

Practical 1

27

Title : Random Variable

(i) Find the mean and variance for the following:-

X	-1	0	1	2
$P(X)$	0.1	0.2	0.3	0.4

Solution:

X	$P(X)$	$X \cdot P(X)$	$E(X)^2$	$[E(X)]^2$
-1	0.1	-0.1	0.1	0.01
0	0.2	0	0	0
1	0.3	0.3	0.3	0.09
2	0.4	0.8	0.16	0.64
$\sum = 1$	$\sum = 1$	$\sum = 2.0$	$\sum [E(X)^2] = 0.74$	

$$\therefore \text{Mean} = E(X) = \sum x_i \cdot P(x) = 1$$

$$\begin{aligned} \therefore \text{Variance} = V(X) &= \sum E(X)^2 - \sum [E(X)]^2 \\ &= 2 - 0.74 \\ &= \underline{\underline{1.24}} \end{aligned}$$

$$\therefore \text{Mean } E(X) = 1 \Delta \text{ Variance is } 1.24$$

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b) $P(X) = \frac{1}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18}$

c) $P(X) = 3, 10, 15, 0.35, 0.25$

Solution:- $X, P(X), X \cdot P(X), E(X)^2, [E(X)]^2$

-1	$\frac{1}{18}$	$-\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
0	$\frac{1}{18}$	0	$\frac{1}{18}$	$\frac{1}{18}$
1	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
2	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
$\Sigma = 1$	$\Sigma = \frac{1}{18}$	$\Sigma = \frac{1}{18}$	$\Sigma = \frac{1}{18}$	$\Sigma = \frac{1}{18}$

$\therefore M_{\text{mean}} = E(X) = \sum x \cdot P(X) = \frac{9}{18}$

$\therefore \text{Variance} = V(X) = \sum E(X)^2 - \sum [E(X)]^2$

$$= \frac{19}{8} - \frac{81}{64}$$

$$\therefore \text{Mean} = E(X) = \sum x \cdot P(X) \\ = 6.05$$

$$\therefore \text{Variance} = V(X) = \sum E(X)^2 - \sum [E(X)]^2 \\ = \frac{83}{64} \\ = 14.85 - 27.7525 \\ = 67.0975$$

$\therefore \text{Mean } E(X) = 9/8 \quad \text{Variance } V(X) = \frac{83}{64}$

$\therefore \text{Mean } E(X) = 6.05 \quad \text{& Variance } V(X) = 67.0975$

Q2) If $P(X)$ is pmf of a random variable X . Find the value of K . Then evaluate mean & variance

Solution:- As $P(X)$ is pmf, then it should satisfy the properties of pmf

- a) $P(X) \geq 0$ for all sample space
- b) $\sum P(X) = 1$

$$X: -1, 0, 1, 2 \\ P(X), \frac{1}{13}, \frac{4}{13}, \frac{1}{13}, \frac{2}{13}$$

$$\therefore \sum P(x) = 1 = \frac{k+1}{13} + \frac{k}{13} + \frac{1}{13} + \frac{k-4}{13}$$

$$1 = k+1 + k+1 + k-4$$

$$13$$

$$13 = 3k - 2$$

$$15 = 3k$$

$$k=5$$

X	P(x)	$x \cdot P(x)$	$E(x)^2$	$[E(x)]^2$
-1	$\frac{6}{13}$	$-\frac{6}{13}$	$\frac{36}{169}$	$\frac{36}{169}$
0	$\frac{5}{13}$	0	0	0
1	$\frac{4}{13}$	$\frac{4}{13}$	$\frac{16}{169}$	$\frac{16}{169}$
2	$\frac{2}{13}$	$\frac{4}{13}$	$\frac{4}{169}$	$\frac{4}{169}$
Σ	1	$\Sigma = \frac{11}{13}$	$\Sigma = \frac{11}{13}$	$\Sigma = \frac{11}{13}$

$$\therefore \text{Mean} = E(x) = \sum x \cdot P(x) = -\frac{3}{13}$$

$$\therefore \text{Variance} = V(x) = \sum E(x)^2 - \overline{[E(x)]^2}$$

$$= \frac{11}{13} - \frac{41}{169}$$

$$= \frac{143 - 41}{169}$$

$$= \frac{102}{169}$$

$$= \underline{\underline{\underline{\underline{1}}}}$$

(3) The pmf of random variable X is given by
 $P(X) : -3, -1, 0, 1, 2, 3, 5, 8$
 $P(X) : 0.1, 0.2, 0.15, 0.2, 0.15, 0.1, 0.15, 0.05, 0.05$

$$\text{Obtain cdf. Find } (a) P(-1 \leq x \leq 2) \quad (b) P(1 \leq x \leq 5) \quad (c) P(x \leq 2) \quad (d) P(x > 2)$$

Solutions:

$$P(X) : -3, -1, 0, 1, 2, 3, 5, 8 \\ P(X) : 0.1, 0.2, 0.15, 0.2, 0.1, 0.15, 0.05, 0.05$$

$$(a) P(-1 \leq x \leq 2) = P(X \leq 2) - P(X \leq -1) + P(X = -1) \\ = P(X_0) - P(X_0) + P(X_0) \\ = 0.75 - 0.3 + 0.2 = 0.25$$

$$(b) P(1 \leq x \leq 5) = P(X \leq 5) - P(X \leq 1) + P(X = 1) \\ = P(X_0) - P(X_0) + P(X_0) \\ = P(S) - P(D) + P(I) \\ = 0.95 - 0.55 + 0.2$$

$$(c) P(X \leq 2) = P(X = -3) + P(X = -1) + P(X = 0) + P(X = 1) \\ = 0.1 + 0.2 + 0.15 + 0.2 + 0.1$$

$$= 0.75$$

$$(d) P(X > 2) = 1 - P(0) + P(0) \\ = 1 - 0.45 + 0.15 \\ = \underline{\underline{\underline{\underline{0.40}}}}$$

