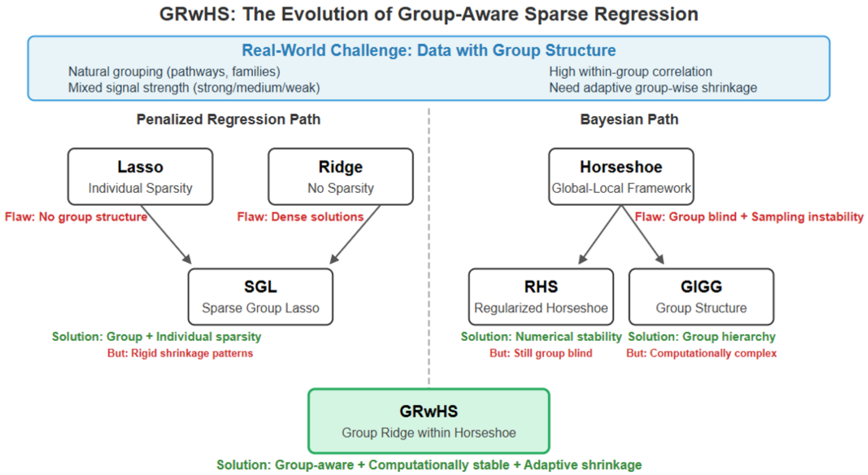
A Trail of "Buts"...



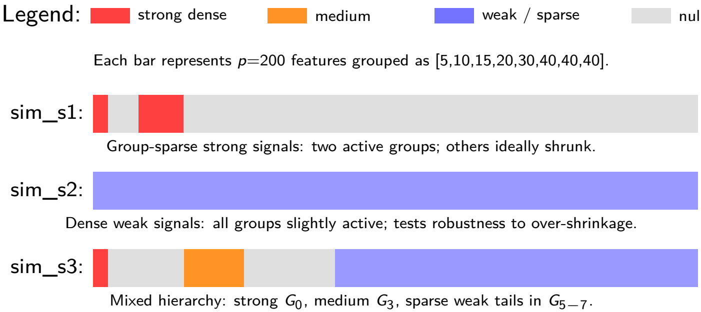
Model Comparison

Table: Structural comparison of sparse regression models

Model	Global	Group	Local Shrinkage
Lasso	α	N/A	$\alpha \sum_j \lvert\beta_j\rvert$
Ridge	α	N/A	$\alpha \sum_j \beta_j^2$
SGL	(α_1, α_2)	$\alpha_2 \sum_g w_g \lVert\beta_g\rVert_2$	$\alpha_1 \sum_j \lvert\beta_j\rvert$
Horseshoe	$\tau \sim \mathrm{C}^+(0, \sigma)$	N/A	$\lambda_j \sim \mathrm{C}^+(0, 1)$
RHS	$\tau \sim \mathrm{C}^+(0, \tau_0)$	N/A	$\lambda_j \sim \mathrm{C}^+(0, 1)$, $\tilde{\lambda}_j^2 = \dfrac{c^2\lambda_j^2}{c^2 + \tau^2\lambda_j^2}$
GIGG	$\tau \sim \mathrm{C}^+(0, \sigma)$	$\gamma_g^2 \sim \mathrm{Gamma}(a_g, 1)$	$\lambda_{gj}^2 \sim \mathrm{InvGamma}(b_g, 1), \gamma_g^2\lambda_{gj}^2 \sim \beta'(a_g, b_g)$
GRwHS	$\tau \sim \mathrm{C}^+(0, \tau_0)$	$\phi_g \sim \mathcal{N}^+(0, \dfrac{\eta^2}{p_g})$	$\lambda_j \sim \mathrm{C}^+(0, 1)$, $\tilde{\lambda}_j^2 = \dfrac{c^2\lambda_j^2}{c^2 + \tau^2\lambda_j^2}$

