Q7) $\frac{dy}{dx} = \sin^2(x-y-2) - x$

Differentiating both sides

$x-y+1 = v$

$\therefore 1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dy}{dx} = \cos^2 x$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + C$

$\tan(x+y-1) = \underline{\underline{x+C}}$

Q8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$\Rightarrow 2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\Rightarrow \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\Rightarrow \frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\Rightarrow \frac{dv}{dx} = \frac{v-1+2v+4}{v+2} = \frac{3v+3}{v+2} = 3\left(\frac{v+1}{v+2}\right)$

$\Rightarrow \int \frac{v+1}{v+1} \, dv + \int \frac{1}{v+1} \, dv = 3x + C$

$\Rightarrow v + \log|v+1| = 3x + C$

$\Rightarrow 2x + 3y + \log|2x+3y+1| = 3x + C$

$\Rightarrow 3y = x - \log|2x+3y+1| + C$

05/02/2020

Practical No. 8

* Using Euler's method. Find the following:

1) $\frac{dy}{dx} = y + e^x - 2$, $y(0)$, $h = 0.5$. Find $y(2)$

2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$. Find $Y(1)$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$. Find $y(1)$

4) $\frac{dy}{dx} = 3x^2 + 1$, $Y(1) = 2$. Find $X(2)$

for $h = 0.5$ & $h = 0.25$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$. Find $y(1.2)$ with h=...

→ Solutions:

Q1) $f(x,y) = y + e^x - 2$

$y(0) = 2$ $h = 0.5$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.2231
2	1	3.2231	3.9414	5.1938
3	1.5	5.1938	7.6755	9.0315

$y(2) = 9.0315$

Q2) Solution:

$\frac{dy}{dx} = 1 + y^2 = f(x,y)$

$y(0) = 0$; $h = 0.2$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.405
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8529	1.2942

$y(1) = 1.2942$

Q3) Solutions:

$f(x,y) = \sqrt{\frac{x}{y}}$; $y(0) = 1$; $h = 0.2$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.472	1.0894
2	0.4	1.0894	0.6059	1.2106
3	0.6	1.2106	0.704	1.3514
4	0.8	1.3514	0.7694	1.5053

$$y(1) = 1.5053$$

Q4) Solutions:

$f(x,y) = 3x^2+1$; $y(1) = 2$; $h = 0.5$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
1	1	2	4	4
2	1.5	4	7.75	7.875

$$y(2) = 7.875$$

Q3) $f(x,y) = \sqrt{xy}+2$; $y(1) = 1$; h

$h = 0.25$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
1	1	2	4	4
2	1.25	3	5.6875	4.4219
3	1.5	4.4219	7.75	6.3594
4	1.75	6.3594	10.1875	8.9063

$$y(1) = 8.9063$$

Q5) $f(x,y) = \sqrt{xy}+2$; $y(1) = 1$, $h = 0.2$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
1	1	1	3	1.6

$$y(1.2) = 1.6$$

Practical No. 9

Topic:- Limits & partial-order derivative

① Evaluate the foll. limits:-

i) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$ ii) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

iii) $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2y^2}$

2) Find f_x, f_y for each of the foll. f:-

i) $f(x,y) = xy\, e^{x^2+y^2}$ ii) $f(x,y) = e^x \cos y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

3) Using definition find values of f_x, f_y at $(0,0)$

for : $f(x,y) = \dfrac{2x}{1+y^2}$

4) Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \dfrac{y^2-xy}{x^2}$ ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

iii) $f(x,y) = \sin(xy) + e^{x+y}$

Q5) Find the linearization of $f(x,y)$ at given point:-

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

→ Solutions:-

Q1)

i) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$\dfrac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$

$\dfrac{64+3+\cancel{1}-\cancel{1}}{4+5}$

$\dfrac{67}{9}$

ii) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$\dfrac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)}$

$\dfrac{1\,(4+0-8)}{2}$

$\dfrac{-4}{2} = \underline{\underline{-2}}$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2y^2}$