Q4) Let $f(x)$ be continuous random variable with pdf

$$= \frac{2x+1}{2}$$

$$= 0$$

otherwise

Obtain cdf of x . ~~Find mean~~

Solution: By definition of cdf

$$F(x) = \int_{-2}^x t dt$$

$$= \int_{-1}^{0.1} \frac{x+1}{2} dx$$

$$= \frac{1}{2} \left(\frac{1}{2} x^2 + 2x \right) \text{ for } -1 < x < 1$$

$$\therefore \text{Cdf is } F(x) = 0 \text{ for } x < -2$$

$$= \frac{1}{18} \left(\frac{1}{2} x^2 + 2x \right)$$

for $-2 < x < 1$

$$= 0 \dots \text{for } x \geq 1$$

Hence cdf is

$$F(x) = 0 \text{ for } -1 < x < 1$$

$$= \frac{1}{18} x^2 + \frac{1}{2} x$$

$$= 0$$

Q5) Let f be continuous random variable w.r.t pdf

$$f(x) = \frac{2x+2}{18}, -2 \leq x \leq 4$$

$$= 0, \text{ otherwise}$$

For c.d.f.

Solution: As per given condition

Practical 2

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Title: Binomial Distribution

- Q.1) An unbiased coin is tossed 4 times. Calculate the probability of obtaining no head, atleast one head & more than one tail.

No head:

> $\text{dbinom}(0, 4, 0.5)$
[1] 0.0625

Atleast one head:

> $1 - \text{dbinom}(0, 4, 0.5)$
[1] 0.9375

More than one tail:

> $\text{pbinom}(1, 4, 0.5, \text{lower.tail} = F)$
[1] 0.9375

- Q.2) The probability that student is accepted to a prestigious college is 0.3. If 5 students apply, what is the probability that atmost 2 are selected.

> $\text{pbinom}(2, 5, 0.3)$
[1] 0.83692

Q3) An unbiased coin is tossed 6 times. The probability of head at any toss = 0.3. Let x be no. of heads that comes up. Cal. $P(x=2)$, $P(x \geq 3)$; $P(1 < x < 5)$

$\gtbinom{2}{2, 6, 0.3}$

[1] 0.324155

$\gtbinom{3}{3, 6, 0.3}$

[2] 0.18522

$\gtbinom{2}{2, 6, 0.3} + \gtbinom{3}{3, 6, 0.3} + \gtbinom{4}{4, 6, 0.3}$

[1] 0.74373

Q4) For $n=10$, $p=0.6$, evaluate binomial probabilities and plot the graphs of pmf & cdf.

$\gtseq(0, 10)$

$\gtdebinom{x, 10, 0.6}$

\gt

[1] 0.0001048576 0.0015728640 0.006168320

0.0424673280 0.1114767360 0.2006581248

0.2808226560 0.2149908480 0.1209323520

0.0403107840 0.0060466176

$\gtplot(x, y, xlab = "Sequence", ylab = "probabilities", pch = 16)$

(5) Generate a random sample of size 10 for a B.D.
 $\rightarrow B(8, 0.3)$. Find the mean & variance of sample.

> $x = rbinom(8, 10, 0.3)$
 [1] 2 2 3 4 3 2 3

> summary(x)

[1] 2.375

> var(x)

[1] 3.0125

(6) The probability of a man hitting the target is $1/4$, if he shoots 10 times. What is the probability that he hits the target exactly 3 times, probability that he hits target atleast one time

> $dbinom(3, 10, 0.25)$

[1] 0.2502823

> $1 - dbinom(1, 10, 0.25)$

[1] 0.8122883

(7) Bits are sent for communication channel in packet of 12. If the probability of bit being corrupted is 0.1. What is the probability of no more than 2 bits are corrupted in a packet

> $pbinom(2, 12, 0.1, \text{lower.tail} = F) + pbinom(2, 12, 0.1)$

[1] 0.3409977

Practical No. 3

"Normal Distribution"

A normal distribution of 100 student with mean = 40
 $\sigma = 15$

Q) Find no. of student whose marks are :-

$$\begin{aligned} & P(X < 30) \\ & P(40 < X < 30) \\ & P(25 < X < 35) \\ & P(X > 60) \end{aligned}$$

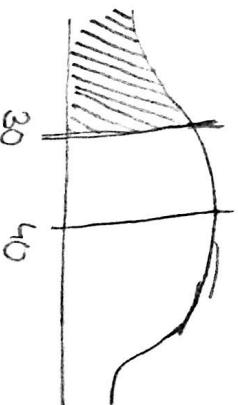
$\Rightarrow P_{\text{norm}}(30, 40, 15)$
 $[E] 0.2524924$

$\Rightarrow P_{\text{norm}}(70, 40, 15) - P_{\text{norm}}(40, 40, 15)$
 $[E] 0.4772499$

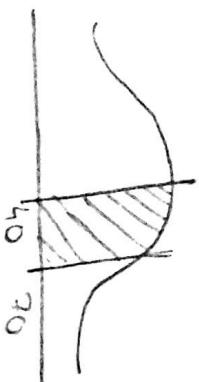
$\Rightarrow P_{\text{norm}}(35, 40, 25) - P_{\text{norm}}(25, 40, 15)$
 $[E] 0.2102861$

$\Rightarrow 1 - P_{\text{norm}}(60, 40, 15)$
 $[E] 0.0912192$

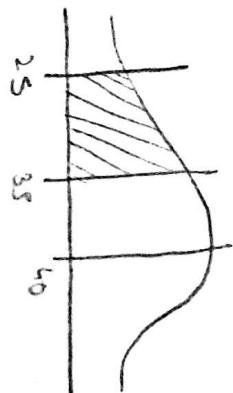
a)



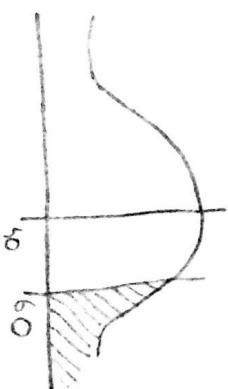
b)



c)

