \mathbf{y}=\begin{Bmatrix} h_1 \\ h_2 \end{Bmatrix}}$$

$$\boxed{\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}+\begin{bmatrix} 0 \\ 0 \end{bmatrix}u}$$

$$\boxed{\begin{aligned} \dot{x}_1 &= \frac{\mathrm{d}h_1}{\mathrm{d}t} = -\frac{g}{A_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)h_1+\frac{1}{\rho A_1 R_1}\Delta p \\ \dot{x}_2 &= \frac{\mathrm{d}h_2}{\mathrm{d}t} = \frac{g}{A_2 R_2}h_1-\frac{g}{A_2 R_3}h_2 \end{aligned}}$$

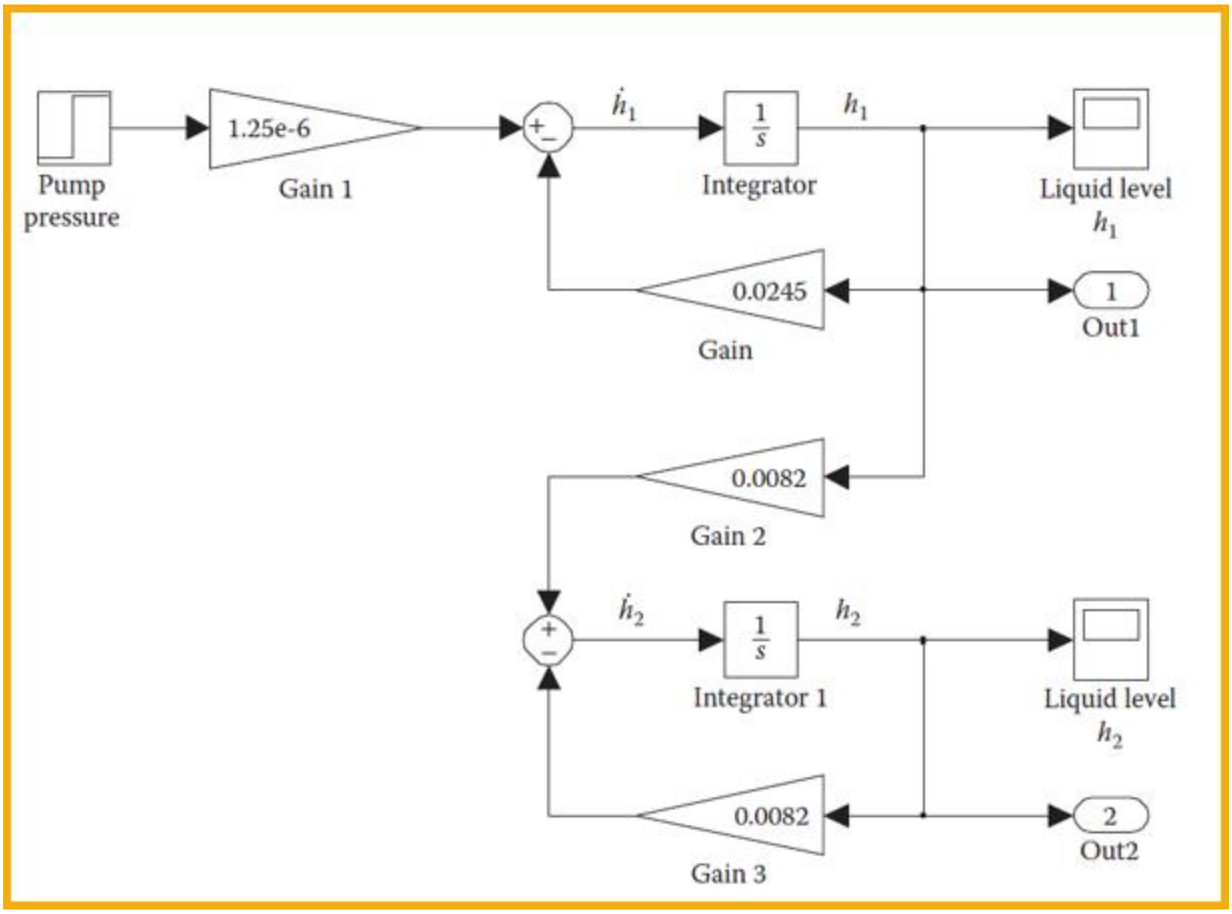
$$\boxed{\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}=\begin{bmatrix} -\dfrac{g}{A_1}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right) & 0 \\ \dfrac{g}{A_2 R_2} & -\dfrac{g}{A_2 R_3} \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}+\begin{bmatrix} \dfrac{1}{\rho A_1 R_1} \\ 0 \end{bmatrix}u}$$