$\dfrac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)(1)}$

$\dfrac{1-1}{1-1}$

0 ... (solⁿ not defined)

Q2) i) $f(x,y) = xye^{x^2+y^2}$

$= x \cdot y \cdot e^{x^2} \cdot e^{y^2}$

$f_x = \dfrac{\partial}{\partial x} x \cdot y \cdot e^{x^2} \cdot e^{y^2}$

$= ye^{y^2} \cdot \dfrac{d}{dx}\, x \cdot e^{x^2}$

$= ye^{y^2}\left[x \dfrac{\partial}{\partial x} e^{x^2} + e^{x^2} \dfrac{\partial}{\partial x} y\right]$

$= ye^{y^2}\left[2x^2 e^{x^2} + e^{x^2}\right]$

$= (2x^2+1)\, ye^{x^2+y^2}$

$f_y = xe^{x^2} \dfrac{\partial}{\partial y}\, y \cdot e^{y^2}$

$= xe^{x^2}\left[y \dfrac{\partial}{\partial y} e^{y^2} + e^{y^2} \dfrac{\partial}{\partial y} y\right]$

$= xe^{y^2}\left[2y^2 e^{y^2} + e^{y^2}\right]$

$= (2y^2+1)\, x e^{x^2+y^2}$

ii) $f(x,y) = e^x - \cos y$

$f_x = \dfrac{\partial}{\partial x} e^x - \cos y$ $\quad f_y = \dfrac{\partial}{\partial y} e^x - \cos y$

$= e^x - \cos y$ $\quad = - e^x \cdot \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$\Rightarrow f_x = \dfrac{\partial}{\partial x}(\cancel{3x}\, x^3y^2 - 3x^2y + y^3 + 1)$

$= 3x^2y^2 - 6xy.$

$\Rightarrow f_y = \dfrac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$= 2x^3y - 3x^2 + 3y^2$

Q3) i) $f(x,y) = \dfrac{2x}{1+y^2}$

$f_x = \dfrac{1}{1+y^2} \dfrac{\partial}{\partial x}(2x)$

$= \dfrac{2}{1+y^2}$

$f_x(0,0) = \frac{2}{1+(0)^2} = \underline{\underline{2}}$

$\therefore f_y = 2x \frac{\partial}{\partial y}\left(\frac{1}{1+y^2}\right)$

$= 2x \times \frac{-1}{(1+y^2)^2} \times 2y$

$= \frac{-4xy}{(1+y^2)^2}$

$f_y(0,0) = \frac{-4\times 0\times 0}{(1+(0)^2)^2} = 0$

Q4) $f(x,y) = \frac{y^2 - xy}{x^2}$

$f_x = \frac{\partial}{\partial x}\left(\frac{y^2 - xy}{x^2}\right)$

$= -2y^2x^{-3} + \frac{y}{x^2}$

$= \frac{y}{x^2} - \frac{2y^2}{x^3}$

$f_y = \frac{2y - x}{x^2}$

$\therefore f_{xx} = \frac{\partial}{\partial x} f_x$

$= \frac{\partial}{\partial x}\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)$

$= \frac{6y^2}{x^4} - \frac{2y}{x^3}$

$\therefore f_{yy} = \frac{\partial}{\partial y} f_y$

$= \frac{2}{x^2}$

$\therefore f_{xy} = \frac{\partial}{\partial y} f_x = \frac{\partial}{\partial y}\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)$

$= \frac{1}{x^2} - \frac{4y}{x^3}$

$\therefore \underline{\underline{f_{xy} = f_{yx}}}$

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\partial}{\partial x}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{\partial}{\partial y}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 0 + 6yx^2 - 0$

