22.10. 1. test!

1)

$\log_7 \sqrt{7} = y$
$7^y = 7^{\frac{1}{2}}$
$y = \frac{1}{2}$

2)

$\log_5 0.04 = y$
$5^y = 0.04$
$5^y = 5^{-2}$
$y = -2$

3)

$\log_{10}(3x^2y^3) = \log_{10} 3 + \log_{10} x^2 + \log_{10} y^3 = \log_{10} 3 + 2\log_{10} x + 3\log_{10} y$

4)

$\log \sqrt[3]{\frac{xy^2}{z^4}} = \log(\frac{xy^2}{z^4})^{\frac{1}{3}} = \frac{1}{3}\log\frac{xy^2}{z^4} = \frac{1}{3}(\log xy^2 - \log z^4) = \frac{1}{3}(\log x + 2\log y - 4\log z)$

5)

$5^{x-1} = 7^{1-x} \quad / \cdot 7^{x-1}$
$(5 \cdot 7)^{x-1} = 7^{1-x+x-1}$
$(5 \cdot 7)^{x-1} = 7^0$
$(5 \cdot 7)^{x-1} = 1$
$35^{x-1} = 35^0$
$x-1 = 0$
$x = 1$

6)

$$(\frac{1}{3})^{2-3x} = 5^x$$
$$3^{-(2-3x)} = 5^x$$
$$3^{3x-2} = 5^x$$
$$(3x-2)\log 3 = x\log 5$$
$$3x\log 3 - 2\log 3 = x\log 5$$
$$3x\log 3 - x\log 5 = 2\log 3$$
$$x(3\log 3 - \log 5) = 2\log 3$$
$$x = \frac{2\log 3}{3\log 3 - \log 5}$$
$$x = \frac{\log 3^2}{\log 3^3 - \log 5}$$
$$x = \frac{\log 9}{\log 27 - \log 5}$$
$$x = \frac{\log 9}{\log \frac{27}{5}}$$

7)

$$4^x + 3^{x+4} = 4^{x+3} - 3^{x+2}$$
$$4^x + 3^x 3^4 = 4^x 4^3 - 3^x 3^2$$
$$4^x - 4^x 4^3 = -3^x 3^2 - 3^x 3^4$$
$$4^x(1 - 4^3) = -3^x(3^2 + 3^4) / \cdot (-1)$$
$$4^x \cdot 63 = 3^x \cdot 90$$
$$(\frac{4}{3})^x = \frac{90}{63}$$
$$(\frac{4}{3})^x = \frac{10}{7}$$
$$x\log\frac{4}{3} = \log\frac{10}{7}$$
$$x = \frac{\log\frac{10}{7}}{\log\frac{4}{3}}$$
$$x = \frac{\log 10 - \log 7}{\log 4 - \log 3}$$

8)

$2^{2x+1} + 2^{x+2} = 16$
$2 \cdot 2^{2x} + 2^2 \cdot 2^x = 2^4 \quad / \div 2$
$2^{2x} + 2 \cdot 2^x = 2^3$
$(2^x)^2 + 2 \cdot 2^x - 2^3 = 0 \quad / a = 2^x$
$a^2 + 2a - 8 = 0$
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-8)}}{2} = \frac{-2 \pm \sqrt{4 + 32}}{2} = \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2}$$
$$a_1 = \frac{-2 + 6}{2} = \frac{4}{2} = 2$$
$a = 2^x$
$2 = 2^x$
$x = 1$

$$a_2 = \frac{-2 - 6}{2} = \frac{-8}{2} = -4$$
$-4 = 2^x$

9)

$x(4 \log 2) = 3 \log 2$
$$x = \frac{3}{4} \log 2$$

10)

$3^{x+2} - 2 \cdot 3^{x+1} + 4 \cdot 3^x = 189$
$3^x \cdot 3^2 - 2 \cdot 3^x \cdot 3 + 4 \cdot 3^x = 189$
$3^x(3^2 - 2 \cdot 3 + 4) = 189$
$3^x \cdot 7 = 189$
$$3^x = \frac{189}{7}$$
$3^x = 9$
$3^x = 3^3$
$x = 3$