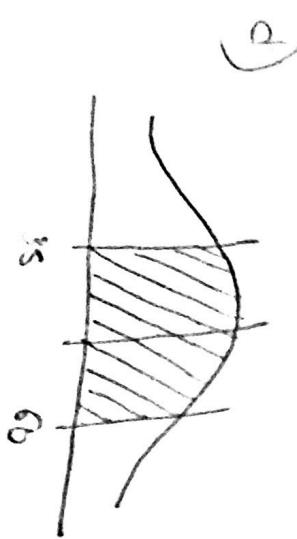


d)

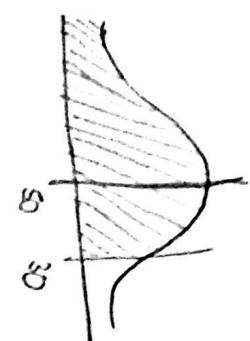
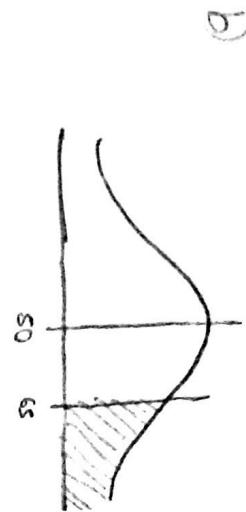


Q3)

(a) If the random variable 'x' follows Normal distribution with mean = 50, $\sigma^2 = 100$



(b)



$$\text{Find :- } P(x \leq 70)$$

$$P(x > 65)$$

$$P(x \leq 32)$$

$$P(35 < x < 60)$$

$$P(20 < x < 30)$$

$$>\text{pnorm}(70, 50, 10)$$

$$[1] 0.9712499$$

$$>\text{pnorm}(65, 50, 10)$$

$$[1] 0.668072$$

$$>\text{pnorm}(32, 50, 10)$$

$$[1] 0.03593032$$

$$>\text{pnorm}(60, 50, 10) - \text{pnorm}(35, 50, 10)$$

$$>\text{pnorm}(30, 50, 10) - \text{pnorm}(20, 50, 10)$$

$$[1] 0.02140023$$

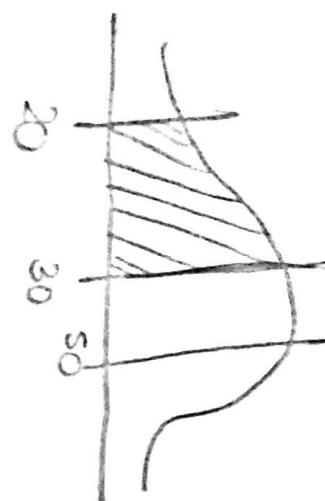
ii

Q3) First min (160, 40). Find k_1 & k_2 such that
 $P(X < k_1) = 0.6$ & $P(X > k_2) = 0.8$

$\rightarrow q_{\text{norm}}(0.6, 160, 20)$
[2] 165.0669

$\rightarrow q_{\text{norm}}(0.8, 160, 20)$
[2] 176.8324

(Q2)
e)



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Practical No. 4

Sample mean & standard deviation gives single population.

- (g) Suppose the food label on the cookie bag states that it has almost 2 gms of saturated fat in a single cookie. In a sample of 35 cookies, it was found that mean amount of saturated fat per cookie is 2.1 gms. Assume that the sample standard deviation is 0.3 at 15% level of significance can be rejected the claim on food label.
 To check whether 'reject' or 'accept', null hypothesis at 95% level of confidence or 5% level of significance.

$$\sigma = 0.3$$

$$n = 35$$

$$\bar{x} = 2.1$$

$$\mu = 2$$

$$H_0 \text{ (null hypothesis)} = \mu \leq 2$$

$$H_1 \text{ (alternate hypothesis)} = \mu > 2$$

$$Z = \frac{\bar{x} - \mu}{\sigma}$$

$$= \frac{2.1 - 2}{\sqrt{n}}$$

$$= \frac{0.1}{\sqrt{35}}$$

$$= 1.0972027$$

- $\therefore \text{P-value} = 1 - \text{Pr}(Z > 2)$
- $= 0.273$

\therefore reject the null hypothesis

$$\text{P-value} < 0.05$$

Accepted alternate hypothesis

(Q2)

A sample of 100 customers was randomly selected. It was found that average spending was 295%. The 50-30 using 0.05 level of significance, would conclude that the amount spent by the customer is more than 250/- whereas the restaurant claims that it is not 250/-.

\Rightarrow Reject the null hypothesis

$\therefore \text{P} < 0.05$

- (Q3)
- A principal at school claims that the IQ is 100 of students. A rank sample of 30 students whose IQ was found to be 112. The S.D. of population is 15. Test the claim of principal.

\Rightarrow Method I:

$$\bar{x} = \frac{295 - 250}{\sqrt{100}} = 4.5$$

- From ($Z, 99, \text{lower tail} = F$)
 $\text{P-value} = 1.3057 \rightarrow 0.13$

- (Q3) A quality control engineer takes a sample of 100 bulbs, having an average life of 470 hours. Assuming population test whether the population mean < 480 hrs at 10% $\rightarrow 0.05$

$$\Rightarrow n = 100, \bar{x} = 470, u = 480, \sigma = 25$$

$$\bar{Z} = \frac{\bar{x} - u}{\frac{\sigma}{\sqrt{n}}}$$

$$\text{P}(\bar{Z} > 2.99, \text{lower tail}) \\ = 6.112576 \approx 0.05$$

\Rightarrow Reject the null hypothesis

\therefore Accept the alternate hypothesis ($H_1 < 480$)

- (Q3)
- A principal at school claims that the IQ is 100 of students. A rank sample of 30 students whose IQ was found to be 112. The S.D. of population is 15. Test the claim of principal.