$= 6x^2y$

$$\therefore f_{xx} = \frac{\partial}{\partial x}(f_x)$$
$$= \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$
$$= 6x + 6y^2 - \frac{4x - 2x^2 + 2}{(x^2+1)^2}$$

$$\therefore f_{yy} = \frac{\partial}{\partial y}(f_y)$$
$$= \frac{\partial}{\partial y}(6x^2 y)$$
$$= \underline{6x^2}$$

$$\therefore f_{xy} = \frac{\partial}{\partial y} f_x$$
$$= \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$
$$= 12xy$$

$$\therefore f_{yx} = \frac{\partial}{\partial x} f_y$$
$$= \frac{\partial}{\partial x}(6x^2 y)$$
$$= 12xy$$

$$\therefore f_{xy} = \underline{f_{yx}}$$

iii) <u>Solution:</u>

$$f(x,y) = \sin(xy) + e^{x+y}$$
$$= \sin(xy) + e^x \cdot e^y.$$

$$f_x = \frac{\partial}{\partial x}\left(\sin(xy) + e^x \cdot e^y\right)$$
$$= y\cos(xy) + e^x \cdot e^y$$

$$f_y = \frac{\partial}{\partial y}\left(\sin(xy) + e^x \cdot e^y\right)$$
$$= x\cos(xy) + e^x \cdot e^y$$

$$\therefore f_{xx} = \frac{\partial}{\partial x} f_x$$
$$= \frac{\partial x}{\partial x}\, y\cos(xy) + e^x \cdot e^y$$
$$= -y^2 \sin(xy) + e^x \cdot e^y$$

$$\therefore f_{yy} = \frac{\partial}{\partial y} f_y$$
$$= \frac{\partial}{\partial y}\left(x\cos(xy) + e^x \cdot e^y\right)$$
$$= -x^2 \cdot \sin^2(xy) + e^x \cdot e^y).$$

$\therefore f_{xy} = \frac{\partial}{\partial y} f_x$

$= \frac{\partial}{\partial y}\left(y\cos(xy) + e^x \cdot e^y\right)$

$= -xy\sin(xy) + \cos(xy) + e^x \cdot e^y$

$\therefore f_{yx} = \frac{\partial}{\partial x} f_y$

$= \frac{\partial}{\partial x}\left(x\cos(xy) + e^x \cdot e^y\right)$

$= -xy\sin(xy) + \cos(xy) + e^x \cdot e^y$

$\therefore f_{xy} = f_{yx}$

Q5) i) $f(x,y) = \sqrt{x^2+y^2}$ $\quad (a,b) = (1,1)$

$f_x = \frac{1}{2\sqrt{x^2+y^2}} \times 2x$ $\quad f_y = \frac{2y}{2\sqrt{y^2+x^2}}$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{2}}$ $\quad = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$

$= \sqrt{2} + \frac{x+y-2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ $\quad (a,b) = (\pi/2, 0)$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 \cdot \sin\frac{\pi}{2}$

$= \frac{2-\pi}{2}$

$f_x = -1 + y\cos x$ $\quad f_y = \sin x$

$f_x(\pi/2, 0) = -1 + 0 \times \cos \pi/2$ $\quad f_y(\pi/2, 0) = \sin \pi/2$

$= -1$ $\quad = 1$

$\therefore L(x,y) = f(\pi/2, 0) + f_x(\pi/2, 0)(x - \pi/2) + f_y(\pi/2, 0)(y-0)$

$= \frac{2-\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1\,(y)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

iii) $f(x,y) = \log x + \log y \quad (a,b) = (1,1)$

$f(1,1) = \log(1) + \log(1)$

$= 0$

$f_x = \frac{1}{x} \qquad f_y = \frac{1}{y}$

$f_x(1,1) = 1 \qquad f(1,1) = 1$

$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 0 + 1(x-1) + 1(y-1)$

$= x + y - 2$

AK
22/11/22

Practical 10

Q1) Find the directional derivative of the given vector at the given points:-

i) $f(x,y) : x + 2y - 3$ at $\bar{u} = 3\hat{i} - \hat{j}$, $a(3,-1)$

ii) $f(x,y) : y^2 - 4x + 1$ at $\bar{u} = \hat{i} + 5\hat{j}$, $a(3,4)$

iii) $f(x,y) : 2x + 3y$ at $\bar{u} = 3\hat{i} + 4\hat{j}$, $a(1,2)$

Q2) Find gradient vector for the following function at the given point:

i) $f(x,y) = x^y + y^x$, $a = (1,1)$

ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$; $a = (1,-1)$

iii) $f(x,y,z) = xyz \cdot e^{x+y+z}$, $a = (1,-1,0)$

Q3) Find the equation of tangent and normal of each of the following curves:

