

## GONIOMETRIE

## Orientovaný úhel

- Dvě polopřímky
  - Počáteční rameno
    - Z bodu P (vrchol úhlu) přes bod A
  - Koncové rameno
    - Z bodu P (vrchol úhlu) přes bod B
- Úhlové jednotky
  - Stupně
    - $\alpha (^{\circ})$
    - $\alpha = 30^{\circ}$
  - Radiány
    - $a \text{ (rad)}$
    - $a = \frac{\pi}{6} \text{ rad}$
- Úhel se periodicky opakuje
  - $\alpha + k \cdot 360^{\circ}, k \in \mathbb{Z}$
  - $a + 2k\pi, k \in \mathbb{Z}$
- Tabulka nejčastějších úhlů

$\alpha (^{\circ})$	0	30	45	60	90	180	270	360
$a \text{ (rad)}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$

- $\sin(\alpha), \cos(\alpha), \operatorname{tg}(\alpha), \operatorname{cotg}(\alpha)$ 
  -



- $\sin \alpha = \frac{\text{Protilehlá}}{\text{Přepona}}$
- $\cos \alpha = \frac{\text{Přilehlá}}{\text{Přepona}}$ 
  - $D(f) = \mathbb{R}$
  - $H(f) = \langle -1; 1 \rangle$
- $\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$
- $\operatorname{cotg} \alpha = \frac{\cos \alpha}{\sin \alpha}$ 
  - $D(f) = \mathbb{R} \setminus \left\{ \frac{(2k+1)\pi}{2}, k \in \mathbb{Z} \right\}$
  - $H(f) = \mathbb{R}$

°	0	30	45	60	90	180	270	360
rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\operatorname{tg} \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-	0	-	0
$\operatorname{cotg} \alpha$	-	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	-	0	-

	$\sin x$	$\cos x$	$\operatorname{tg} x$ $\operatorname{cotg} x$
$\frac{2}{3} \mid \frac{1}{4}$	$\frac{+}{-} \mid \frac{+}{-}$	$\frac{-}{-} \mid \frac{+}{+}$	$\frac{-}{+} \mid \frac{+}{-}$

## CYKLOMETRICKÉ FUNKCE

$$y = \arcsin(x) \quad \leftrightarrow \quad x = \sin(y)$$

## GONIOMETRICKÉ ROVNICE

$$\begin{aligned} \sin(x) &= a & \cos(x) &= a & a &\in \langle -1; 1 \rangle \\ \operatorname{tg}(x) &= b & \operatorname{cotg}(x) &= b & b &\in \mathbb{R} \\ && && \text{! NEKONEČNĚ MNOHO ŘEŠENÍ !} \end{aligned}$$

1)

$$\sin x = 0$$

$$x_1 = 0$$

$$x_2 = \pi$$

$$x_3 = 2\pi$$

⋮

$$x \in \{k \cdot \pi, k \in \mathbb{Z}\}$$

2)

$$\cos x = 0$$

$$x_1 = \frac{\pi}{2}$$

$$x_2 = \frac{3\pi}{2}$$

$$x_3 = \frac{5\pi}{2}$$

⋮

$$x \in \left\{ \frac{(2k+1)\pi}{2}, k \in \mathbb{Z} \right\}$$

3)

$$\operatorname{tg} x = 0$$

$$\frac{\sin \alpha}{\cos \alpha} = 0$$

$$\sin x = 0$$

⋮

$$x \in \{k \cdot \pi, k \in \mathbb{Z}\}$$

4)

$$\operatorname{cotg} x = 0$$

$$\frac{\cos \alpha}{\sin \alpha} = 0$$

$$\cos x = 0$$

⋮

$$x \in \left\{ \frac{(2k+1)\pi}{2}, k \in \mathbb{Z} \right\}$$

5)

$$\sin x = 1$$

$$x_1 = \frac{\pi}{2}$$

$$x_2 = \frac{5\pi}{2}$$

$$x_3 = \frac{9\pi}{2}$$

⋮

$$x \in \left\{ \frac{(4k+1)\pi}{2}, k \in \mathbb{Z} \right\}$$

6)

$$\sin x = -1$$

$$x_1 = \frac{3\pi}{2}$$

$$x_2 = \frac{7\pi}{2}$$

$$x_3 = \frac{11\pi}{2}$$

⋮

$$x \in \left\{ \frac{(4k+3)\pi}{2}, k \in \mathbb{Z} \right\}$$

7)

$$\cos x = 1$$

$$x_1 = 0$$

$$x_2 = 2\pi$$

$$x_3 = 4\pi$$

$\vdots$

$$x \in \{2k\pi, k \in \mathbb{Z}\}$$

8)

$$\cos x = -1$$

$$x_1 = \pi$$

$$x_2 = 3\pi$$

$$x_3 = 5\pi$$

$\vdots$

$$x \in \{(2k + 1)\pi, k \in \mathbb{Z}\}$$

9)

$$\sin x = \frac{1}{2}$$

$$x_1 = \frac{\pi}{6}$$

$$x_2 = \frac{5\pi}{6}$$

$$x_3 = \frac{13\pi}{6}$$

$\vdots$

$$x \in \left\{ \frac{(12k + 1)\pi}{6}, k \in \mathbb{Z} \right\} \cup \left\{ \frac{(12k + 5)\pi}{6}, k \in \mathbb{Z} \right\}$$

10)

$$\cos x = \frac{1}{2}$$

$$x_1 = \frac{\pi}{3}$$

$$x_2 = \frac{5\pi}{3}$$

$$x_3 = \frac{7\pi}{3}$$

$\vdots$

$$x \in \left\{ \frac{(6k + 1)\pi}{3}, k \in \mathbb{Z} \right\} \cup \left\{ \frac{(6k + 5)\pi}{3}, k \in \mathbb{Z} \right\}$$