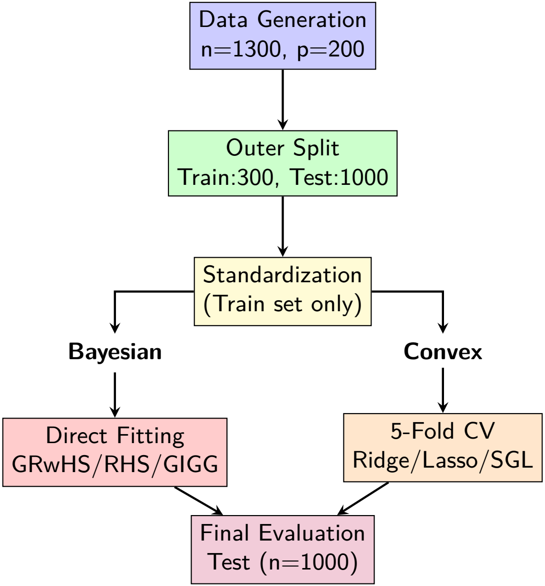
Synthetic Scenarios

- Design: n=1300, Train/test: 300/1000, 5-fold CV, p=200, G=8 groups [5,10,15,20,30,40,40,40]
- Correlation: group block (ρ_{in}=0.7, ρ_{out}=0.1); Signal-to-noise ratio(SNR) $\in \{0.1, 0.5, 1, 3\}$

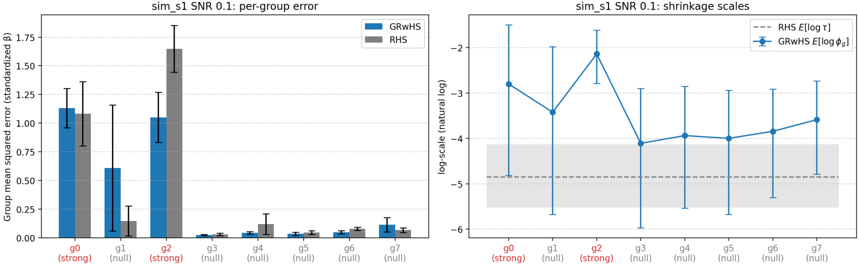


All share identical design, standardization, and evaluation pipeline (RMSE, group edf, posterior shrinkage).