c. **Modelul Simulink**

Varianta 1 (Diagrama bloc)

$$\frac{dh_1}{dt} = -0.0245h_1 + 1.25\times10^{-6}\Delta p$$

$$\frac{dh_2}{dt} = 0.0082h_1 - 0.0082h_2$$

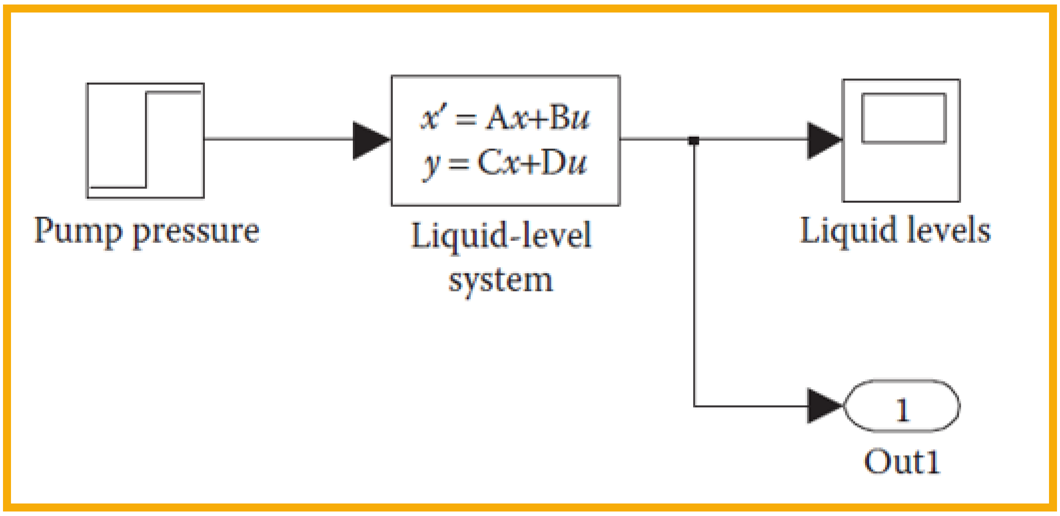


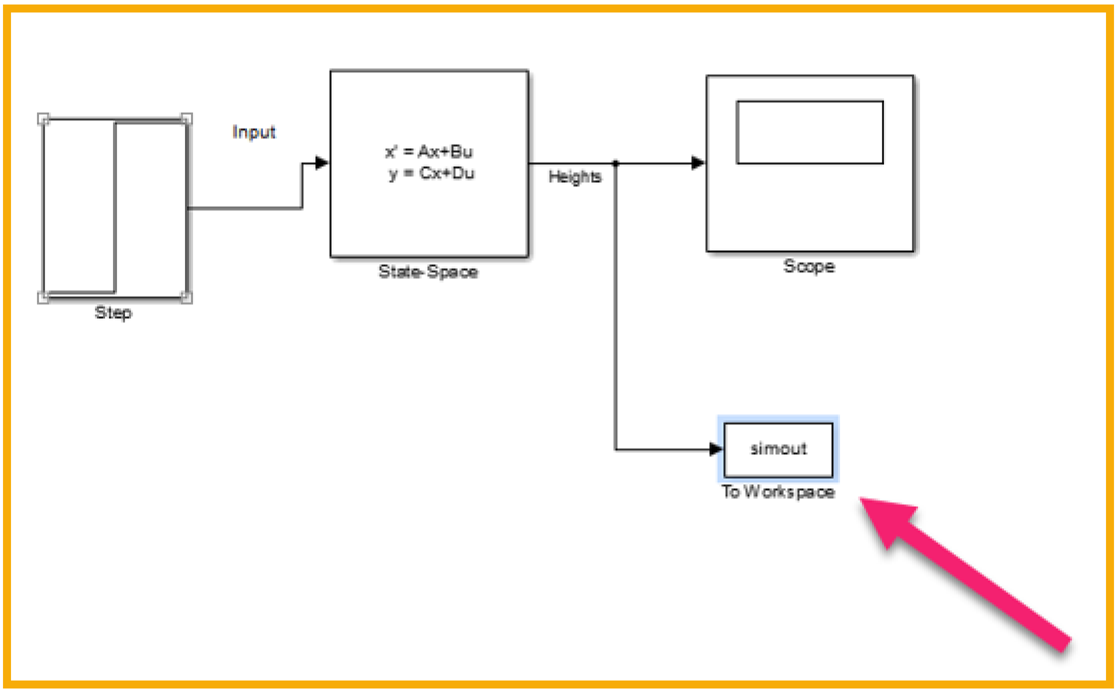

Scope
File
Tools
View
Simulation
Help
7
6
5
4
3
2
1
0
0
100
200
300
400
