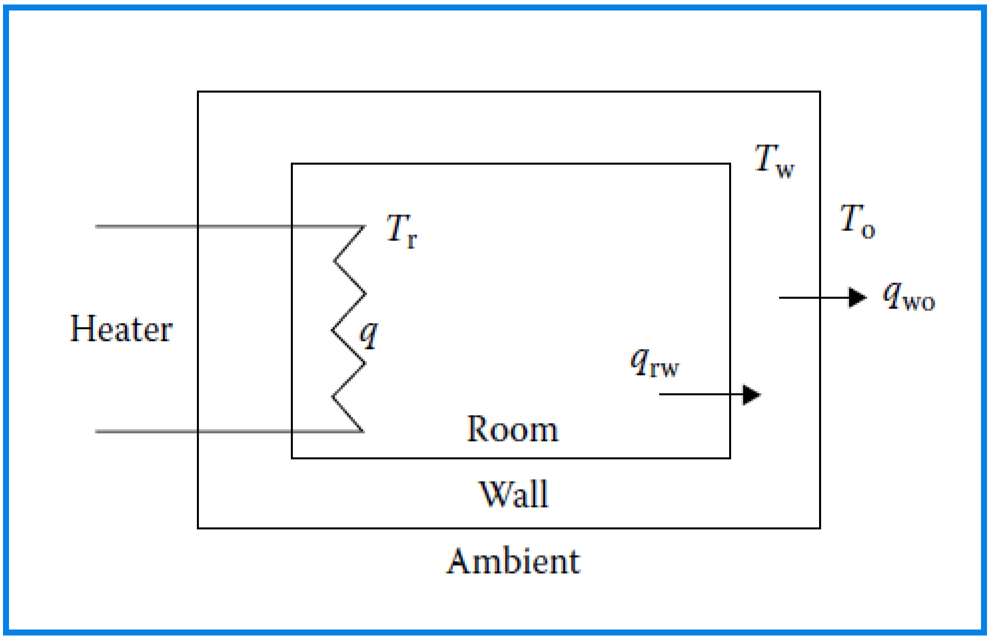
MODELARE ŞI SIMULARE

LABORATOR NR. 9 – SISTEME TERMICE (2), MATLAB (6), SIMULINK (8)

9.1 Figura de mai jos prezintă o cameră încălzită electric. Temperatura din cameră este T_r, T_w este temperatura pereţilor. Dacă temperatura de afară este T_o, deduceţi modelul dinamic pe stare (T_r, T_w fiind variabilele de stare şi cele două ieşiri ale modelului). *q* este rata transferului termic de la încălzitor.



Rata transferului de caldura dinspre cameră spre pereţi:

$$\boxed{q_{rw} = \frac{T_r - T_w}{R_r}}$$

Rata transferului de caldura dinspre pereţi spre exterior:

$$\boxed{q_{wo} = \frac{T_w - T_o}{R_w}}$$

CONSERVAREA ENERGIEI, DEDUCEREA ECUATIILOR DINAMICE

1

$$q - q_{rw} = C_1 \frac{dT_r}{dt}$$

Conservarea energiei (camera)

2

$$q - \frac{T_r - T_w}{R_r} = C_1 \frac{dT_r}{dt}$$

3

$$C_1 \overset{o}{T_r} + \frac{T_r - T_w}{R_r} = q$$

4

$$q_{rw} - q_{wo} = C_2 \frac{dT_w}{dt}$$

Conservarea energiei (perete)

5

$$\frac{T_r - T_w}{R_r} - \frac{T_w - T_o}{R_w} = C_2 \frac{dT_w}{dt}$$

6

$$C_2 \overset{o}{T_w} - \frac{T_r}{R_r} + \left(\frac{1}{R_r} + \frac{1}{R_w}\right) T_w = \frac{T_o}{R_w}$$

$$\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{bmatrix} \dot{T}_r \\ \dot{T}_w \end{bmatrix} + \begin{bmatrix} \frac{1}{R_r} & -\frac{1}{R_r} \\ -\frac{1}{R_r} & \frac{1}{R_r} + \frac{1}{R_w} \end{bmatrix} \begin{bmatrix} T_r \\ T_w \end{bmatrix} = \begin{bmatrix} q \\ \frac{T_o}{R_w} \end{bmatrix}$$

$$\begin{bmatrix} \dot{T}_r \\ \dot{T}_w \end{bmatrix} = \begin{bmatrix} \frac{q}{C_1} \\ \frac{T_o}{C_2 R_w} \end{bmatrix} - \begin{bmatrix} \frac{1}{C_1 R_r} & -\frac{1}{C_1 R_r} \\ -\frac{1}{C_2 R_r} & \frac{1}{C_2 R_r} + \frac{1}{C_2 R_w} \end{bmatrix} \begin{bmatrix} T_r \\ T_w \end{bmatrix}$$

