Množiny

Označujeme velkými písmeny abecedy $A, B, C, \ldots$

Symbol „patří“ (je prvkem) $\in$

Zápis výpisem prvků

$\{1,2,3\}$ ← nezáleží na pořadí

Zápis společnou vlastností

$\{x \mid \varphi(x)\}$

$\{n^2;\ n \in \mathbb{N}\}$

$\{k \mid \exists n \in \mathbb{N}\colon k = 2^n\}$

Prázdná množina

$\emptyset$

$\{\,\}$

Mohutnost množiny

$A = \{1,2,7,3,0\} \rightarrow |A| = 5$

Potenční množina

$P(X),\ X = \{2, \{\emptyset\}\}$

$P(X) = \{\emptyset, \{2\}, \{\{\emptyset\}\}, \{2, \{\emptyset\}\}\}$

$!\ \emptyset \neq \{\emptyset\}$

Triviální podmnožiny

Vždy 2

$A = \{1,2\} \rightarrow \emptyset, \{1,2\}$

Sjednocení

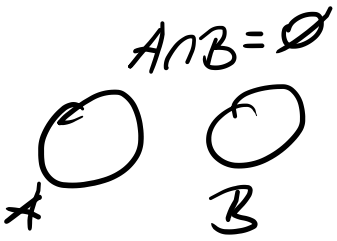
$A = \{1,2\}, B = \{3,4\} \rightarrow A \cup B = \{1,2,3,4\}$

Průnik

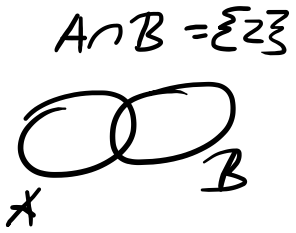
$A = \{1,2\}, B = \{3,4\} \rightarrow A \cap B = \emptyset$

Vennovy diagramy

$A = \{1,2\}, B = \{3,4\}$



$A = \{1,2\}, B = \{2,3\}$



Matematická Indukce

$2|2n+2$

1. n = 1
   $2|2n+2$
   $2|2\cdot 1+2$
   $2|2+2$
   $2|4$ ✓
2. n = n_0
   $2|2n_0+2$
3. n = n_0+1
   $2|2(n_0+1)+2$
   $2|2n_0+2+2$
   $2|2$ ✓ □

$3|n \Rightarrow 3|n^2$

1. n = 3
   $3|3 \Rightarrow 3|3^2$
   $3|3 \rightarrow 3|9$ ✓
2. n = n_0
   $3|n_0 \Rightarrow 3|n_0^2$
3. n = n_0+3
   $3|n_0+3 \Rightarrow 3|(n_0+3)^2$
   $3|n_0+3 \Rightarrow 3|n_0^2+6n_0+9$
   $3|3 \Rightarrow 3|6n_0+9$
   $3|3 \Rightarrow 3|3(2n_0+3)$
   $\frac{3(2n_0+3)}{3} = 2n_0+3$ ✓ □

$X = \{1,\{0\}\}$

$P(X) = \{\emptyset,\{1\},\{\{0\}\},\{1,\{\emptyset\}\}\}$

$Y = \{\{a,2\},0\}$

$P(Y) = \{\emptyset,\{\{a,2\}\},0,\{\{a,2\},0\}\}$

$\emptyset \subseteq \emptyset$ ✓

$\emptyset \in \emptyset$ ✗

$\{a\} \in \{a,b,c\}$ ✗

$\{a\} \in \{\{a,b\},c\}$ ✗

$\{a,b\} \subseteq \{\{a,b\},a\}$ ✓

$\{a,b\} \subseteq \{\{a,b\},a,b\}$ ✓

$A \in P(A)$ ✓

$2^n \geq 2n; n \in \mathbb{N}$

1. n = 1
   $2^1 \geq 2\cdot 1$
   $2 \geq 2$ ✓
2. n = n_0
   $2^{n_0} \geq 2n_0$
3. n = n_0+1
   $2^{(n_0+1)} \geq 2(n_0+1)$
   $2^{n_0+1} \geq 2n_0+2$
   $2\cdot 2^{n_0} \geq 2n_0+2$
   $2^{n_0}+2^{n_0} \geq 2n_0+2$
   $2^{n_0} \geq 2$ ✓ □

$4|2n^2 + 2n, n \in \mathbb{N}$

1. n = 1
   $4|2 \cdot 1^2 + 2 \cdot 1$
   $4|2 \cdot 1 + 2 \cdot 1$
   $4|2 + 2$
   $4|4$ ✓
2. n = n_0
   $4|2n_0^2 + 2n_0$
3. n = $n_0 + 1$
   $4|2 \cdot (n_0 + 1)^2 + 2 \cdot (n_0 + 1)$
   $4|2 \cdot (n_0^2 + 2n_0 + 1) + 2 \cdot (n_0 + 1)$
   $4|2n_0^2 + 2n_0 + 2n_0 + 2 + 2n_0 + 2$
   $4|4n_0 + 4$
   $4|4(n_0 + 1)$
   $\frac{4(n_0 + 1)}{4}$ □

$7|2^{n+2} + 3^{2n+1}, n \in \mathbb{N}$

1. n = 1
   $7|2^{1+2} + 3^{2+1}$
   $7|2^3 + 3^3$
   $7|8 + 27$
   $7|35$ ✓
2. n = n_0
   $7|2^{n_0+2} + 3^{2n_0+1}$
3. n = $n_0 + 1$
   $7|2^{n_0+2+1} + 3^{2(n_0+1)+1}$
   $7|2^{n_0+2+1} + 3^{2n_0+1+2}$
   $7|2 \cdot 2^{n_0+2} + 3^2 \cdot 3^{2n_0+1}$
   $7|2 \cdot 2^{n_0+2} + 9 \cdot 3^{2n_0+1}$
   $7|2^{n_0+2} + 2^{n_0+2} + 3^{2n_0+1} + 3^{2n_0+1} + 7 \cdot 3^{2n_0+1}$
   $7|7 \cdot 3^{2n_0+1}$
   $\frac{7 \cdot 3^{2n_0+1}}{7}$ □

$\left(1 + \frac{1}{3}\right)^n \geq 1 + \frac{n}{3}; n \in \mathbb{N}$

1. n = 1
   $\left(1 + \frac{1}{3}\right)^1 \geq 1 + \frac{1}{3}$
   $1 + \frac{1}{3} \geq 1 + \frac{1}{3}$ ✓
2. n = n_0
   $\left(1 + \frac{1}{3}\right)^{n_0} \geq 1 + \frac{n_0}{3}$
3. n = $n_0 + 1$
   $\left(1 + \frac{1}{3}\right)^{n_0+1} \geq 1 + \frac{n_0 + 1}{3}$
   $\left(1 + \frac{1}{3}\right) \cdot \left(1 + \frac{1}{3}\right)^{n_0} \geq 1 + \frac{n_0}{3} + \frac{1}{3}$
   $\left(1 + \frac{1}{3}\right)^{n_0} + \frac{\left(1 + \frac{1}{3}\right)^{n_0}}{3} \geq 1 + \frac{n_0}{3} + \frac{1}{3}$
   $\frac{\left(1 + \frac{1}{3}\right)^{n_0}}{3} \geq \frac{1}{3}$
   $\left(1 + \frac{1}{3}\right)^{n_0} \geq 1$ □