500
600
700
800
900
1000
Ready
Sample based
T=1000.000

Varianta 2 (modelul pe stare)

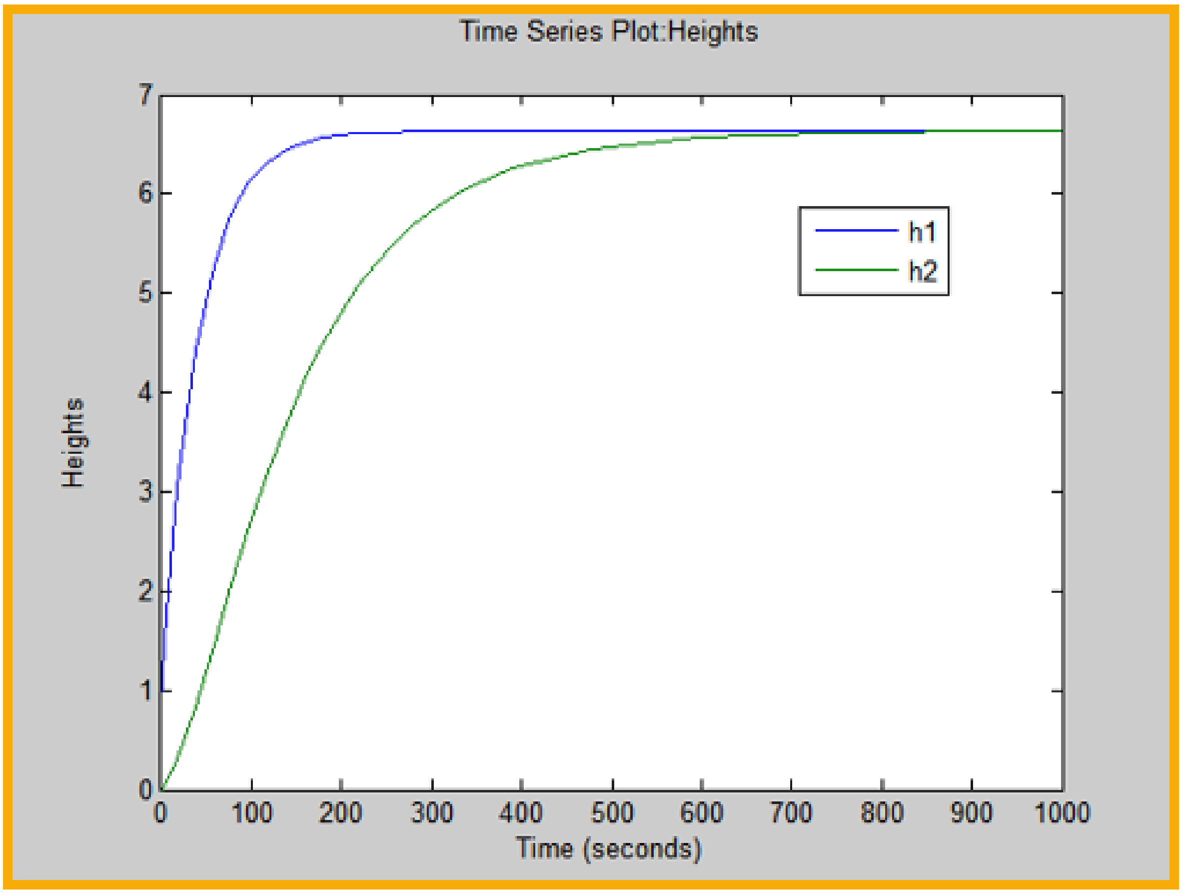
$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} -0.0245 & 0 \\ 0.0082 & -0.0082 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 1.25 \times 10^{-6} \\ 0 \end{bmatrix} u$$