Se cunosc:

C1=0.5;
C2=1.5;

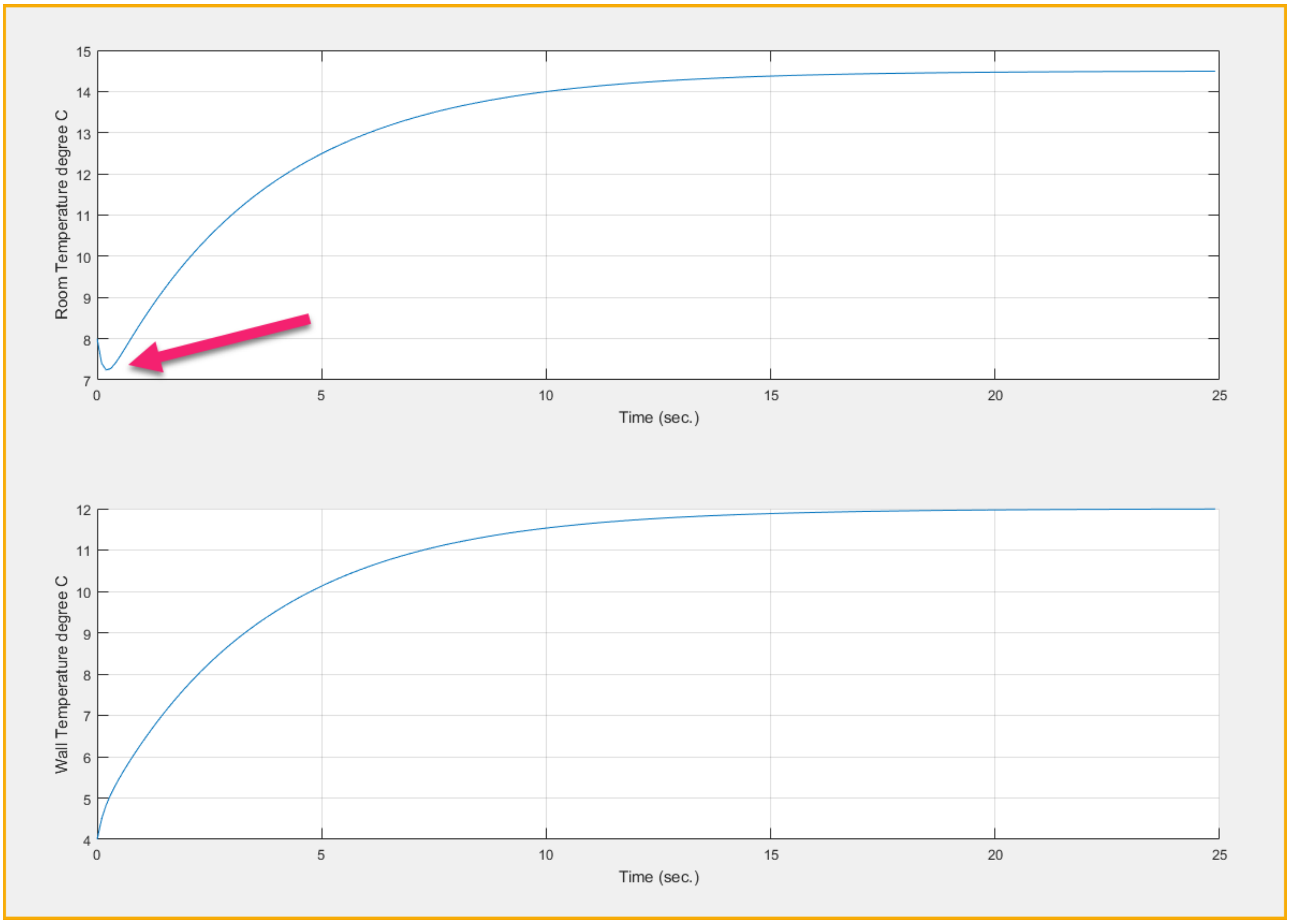
Rr=0.5;
Rw=1.8;

Tr_initial=8;
Tw_initial=4;

To=3;
q=5;

Construiti diagrama Simulink pentru simularea sistemului de mai sus

Cele doua iesiri:



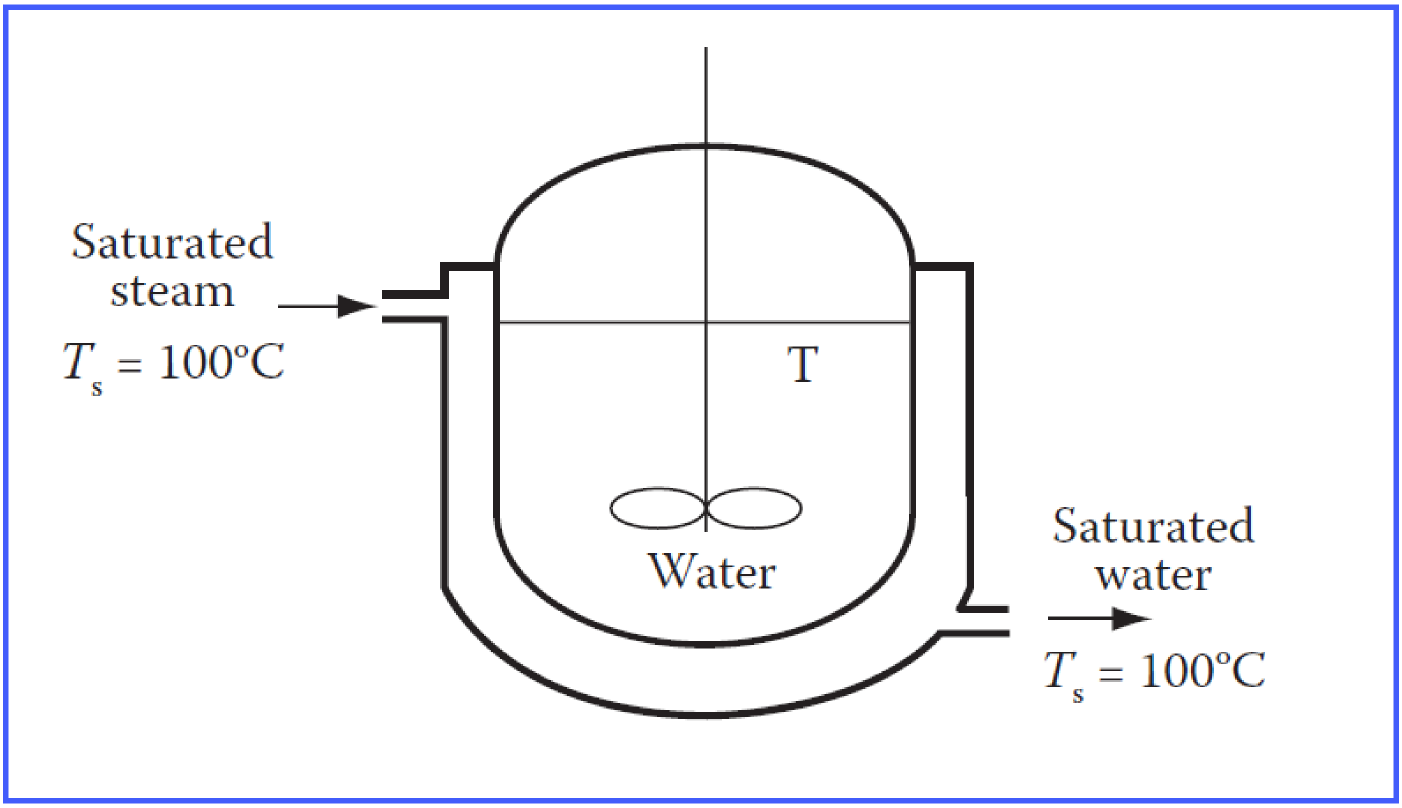
Matlab – vezi codul *.mlx complet postat pe Moodle si cele doua programe similare din laboratorul nr. 2.

```
clear all;
C1=0.5;
C2=1.5;
Rr=0.5;
Rw=1.8;
Tr=8;
Tw=4;
To=3;
q=5;
A=[1/(C1*Rr) -1/(C1*Rr)
-1/(C2*Rr) (1/(C2*Rr) )+(1/(C2*Rw) )];
B=[1/C1 0
0 1/(C2*Rw)];
X=[Tr; Tw;];
dt=0.1; % step size
t=0; % Initial time
tsim=25; % Simulation time
n=round(tsim-t)/dt;
for i=1:n
    X1(i,:)=[X' t];
    dX=-A*X+B*[q; To];
    X=X+dt*dX;
    t=t+dt;
end
subplot(2,1,1)
plot(X1(:,3),X1(:,1) )
xlabel('Time (sec.)')
ylabel('Room Temperature degree C')
subplot(2,1,2)
plot(X1(:,3),X1(:,2) )
xlabel('Time (sec.)')
ylabel('Wall Temperature degree C')
```

