Vietovi vzorce

$$a\mathrm{x}^2 + bx + c = 0 \quad \longrightarrow \quad x_1 + x_2 = \frac{-b}{a}$$

$$\longrightarrow \quad x_1 \cdot x_2 = \frac{c}{a}$$

Procvičování na písemku

1)

$$\frac{1}{3^x} = \frac{1}{\sqrt{3}} \cdot \sqrt[6]{27^{3\text{-}3x}} \cdot \left(\frac{1}{9}\right)^{x+3}$$

$$3^{\text{-}x} = 3^{\text{-}\frac{1}{2}} \cdot (27^{3\text{-}3x})^{\frac{1}{6}} \cdot (3^{\text{-}2})^{x+3}$$

$$3^{\text{-}x} = 3^{\text{-}\frac{1}{2}} \cdot \left(3^{3\cdot(3-3x)}\right)^{\frac{1}{6}} \cdot 3^{-2(x+3)}$$

$$3^{\text{-}x} = 3^{\text{-}\frac{1}{2}} \cdot (3^{3\text{-}3x})^{\frac{1}{2}} \cdot 3^{-2x\text{-}6}$$

$$3^{\text{-}x} = 3^{\text{-}\frac{1}{2}} \cdot 3^{\frac{3\text{-}3x}{2}} \cdot 3^{-2x\text{-}6}$$

$$-x = -\frac{1}{2} + \frac{3\text{-}3x}{2} - 2x - 6$$

$$-2x = -1 + 3 - 3x - 4x - 12$$
$$-3x - 4x + 2x = 1 - 3 + 12$$
$$-5x = 10$$
$$x = -2$$

2)

$$\left(\frac{4}{9}\right)^{\mathrm{x}} \cdot \left(\frac{27}{8}\right)^{\mathrm{x}\text{-}1} = \frac{\log 4}{\log 8}$$

$$\left(\frac{2}{3}\right)^{2\mathrm{x}} \cdot \left(\frac{3}{2}\right)^{3(x-1)} = \frac{\log 2^2}{\log 2^3}$$

$$\left(\frac{2}{3}\right)^{2\mathrm{x}} \cdot \left(\frac{2}{3}\right)^{-3(x-1)} = \frac{2 \cdot \log 2}{3 \cdot \log 2}$$

$$\left(\frac{2}{3}\right)^{2x-3(x-1)} = \frac{2}{3}$$

$$2x - 3(x-1) = 1$$
$$2x - 3x + 3 - 1 = 0$$
$$-x + 2 = 0$$
$$x = 2$$

3)

$\log\left(\mathrm{x}^{\log \mathrm{x}}\right) = 1$
$\log \mathrm{x} \cdot \log \mathrm{x} = 1$
$(\log \mathrm{x})^2 = 1$
$\log \mathrm{x} = \pm 1$
$\mathrm{x}_1 = 10^1 = 10 \longleftarrow$ vyhovuje zkoušce
$\mathrm{x}_2 = 10^{-1} = \frac{1}{10} \leftarrow$ vyhovuje zkoušce

4)

$(\log \mathrm{x})^{\log \mathrm{x}} = 1$
$\log\left[(\log \mathrm{x})^{\log \mathrm{x}}\right] = \log 1$
$\log \mathrm{x} \cdot \log[\log(x)] = 0$

$\log \mathrm{x}_1 = 0$
$\mathrm{x}_1 = 1 \longleftarrow \mathit{NEVYHOVUJE\ zkoušce}$

$\log(\log \mathrm{x}_2) = 0$
$\log \mathrm{x}_2 = 10^0$
$\log \mathrm{x}_2 = 1$
$\mathrm{x}_2 = 10^1$
$\mathrm{x}_2 = 10 \longleftarrow \mathit{vyhovuje\ zkoušce}$

5)

$\sin \mathrm{x} + \sin 2\mathrm{x} = 0$
$\sin \mathrm{x} + 2 \cdot \sin \mathrm{x} \cdot \cos \mathrm{x} = 0$
$\sin \mathrm{x} \cdot (1 + 2 \cdot \cos \mathrm{x}) = 0$

$\sin \mathrm{x}_1 = 0$
$\mathrm{x}_1 = \pi + k\pi$

$1 + 2 \cdot \sin \mathrm{x}_2 = 0$
$\sin \mathrm{x}_{2,3} = -\frac{1}{2}$
$\mathrm{x}_2 = \frac{7\pi}{6} + 2k\pi$
$\mathrm{x}_3 = \frac{11\pi}{6} + 2k\pi$

6)

$$\sin 2\mathrm{x} \cdot \cos \mathrm{x} + \sin^2 \mathrm{x} = 1$$
$$2 \cdot \sin \mathrm{x} \cdot \cos^2 \mathrm{x} + \sin^2 \mathrm{x} = 1$$
$$2 \cdot \sin \mathrm{x} \cdot \cos^2 \mathrm{x} + 1 - \cos^2 \mathrm{x} = 1$$
$$2 \cdot \sin \mathrm{x} \cdot \cos^2 \mathrm{x} - \cos^2 \mathrm{x} = 0$$
$$\cos^2 \mathrm{x} \cdot (2 \sin \mathrm{x} - 1) = 0$$

$$\cos^2 \mathrm{x}_1 = 0$$
$$\cos \mathrm{x}_1 = 0$$
$$\mathrm{x}_1 = \frac{\pi}{2} + 2k\pi$$

$$2 \sin \mathrm{x}_{2,3} - 1 = 0$$
$$\sin \mathrm{x}_{2,3} = \frac{1}{2}$$
$$\mathrm{x}_2 = \frac{\pi}{6} + 2k\pi$$
$$\mathrm{x}_3 = \frac{5\pi}{6} + 2k\pi$$

7)

$$2 \sin 2\mathrm{x} - 2 \cos 2\mathrm{x} = 2$$
$$\sin 2\mathrm{x} - \cos 2\mathrm{x} = 1$$
$$2 \cdot \sin \mathrm{x} \cdot \cos \mathrm{x} - (\cos^2 \mathrm{x} - \sin^2 \mathrm{x}) = 1$$
$$2 \cdot \sin \mathrm{x} \cdot \cos \mathrm{x} - (\cos^2 \mathrm{x} - 1 + \cos^2 \mathrm{x}) = 1$$
$$2 \cdot \sin \mathrm{x} \cdot \cos \mathrm{x} - \cos^2 \mathrm{x} + 1 - \cos^2 \mathrm{x} = 1$$
$$2 \cdot \sin \mathrm{x} \cdot \cos \mathrm{x} - 2 \cos^2 \mathrm{x} = 0$$
$$\sin \mathrm{x} \cdot \cos \mathrm{x} - \cos^2 \mathrm{x} = 0$$
$$\cos \mathrm{x} \cdot (\sin \mathrm{x} - \cos \mathrm{x}) = 0$$

$$\cos \mathrm{x}_1 = 0$$
$$\mathrm{x}_1 = \frac{\pi}{2} + k\pi$$

$$\sin \mathrm{x}_2 = \cos \mathrm{x}_2$$
$$\mathrm{x}_2 = \frac{\pi}{4} + \mathrm{k}\pi$$