1)

$(i+1)(i-1)+(2i+1)^2=$
$i^2-1+4i^2+4i+1=$
$-1-1-4+4i+1=$
$-5+4i$

$i^1=i$

$i^2=-1$

$i^3=-i$

$i^4=1$

2)

$49(2-i\sqrt{3})^{-2}=$
$\frac{49}{(2-i\sqrt{3})^2}=$
$\frac{49}{4-4i\sqrt{3}+3i^2}=$
$\frac{49}{4-4i\sqrt{3}-3}=$
$\frac{49}{1-4i\sqrt{3}}=$
$\frac{49}{1-4i\sqrt{3}}\cdot\frac{1+4i\sqrt{3}}{1+4i\sqrt{3}}=$
$\frac{49+49\cdot 4i\sqrt{3}}{1+16\cdot 3}=$
$\frac{49+49\cdot 4i\sqrt{3}}{49}=$
$1+4i\sqrt{3}$

3)

$\frac{2+i}{i}+\frac{i}{i+1}-\frac{2i+1}{i-1}=$
$\frac{(2+i)(i^2-1)+i^2(i-1)-i(2i+1)(i+1)}{i(i^2-1)}=$
$\frac{-2-2-i-i-i+1-(2i+1)(i-1)}{i(i^2-1)}=$
$\frac{-4-2i-i+1+2i+2+1-i}{-2i}=$
$\frac{-2i}{-2i}=$
1

4)

$1 + \mathrm{i}^2 + \mathrm{i}^3 + \mathrm{i}^4 + \mathrm{i}^{-3} =$
$1 - 1 + (-i) + 1 + \left(-\frac{1}{\mathrm{i}}\right) =$
$1 - i - \frac{1}{\mathrm{i}} =$
$1 - i - \frac{1}{\mathrm{i}} \cdot \frac{\mathrm{i}}{\mathrm{i}} =$
$1 - i + \frac{\mathrm{i}}{1} =$
$1 - i + i =$
1

5)

$(1 + i)^8 =$
$((1 + i)^2)^4 =$
$(1 + 2\mathrm{i} - 1)^4 =$
$(2\mathrm{i})^4 =$
$16\mathrm{i}^4 =$
16

Výpočet goniometrického tvaru

6)

$z = \sqrt{3} + i$
$x = \sqrt{3}, \qquad y = 1$
$|\mathrm{z}| = \sqrt{\mathrm{x}^2 + \mathrm{y}^2}$
$|\mathrm{z}| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$
$\sin\varphi = \frac{\mathrm{y}}{|\mathrm{z}|} = \frac{1}{2}$
$\cos\varphi = \frac{\mathrm{x}}{|\mathrm{z}|} = \frac{\sqrt{3}}{2}$
$\varphi = 30° = \frac{\pi}{6}$
$z = |\mathrm{z}| \cdot (\cos\varphi + i \cdot \sin\varphi)$
$z = 2\left(\cos\frac{\pi}{6} + \mathrm{i} \cdot \sin\frac{\pi}{6}\right)$

7)

$z = -1 - i$
$x = -1, y = -1$
$|z| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$
$\sin\varphi = \frac{-1}{\sqrt{2}}$
$\cos\varphi = \frac{-1}{\sqrt{2}}$
$\varphi = \frac{5\pi}{4}$
$z = \sqrt{2} \cdot \left(\cos\frac{5\pi}{4} + \mathrm{i} \cdot \sin\frac{5\pi}{4}\right)$

8)

$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
$x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$
$|z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$ <- Velikost nemůže být záporná, proto -1 NENÍ řešením
$\sin\varphi = \frac{\sqrt{3}}{2}$
$\cos\varphi = \frac{1}{2}$
$\varphi = \frac{\pi}{3}$
$z = 1\left(\cos\frac{\pi}{3} + i \cdot \sin\frac{\pi}{3}\right)$
$z = \cos\frac{\pi}{3} + i \cdot \sin\frac{\pi}{3}$

9)

$(1+\mathrm{i})^6 =$

9.1)

$((1+i)^2)^3 = (2i)^3 = 8\mathrm{i}^3 = -8i$

9.2)

$1+i = \sqrt{2}\cdot\left(\cos\frac{\pi}{4} + i\cdot\sin\frac{\pi}{4}\right)$

$(1+i)^6 = \left(\sqrt{2}\right)^6\cdot\left(\cos\frac{\pi}{4} + i\cdot\sin\frac{\pi}{4}\right)^6 =$

$8\left(\cos\frac{6\pi}{4} + i\cdot\sin\frac{6\pi}{4}\right) =$

$8\left(\cos\frac{3\pi}{2} + i\cdot\sin\frac{3\pi}{2}\right) =$

$8\big(0 + i\cdot(-1)\big) =$

$-8i$