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

ii) ~~$3xyz - x - y + z = -4$ at $(1,-1,2)$~~

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

Q4) Find the equation of tangent and normal of each of the following curves:-

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2,-1,0)$

(ii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

Q5) Find the local maximum & minimum for the following function:-

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

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→ Solutions:-

i) $\bar{u} = 3\hat{i} - \hat{j}$

$\therefore \hat{u} = \dfrac{\bar{u}}{|\bar{u}|} = \dfrac{1}{3^2+(-1)^2}(3\hat{i}-\hat{j})$

$= \dfrac{1}{\sqrt{10}}(3\hat{i}-\hat{j})$

$= \dfrac{3}{\sqrt{10}}, \dfrac{-1}{\sqrt{10}}$

$a = (1,-1)$

$\therefore f(a) = 1 + 2(-1) - 3$

$= 1 + (-2) - 3$

$= \underline{\underline{-4}}$

$\therefore f(a+hu) = f\left((1,-1) + h\left(\dfrac{3}{\sqrt{10}}, \dfrac{-1}{\sqrt{10}}\right)\right)$

$= f\left(\left(1 + \dfrac{3}{\sqrt{10}}h\right), \left(-1 - \dfrac{h}{\sqrt{10}}\right)\right)$

$= 1 + \dfrac{3}{\sqrt{10}}h + 2\left(-1 - \dfrac{h}{\sqrt{10}}\right) - 3$

$= 1 + \dfrac{3}{\sqrt{10}}h - 2 - \dfrac{2h}{\sqrt{10}} - 3$

$= \underline{\underline{\dfrac{h}{\sqrt{10}} - 4}}$

$\therefore D_u f(a) = \lim\limits_{h\to 0} \dfrac{f(a+hu) - f(a)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{h}{\sqrt{10}} - 4 - (-4)}{h}$

$= \dfrac{1}{\sqrt{10}} \lim\limits_{h\to 0} \dfrac{h}{h}$

$= \underline{\underline{\dfrac{1}{\sqrt{10}}}}$

ii) Solutions:-

$f(x,y) = y^2 - 4x + 1 \quad ; \quad a = \left(\overset{3,\,4}{\cancel{4},5}\right)$

$\bar{u} = \hat{i} + 5\hat{j}$

$\hat{u} = \dfrac{\bar{u}}{|\bar{u}|} = \dfrac{\hat{i}+5\hat{j}}{\sqrt{1^2+5^2}} = \dfrac{1}{\sqrt{26}}(\hat{i}+5\hat{j})$

$\therefore \hat{u} \left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)$

$f(a) = (4)^2 - 4(3) + 1$

$= 16 - 12 + 1$

$= \underline{\underline{5}}$

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$\therefore f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$

$= f\left(\left(3 + \frac{h}{\sqrt{26}}\right), \left(4 + \frac{5h}{\sqrt{26}}\right)\right)$

$\left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= \frac{25h^2}{26} - \frac{36h}{\sqrt{26}} + 5$

$\therefore D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{\frac{25h^2}{26} - \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$= \lim_{h\to 0} \frac{h\left(\frac{25h}{26} - \frac{36}{\sqrt{26}}\right)}{h}$

$= \lim_{h\to 0} \frac{25h}{26} - \frac{36}{\sqrt{26}}$

$= \frac{36}{\sqrt{26}}$

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iii) $f(x,y) : 2x+3y$ at $\bar{u} = 3\hat{i} + 4\hat{j}$, $a\,(1,2)$

$\bar{u} = 3\hat{i} + 4\hat{j}$

$\therefore \hat{u} = \frac{\bar{u}}{u} = \frac{1}{\sqrt{3^2+4^2}}(3\hat{i} + 4\hat{j})$