9.2 Se dă schimbătorul de căldura de mai jos. Se cunosc:

$$V = 30m^3, T(0) = T_0 = 25℃, A = 80m^2, U = 500\frac{J}{m^2 * s * ℃},$$
$$\rho = 1000\frac{kg}{m^3}; \qquad c = 4200\frac{J}{kg * ℃}$$

Se cere să se afle după cât timp va ajunge temperatura apei din rezervor la 50 grade Celsius.



CONSERVAREA ENERGIEI

1

$$\dot{q}_{\text{in}} = UA(T_S - T)$$

2

$$mC\frac{dT}{dt} = UA(T_S - T)$$

3

$$T(0) = T_0 = 25°\text{C}.$$

4

$$\alpha = UA/mC$$

5

$$\frac{dT}{dt} = -\alpha(T - T_S)$$

6

$$\ln\left(\frac{T - T_S}{T_0 - T_S}\right) = -\alpha t$$

$$\alpha = \ ?$$

$$\alpha = \frac{UA}{\rho VC} = \left(\frac{500\ \mathrm{J}}{\mathrm{s}\cdot\mathrm{m}^2\cdot{}^\circ\mathrm{C}}\right)\left(\frac{80\ \mathrm{m}^2}{1}\right)\left(\frac{\mathrm{m}^3}{1000\ \mathrm{kg}}\right)\left(\frac{1}{30\ \mathrm{m}^3}\right)\left(\frac{{}^\circ\mathrm{C}\cdot\mathrm{kg}}{4200\ \mathrm{J}}\right) = 0.00032\ \mathrm{s}^{-1}$$