\Rightarrow Method II:

$$H_0: u = 100$$

$$H_1: u < 100$$

$$\bar{Z} = \frac{\bar{x} - u}{\frac{\sigma}{\sqrt{n}}} = \frac{112 - 100}{\frac{15}{\sqrt{30}}} = 4.38178$$

• P-value = 5.88567×10^{-6}

• Project the null hypothesis = claim of mind.

$$\boxed{u = 100}$$

\Rightarrow Method 2:

$$H_0: \mu = 100$$

$$H_1: \mu \neq 100$$

$$\bullet P\text{-value} = 2(1 - \text{pnorm}(\text{abs}(2)))$$

$$= 1.977134 \times 10^{-5}$$

Reject the null hypothesis

$$\therefore P\text{-value} < 0.05$$

* Single Population Proportion:-

• It is believed that coin is fair. The coin is

tossed 50 times, 25 times head. Indicate whether

the coin is fair or not at 95% L.O.C.
↳ Probability of sample

$$Z = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} - \text{probability of population}$$

$$P_0 = \begin{cases} P = P_0 \\ P \neq P_0 \end{cases}$$

$$Z = \frac{(P - P_0)}{\sqrt{P_0(1-P_0)/n}} (\text{sqrt}(P_0(1-P_0)/n))$$

$$Z = 1.2645$$

$$P_0 = 0.5 \quad , q_0 = 1 - P_0 = 0.5$$

$$P = \frac{28}{50} = 0.7 \quad , u = 28$$

$$Z = \frac{0.7 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{50}}} = 0.07$$

$$\bullet H_0: \mu = 0.5$$

$$\bullet H_1: \mu \neq 0.5$$

$$\bullet P\text{-value} = 2(1 - \text{pnorm}(\text{abs}(2)))$$

$$= 0.01141209$$

Reject the null hypothesis

$$\therefore P < 0.05$$

• Accept the alternate hypothesis

↳ In a hospital 480 females & 520 males are born in a week to confirm male & female are equal in sex.

$$Z = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \Rightarrow P = \frac{520}{1000} = 0.52 \quad P_0 = 0.5$$

$$q_0 = 0.5 \quad n = 1000$$

$$P_0 = \begin{cases} P = P_0 \\ P \neq P_0 \end{cases}$$

$$Z = \frac{(P - P_0)}{\sqrt{P_0(1-P_0)/n}} (\text{sqrt}(P_0(1-P_0)/n))$$

$$Z = 1.2645$$

$$\bullet P\text{-value} = 2 \times (1 - \text{pnorm}(\text{abs}(2)))$$

$$= 0.260506$$

• Project the null hypothesis, < 0.5

• Accept the alternate hypothesis, i.e. $P \neq 100$

(Q2) In a big city 325 users of 600 users are found to be self employed. Conclusion is that maximum users in city are self employed.

$$Z = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \quad P = 0.541607$$

$$P = \frac{325}{600} = 0.541607 \quad P_0 = 0.5 \quad n = 600$$

$$Z = \frac{(0.541607 - 0.5)}{\sqrt{(0.5 * 0.5) / 600}}$$

$$H_0 = [P = 0.5] \quad H_1 = [P \neq 0.5]$$

$$Z = [(0.541607 - 0.5) / \sqrt{(0.5 * 0.5) / 600}]$$

$$Z = 2.03975$$

$$\text{P-value} = 2 \times (1 - \text{Pr}(Z < 2.03975))$$

i. Reject the null hypothesis

ii. P-value < 0.5

iii. Accept the alternate hypothesis
i.e. $P \neq P_0$

$$Z = \frac{P_1 - P_0}{\sqrt{\frac{P_0(1-P_0)}{n+m}}} \quad P_1 = 0.125 \quad P_0 = 0.2 \quad n = 400 \quad m = 600$$

$$Z = \frac{0.125 - 0.2}{\sqrt{(0.2 * 0.8) / 1000}} = -0.68346$$

Reject the null hypothesis: P-value < 0.5

Accept the alternate hypothesis: P-value ≠ 0.2

* formula:

$$Z = \frac{P_1 - P_0}{\sqrt{\frac{P_0(1-P_0)}{n+m}}} \quad \text{where } P = \frac{P_1 n + P_2 m}{n+m}$$

- (Q) In an election campaign a telephone of 800 registered registered were ~~showed~~ P-value 650. Second poll opinion 620 of 1000 registered voters favoured the candidate at 0.5% L.O.C
is there sufficient evidence that probability has decreased

$$n = 800, P_1 = 600/800 = 0.575 \\ m = 1000, P_2 = 620/1000 = 0.52$$

(Q3) Experience shows that 20% of manufactured products are of top quality. In 1 day, production of 400 articles only 50 are of top quality. Test the hypothesis that experience of 10% of manufactured products.

$$P = (0.575 * 800 + 0.52 * 1000) / 1800$$

$$\boxed{P = 0.544}$$

$$Z = 0.011215 \frac{92}{\underline{\underline{92}}}$$

$$H_0: P = 0.544$$

$$H_1: P < 0.544$$

$$P_{\text{value}} = 2 \times (1 - \text{norm}(\text{abs}(Z)))$$

$$P_{\text{value}} = 0.9991 \underline{\underline{0553}}$$

∴ Reject the null hypothesis

$$\therefore P_{\text{value}} > 0.5$$

$$\therefore \text{Accept } P = 0.544$$

(Q2) From a consignment A 100 articles are drawn

244 were found defective from consignment

200 samples are drawn out of which 50

are defective. Test whether the proportion of

defective items in 2 consignments are

significantly different.

$$\rightarrow H_0: P_1 = P_2$$

$$H_1: P_1 \neq P_2$$

$$P_1 = \frac{50}{200} = 0.25$$

$$n_2 = 200$$

$$\begin{aligned} P_2 &= 30 / 200 = 0.15 \\ P &= \frac{(P_1 n_1 + P_2 n_2)}{n_1 + n_2} \\ P &= \frac{(0.22 * 200 + 0.15 * 200)}{400} \end{aligned}$$

$$Z = \frac{(0.22 + (0.15 * 0.815) * (21200)) - 0.00388476}{\sqrt{0.00388476}}$$

$$P_{\text{value}} = 0.9969018$$

$$\therefore P_{\text{value}} > P$$

∴ Accept the null hypothesis

$$\text{i.e. } P_1 = P_2$$

Note

Practical 5

Title: Chi-square Test

- (a) Use the following data to test whether the attribute condition of home & child are independent.