$= \frac{1}{\sqrt{25}}(3\hat{i} + 4\hat{j})$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$\therefore f(a) = 2(1) + 3(2)$

$= 2 + 6$

$= 8$

$\therefore f(a+hu) = f\left((1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$

$= f\left(\left(1 + \frac{3h}{5}\right), \left(2 + \frac{4h}{5}\right)\right)$

$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

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$\therefore D_u f(a) = \lim_{h\to 0} \dfrac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \dfrac{\frac{18h}{5} + 8 - 8}{h}$

$= \lim_{h\to 0} \dfrac{\frac{18h}{5}}{h}$

$= \dfrac{18}{5} \lim_{h\to 0} \dfrac{h}{h}$

$= \dfrac{18}{5}$

(52) i) $f(x,y) = x^y + y^x$ at $a(1,1)$

$f_x = \dfrac{\partial}{\partial x}(x^y + y^x)$

$f_x = yx^{y-1} + y^x \cdot \log y$

$f_y = \dfrac{\partial}{\partial y} x^y + y^x$

$f_y = xy^{x-1} + x^y \cdot \log x$

$\nabla f(x,y) = (f_x, f_y)$

$\nabla f(x,y) = (yx^{x-1} + y^x \log y ; xy^{x-1} + x^y \log x)$

$\nabla f(1,1) = (1(1)^{1-1} + 1^1 \log 1, (1)^{1-1} + 1^1 \log 1)$

$\nabla f(1,1) = (1,1)$

ii) $f(x,y) = (\tan^{-1} x) y^2$ at $(1,-1)$

$f_x = \dfrac{d}{dx}(\tan^{-1} x) y^2$

$= \dfrac{y^2}{1+x^2}$

$f_y = \dfrac{\partial}{\partial y}(\tan^{-1} x) y^2$

$= 2y \cdot \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\dfrac{y^2}{1+x^2}, 2y \cdot \tan^{-1} x\right)$

$\nabla f(1,-1) = \left(\dfrac{1^2}{1+1^2}, 2(-1) \cdot \tan^{-1}(1)\right)$

$= \left(\dfrac{1}{1+1}, -2 \times \dfrac{\pi}{4}\right)$

$= \left(\dfrac{1}{2}, \dfrac{-\pi}{2}\right)$

$\therefore \nabla f(1,-1) = \dfrac{1}{2}, \dfrac{-\pi}{2}$

iii) $f(x,y,z) = xyz - e^{x+y+z}$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\therefore \nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla f(1,-1,0) = (-1\times 0 - e^{-1+1+0}, 1(0) - e^{-1+1+0}, 1(-1) - e^{-1+1+0})$

$= (-1, -1, -2)$

Q3) i) $x^2 \cos y + e^{xy} - 2 = 0$

$f(x) = 2x\cos y + y e^{xy}$

$f(y) = -x^2 \sin y + x e^{xy}$

$f(x)(x - x_0) + f_y(y - y_0) = 0$

$\rightarrow (2x\cos y + y e^{xy})(x-1) + (-x^2 \sin y + x e^{xy})(y-0) =$

$\rightarrow 2x^2 \cos y + xy e^{xy} - 2x\cos y - y e^{xy} - (x^2 \sin y - x e^{xy} y)$

ii) $f(x,y) = x^2 + y^2 - 2x + 3y = 0$

$f_x = 2x - 2$ $\quad f_x(2,-2) = 2$

$f_y = 2y + 3$ $\quad f_y(2,-2) = -1$

Tangent = $f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(x-x_0) = 0$

$2(x-2) + 1(y+2) = 0$

$2x - 4 + y + 2 = 0$

$2x + y - 2 = 0$

Normal:- $x - 2y + d = 0$

$2 - 2(-2) + d = 6$

$d = 2$

$\therefore x - 2y + 2 = 0$

Q4 iii) $f(x,y,z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x + z$ $\quad f_x(x_0,y_0,z_0) = 2(2) + 0 = 4$