Experimental Design: Nested Data Splitting



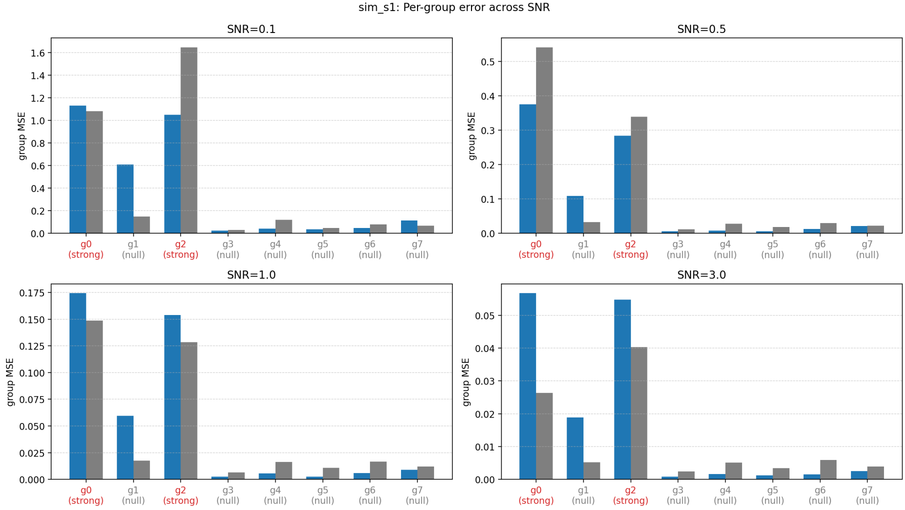
Results: sim_s1



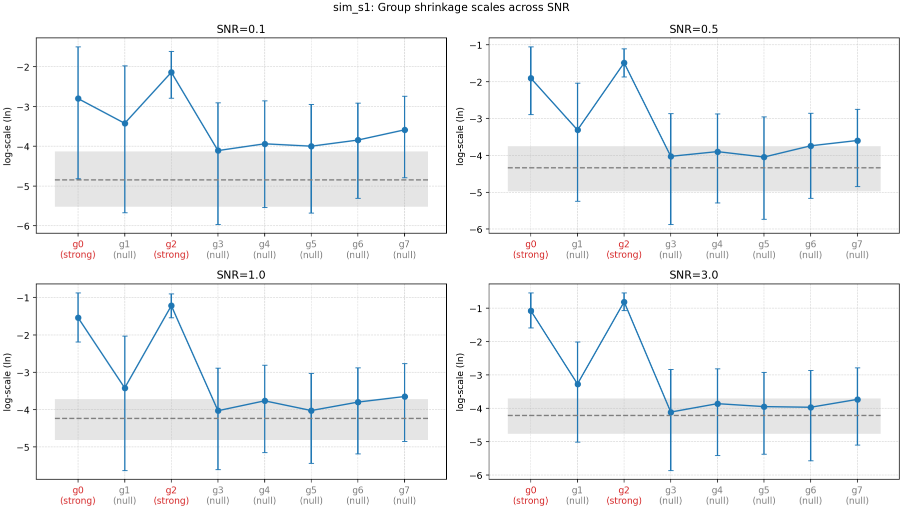
Left: GRwHS lowers error in the strong (g2) groups while matching RHS on weak/null groups.

Right: GRwHS learns larger E[log ϕ_g] for signal groups, whereas RHS uses one global τ (dashed).
This group-aware shrinkage releases true signal and preserves shrinkage on noise, explaining the error gains.

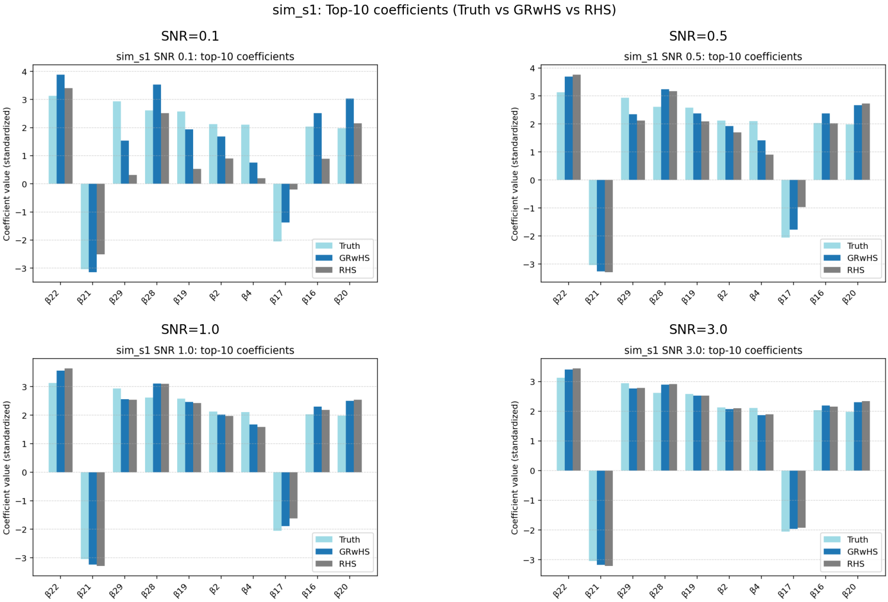
Results: sim_s1(Group-sparse Strong)