Condition of Home	Own	Don't Own	No. of dice
Chair	70	30	1
80	20	3	25
35	45	5	18
		6	10
		22	22
		15	15

H_0 = Both are independent, H_1 = Both are dependent

$$\begin{aligned} >& x = c(70, 80, 35) \\ >& y = c(50, 20, 45) \\ >& z = \text{chisq.test}(x, y) \end{aligned}$$

$[z]$

24
20
25
35

$[z] 20$

$$\chi^2 = \sum ((\text{obs}) - \text{exp})^2 / (\text{exp})$$

$$\text{pchisq}(\chi^2, \text{df} = \text{length}(\text{obs}) - 1)$$

χ^2

- Project null hypothesis
Both are dependent

Test the hypothesis that the dice is unbiased.

$\therefore H_0$ = dice is unbiased
 H_1 = dice is biased

$$\begin{aligned} >& \text{obs} = c(30, 25, 15, 10, 22, 15) \\ >& \text{exp} = \text{sum}(\text{obs}) / \text{length}(\text{obs}) \end{aligned}$$

exp

$$\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{obs} & 30 & 25 & 15 & 10 & 22 & 15 \\ \text{exp} & 20 & 20 & 20 & 20 & 20 & 20 \end{array}$$

$$\chi^2 = \sum ((\text{obs}) - \text{exp})^2 / (\text{exp})$$

- χ^2
Accept the null hypothesis
dice is unbiased.

(Q3) A IQ test was conducted & the students obtained before & after training, the results are following

Following

Before

110

120

118

120

123

132

125

121

126

121

After

110

120

125

123

132

125

121

Before 40
After 45

(Q4) online Graduate 20 Undergraduate (U.G.) 25

Test whether there is a change in IQ after the training

H_0 = No change in IQ

$\therefore H_1$ = Change in IQ

$a_{2 \times 5} (120, 118, 125, 136, 121)$
 $b_{2 \times 5} (110, 120, 125, 132, 125)$
 $2 = \text{sum}((a - b)^2 / 2)$

data: 2

$$\begin{aligned} X &= c(20, 40, 25, 5) \\ 2 &= \text{matrix}(X, nrow=2) \\ \Rightarrow \chi^2 &= \text{chisq.test}(2) \end{aligned}$$

$$\chi^2 = 18.5, \text{ df} = 1, \text{ pvalue} = 2.157 \times 10^{-4.5}$$

Reject null hypothesis

Both are independent.

\therefore There is change in IQ after training.

Q8) A dice is tossed 180 times.

No. of faces	Freq
1	20
2	30
3	35
4	40
5	12
6	43

Test the hypothesis that dice is unbiased

$H_0 \rightarrow$ dice is biased

$H_1 \rightarrow$ dice is unbiased

$\chi^2 \text{ test}(x^2)$
 $\text{data: } 2$

$\chi^2 - \text{square} = 23.933$, df = 5, p-value = 0.000

• Reject null hypothesis

• Dice is unbiased.

Very

Practical 6: t-test

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Title: t-test

Q) Let $x \in [3366, 3337, 3361, 3410, 3316, 3357, 3348, 3356, 3376, 3382, 3377, 3355, 3408, 3401, 3398, 3424, 3383, 3374, 3384, 3374]$.

Write the R command for following test hypothesis.

- ① $H_0: \mu = 3400 ; H_1: \mu \neq 3400$
- ② $H_0: \mu = 3400 ; H_1: \mu > 3400$
- ③ $H_0: \mu = 3400 ; H_1: \mu < 3400$

at 95% level of confidence. Also check at 97% level of confidence

① $H_0: \mu = 3400$

$H_1: \mu \neq 3400$

$> x = c(3366, 3337, 3361, 3410, 3316, 3357, 3348, 3356, 3376, 3382, 3377, 3355, 3408, 3401, 3398, 3423, 3383, 3374, 3384, 3374)$

$> t.test(x, mu=3400, alternative="two.sided", conf.level=0.95)$

One sample t-test

data: x

$t = -5.5865, df = 19, p-value = 0.0002528$

alternative hypothesis: true mean is not equal to 3400

95 percent confidence level: 3361.797 3386.103

Sample estimates:
Mean of x : 3373.95

\therefore Reject H_0

\therefore Accept H_1

$> t\text{-test} (\bar{x}, \mu = 3400, \text{alter} = \text{"two-sided"},$
conf. level = 0.97)

One sample t-test

data: x

$t = -4.4865, df = 19, p\text{-value} = 0.00025$

alternative hypothesis: true mean is not equal
to 3400

97 percent confidence level: 3360.33 3387.57

Sample estimates

Mean of x : 3373.95

\therefore Reject H_0

\therefore Accept H_1

② $H_0: \mu = 3400$

$H_1: \mu > 3400$

$> t\text{-test} (\bar{x}, \mu = 3400, \text{alter} = \text{"greater"}, \text{conf.}$
level = 0.95)

One sample t-test

data: x

$t = -4.4865, df = 19, p\text{-value} = 0.9999$

alternative hypothesis: true mean is greater than
3400

3363.95

Sample estimates

Mean of x : 3373.95

\therefore Accept H_0

$>t\text{-test}(\text{x}, \text{mu}=3400, \text{alter}=\text{"greater"}, \text{conf.level}=0.9)$
 One sided t-test

data: x

$t = -4.4865, df = 19, p\text{-value} = 0.9999$

alternative hypothesis: true mean is greater than 3400
 33.67337 Inf

Sample estimates:

Mean of x: 33.73.95

$\therefore \text{Accept } H_0$

$\exists H_0: M = 3400$

$H_1: M \neq 3400$

$>t\text{-test}(\text{x}, \text{mu}=3400, \text{alter}=\text{"less"}, \text{conf.level}=0.95)$
 One sided t-test

data: x

$t = -4.4865, df = 19, p\text{-value} = 0.0001265$

alternative hypothesis: true mean is less than 3400

95 percent confidence level

-Inf 33.83.99

Sample estimates:

Mean of x:
 33.73.95

$\therefore \text{Reject } H_0$

$\therefore \text{Accept } H_1$

$>t\text{-test}(\text{x}, \text{mu}=3400, \text{alter}=\text{"less"}, \text{conf.level}=0.97)$

One sided t-test

Data: sc

t = -4.04865, df = 19, p-value = 0.0001263

alternative hypothesis: true mean is less than 340,
97 percent confidence level

-Inf 338 5.563

Sample estimates:

Mean of sc:

3373.95

∴ Reject H₀

.∴ Accept H₁

Q2) Below are the data of gain in weights of 24 different data A & B.

Data A: 25, 32, 30, 43, 24, 14, 32, 24, 31, 31, 35, 25

Data B: 44, 34, 22, 10, 42, 31, 40, 30, 32, 35, 18, 21

∴ H₀: a - b = 0

∴ H₁: a - b ≠ 0

>Data A: {25, 32, 30, 43, 24, 14, 32, 24, 31, 31, 35, 25}

Data B: {44, 34, 22, 10, 42, 31, 40, 30, 32, 35, 18, 21}

Paired t-test (a, b, paired = T, alter = "two-sided", conf. level = 0.95)

Paired t-test

data: a and b

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence level

$$-14.0267330 \quad 7.933997$$

Sample estimates:

Mean of the differences:

$$-3.166667$$

∴ Accept H_0

∴ There is no difference in weights

(iii) 11 students gave the test after 1 month, they again gave the test after coaching, do the marks give evidence that students have benefitted by coaching.

$$E_1: 23, 20, 19, 21, 28, 20, 18, 17, 23, 16, 19$$

$$E_2: 25, 19, 22, 18, 20, 22, 20, 20, 23, 20, 17$$

test at 99% level of ~~confidence~~ confidence.

~~$$E_1 = \{23, 20, 19, 21, 18, 20, 18, 17, 23, 16, 19\}$$~~

~~$$E_2 = \{25, 19, 22, 18, 20, 22, 20, 20, 23, 20, 17\}$$~~

∴ $H_0: E_1 = E_2$

∴ $H_1: E_1 < E_2$

t-test (E_1, E_2 , paired = T, alter = "less", conf.level = 0.99)

Q1

Paired t-test

Data: E_1 and E_2

alternative hypothesis:

$t = -1.4832$, $df = 10$, p-value = 0.0855

alternative hypothesis: true mean difference in means less than 0

99 percent confidence level.

-Inf 0.863333

Sample estimates:

Mean of the differences:

-1

∴ Accept H_0

Q4) Two drugs for BP was given and data was collected

$D_1: 0.7, -1.8, -0.2, -1.2, -0.1, 3.4, 3.7, 0.8, 0.2$

$D_2: 1.9, 0.8, 1.1, 0.1, -0.1, 4.4, 5.5, 1.6, 4.6, 3.4$

The two drugs have same effect, check whether two drugs have same effect on patient or not

$H_0: D_1 = D_2$

$H_1: D_1 \neq D_2$

$\Rightarrow D_1 = c(0.7, -1.6, -0.2, -1.2, -0.1, 3.4, 3.7, 0.8, 0.2)$

$\Rightarrow D_2 = c(1.9, 0.8, 1.1, 0.1, -0.1, 4.4, 5.5, 1.6, 4.6, 3.4)$

\Rightarrow t-test (D_1, D_2 , alter = "two.sided", paired = T, conf.level = 0.95)

Paired t-test

data: D₁ and D₂

$t = -4.0621$, df = 9, p-value = 0.002833
alternative hypothesis: true difference in means
is not equal to 0

95 percent confidence level

Mean of the difference:

-10.58

∴ Reject H₀

∴ Accept H₁

If there is difference in salaries for the same job in 2 different countries.

CA: 53000, 49958, 51974, 44366, 50470, 36963
CB: 62490, 58850, 49495, 52263, 47674, 43552

∴ H₀: CA = CB

∴ H₁: CA ≠ CB

> CA = c(53000, 49958, 51974, 44366, 50470, 36963)

> CB = c(62490, 58850, 49495, 52263, 47674, 43552)

✓ t-test (CA, CB, paired = T, alter = "two.sided",
conf.level = 0.95)

Paired t-test

data: CA and CB

$t = -40.4569$, df = 5, p-value = 0.00666

alternative hypothesis : true difference in means
not equal to 0
95 percent confidence level
 -10404.821 -2792.896

Sample estimates:

Mean of the differences:

$$-6598.833$$

- ∴ Reject H_0
- ∴ Accept H_1

Practical 7 (F-test)

53

Life expectancy in 10 regions in India in 1990 & 2000 are given below:

Test whether the variances at the two times are same.

$$H_0: \sigma^2(y_1) = \sigma^2(y_2)$$

$$H_1: \sigma^2(y_1) \neq \sigma^2(y_2)$$

1990: 37, 39, 36, 42, 45, 44, 46, 49, 50, 51, 52

2000: 44, 45, 47, 43, 42, 49, 50, 41, 52, 59

$$\rightarrow y_1 = c(37, 39, 36, 42, 45, 44, 46, 49, 50, 52)$$

$$\rightarrow y_2 = c(44, 45, 47, 43, 42, 49, 50, 41, 52, 59)$$

$$\rightarrow \text{var.test}(y_1, y_2)$$

F test to compare two variances

Data: y_1 and y_2

~~$F = 3.4085$, num df = 10, denom df = 10, p-value = 0.06609~~

~~alternative hypothesis: true ratio of variances is not equal to 1~~

~~95 percent confidence level:~~

~~0.9170663 12.6688513~~

Sample estimates:

ratio of variances

3.408545

∴ Accept H_1

Q2) For the foll. data of the two samples. Test whether the variance of the two sample are same.