$f_y = -2z + 3$ $\quad f_y(x_0,y_0,z_0) = -2(0) + 3 = 3$

$f_z = -2y + x$ $\quad f_z(x_0,y_0,z_0) = -2(1) + 2 = 0$

$\therefore$ Tangent : $f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$

$4(x-2)+3(y-1)+0(z-0)$

$4x-8+3y-3+0=0$

$4x+3y-11=0$

$\therefore$ Normal : $\dfrac{x-x_0}{f_x(x_0,y_0,z_0)}=\dfrac{y-y_0}{f_y(x_0,y_0,z_0)}=\dfrac{z-z_0}{f_z(x_0,y_0,z_0)}$

$=\dfrac{x-2}{4}=\dfrac{y-1}{1}=\dfrac{z-0}{0}$

Q5)

i) $f(x,y):3x^2+y^2-3xy+6x-4y$

$f_x=6x-3y+6$

$f_y=2y-3x-4$

$f_x=0 \qquad f_y=0$

$6x-3y=-6 \qquad 3x-2y=-4$

$\therefore y=2$

$\therefore x=0$

$\therefore (x,y)=(0,2)$ is a root

$r=f_{xx}=6$

$s=f_{xy}=-3$

$t=f_{yy}=2$

$rt-s^2=6(2)-(-3)^2=12-9$

$=3>0$

$r>0$

$\therefore$ 'f' is maximum at $(0,2)$

$f(0,2)=3(0)^2+(2)^2(-3(0)(2))+6(0)-4(2)$

$=4+6-0+0-8$

$=-4$

iii) $f(x,y): 2x^4+3x^2y-y^2$

$f_x=8x^3+6xy$

$f_y=-2y+3x^2$

$f_x=0 \qquad f_y=0$

$8x^3+6xy=0 \qquad -2y+3x^2=0$

$x(8x^2+6y)=0 \qquad -2y+3(0)^2=0$

$x=0 \qquad \therefore y=0$

$(x,y)=(0,0)$ is a root

$8x^2+6y=0$

$x^2=\dfrac{-2}{3}y$

$f_y:-2y-\dfrac{2}{3}\times 3y=0$

$\therefore -4y=0$

$y=0$

$x^2=0$

$\therefore x=0$

$\therefore (x,y)=(0,0)$ is the only root

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$r = f_{xx} = 24x^2 + 6y = 0$
$s = f_{xy} = 6x = 0$
$t = f_{yy} = -2 + 0 = -2$

$r = 0$
$rt - s^2 = (0)(0) - (2)^2$
$\therefore rt - s^2 = -4 < 0$

$\therefore (0,0)$ is the saddle point

(Q4) ii) $f(x,y,z) = 3xyz - x - y + z + 4 = 0$
$f_x = 3yz - 1 \quad f_x(x_0,y_0,z_0) = 3(-1)(2) - 1 = -7$
$f_y = 3xz - 1 \quad f_y(x_0,y_0,z_0) = 3(1)(2) - 1 = 5$
$f_z = 3xy + 1 \quad f_z(x_0,y_0,z_0) = 3(1)(-1) + 1 = -2$

$\therefore$ Tangent: $f_x(x_0,y_0,z_0)(x - x_0) + f_y(x_0,y_0,z_0)(y$
$+ f_z(x_0,y_0,z_0)(z - z_0) = 0$
$\therefore -7(x-1) + 5(y+1) - 2(z-2) = 0$
$\therefore -7x + 7 + 5y + 5 - 2z + 4 = 0$
$\therefore -7x + 5y - 2z + 16 = 0$
$7x - 5y + 2z - 16 = 0$

Normal: $\dfrac{x - x_0}{f_x(x_0,y_0,z_0)} = \dfrac{y - y_0}{f_y(x_0,y_0,z_0)} = \dfrac{z - z_0}{f_z(x_0,y_0,z_0)}$

$\Rightarrow \dfrac{x-1}{7} = \dfrac{y+1}{-5} = \dfrac{z-2}{2}$

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