11)

$16 \cdot 2^{x+1} = 4 \cdot 8^x$
$2^4 \cdot 2^x \cdot 2^1 = 2^2 + 2^3 \cdot 2^x$
$2^{x+5} = 2^{3x+2}$
$x + 5 = 3x + 2$
$2x = 3$
$$x = \frac{3}{2}$$

12)

$3^{x+1} + 3^{x+2} + 3^{x+3} = 39$
$3^x \cdot 3 + 3^x \cdot 3^2 + 3^x \cdot 3^3 = 39$
$3^x \cdot (3 + 3^2 + 3^3) = 39$
$3^x \cdot 39 = 39$
$3^x = 1$
$3^x = 3^0$
$x = 0$

13)

$5^{x+1} + 5^{x-1} = 26$
$5^{x+1} + 5^{x-1} = 5^2 + 5^0$
$x + 1 + x - 1 = 2 + 0$
$2x = 2$
$x = 1$

14)

$2^{x+1} + 2^{x-1} + 2^{x+3} = \frac{21}{8}$
$2^{x+1} + 2^{x-1} + 2^{x+3} = \frac{2^4 + 2^2 + 2^0}{2^3}$
$2^{x+1} + 2^{x-1} + 2^{x+3} = (2^4 + 2^2 + 2^0) \cdot 2^{-3}$
$2^{x+1} + 2^{x-1} + 2^{x+3} = 2^1 + 2^{-1} + 2^{-3}$
$x + 1 + x - 1 + x + 3 = 1 - 1 - 3$
$3x + 3 = -3$
$3x = -6$
$x = -2$

15)

$4^{2x} - 6 \cdot 4^x + 8 = 0$
$(4^x)^2 - 6 \cdot 4^x + 8 = 0 \quad / \; a = 4^x$
$a^2 - 6a + 8 = 0$
$a = \frac{6 \pm \sqrt{6^2 - 4 \cdot 8}}{2} = \frac{6 \pm \sqrt{36\text{-}32}}{2} = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2}$
$a_1 = \frac{6 + 2}{2} = \frac{8}{2} = 4$
$a = 4^x$
$4 = 4^x$
$x = 1$
$a_2 = \frac{6\text{-}2}{2} = \frac{4}{2} = 2$
$a = 4^x$
$2 = 4^x$
$x = \frac{1}{2}$

$\log_a x = b$
$a^b = x$

$\log_a f(x) = \log_a g(x)$
$f(x) = g(x)$

16)

$\log 8 - \log 2 = \frac{\log(2x-2)}{2} \quad / \cdot 2$
$2 \cdot (\log 8 - \log 2) = \log(2x-2)$
$2 \cdot \log\frac{8}{2} = \log(2x-2)$
$2 \cdot \log 4 = \log(2x-2)$
$\log 4^2 = \log(2x-2)$
$4^2 = 2x-2$
$16 = 2x-2$
$2x = 18$
$x = 9$

17)

$\frac{\log(3x-1)}{3} = \log 6 - \log 3 \quad / \cdot 3$
$\log(3x-1) = 3 \cdot (\log 6 - \log 3)$
$\log(3x-1) = 3 \cdot \left(\log\frac{6}{3}\right)$
$\log(3x-1) = 3 \cdot \log 2$
$\log(3x-1) = \log 2^3$
$3x-1 = 2^3$
$3x-1 = 8$
$3x = 9$
$x = 3$

18)

$\log\left(x^{\log x}\right) = 1$
$\log x \cdot \log x = 1$
$(log\, x)^2 = 1/\sqrt{\ }$
$\log x = \pm 1$
$x_1 = 10$
$x_2 = \frac{1}{10}$

19)

$(\log x)^{\log x} = 1$
$\log((\log x)^{\log x}) = \log 1$
$\log x \cdot \log(\log x) = \log 1$
$\log x \cdot \log(\log x) = 0$
$\log x_1 = 0$
$x_1 = 1$
2:
$\log(\log x_2) = 0$
$\log x_2 = 10^0$
$\log x_2 = 1$
$x_2 = 10$

20)