$$\ln\left(\frac{50-100}{25-100}\right) = -0.00032t$$

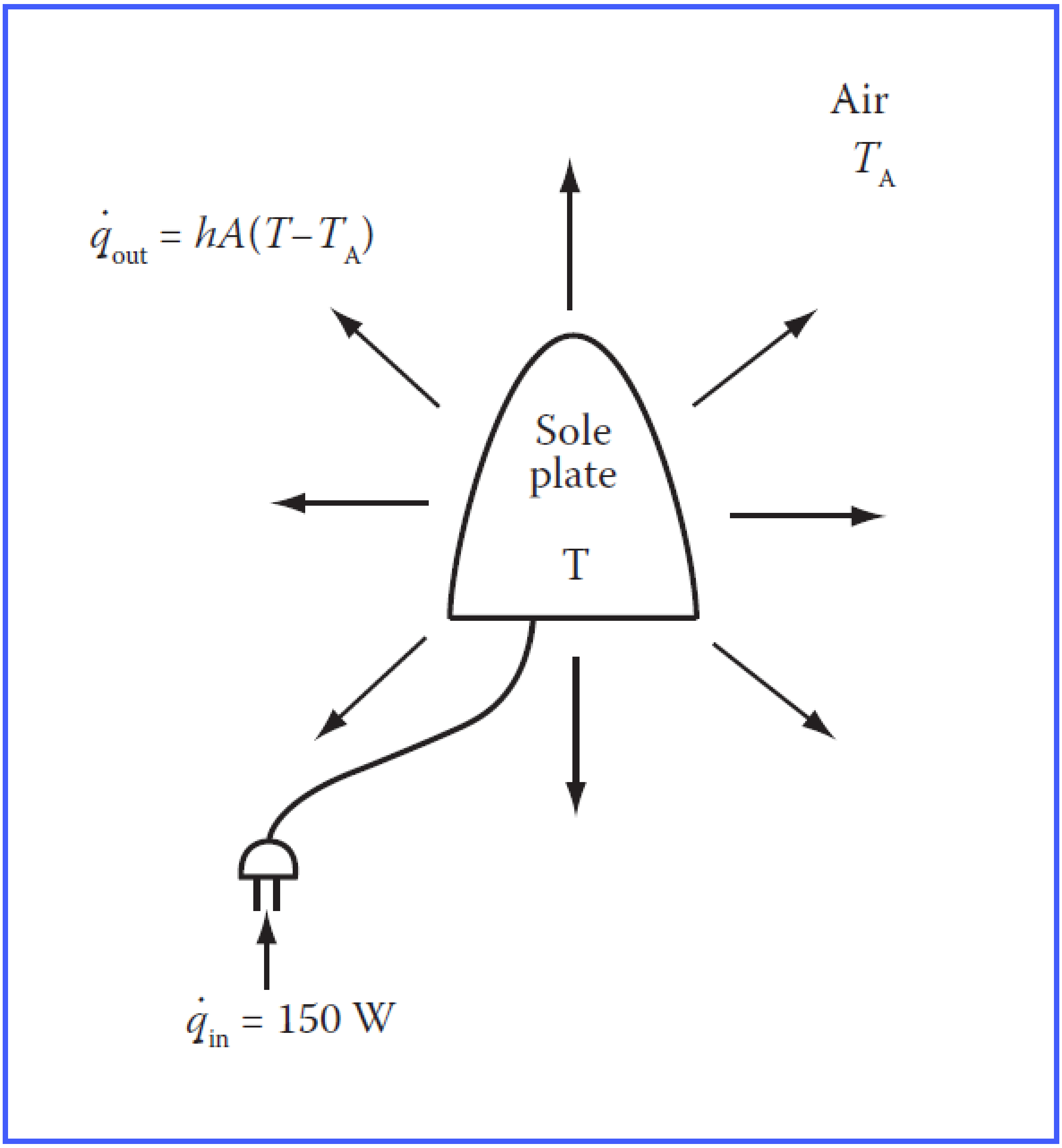
t=1267 s

9.3 Se dă sistemul de mai jos, un fier de călcat cu puterea de 150 W. Se cunosc:

$$m = 1.75\ kg, A = 0.05m^2, c = 450\ \frac{J}{kg * {}^\circ C}, h = 20\ \frac{J}{m^2 * s * {}^\circ C}$$

a. **Care este temperatura finală a fierului cand va fi alimentat?**

b. **În cât timp va atinge acesta temperatura de 100 grade?**



$$\boxed{mC\frac{dT}{dt} = \dot{q}_{\text{in}} - hA(T - T_A)}$$

$$\boxed{T(0) = T_0 = 25°\text{C}.}$$

$$\boxed{T = T_{ss} \qquad dT/dt = 0.}$$

ECUATIA DINAMICA SI CONDITIA DE REGIM STATIONAR

1

$$mC\frac{dT}{dt} = \dot{q}_{in} - hA(T - T_A)$$

Conservarea energiei

2

$$T(0) = T_0 = 25°C.$$

Conditia initiala

3

$$T = T_{ss} \qquad dT/dt = 0.$$

Conditia de regim stationar

4

$$T_{ss} = \frac{\dot{q}_{in}}{hA} + T_A$$

Valoarea de regim stationar

5

$$T_{ss} = \left(\frac{150\ \mathrm{J}}{\mathrm{s}}\right)\left(\frac{\mathrm{s}\cdot\mathrm{m}^2\cdot°\mathrm{C}}{20\ \mathrm{J}}\right)\left(\frac{1}{0.05\ \mathrm{m}^2}\right) + 25°\mathrm{C} = 175°\mathrm{C}$$

Inlocuim valorile date

b.

REZOLVAREA ECUATIEI DIFERENTIALE

1

$$\frac{mC}{hA}\frac{dT}{dt}=\frac{\dot{q}_{\text{in}}}{hA}+T_A-T$$

Ecuatia de regim dinamic

2

$$\alpha = hA/mC$$

Notatie

3

$$\beta=\frac{\dot{q}_{\text{in}}}{hA}+T_A$$

Notatie

4

$$\frac{1}{\alpha}\frac{dT}{dt}=\beta-T$$

Ecuatia de regim dinamic

5

$$\int\frac{dT}{T-\beta}=\int-\alpha dt$$

Separam variabilele

6

$$\ln\left(\frac{T-\beta}{T_0-\beta}\right)=-\alpha t$$

Solutia ecuatiei

$$\alpha = \frac{hA}{mC} = \left(\frac{20\ \mathrm{J}}{\mathrm{s}\cdot\mathrm{m}^2\cdot{}^\circ\mathrm{C}}\right)\left(\frac{0.05\ \mathrm{m}^2}{1.75\ \mathrm{kg}}\right)\left(\frac{{}^\circ\mathrm{C}\cdot\mathrm{kg}}{450\ \mathrm{J}}\right) = 0.00127\ \mathrm{s}^{-1}$$

$$t = \frac{-1}{\alpha}\ln\left(\frac{T-\beta}{T_0-\beta}\right) = \frac{-1}{0.00127}\ln\left(\frac{100-175}{25-175}\right) = 546\ \mathrm{s}$$

Diagrama Simulink?

Cod Matlab?