a: (25, 28, 26, 22, 22, 29, 31, 31, 26, 31)
 b: (30, 25, 31, 32, 23, 25, 36, 25, 31, 32, 32, 21, 31, 38, 24)

$$H_0: \sigma(a) = \sigma(b)$$

$$H_1: \sigma(a) \neq \sigma(b)$$

> a=c(25, 28, 26, 22, 22, 29, 31, 31, 26, 31)
 > b=c(30, 25, 31, 32, 23, 25, 36, 25, 31, 32, 32, 21, 31, 38, 24)

> Shapiro.test(a)

Shapiro-Wilk normality test

data: a

W = 0.89253, P-value = 0.181

> Shapiro.test(b)

Shapiro-Wilk normality test

data: b

W = 0.92642, P-value = 0.2411

> var.test(a, b)

F test to compare two variances
data: a and b

$F = 0.60558$, num df = 9, denom df = 14, p-value = 0.4535

alternative hypothesis: true ratio of variances is not equal to 1

95 percent confidence interval:

0.1886943 2.2999517

Sample estimates:

ratio of variances

0.6055767

∴ Accept H_0

- For the foll. data, test the hypothesis for:
- ① Equality of 2 population mean
 - ② Equality of 2 population variance

I: 175, 168, 148, 190, 181, 185, 175, 200

II: 180, 170, 153, 180, 179, 183, 187, 205

$$H_0: \sigma(I) = \sigma(II)$$

$$H_1: \sigma(I) \neq \sigma(II)$$

$$H_0: m(I) = m(II)$$

$$H_1: m(I) \neq m(II)$$

~~I = c(175, 148, 168, 190, 181, 185, 175, 200)~~

~~II = c(180, 170, 153, 180, 179, 183, 187, 205)~~

2.2

> shapiro.test (I)

Shapiro-Wilk normality test

data: I

W = 0.96943 , p-value = 0.8943

> shapiro.test (II)

Shapiro-Wilk normality test

data: II

W = 0.96112 , p-value = 0.8283

for now

① > t.test (I, II)

Welch Two Sample t-test

data: I and II

t = -0.32965 , df = 11.979 , p-value = 0.747

alternative hypothesis : true difference is not equal to 0

95 percent confidence interval:

-21.74526 16.03098

Sample estimates.

Mean of x and y

176.7143 179.5714

∴ Reject H_0

∴ Accept H_2

② > var.test (I, II)

Data: I and II
F test to compare two variances

$F = 1.087$, num df = 6, denom df = 6, p-value = 0.9218
alternative hypothesis: true ratio of variances is not equal to 1

95 Percent confidence Interval:

0.1867998 6.3268267

Sample estimates:

Ratio of variances:

1.087129

i) The following prices of commodity in the sample of shops selected at random from 2 different cities

City A = 74.10, 77.70, 75.35, 74, 73.80, 79.30,
75.80, 76.80, 79.10, 76.40

City B = 70.80, 75.90, 76.20, 72.80, 78.10,
73.70, 69.80, 81.20

$x = c(74.10, 77.70, 75.35, 74, 73.80, 79.30,$
 $75.80, 76.80, 79.10, 76.40)$

$y = c(70.80, 75.90, 76.20, 72.80, 78.10,$
 $73.70, 69.80, 81.20)$

→ Shapiro-test (x)

p-value = 0.6559

∴ data is normal

→ Shapiro-test (y)

p-value = 0.9304

∴ data is normal

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

P-value = 0.04249

2 variances are not equal

∴ Reject H_0

∴ Accept H_1

$$H_0: \bar{X}_1 = \bar{X}_2$$

$$H_1: \bar{X}_1 \neq \bar{X}_2$$

> t-test (x, var.equal = R)

P-value = 3.488e-16

> t-test (y, var.equal = F)

P-value = 1.46e-10

∴ Accept H_0

Q5) Prepare a .csv file in excel. Import the file in R and apply the test to check the equality of variances of 2 dots.

Obs 1: 10, 15, 17, 11, 16, 20

$$H_0: \sigma_1^2 = \sigma_2^2$$

Obs 2: 15, 14, 16, 11, 12, 19

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

3 data = read.csv (file.choose(), header = T)

3 data

Obs..1

Obs..2

10

15

15

14

17

16

11

11

16

12

25?

18

> attach(data)
> mean(obs..1)

15.83333

> var.test(obs..1, obs..2)

p-value = 0.5717

: Accept H_0

Revised
Answer

P.G Practical No. 8

Non-Parametric Test

- (Q1) The times of failure (in hours) of 10 randomly selected 9V batteries of a certain company are as follows
- 28.9, 15.2, 28.7, 72.5, 48.6, 52.4, 37.6, 49.5,
62.1, 54.5

Test the hypothesis that population median is less than 63.

Solution: $H_0: \text{Median} = 63$

$H_1: \text{median} < 63$

1) $\text{dec} (28.9, 15.2, 28.7, 72.5, 48.6, 52.4, 37.6, 49.5, 62.1, 54.5)$

2) $S_n = \text{length } (\text{which } x < 63)$

3) $S_p = \text{length } (\text{which } x > 63)$

4) $n = S_p + S_n$

5) $q \text{ binom}(0.05, n, 0.5)$

6) $S_n = 9$

Since $q \text{ binom} \leq S_n$, H_0 is accepted

7) $p \text{ binom}(S_n, n, 0.5)$
 0.9990235

Since $p \text{ binom} > 0.5$, Accept H_0

∴ H_0 is accept

(b) Data of weights of 40 students is given. Test the hypothesis that the median is 50 against it is greater than 50.