$5 \cdot \log \sqrt[3]{x} - 4 \cdot \log \sqrt[6]{x} + \frac{1}{2} \cdot \log x^8 = 9 - \log x^5$
$5 \cdot \log x^{\frac{1}{3}} - 4 \cdot \log x^{\frac{1}{6}} + \frac{1}{2} \cdot \log x^8 = 9 - \log x^5$
$5 \cdot \frac{1}{3} \cdot \log x - 4 \cdot \frac{1}{6} \cdot \log x + 8 \cdot \frac{1}{2} \cdot \log x = 9 - 5 \cdot \log x$
$\frac{5}{3} \cdot \log x - \frac{4}{6} \cdot \log x + \frac{8}{2} \cdot \log x = 9 - 5 \cdot \log x$
$\frac{5}{3} \cdot \log x - \frac{4}{6} \cdot \log x + 4 \cdot \log x + 5 \cdot \log x = 9$
$\left(\frac{5}{3} - \frac{4}{6} + 4 + 5\right) \cdot \log x = 9$
$10 \cdot \log x = 9$
$\log x = \frac{9}{10}$
$x = 10^{\frac{9}{10}}$

21)

$\log_7 \frac{x^2 + 1}{x^2\text{-}1} = 1$
$\log_7 \frac{x^2 + 1}{x^2\text{-}1} = \log_7 7$
$\frac{x^2 + 1}{x^2\text{-}1} = 7$
$x^2 + 1 = 7x^2 - 7$
$6x^2 = 8$
$x^2 = \frac{4}{3}$
$x = \pm\frac{2}{\sqrt{3}}$

22)

$$\frac{\log x}{\log(x-2)} = \frac{\log 9}{\log 3} \quad / \cdot \log(x-2) \quad \text{Pod: } x > 2$$
$$\log x = \frac{\log 9}{\log 3} \cdot \log(x-2)$$
$$\log x = \frac{2\log 3}{\log 3} \cdot \log(x-2)$$
$$\log x = 2 \cdot \log(x-2)$$
$$\log x = \log(x-2)^2$$
$$x = (x-2)^2$$
$$x = x^2 - 4x + 4$$
$$x^2\text{-}5x + 4 = 0$$
$$x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 4}}{2} = \frac{5 \pm \sqrt{25\text{-}16}}{2} = \frac{5 \pm \sqrt{9}}{2} = \frac{5 \pm 3}{2}$$
$$x_1 = \frac{5+3}{2} = \frac{8}{2} = 4$$
$$x_2 = \frac{5\text{-}3}{2} = \frac{2}{2} = 1$$

23)

$$\log_2\left(\log_3\left(\log_{\frac{1}{2}}(x)\right)\right) = 0 \quad Pod: x > 0$$
$$\log_3\left(\log_{\frac{1}{2}}(x)\right) = 1$$
$$\log_{\frac{1}{2}}(x) = 3$$
$$x = \left(\frac{1}{2}\right)^3$$
$$x = \frac{1}{8}$$

24)

$$\log_5(x^2 + 2x) = \log_5(-3x) \quad Pod: x > 0$$
$$x^2 + 2x = -3x$$
$$x^2 + 5x = 0$$
$$x(x+5) = 0$$
$$x_1 = 0$$
$$x_2 = -5$$

25)

$\log_8 \sqrt{x+30} + \log_8 \sqrt{x} = 1 \quad Pod: x > 0$

$\log_8\left(\sqrt{x+30} \cdot \sqrt{x}\right) = \log_8 8$

$\sqrt{x+30} \cdot \sqrt{x} = 8 \quad / \ ^2$

$x(x+30) = 64$

$x^2 + 30x = 64$

$x^2 + 30x - 64 = 0$

$$x = \frac{-30 \pm \sqrt{(-30)^2 - 4 \cdot (-64)}}{2} = \frac{-30 \pm \sqrt{900+256}}{2} = \frac{-30 \pm \sqrt{1156}}{2} = \frac{-30 \pm 34}{2}$$

$$x_1 = \frac{-30+34}{2} = \frac{40}{2} = 2$$

$$x_2 = \frac{-30-34}{2} = \frac{-64}{2} = -32$$