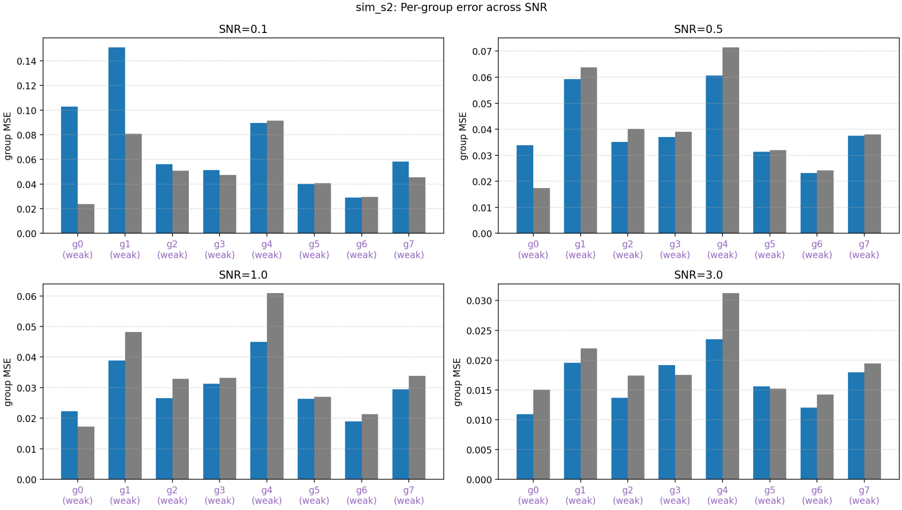
Results: sim_s1(Group-sparse Strong)



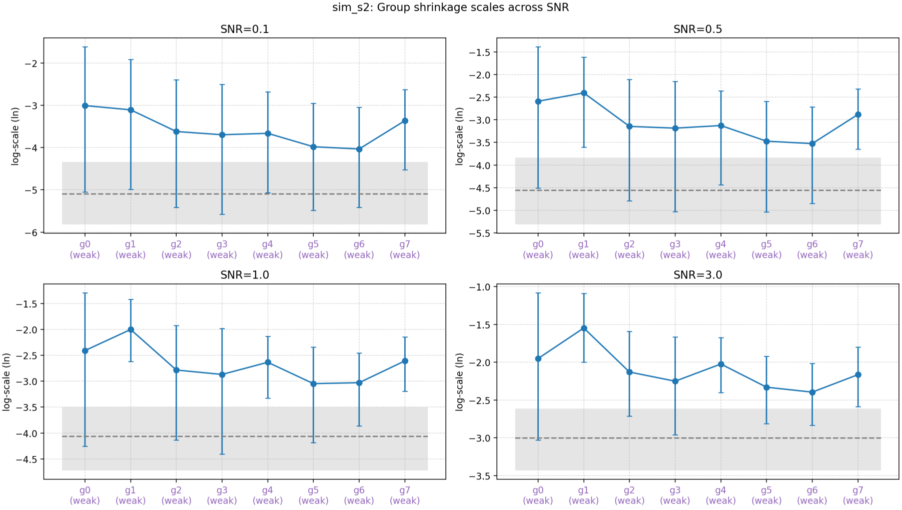
Results: sim_s1(Group-sparse Strong)



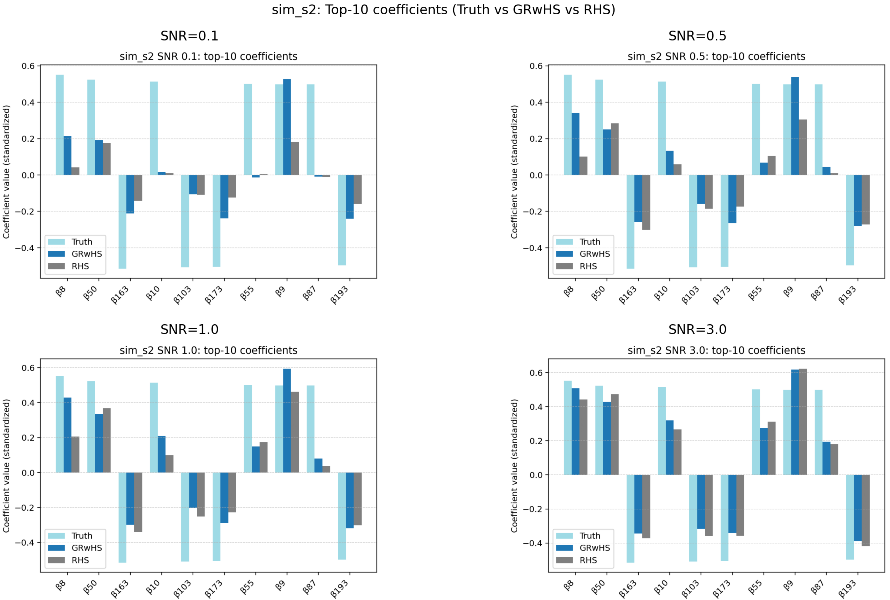
Results: sim_s2(Dense Weak)