Solution:

$$H_0: \text{median} = 50$$

$$H_1: \text{median} > 50$$

$$\begin{aligned} > x = c(46, 49, 52, 64, 46, 67, 54, 48, 49, 61, 57, \\ & 50, 48, 65, 61, 66, 54, 50, 48, 49, 62, \\ & 47, 49, 47, 55, 59, 63, 53, 56, 57, \\ & 49, 60, 64, 53, 50, 48, 51, 52, 54) \end{aligned}$$

$$> S_n = \text{length}(\mathbf{x} [x < 50])$$

$$> S_p = \text{length}(\mathbf{x} [x > 50])$$

$$> n = S_p + S_n$$

$$> q_{\text{binom}} (0.05, n, 0.5)$$

14

$$> S_n$$

13

Since $q_{\text{binom}} > S_n$, Reject H_0

$\therefore H_0$ is rejected.

- Q) The median age of tourist visiting the central place is to be 41 years. A random sample of 17 tourists is given. Use the sign test to check the claim.

Solution: $H_0: \text{median} = 41$

$$H_1: \text{median} \neq 41$$

$$\rightarrow x = c(25, 24, 48, 52, 57, 39, 45, 36, 30, 49, 28, 39, 44, 63, 32, 65, 42)$$

$$\begin{aligned} & S_p = \text{length} \{ \text{which } (x > 41) \} \\ & S_n = \text{length} \{ \text{which } (x < 41) \} \end{aligned}$$

$$\text{qbinom}(0.05, n, 0.5)$$

$$\begin{matrix} 5 \\ S_n \end{matrix}$$

$$\begin{matrix} 8 \\ 8 \end{matrix}$$

#

Since $\text{qbinom} < S_n$, H_0 is accepted

∴ Median $= 41$

∴ H_0 is accepted

Q) The time in minutes that a patient have to wait for consultation is recorded.
 Use Wilcoxon signed test to test which level of significance if median is more than 20 at 5%.

Solution: $H_0: \text{median} > 20$
 $H_1: \text{median} < 20$

$\rightarrow x = c(15, 17, 24, 25, 20, 21, 32, 28, 32, 25, 24, 26)$

$\rightarrow \text{Wilcox.test}(x, \text{alternative} = \text{"less"})$
 0.6999

\therefore Accept H_0

Q) The weight of person before and after quitting smoking is given. Check whether the weight increases after he stops smoking. Using Wilcoxon test at 5% level of significance.

~~$b = c(65, 75, 75, 62, 72)$~~
 ~~$a = c(72, 82, 72, 66, 74)$~~

$\rightarrow x_2 = b - a$

x

"7 -7 3 -4 -1

8.3.

> Wilcox.t test (x_c , $\mu_0 = 0$)
p-value = 0.1756

Since pvalue > 0.05

∴ Accept H_0

Next:

Practical 9 (ANOVA Table)

The foll. data gives the effect of three treatments

Treatment 1	Treatment 2	Treatment 3
2	10	10
3	8	13
7	7	14
2	5	13
6	10	15

$$t_1 = c(2, 3, 7, 2, 6)$$

$$t_2 = c(10, 8, 7, 5, 10)$$

$$t_3 = c(10, 13, 14, 13, 15)$$

H_0 : Treatments are equally effective

H_1 : Treatments are not equally effective

$d = \text{data.frame}(t_1, t_2, t_3)$

$e = \text{stack}(d)$

$\text{oneway.test(values ~ ind, data = e)}$

p-value 0.0006282

There is enough evidence to reject H_0 .

Q2) The foll. is life of tyres of 4 brands

A	B	C	D
20	19	21	15
23	15	19	14
18	17	22	16
17	20	17	18
22	16	20	14
24	17		16

→ Test the hypothesis so that the avg life of all brands is same

Solution H_0 : Life of all brands of tyres is same
 H_1 : Life of all brands of tyres is different

- ✓ A = c (20, 13, 18, 17, 22, 24)
- ✓ B = c (14, 15, 17, 20, 16, 12)
- ✓ C = c (21, 19, 22, 17, 20)
- ✓ D = c (15, 14, 16, 18, 14, 16)
- ✓ Z = list ($P = A$, $Q = B$, $R = C$, $S = D$)
- ✓ R = stock (Z)
- ✓ One way test (values ~ ind, data = k)

$$\text{p-value} = 0.01762$$

∴ H_0 is rejected

- Q3) Three brands of wax applied for protection of car and no. of days of protection were noted. Test whether these are equally effective.
- H_0 : Equally Effective H_a : Not equally effective
- $w_1 = c(44, 45, 46, 47, 48, 49)$
- $w_2 = c(40, 42, 51, 52, 55)$
- $w_3 = c(50, 53, 58, 59)$
- $u = \text{list}(t_2 w_1, f_2 w_2, h = w_3)$
- $q_2 \text{ stack}(u)$
- One-way test (values ~ ind, data = i)

$$\text{P-value} = 0.3822$$

We have enough evidence to reject H_0 .

- Q4) The experiment was conducted on 8 persons & the observation was noted

I : No exercise

II : 20 mins of exercise

III : 60 mins of exercise

Test the hypothesis that all groups have equal results on exercise

H_0 : The results are equal

H_a : The results are not equal

- $I = c(23, 26, 51, 48, 58, 37, 29, 44)$
- $II = c(22, 27, 29, 39, 46, 48, 49, 65)$
- $III = c(59, 66, 38, 49, 56, 60, 56, 62)$

58

> z1 = list($r_1 = I$, $r_2 = II$, $r_3 = III$)
> stk2 stack(z1)
> oneway.test(values ~ ind, data = stk)

$$p\text{-value} = 0.01633$$

$\therefore H_0$ is rejected

reject