$$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$



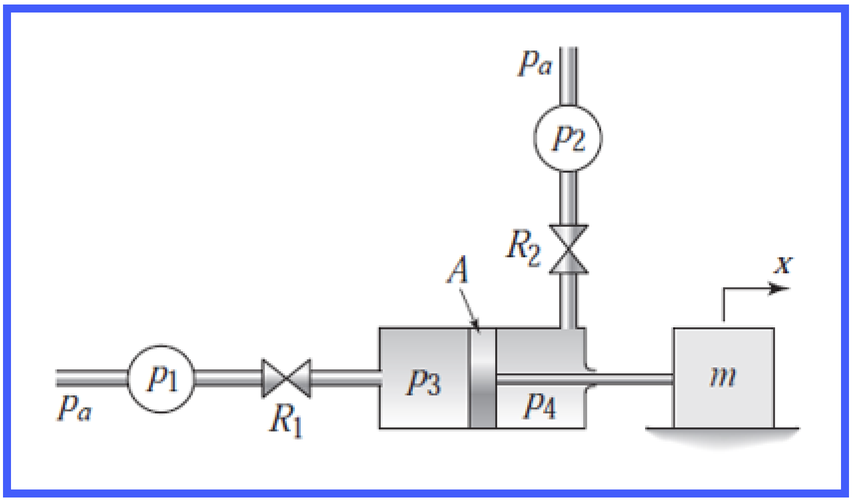


Input
x' = Ax+Bu
y = Cx+Du
Heights
State-Space
Step
Scope
simout
To Workspace



Time Series Plot:Heights
Heights
Time (seconds)
h1
h2
0
1
2
3
4
5
6
7
0
100
200
300
400
500
600
700
800
900
1000

5.5 Dispozitivul de mai jos mişcă masa *m* ca răspuns la sursele de presiune p_1 şi p_2. Fluidul este incompresibil, rezistenţele sunt liniare şi masa pistonului este inclusă în *m*. Deduceţi ecuaţia de mişcare pentru masa *m*.



$$q_{m_1} = \frac{1}{R_1}(p_1 + p_a - p_3)$$

$$q_{m_2} = \frac{1}{R_2}(p_4 - p_2 - p_a)$$

Din conservarea masei:

$$q_{m_1} = q_{m_2} \qquad q_{m_1} = \rho A \dot{x}$$

Deci, din aceste 4 ecuaţii de mai sus, obţinem:

$$p_1 + p_a - p_3 = R_1 \rho A \dot{x}$$

$$p_4 - p_2 - p_a = R_2 \rho A \dot{x}$$

Adunăm ultimele două ecuaţii:

$$\boxed{p_4 - p_3 = p_2 - p_1 + (R_1 + R_2)\rho A\dot{x}}$$

Din legea lui Newton:

$$\boxed{m\ddot{x} = A(p_3 - p_4)}$$

Deci:

$$\boxed{m\ddot{x} + (R_1 + R_2)\rho A^2\dot{x} = A(p_1 - p_2)}$$

PRO 5.6 Implementați diagramele Simulink pentru modelele de la problema 5.1, folosind blocurile Simulink denumite *Matlab function* și *Matlab interpreted function*.