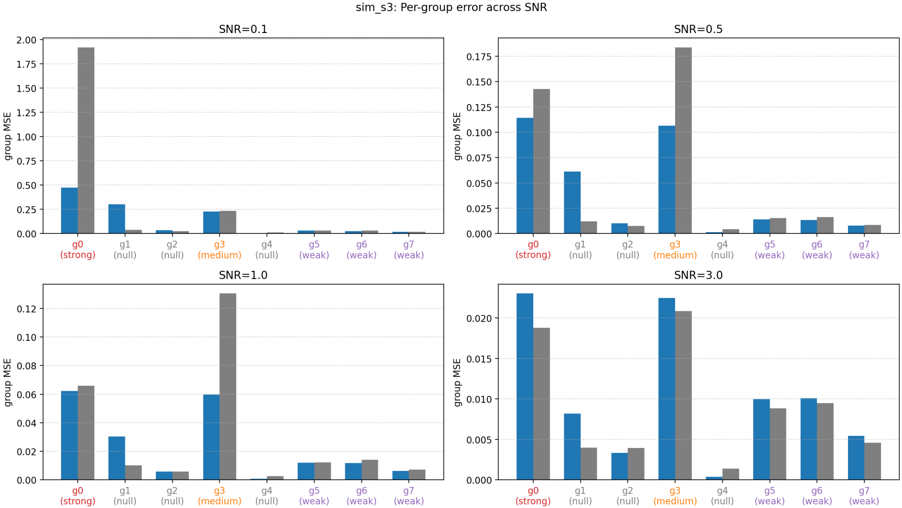
Results: sim_s2(Dense Weak)



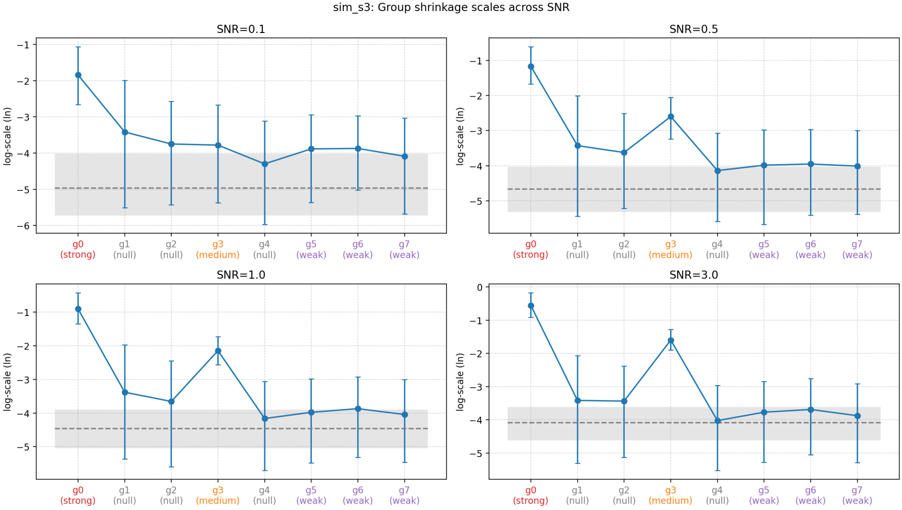
Results: sim_s2(Dense Weak)



Results: sim_s3(Mixed Hierarchy)



Results: sim_s3(Mixed Hierarchy)



Results: sim_s3(Mixed Hierarchy)